Prospects for aTGC constraints at the LHC and at e^+e^- colliders and the role of polarization

Rafiqul Rahaman

RECAPP, HRI, Prayagraj, India

ECFA Workshop — DESY, Hamburg

October 6, 2022

Outline

Outline

Introduction

- The Standard Model (SM) of particle physics is highly successful theory which describe the governing principle of elementary constituents of matter and their interactions.
- However, the SM has issues within the theoretical framework (the hierarchy of mass, strong *CP* problem) and also the inability to address neutrino oscillation, dark matter, baryogenesis and many more.
- New physics has been postulated to cure those, predicting new particles and symmetry.
- Sadly, nothing found beyond the SM at the current reach of energy ($\mathscr{O}(10)$ TeV).

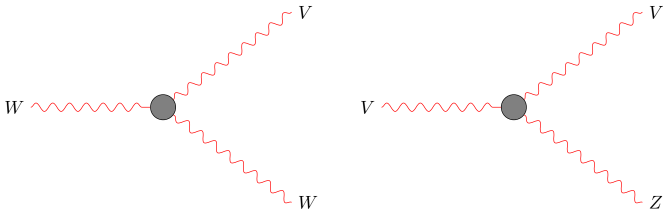
Introduction

- One could expect that the new physics scale is too heavy to be directly explored at the LHC at the current energy range.
- They may leave footprints in the available energy range. They will modify the structure of SM interactions or bring some new interactions.
- These can be modelled by effective filed theory (**EFT**) through higher dimensional operators,

$$\mathscr{L}_{eft} = \mathscr{L}_{SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathscr{O}_i^{(6)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4} \mathscr{O}_i^{(8)} + \dots \quad .$$

Outline

Anomalous triple gauge boson couplings (**aTGC**)



$$V = Z/\gamma$$

- Possible triple gauge boson self interactions in electro weak (EW) theory:

$$\text{Present in the SM} \; : \; WWZ,\; WW\gamma \;\; (WWV),$$
$$\text{Not present in the SM} \; : \; ZZZ,\; ZZ\gamma,\; Z\gamma\gamma \; (ZVV).$$

aTGC operators

- The operators contributing to WWV couplings are

$$\mathcal{O}_{WWW} = \mathrm{Tr}[W_{\mu\nu}W^{\nu\rho}W_{\rho}^{\mu}], \qquad \mathcal{O}_{\widetilde{WWW}} = \mathrm{Tr}[\widetilde{W}_{\mu\nu}W^{\nu\rho}W_{\rho}^{\mu}],$$

$$\mathcal{O}_{W} = (\mathscr{D}_{\mu}\Phi)^{\dagger}W^{\mu\nu}(\mathscr{D}_{\nu}\Phi), \qquad \mathcal{O}_{\widetilde{W}} = (\mathscr{D}_{\mu}\Phi)^{\dagger}\widetilde{W}^{\mu\nu}(\mathscr{D}_{\nu}\Phi).$$

$$\mathcal{O}_{B} = (\mathscr{D}_{\mu}\Phi)^{\dagger}B^{\mu\nu}(\mathscr{D}_{\nu}\Phi),$$

K. Hagiwara, *Nucl. Phys.* **B282** (1987) 253–307

- The operator for ZVV couplings are.

$$\begin{aligned}
\mathcal{O}_{BW} &= i\Phi^{\dagger}B_{\mu\nu}W^{\mu\rho}\{\mathscr{D}_{\rho},\mathscr{D}^{\nu}\}\Phi,\\
\mathcal{O}_{WW} &= i\Phi^{\dagger}W_{\mu\nu}W^{\mu\rho}\{\mathscr{D}_{\rho},\mathscr{D}^{\nu}\}\Phi,\\
\mathcal{O}_{BB} &= i\Phi^{\dagger}B_{\mu\nu}B^{\mu\rho}\{\mathscr{D}_{\rho},\mathscr{D}^{\nu}\}\Phi,\\
\mathcal{O}_{\widetilde{B}W} &= i\Phi^{\dagger}\widetilde{B}_{\mu\nu}W^{\mu\rho}\{\mathscr{D}_{\rho},\mathscr{D}^{\nu}\}\Phi.
\end{aligned}$$

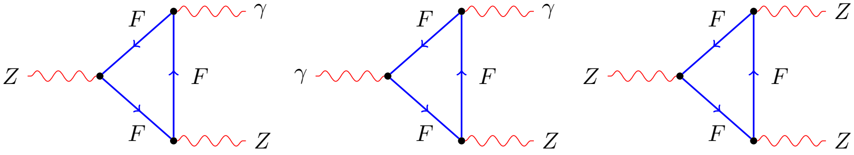
$$\begin{aligned}
\mathscr{D}_{\mu} &= \partial_{\mu} + \frac{i}{2}g\tau^{I}W_{\mu}^{I} + \frac{i}{2}g'B_{\mu}\\
W_{\mu\nu} &= \frac{i}{2}g\tau^{I}(\partial_{\mu}W_{\nu}^{I} - \partial_{\nu}W_{\mu}^{I} + g\epsilon_{IJK}W_{\mu}^{J}W_{\nu}^{K}),\\
B_{\mu\nu} &= \frac{i}{2}g'(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}),\ \ \widetilde{V}^{\mu\nu} = 1/2\epsilon^{\mu\nu\rho\sigma}V_{\rho\sigma}
\end{aligned}$$

Gounaris *et al.* Phys. Rev. D **61**, 073013 (2000)

Contribution can arise from MSSM, Little Higgs model, 2HDM, NCSM, UED, Georgi-Machacek model.

Toy model :

$$\mathscr{L}_{VF\bar{F}} = -eQ_F A^\mu \bar{F}\gamma_\mu F - \frac{e}{2s_w c_w} Z^\mu \bar{F}\left(\gamma_\mu g_{VF} - \gamma_\mu\gamma_5 g_{AF}\right) F$$

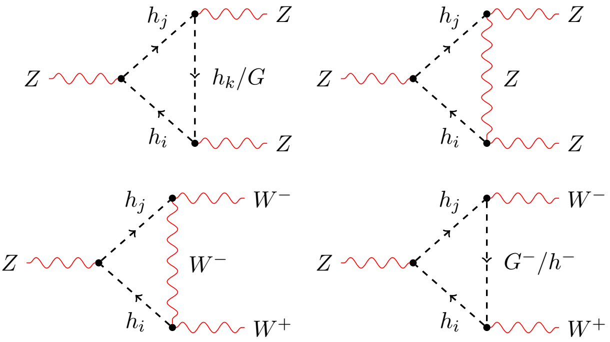


Gounaris *et* al. Phys. Rev. D **61**, 073013 (2000)

BSM contribution to anomalous couplings — 2HDM

$$\phi_{i(=1,2)} = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}, \quad \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tan\beta = \frac{v_2}{v_1}, v = \sqrt{v_1^2 + v_2^2}$$

$$H_1 = \begin{pmatrix} -iG^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(R + iI\right) \end{pmatrix}$$



Corbett *et* al. Phys. Rev. D97 no. 11, (2018) 115040,

Grzadkowski *et* al. JHEP 05 (2016) 025, Bélusca-Maïto *et* al. JHEP 04 (2018) 002

Outline

Spin density matrix and polarization parameters

$$\rho_{1/2}(\lambda,\lambda') = \frac{1}{2}\left[\mathbb{I}_{2\times 2} + \vec{p}\cdot\vec{\sigma}\right],$$

$$\rho_{1}(\lambda,\lambda') = \frac{1}{3}\left[\mathbb{I}_{3\times 3} + \frac{3}{2}\vec{p}\cdot\vec{S} + \sqrt{\frac{3}{2}}T_{ij}\left(S_iS_j + S_jS_i\right)\right]$$

Spin density matrix and polarization parameters

$$\rho_{1/2}(\lambda,\lambda') = \frac{1}{2}\left[\mathbb{I}_{2\times 2} + \vec{p}\cdot\vec{\sigma}\right],$$

$$\rho_{1}(\lambda,\lambda') = \frac{1}{3}\left[\mathbb{I}_{3\times 3} + \frac{3}{2}\vec{p}\cdot\vec{S} + \sqrt{\frac{3}{2}}T_{ij}\left(S_iS_j + S_jS_i\right)\right]$$

$$P_{1/2}(\lambda,\lambda') = \frac{1}{2}\begin{bmatrix} 1+p_z & p_x - ip_y \\ p_x + ip_y & 1-p_z \end{bmatrix}$$

$$P_{1}(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3}+\frac{p_z}{2}+\frac{T_{zz}}{\sqrt{6}} & \frac{p_x-ip_y}{2\sqrt{2}}+\frac{T_{xz}-iT_{yz}}{\sqrt{3}} & \frac{T_{xx}-T_{yy}-2iT_{xy}}{\sqrt{6}} \\ \frac{p_x+ip_y}{2\sqrt{2}}+\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{2T_{zz}}{\sqrt{6}} & \frac{p_x-ip_y}{2\sqrt{2}}-\frac{T_{xz}-iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx}-T_{yy}+2iT_{xy}}{\sqrt{6}} & \frac{p_x+ip_y}{2\sqrt{2}}-\frac{T_{xz}+iT_{yz}}{\sqrt{3}} & \frac{1}{3}-\frac{p_z}{2}+\frac{T_{zz}}{\sqrt{6}} \end{bmatrix}$$

F. Boudjema and R. K. Singh JHEP **0907**, 028 (2009)

Angular distribution of decayed fermion (spin-1)

$$\frac{1}{\sigma}\,\frac{d\sigma}{d\Omega_f} = \frac{2S+1}{4\pi}\sum_{\lambda,\lambda'}\underbrace{P_1(\lambda,\lambda')}_{Production}\times\underbrace{\Gamma_1(\lambda,\lambda')}_{Decay},$$

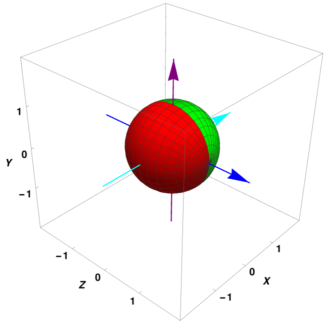
$$\begin{aligned}
= \;& \frac{3}{8\pi}\Bigg[\left(\frac{2}{3}-(1-3\delta)\,\frac{T_{zz}}{\sqrt{6}}\right)+\alpha\;p_z\cos\theta+\sqrt{\frac{3}{2}}(1-3\delta)\;T_{zz}\cos^2\theta \\
+ \;& \left(\alpha\;p_x+2\sqrt{\frac{2}{3}}(1-3\delta)\;T_{xz}\cos\theta\right)\sin\theta\;\cos\phi \\
+ \;& \left(\alpha\;p_y+2\sqrt{\frac{2}{3}}(1-3\delta)\;T_{yz}\cos\theta\right)\sin\theta\;\sin\phi \\
+ \;& (1-3\delta)\left(\frac{T_{xx}-T_{yy}}{\sqrt{6}}\right)\sin^2\theta\cos(2\phi) \\
+ \;& \sqrt{\frac{2}{3}}(1-3\delta)\;T_{xy}\;\sin^2\theta\;\sin(2\phi)\Bigg]
\end{aligned}$$

F. Boudjema and R. K. Singh JHEP **0907**, 028 (2009)

Polarization parameters from asymmetry

$$A_x = \frac{1}{\sigma}\left[\int_{\theta=0}^{\pi}\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\sigma}{d\Omega_f} d\Omega_f - \int_{\theta=0}^{\pi}\int_{\phi=\frac{\pi}{2}}^{3\frac{\pi}{2}} \frac{d\sigma}{d\Omega_f} d\Omega_f\right] = \frac{3\alpha p_x}{4},$$

$$\equiv \frac{\sigma(\cos\phi > 0) - \sigma(\cos\phi < 0)}{\sigma(\cos\phi > 0) + \sigma(\cos\phi < 0)} \equiv \frac{\text{Green Region} - \text{Red Region}}{\text{Green Region} + \text{Red Region}}.$$



Rahaman, Singh, *Eur. Phys. J.* **C76** no. 10, (2016) 539

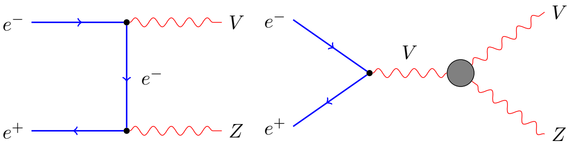
Polarization parameters from asymmetry

$$
\begin{aligned}
A_y &\equiv \frac{\sigma(\sin\phi > 0) - \sigma(\sin\phi < 0)}{\sigma(\sin\phi > 0) + \sigma(\sin\phi < 0)} = \frac{3\alpha p_y}{4}, \\
A_z &\equiv \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)} = \frac{3\alpha p_z}{4}, \\
A_{xy} &\equiv \frac{\sigma(\sin 2\phi > 0) - \sigma(\sin 2\phi < 0)}{\sigma(\sin 2\phi > 0) + \sigma(\sin 2\phi < 0)} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)\,T_{xy}, \\
A_{xz} &\equiv \frac{\sigma(\cos\theta\cos\phi > 0) - \sigma(\cos\theta\cos\phi < 0)}{\sigma(\cos\theta\cos\phi > 0) + \sigma(\cos\theta\cos\phi < 0)} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)\,T_{xz}, \\
A_{yz} &\equiv \frac{\sigma(\cos\theta\sin\phi > 0) - \sigma(\cos\theta\sin\phi < 0)}{\sigma(\cos\theta\sin\phi > 0) + \sigma(\cos\theta\sin\phi < 0)} = \frac{2}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)\,T_{yz}, \\
A_{x^2-y^2} &\equiv \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)} = \frac{1}{\pi}\sqrt{\frac{2}{3}}(1-3\delta)\,(T_{xx} - T_{yy}), \\
A_{zz} &\equiv \frac{\sigma(\sin 3\theta > 0) - \sigma(\sin 3\theta < 0)}{\sigma(\sin 3\theta > 0) + \sigma(\sin 3\theta < 0)} = \frac{3}{8}\sqrt{\frac{3}{2}}(1-3\delta)\,T_{zz}.
\end{aligned}
$$

Rahaman, Singh, *Eur. Phys. J.* **C76** no. 10, (2016) 539

Outline

CP-even and CP-odd distinction



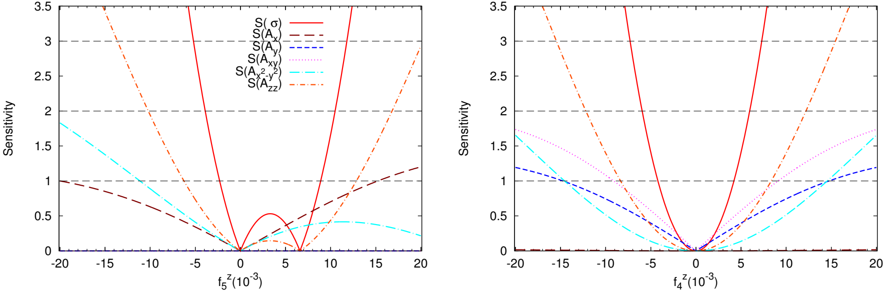
$$V = Z/\gamma$$

$$\mathscr{L}_{ZVV} = \frac{e}{m_Z^2}\Bigg[\; - \; \left[f_4^{\gamma}(\partial_\mu F^{\mu\beta}) + f_4^{Z}(\partial_\mu Z^{\mu\beta})\right] Z_\alpha(\partial^\alpha Z_\beta)$$
$$+ \; \left[f_5^{\gamma}(\partial^\sigma F_{\sigma\mu}) + f_5^{Z}(\partial^\sigma Z_{\sigma\mu})\right] \widetilde{Z}^{\mu\beta} Z_\beta$$
$$- \; \left[h_1^{\gamma}(\partial^\sigma F_{\sigma\mu}) + h_1^{Z}(\partial^\sigma Z_{\sigma\mu})\right] Z_\beta F^{\mu\beta}$$
$$- \; \left[h_3^{\gamma}(\partial_\sigma F^{\sigma\rho}) + h_3^{Z}(\partial_\sigma Z^{\sigma\rho})\right] Z^\alpha \widetilde{F}_{\rho\alpha} \Bigg].$$

Gounaris *et* al. Phys. Rev. D **61**, 073013 (2000)

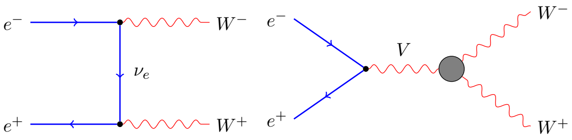
$$\text{Sensitivity: } \mathscr{S}\mathscr{O}(\vec{f}) = \frac{|\mathscr{O}(\vec{f}) - \mathscr{O}(\vec{f}=0)|}{\delta\mathscr{O}},$$

Figure: $e^+e^- \to ZZ \to l^+l^-q\bar{q}$, $\sqrt{s} = 500$ GeV, $\mathscr{L} = 100$ fb^{-1}



Rahaman, Singh, *Eur. Phys. J.* **C76** no. 10, (2016) 539

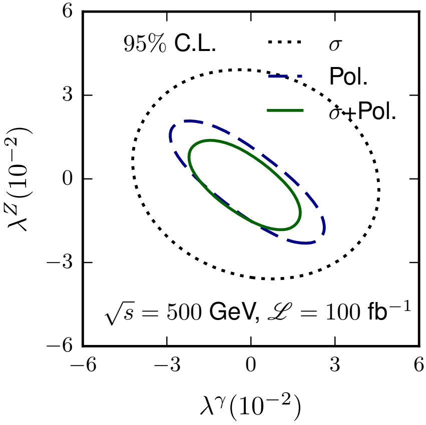
Constraining aTGC



$$V = Z/\gamma$$

$$\begin{aligned}\mathscr{L}_{WWV} = ig_{WWV}\Big[\quad & + \quad \Delta g_1^V (W^+_{\mu\nu} W^{-\mu} - W^{+\mu} W^-_{\mu\nu}) V^\nu \\ & + \quad i g_4^V W^+_\mu W^-_\nu (\partial^\mu V^\nu + \partial^\nu V^\mu) \\ & - \quad i g_5^V \epsilon^{\mu\nu\rho\sigma} (W^+_\mu \partial_\rho W^-_\nu - \partial_\rho W^+_\mu W^-_\nu) V_\sigma \\ & + \quad \frac{\lambda^V}{m_W^2} W^{+\nu}_\mu W^{-\rho}_\nu V^\mu_\rho + \frac{\widetilde{\lambda^V}}{m_W^2} W^{+\nu}_\mu W^{-\rho}_\nu \widetilde{V}^\mu_\rho \\ & + \quad \Delta\kappa^V W^+_\mu W^-_\nu V^{\mu\nu} + \widetilde{\kappa^V} W^+_\mu W^-_\nu \widetilde{V}^{\mu\nu} \Big].\end{aligned}$$

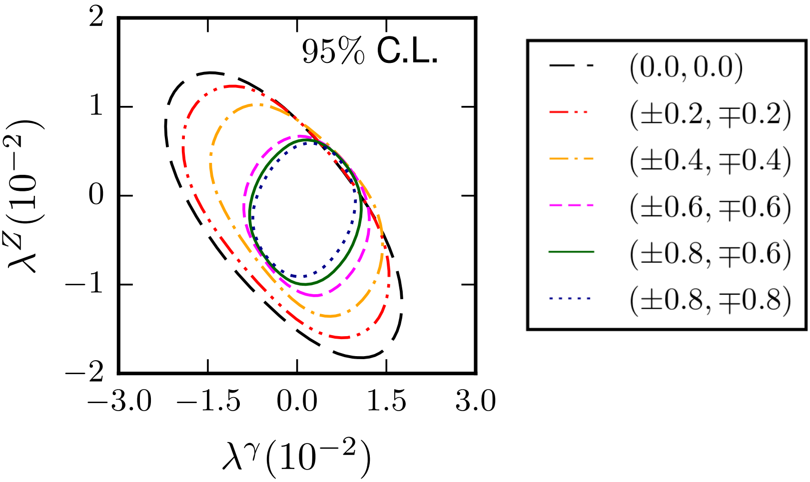
$$e^+e^- \to W^+W^- \to l^-\bar{\nu}_l q\bar{q}'$$



Multi parameter variations

Rahaman, Singh, *Phys. Rev. D* **101** no. 7, (2020) 075044

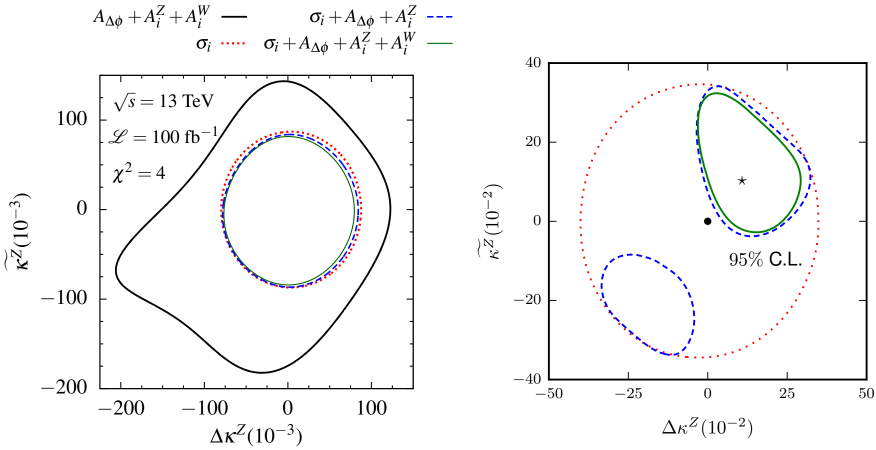
$e^+e^- \to W^+W^- \to l^-\bar{\nu}_l q\bar{q}'$, $\sqrt{s} = 500$ GeV, $\mathscr{L} = 100$ fb^{-1}



Rahaman, Singh, *Phys. Rev. D* **101** no. 7, (2020) 075044

Extraction of parameters

Figure: $pp \to 3l + \not{E}_T (W^{\pm}Z)$, $\sqrt{s} = 13$ TeV, $\mathscr{L} = 100$ fb^{-1}



$(\bullet) \equiv$ SM

$(\star) \equiv \{\Delta g_1^Z, \lambda^Z, \Delta\kappa^Z, \widetilde{\lambda^Z}, \widetilde{\kappa^Z}\} = \{0.01, 0.01, 0.1, 0.01, 0.1\}$

Rahaman, Singh, *JHEP* **04** (2020) 075

Outline

Summary

- The polarization observables help to distinguish between CP-even and CP-odd couplings.
- They help to constrain anomalous couplings.
- Beam polarization at e^+e^- collider improve limits on anomalous couplings.
- Polarization observables are instrumental in extracting the values of anomalous couplings if a deviation from the SM is observed at the LHC.
- Spin-spin correlations can be added to further improve the limits.

Thank you

Backup slides

Relation of form factors with $SU(2) \times U(1)$ operators

$$
\begin{aligned}
\Delta g_1^Z &= c_W \frac{m_Z^2}{2\Lambda^2} \quad , \\
g_4^V &= g_5^V = \Delta g_1^\gamma = 0 \quad , \\
\lambda^\gamma &= \lambda^Z = \lambda^V = c_{WWW} \frac{3g^2 M_W^2}{2\Lambda^2} \quad , \\
\widetilde{\lambda^\gamma} &= \widetilde{\lambda^Z} = \widetilde{\lambda^V} = c_{\widetilde{WWW}} \frac{3g^2 M_W^2}{2\Lambda^2} \quad , \\
\Delta\kappa^\gamma &= (c_W + c_B) \frac{M_W^2}{2\Lambda^2} \quad , \\
\Delta\kappa^Z &= (c_W - c_B \tan^2\theta_W) \frac{M_W^2}{2\Lambda^2} \quad , \\
\widetilde{\kappa^\gamma} &= c_{\widetilde{W}} \frac{M_W^2}{2\Lambda^2} \quad , \\
\widetilde{\kappa^Z} &= -c_{\widetilde{W}} \tan^2\theta_W \frac{M_W^2}{2\Lambda^2} \quad .
\end{aligned}
$$

$$
\begin{aligned}
&\Delta g_1^Z = \Delta\kappa^Z + \tan^2\theta_W \Delta\kappa^\gamma \quad , \\
&\widetilde{\kappa^Z} + \tan^2\theta_W \widetilde{\kappa^\gamma} = 0 \quad .
\end{aligned}
$$

Relation of form factors with $SU(2) \times U(1)$ operators

$$
\begin{aligned}
f_5^Z &= 0, \\
f_5^\gamma &= \frac{v^2 m_Z^2}{4 c_w s_w} \frac{C_{\widetilde{B}W}}{\Lambda^4}, \\
f_4^Z &= \frac{m_Z^2 v^2 \left({c_w}^2 C_{WW} + 2 c_w s_w C_{BW} + 4 {s_w}^2 C_{BB} \right)}{2 c_w s_w \Lambda^4}, \\
f_4^\gamma &= -\frac{m_Z^2 v^2 \left(-c_w s_w C_{WW} + C_{BW} \left({c_w}^2 - {s_w}^2 \right) + 4 c_w s_w C_{BB} \right)}{4 c_w s_w \Lambda^4}
\end{aligned}
$$

$$
\begin{aligned}
h_3^Z &= \frac{v^2 m_Z^2}{4 c_w s_w} \frac{C_{\widetilde{B}W}}{\Lambda^4}, \\
h_4^Z &= h_3^\gamma = h_4^\gamma = h_2^Z = h_2^\gamma = 0, \\
h_1^Z &= \frac{m_Z^2 v^2 \left(-c_w s_w C_{WW} + C_{BW} \left({c_w}^2 - {s_w}^2 \right) + 4 c_w s_w C_{BB} \right)}{4 c_w s_w \Lambda^4}, \\
h_1^\gamma &= -\frac{m_Z^2 v^2 \left({s_w}^2 C_{WW} - 2 c_w s_w C_{BW} + 4 {c_w}^2 C_{BB} \right)}{4 c_w s_w \Lambda^4},
\end{aligned}
$$

$$
f_5^\gamma = h_3^Z \qquad \text{and} \qquad h_1^Z = -f_4^\gamma .
$$

Production density matrix and polarization parameters

$$\rho_1(\lambda,\lambda') = \frac{1}{3}\left[I_{3\times 3} + \frac{3}{2}\vec{p}\cdot\vec{S} + \sqrt{\frac{3}{2}}T_{ij}\left(S_iS_j + S_jS_i\right)\right],$$

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\; S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},\; S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$\vec{p} = \{p_x, p_y, p_z\},\quad T = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ & T_{yy} & T_{yz} \\ & & T_{zz} \end{pmatrix},\; \sum_i T_{ii} = 0.$$

$$P(\lambda,\lambda') = \begin{bmatrix} \frac{1}{3} + \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} + \frac{T_{xz} - iT_{yz}}{\sqrt{3}} & \frac{T_{xx} - T_{yy} - 2iT_{xy}}{\sqrt{6}} \\ \frac{p_x + ip_y}{2\sqrt{2}} + \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{2T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} - \frac{T_{xz} - iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx} - T_{yy} + 2iT_{xy}}{\sqrt{6}} & \frac{p_x + ip_y}{2\sqrt{2}} - \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} \end{bmatrix}.$$

F. Boudjema and R. K. Singh JHEP **0907**, 028 (2009)

The decay density matrix of a spin-1 particle

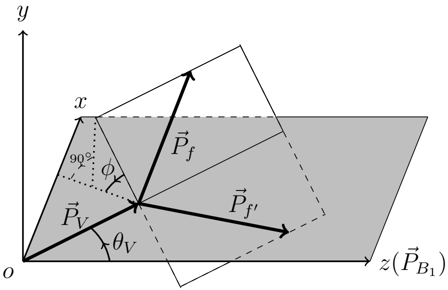
$$\Gamma_1(\lambda, \lambda') =$$

$$\begin{bmatrix} \frac{1+\delta+(1-3\delta)\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{i\phi} & (1-3\delta)\frac{(1-\cos^2\theta)}{4}\ e^{i2\phi} \\ \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{-i\phi} & \delta+(1-3\delta)\frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{i\phi} \\ (1-3\delta)\frac{(1-\cos^2\theta)}{4}\ e^{-i2\phi} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}}\ e^{-i\phi} & \frac{1+\delta+(1-3\delta)\cos^2\theta-2\alpha\cos\theta}{4} \end{bmatrix}$$

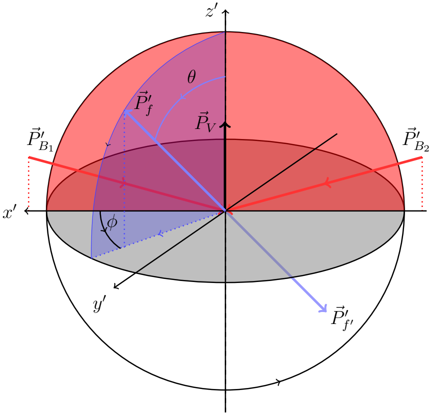
F. Boudjema and R. K. Singh JHEP **0907**, 028 (2009)

- $Vff' : \gamma^\mu \left(C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2} \right)$.
- $\delta \to 0$ for $m_f \to 0$ and $\alpha \to \frac{\left(C_R^f\right)^2 - \left(C_L^f\right)^2}{\left(C_R^f\right)^2 + \left(C_L^f\right)^2}$.

Lab frame and rest frame momentum configuration



Lab frame



Rest frame

Sensitivity of observables to aTGC

Sensitivity of an observable $\mathscr{O}$ dependent on parameter $\vec{f}$ is defined as

$$\mathscr{S}(\mathscr{O}(\vec{f})) = \frac{|\mathscr{O}(\vec{f}) - \mathscr{O}(\vec{f}=0)|}{\delta\mathscr{O}}$$

$$\delta\mathscr{O} = \sqrt{(\delta\mathscr{O}_{stat.})^2 + (\delta\mathscr{O}_{sys.})^2}$$

For Asymmetry the error is

$$\delta A = \sqrt{\frac{1-A^2}{\mathscr{L}\sigma} + \epsilon_A^2},$$

$\mathscr{L}$ being the integrated luminosity.

For the cross section the error is

$$\delta\sigma = \sqrt{\frac{\sigma}{\mathscr{L}} + (\epsilon_\sigma\sigma)^2}.$$

Here ϵ_A and ϵ_σ are the systematic fractional errors in A and σ, respectively.

Observables with parametric dependence

$$e^+e^- \to ZZ$$

Observables	Linear terms	Quadratic terms
σ	f_5^Z, f_5^γ	$(f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z$
$\sigma \times A_x$	f_5^γ, f_5^Z	–
$\sigma \times A_y$	f_4^γ, f_4^Z	–
$\sigma \times A_{xy}$	f_4^Z, f_4^γ	$f_4^Z f_5^\gamma, f_4^\gamma f_5^Z, f_4^\gamma f_5^\gamma, f_4^Z f_5^Z$
$\sigma \times A_{x^2-y^2}$	f_5^Z, f_5^γ	$(f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z$
$\sigma \times A_{zz}$	f_5^Z, f_5^γ	$(f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z$

MCMC algorithm

1. 1^{st} point in chain: $\vec{f}_1 \in Uniform[\vec{f}_{min}, \vec{f}_{max}]$ and calculate likelihood, $\mathscr{L}(\vec{f}_1)$.
2. Set Standard deviation $\delta\vec{f} = \frac{\vec{f}_{max} - \vec{f}_{min}}{h}$.
3. Generate 2^{nd} point in chain: $\vec{f}_2 \in Gaus[\vec{f}_1, \delta\vec{f}]$ and calculate $\mathscr{L}(\vec{f}_2)$.
4. Accept nearby points:
   DO
   IF $Uniform[0,1] < \frac{\mathscr{L}(\vec{f}_2)}{\mathscr{L}(\vec{f}_1)}$
   $\vec{f}_1 = \vec{f}_2$.
   Write w, $\mathscr{L}(\vec{f}_2)$, $\vec{f}_2, \mathscr{O}_i(\vec{f}_2)$.
   $w = 1$.
   ELSE $w = w + 1$
   .....Continue.
5. `GetDist` chain: w, $-\log[\mathscr{L}(\vec{f})]$, $\vec{f}$, $\mathscr{O}_i(\vec{f})$
6. Obtain correlations, BCI, etc.

End of backup slides