Interplay of periodic dynamics and noise: insights from a simple adaptive system

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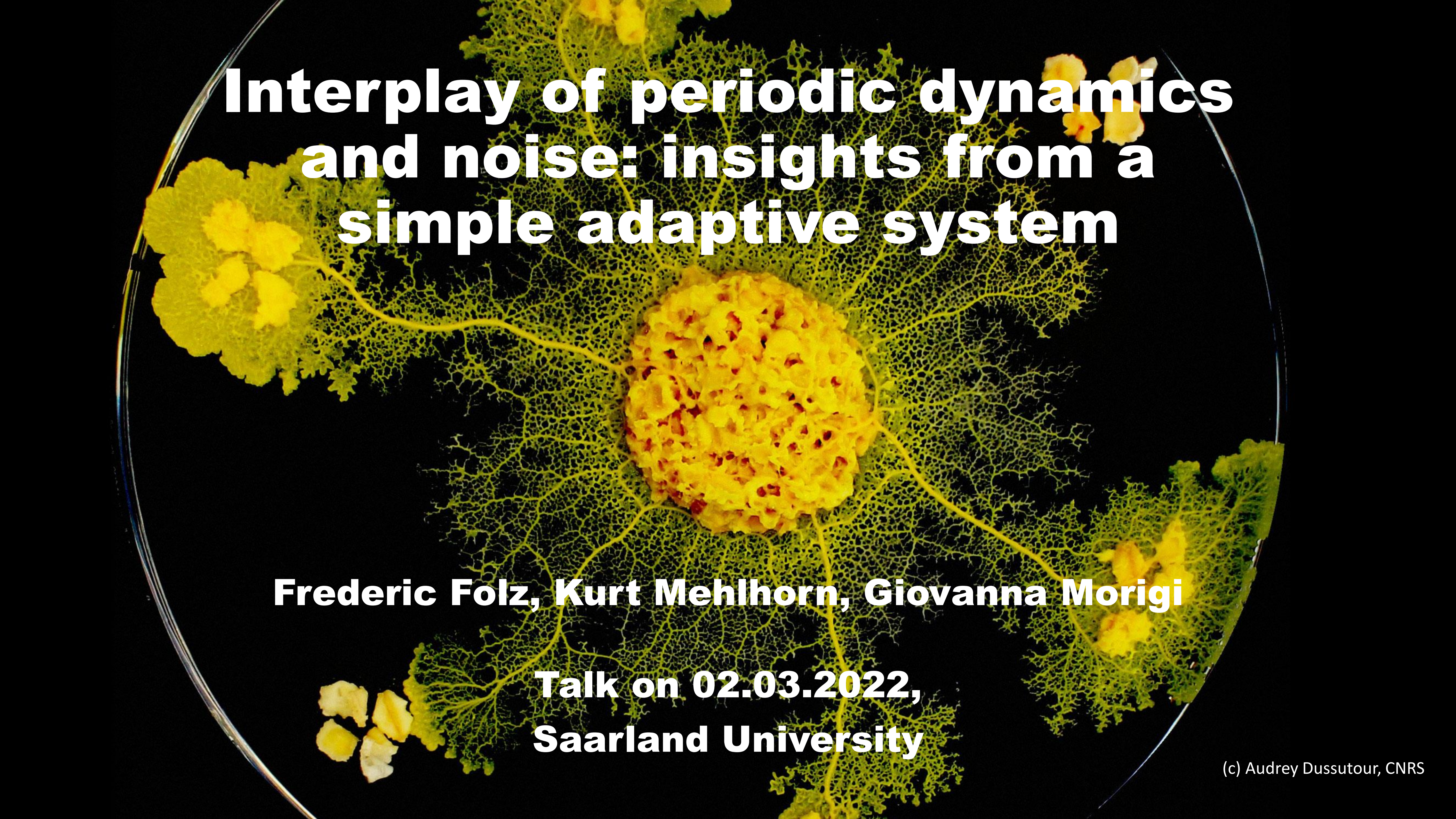
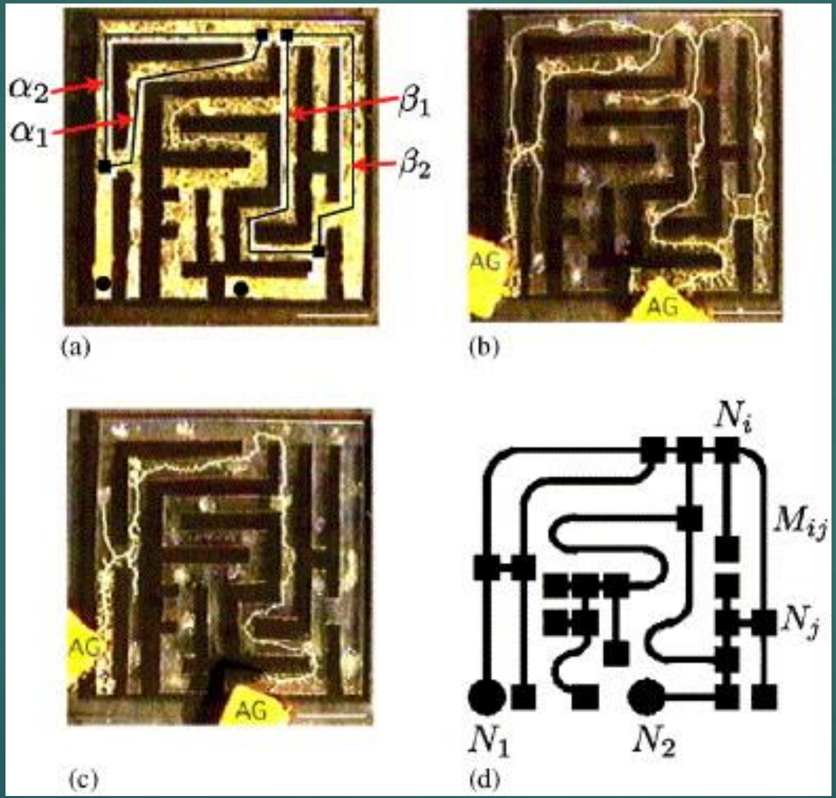
Saarland University



Food search



Tero et al., Journal of Theoretical Biology, Volume 244, Issue 4, 2007, Pages 553-564

Fig.: Different steps in selection of shortest path.

Content

Noise

- stochastic fluctuations of physical variables
- unavoidable in open physical systems
- often considered destructive
- can have non-trivial effects on dynamics
    - noise-induced phase transitions
    - noise-induced oscillations
    - noise-induced resonances

Mathematical model

- Adaptation of conductivity:

$$\frac{\partial D_i}{\partial t} = f\left(\frac{D_i}{D_1 + D_2}\right) - (\gamma + \Phi_i(t))D_i + \alpha\xi_i(t)$$

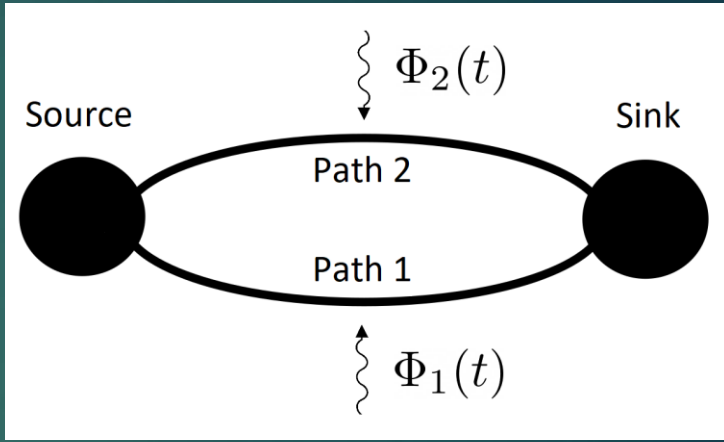
- Interaction function:

$$f(Q_i) = (1+\epsilon)\left(\frac{Q_i^\mu}{\epsilon + Q_i^\mu}\right)$$

- Gaussian white noise:

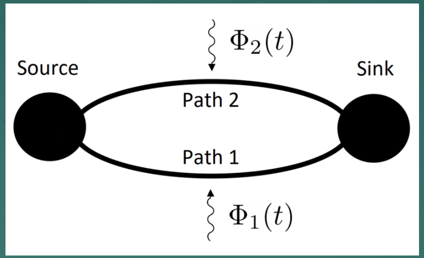
$$\langle \xi_i(t)\xi_j(t')\rangle = \delta_{i,j}\delta(t-t')$$

$$\langle \xi_i \rangle = 0$$

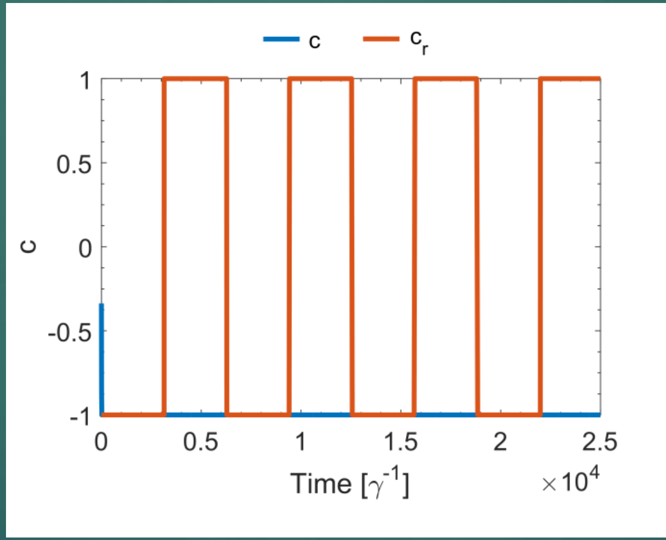


Meyer et al., The role of noise in self-organized decision making by the true slime mold Physarum polycephalum, 2017, PLOS ONE 12(3): e0172933.

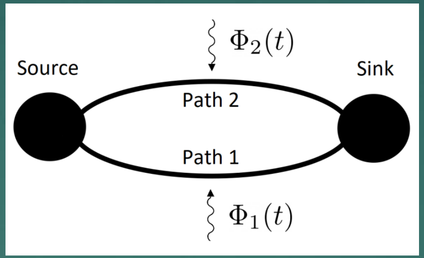
Symmetric configuration



$$c = \frac{D_1 - D_2}{D_1 + D_2}$$



Symmetric configuration



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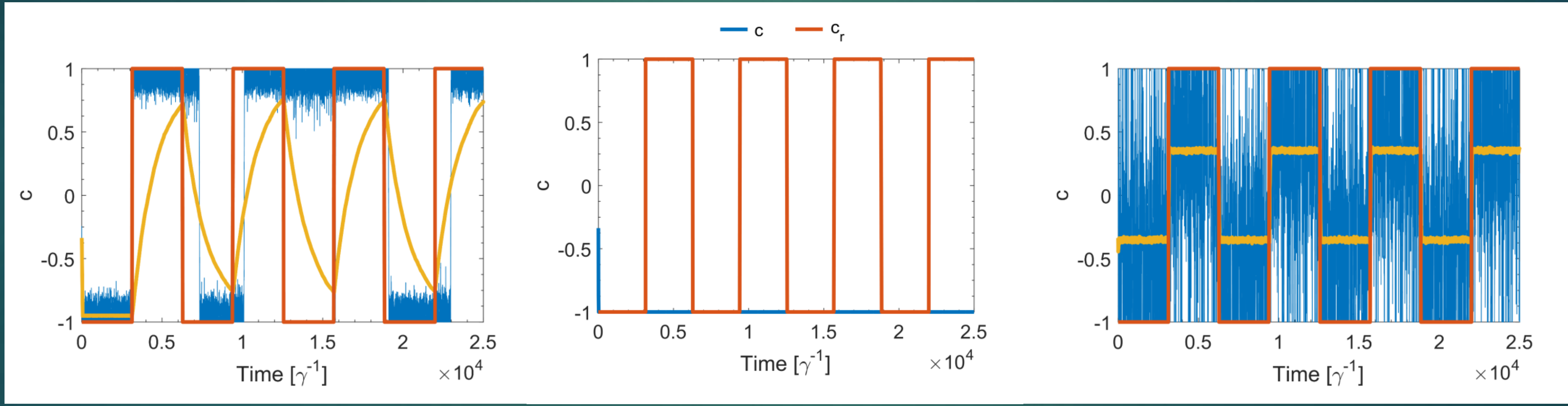


Symmetric configuration

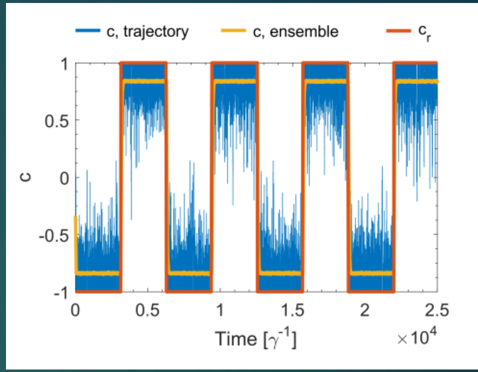
- Normalized correlation function:

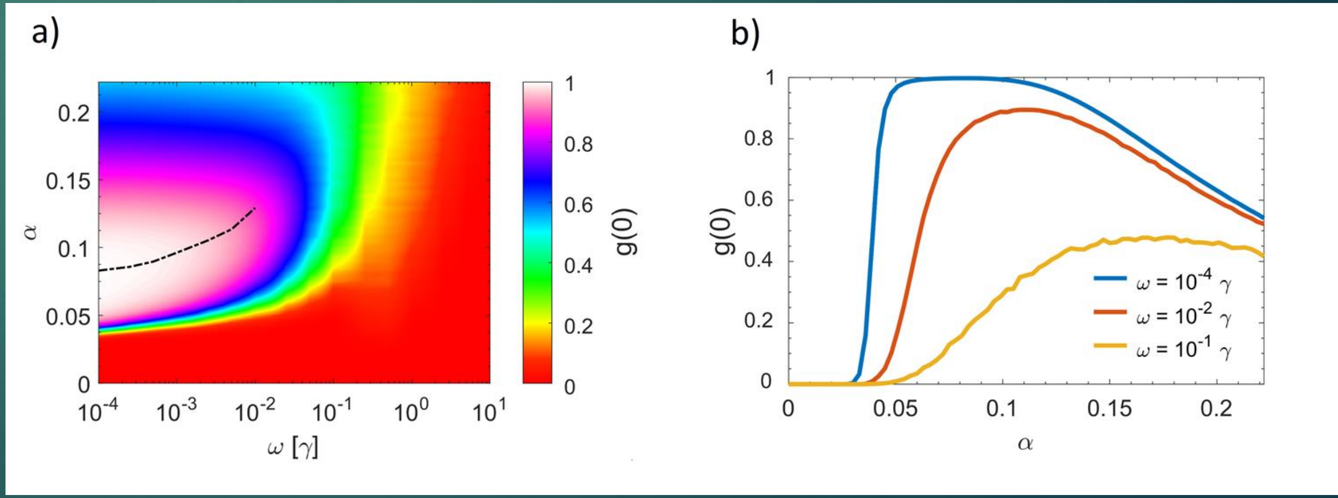
$$g(\tau) = \frac{2}{t_{\text{end}}} \int_0^{t_{\text{end}}} \left(\theta(c(t)) - \frac{1}{2}\right) c_r(t-\tau)\,dt$$

- Stochastic resonance:

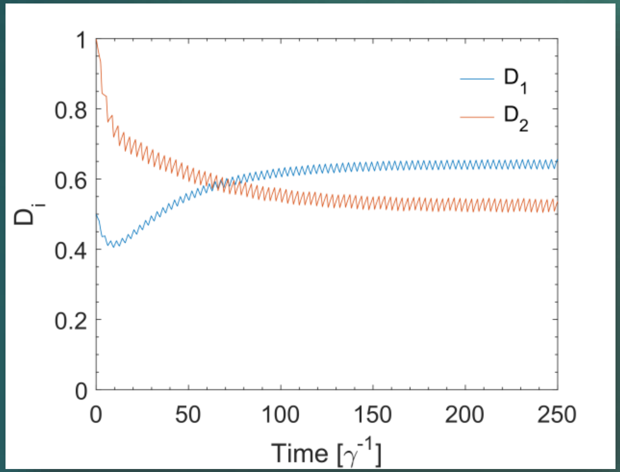
$$t_s = \frac{\pi}{\omega}$$

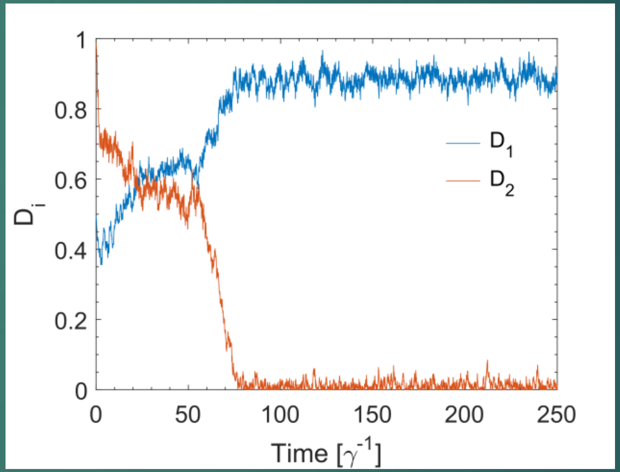
- Adaptation maximized at finite noise strength

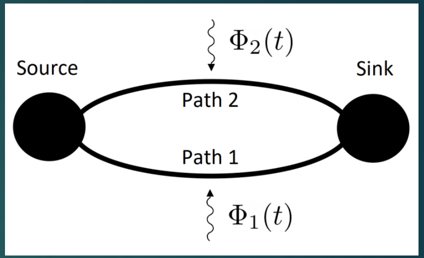




Asymmetric configuration



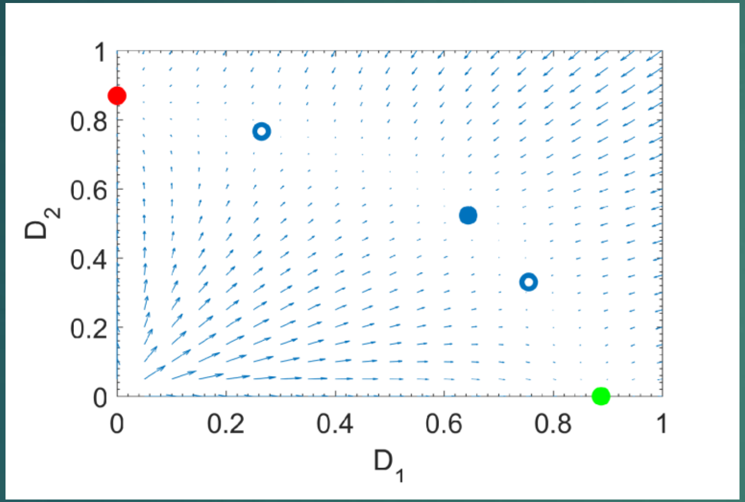


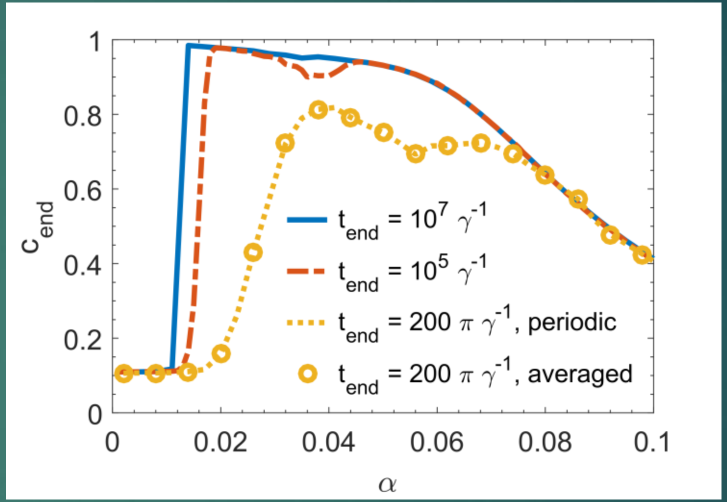


$$\langle\Phi_1\rangle = \frac{5}{6}\langle\Phi_2\rangle$$

- Dissipation minimization at finite noise strength
- Convergence speed improved due to noise

Asymmetric configuration



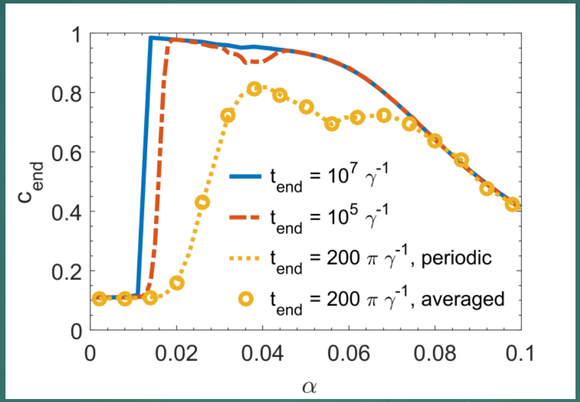


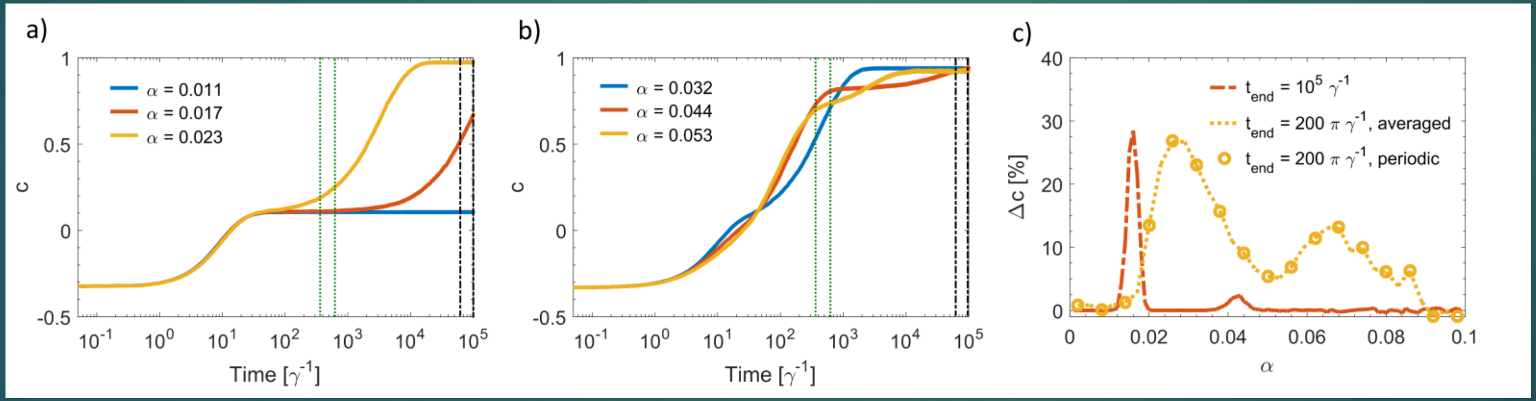
$$0 = f\left(\frac{D_i^*}{D_1^* + D_2^*}\right) - (\gamma + \langle\Phi_i\rangle)D_i^*$$

$$c = \frac{D_1 - D_2}{D_1 + D_2}$$

- Optimal noise for dissipation minimization

Asymmetric configuration





- Noise-induced metastable configurations

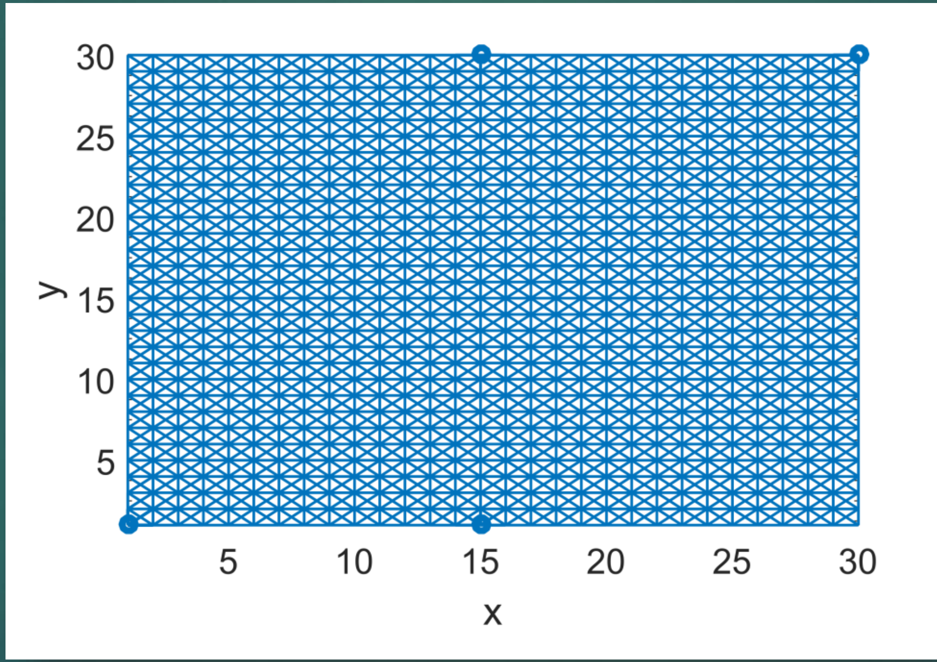
Conclusion

- Stochastic resonance enables the system to adapt
- Convergence speed is influenced by noise
- Occurrence of noise-induced metastable configurations
- Mapping to a one-dimensional system yields very good agreement

Outlook

- Extending the analysis to network dynamics
  - Transitions between network topologies
  - Robustness and transport efficiency of networks
- New algorithms for network growth under dynamical constraints
  - Reaction to time-dependent stimuli
  - Robustness against changes in the environment

Creating efficient transport networks



Creating efficient transport networks

- Flow of sol between neighboring nodes:

$$Q^i_{u,v} = \frac{D_{u,v}}{L_{u,v}}(p^i_u - p^i_v)$$

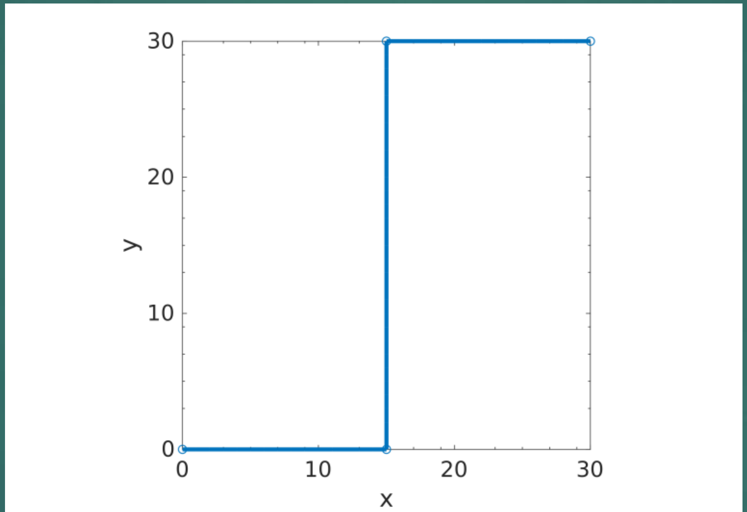
- Adaptation of tube diameter:

$$\frac{\partial D_{u,v}}{\partial t} = f\left(Q^1_{u,v}, \dots, Q^k_{u,v}\right) - \gamma_{u,v}(t) D_{u,v} + \tilde{\alpha}\xi_{u,v}(t)$$

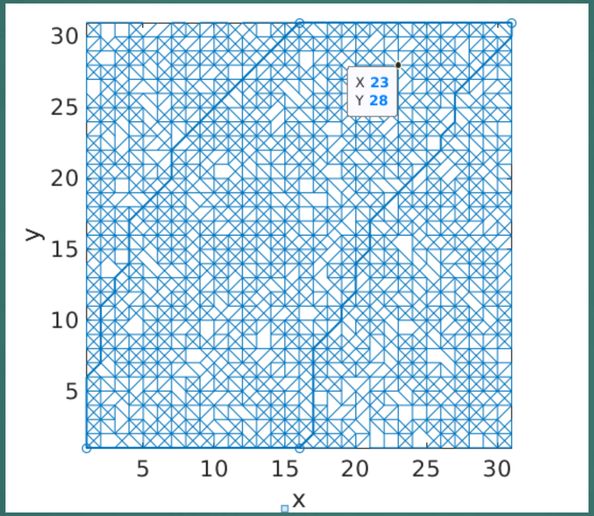
- Sol conservation:

$$\sum_{v \in E_u} Q^i_{u,v} = \begin{cases} 0 & \text{, if } u \text{ is a transit node of commodity } i \\ -I_0 & \text{, if } u \text{ is a source node of commodity } i \\ I_0 & \text{, if } u \text{ is a sink node of commodity } i \end{cases}, \forall i \in \{1, \dots, k\},$$

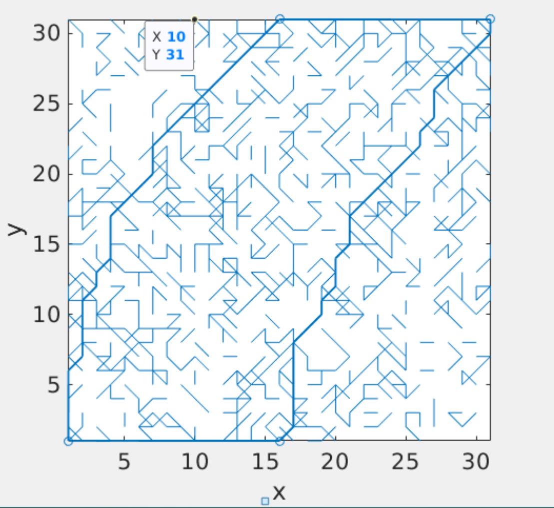
Creating efficient transport networks



Creating efficient transport networks



Creating efficient transport networks



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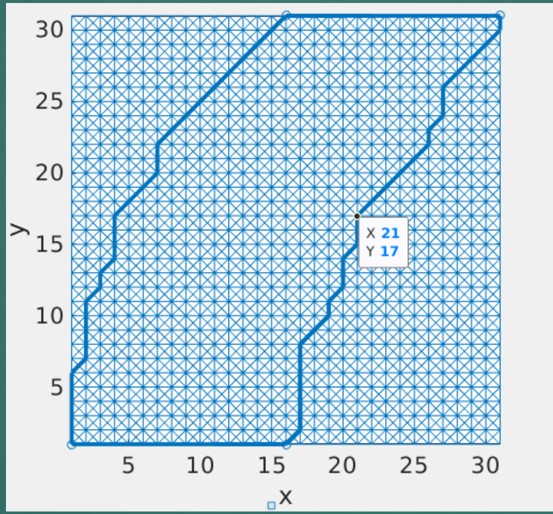

Creating efficient transport networks