Computation of non-singlet meson screening masses from low to high temperature

JHEP 04 (2022) 034, PoS (LATTICE 2021) 190

Mattia Dalla Brida, Leonardo Giusti, Tim Harris, *Davide Laudicina*, Michele Pepe

Università di Milano-Bicocca, Milan, Italy

May 30, 2022

Outline

High temperature QCD

Compact temporal extent $\rightarrow$ Matsubara formalism

$$\omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ 2\pi(n+\frac{1}{2})T & \text{fermions} \end{cases} \quad \text{with} \quad n \in \mathbb{Z}$$

At high temperature

- Heavy gauge non-zero modes decouple $\rightarrow$ only constant modes in x_0 survive
- Fermion fields are heavy static fields with mass $\sim \pi T$

The theory undergoes dimensional reduction

$$\downarrow$$

Effective Field Theory approach

M. Laine and A. Vuorinen, vol. 925, Springer (2016)

Electrostatic QCD

- At large T

Field content made of Matsubara zero-modes of the gauge fields. Quark fields decouple due to mass $\sim \pi T$.

$$S_{\mathsf{EQCD}} = \frac{1}{g_E^2} \int d^3x \, \frac{1}{2} \mathrm{Tr}\left[F_{ij}F_{ij}\right] + \mathrm{Tr}\left[(D_j A_0)(D_j A_0)\right] + m_E^2 \mathrm{Tr}\left[A_0^2\right] + \ldots$$

with the low energy constants $g_E^2 \sim g^2 T$, $m_E \sim gT$ obtained by perturbative matching with QCD.

- Three-dimensional Yang-Mills theory with field strength tensor F_{ij}
- A_0 behaves as a three-dimensional scalar field in the adjoint representation with mass m_E

T. Appelquist and R.D. Pisarski, Phys. Rev. D 23 (1981) 2305

Magnetostatic QCD

- At asymptotically large T

The scalar field is a heavy field with mass m_E and decouples. The theory reduces to a three-dimensional Yang-Mills theory

$$S_{\mathsf{MQCD}} = \frac{1}{g_E^2} \int d^3x \frac{1}{2} \,\mathrm{Tr}\,[F_{ij}F_{ij}] + \ldots$$

This theory is confining and needs to be solved non-perturbatively.

- g_E is the only energy scale of the theory $\rightarrow$ all dimensionful quantities are proportional to $g^2 T$
- Scale hierarchy $g_E^2/\pi \ll m_E \ll \pi T$ arises

A.D. Linde, Phys. Lett. B 96 (1980) 289

Fermionic sector

The quarks fields are heavy, static fields with effective action (by writing the fermion field $\psi = (\chi, \phi)$)

$$S_q = \int d^3x \, i\chi^\dagger \left[M - g_E + D_3 - \frac{1}{2M}\left(D_k^2 + \frac{g_E}{4i}\left[\sigma_k, \sigma_l\right] F_{kl} \right) \right] \chi + i\phi^\dagger \left[M - g_E - D_3 - \frac{1}{2M}\left(D_k^2 + \frac{g_E}{4i}\left[\sigma_k, \sigma_l\right] F_{kl} \right) \right] \phi + \dots,$$

where for the lightest mode $M \sim \pi T$.

- In three dimensions chiral symmetry is enlarged and the mass term breaks the symmetry down to the four-dimensional chiral group
- By power counting the spin-dependent term is $O(g^4)$.

M. Laine and M. Vepsäläinen, JHEP 02 (2004) 004

Screening masses

Screening mass m characterizes the large distance behaviour of fermionic bilinears.

$$C_{\mathcal{O}}(x_3) = \int dx_0 dx_1 dx_2 \, \langle O(x) O(0) \rangle \overset{x_3 \to \infty}{=} Ae^{-m x_3} + \ldots$$

- Related to the response of the plasma when a charge with quantum numbers carried by O is put in the system
- Carries information about symmetry restorations of the plasma
- Ideal lattice observable (RGI, static, no signal-to-noise ratio problem at high T)
- Check of the EFT prediction

Non-singlet screening masses in the EFT

Obtained at one-loop by solving the Schrödinger equation in 3-dimensions with static potential

$$V(r) \sim g_E^2 \ln r + g_E^4 r + O(g_E^6 r)\,.$$

The one-loop expression reads

$$m_{PT} \;=\; 2\pi T + \frac{g_E^2}{3\pi}(1 + 0.93878278) \;=\; 2\pi T(1 + 0.032739961\, g^2)\,.$$

- Scale hierarchy puts a constraint on the validity of the perturbative approach to the EFT
- Non-perturbative terms arising at $O(g^3)$ from the string term in the static potential

M. Laine and M. Vepsäläinen, JHEP 02 (2004) 004

T.H. Hansson and I. Zahed, Nucl. Phys. B 374 (1992) 277

Lattice QCD at very high temperatures

The problem

Renormalize the theory at very high temperature using a hadronic scheme would require to accommodate on a single lattice two scales that differ by orders of magnitude

$$a \ll \frac{1}{T} \ll \frac{1}{M_{\mathrm{had}}} \ll L$$

$$\downarrow$$

high resolution and large volumes required

The solution

Finite-volume coupling are defined at the scale $\mu = 1/L$ and by using step-scaling technique a wide range of scales can be explored in a fully non-perturbative way.

L. Giusti and M. Pepe, Phys. Lett. B 769 (2017) 385

Finite-volume coupling

Finite-volume coupling: finite size effects are no more a source of systematic error but part of the definition itself.

$$\bar{g}^2_{\mathsf{SF}}(L) \quad \text{at} \quad \mu = 1/L. \qquad \text{in a massless scheme}$$

Renormalize the theory by fixing the value of the renormalized coupling constant at fixed lattice spacing to be

$$\bar{g}^2_{\mathsf{SF}}(g_0^2, a\mu) \;=\; \bar{g}^2_{\mathsf{SF}}(\mu)\,, \qquad a\mu \ll 1\,.$$

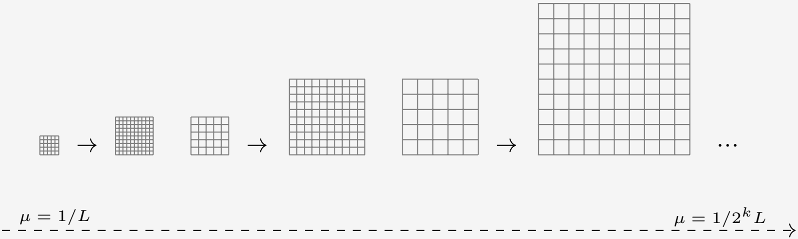
This fixes the **Lines of Constant Physics (LCP)**, i.e. the dependence of the bare coupling on the lattice spacing, for values of a at which the scale μ can be accommodated.

M. Lüscher et al., Nucl. Phys. B 359 (1991) 221

Step-scaling function

- Step-scaling function used to reach very high energy scale

$$g^2_{\mathsf{SF}}(L) \quad \Longrightarrow \quad \sigma(u) = g^2(2L)\Big|_{g^2(L)=u} \quad \Longrightarrow \quad u_k = \sigma(u_{k-1})$$



- The overall scale is fixed at low energy by some suitable dimensionful hadronic quantity $(r_0, f_\pi, f_{\pi K})$

ALPHA collaboration, Phys. Rev. Lett. 117 (2016) 182001

ALPHA collaboration, Eur. Phys. J. C 78 (2018) 387

M. Bruno et al.,Phys. Rev. D 95 (2017) 074504

Exploring different scales

Renormalized coupling at a given scale

$$\bar{g}^2_{\mathsf{SF}}(\mu_0) = 2.0120 \longleftrightarrow \mu_0 = 4.30(11)\mathrm{GeV}$$

Non-perturbative relation between renormalized coupling and energy scale

$$\ln\left(\frac{\mu}{\mu_0}\right) = \int_{\bar{g}_{\mathsf{SF}}(\mu_0)}^{\bar{g}_{\mathsf{SF}}(\mu)} \frac{dg}{\beta_{\mathsf{SF}}(g)}$$

Bare coupling extracted for each L/a with the interpolating formula

$$\frac{1}{g^2_{\mathsf{SF}}} = \frac{1}{g_0^2} + \sum_{k=0}^{n_p} c_k g_0^{2k}$$

Same strategy applied in the low energy regime with GF coupling.

ALPHA collaboration, Phys. Rev. D 95 (2017) 014507

From finite volume scheme to finite temperature

Finite volume scheme

Finite volume coupling with SF boundary conditions

$\downarrow$

Step-scaling function

$\downarrow$

Matching with hadronic scheme ($M_{\rm had}$)

$\downarrow$

Known relation between

$$M_{\rm had} \Longleftrightarrow L_{\sf SF} \Longleftrightarrow g_{\sf SF}(L_{\sf SF})$$

Finite Temperature

Finite temporal extent L_0 with periodic boundary conditions

$\downarrow$

Temperature fixed by imposing the relation $1/T = L_0 = L_{\sf SF} = 1/\mu$

$\downarrow$

Known relation between

$$T \Longleftrightarrow g_{\sf SF}(L_0)$$

No large volumes needed

Finite volume effects

Define the finite-volume residue due to the compactification in the 1-direction

$$\mathcal{I}(x_3) \equiv \left[1 - \lim_{L_1 \to \infty}\right] C_O(x_3)$$

Using spectral decomposition and inserting a set of states $|n\rangle$, this can be written in terms of a matrix element $G_n(x_3)$

$$\mathcal{I}(x_3) = \sum_{n|_{\text{1-particle states}}} e^{-LE_n} \{G_n(x_3) - G_0(x_3)\}$$

where the 1-particle states have $M_{\text{gap}} \leq E_n \leq \pi T$ with $M_{\text{gap}} \sim g^2 T$.

- Finite volume effects exponentially suppressed as $M_{\text{gap}} L \to \infty$

M.T. Hansen, A. Patella, Phys. Rev. Lett 123 (2019) 172001

Lattice set-up

- HMC algorithm
- $N_f = 3$ quarks in the **chiral limit**
- O(a)-improved Wilson fermions
- four different lattice resolutions: $L_0/a = 4, 6, 8, 10$
- large volumes to keep finite volume effects under control $(LT \sim 20 - 50)$
- 12 values of the temperature in the range $1.167 - 164.6$ GeV
- restricted to $Q = 0$ topology sector $(\chi_Q \sim T^{-8})$
- shifted boundary conditions
- distance-preconditioning

	$\bar{g}^2_{\mathrm{SF}}(\mu = T\sqrt{2})$	T (GeV)
T_0	-	164.6(5.6)
T_1	1.11000	82.3(2.8)
T_2	1.18446	51.4(1.7)
T_3	1.26569	32.8(1.0)
T_4	1.3627	20.63(63)
T_5	1.4808	12.77(37)
T_6	1.6173	8.03(22)
T_7	1.7943	4.91(13)
T_8	2.0120	3.040(78)
	$\bar{g}^2_{\mathrm{GF}}(\mu = T/\sqrt{2})$	T (GeV)
T_9	2.7359	2.833(68)
T_{10}	3.2029	1.821(39)
T_{11}	3.8643	1.167(23)

Shifted boundary conditions - I

Thermal theory defined imposing periodic (anti-periodic for fermions) boundary conditions in the spatial direction and shifted boundary conditions in the temporal extent.

$$\begin{aligned} U_\mu(x_0 + L_0, \boldsymbol{x}) &= U_\mu(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi}) \\ \psi(x_0 + L_0, \boldsymbol{x}) &= -\psi(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})\,. \end{aligned}$$

Theory equivalent to the same theory with usual periodic (anti-periodic) boundary conditions but with temporal extent $L_0' = L_0\sqrt{1+\boldsymbol{\xi}^2}$ and temperature

$$T = \frac{1}{L_0\sqrt{1+\boldsymbol{\xi}^2}}\,.$$

In this work $\boldsymbol{\xi} = (1, 0, 0)$

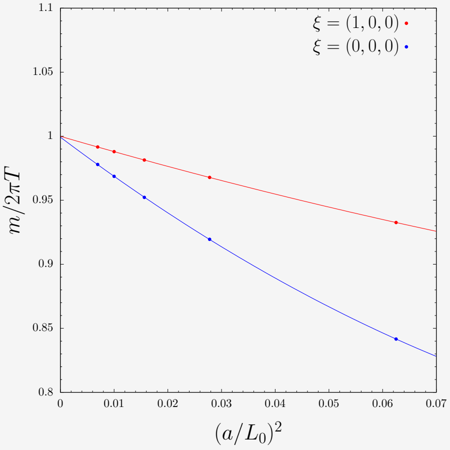
L. Giusti and H.B. Meyer, JHEP 11 (2011) 087

L. Giusti and H.B. Meyer, Phys. Rev. Lett. 106 (2011) 131601

L. Giusti and H.B. Meyer, JHEP 01 (2013) 140

Shifted boundary conditions - II

- Efficient set-up for several problems
- Not crucial for this work, but **milder discretization effects**



L. Giusti and M. Pepe, Phys. Rev. Lett. 113 (2014) 031601

M. Bresciani et al., arXiv:2203.14754

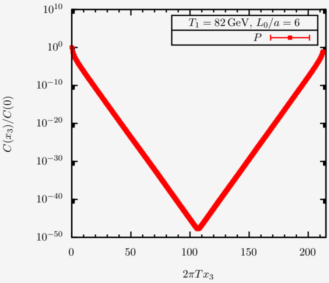
Screening correlation functions

Screening correlators on the lattice computed along 3-direction

$$C_O(x_3 - y_3) = -\frac{a^3}{2} \sum_{x_0, x_1, x_2} \langle \mathrm{Tr}\left[\Gamma_O D^{-1}(x,y) \Gamma_O \gamma_5 D^{\dagger -1}(x,y)\gamma_5\right]\rangle \ ,$$

with trace over color and spin indices and $\Gamma_O = (\gamma_5, \mathbb{I}, \gamma_2, \gamma_2\gamma_5)$ corresponding to

$$\begin{aligned} P^a &= \bar{\psi}\gamma_5 T^a \psi \\ S^a &= \bar{\psi} T^a \psi \\ V_2^a &= \bar{\psi}\gamma_2 T^a \psi \\ A_2^a &= \bar{\psi}\gamma_2\gamma_5 T^a \psi \end{aligned}$$



- Only connected contribution

Distance preconditioning

The problem

- $D^{-1}(x, y)$ suppressed at large distances $|x - y|$ due to $\omega_0 = \pi T$
- global stopping criterion not reliable at large distances

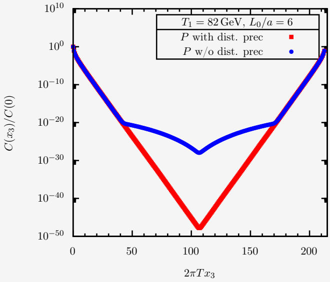
The solution: Distance preconditioned version of the Dirac equation

$$\tilde{D}\tilde{\psi} = \tilde{\eta} \qquad \text{with} \qquad \begin{cases} \tilde{D} = M^{-1} D M \\ \tilde{\psi} = M^{-1}\psi \\ \tilde{\eta} = M^{-1}\eta \end{cases}$$

where

$$M(x, y) = \cosh\left[m_M(x_3 - y_3 - L/2)\right].$$

With $m_M \sim \pi T$ all the components of $\tilde{\psi}$ have the same magnitude.



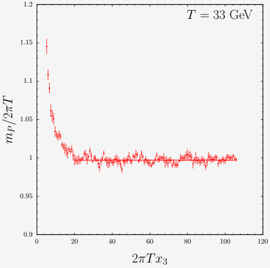
G.M. de Divitiis et al., Phys. Lett. B 692 (2010) 157

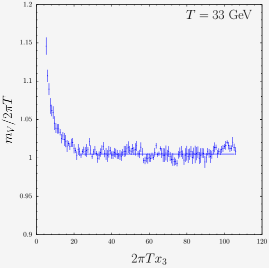
Effective mass

At large separation, the effective mass reads

$$m_O(x_3) \;=\; \frac{1}{a}\text{arcosh}\left[\frac{C_O(x_3+a)+C_O(x_3-a)}{2C_O(x_3)}\right]$$

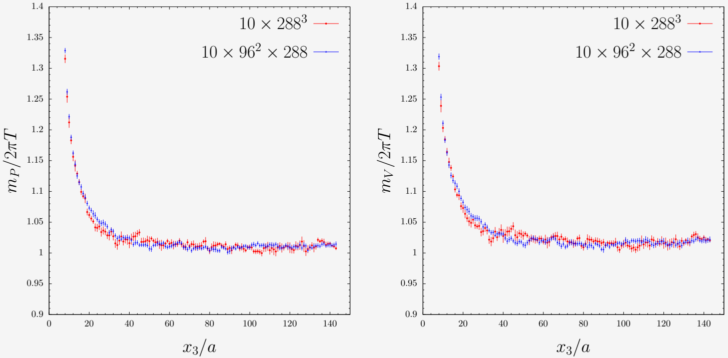
Large spatial extents ($L/a = 288$) guarantee very long plateaux
➔ excited states contribution is negligible in the fit region.





Finite volume effects

- expected to be suppressed for very large LT
- explicit check on volumes with smaller transverse spatial extent $(L/a = 96, 144)$ at the highest and lowest temperatures



➢ Negligible finite volume effects within statistical precision

Continuum limit - I

- Symanzik effective theory predicts the leading lattice artefacts to be $O(a^2)$.
- Convergence to the continuum limit accelerated with the tree-level improved definition

$$m_O \to m_O - \left[m_O^{\text{free}} - 2\pi T \right]$$

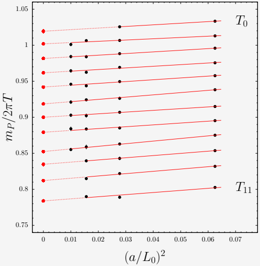
- m_O^{free} analytically computed at fixed lattice spacing a/L_0 in the infinite spatial-volume limit
- Spin-independent tree-level contribution

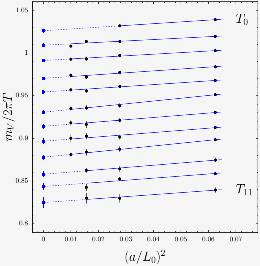
L_0/a	$m^{\text{free}}/2\pi T$
4	0.932614077...
6	0.967811412...
8	0.981401809...
10	0.987944825...

Continuum limit - II

With the improved definition we fit linearly in $(a/L_0)^2$ with fit ansatz

$$m_O = m_O^{\text{cont}} + const \times \left(\frac{a}{L_0}\right)^2$$



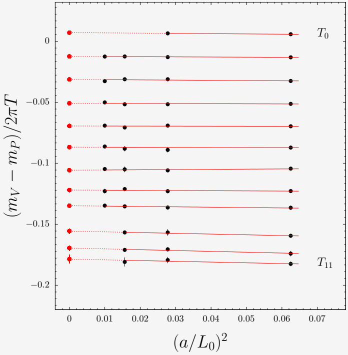


- high precision: few **per mille-accuracy** in the continuum limit
- $\chi^2/\text{dof} \sim 1$ in almost every extrapolation

Continuum limit - III

Spin-dependence analysis by studying mass-splitting between the vector and the pseudoscalar channels

- coefficient of $(a/L_0)^2$ compatible with zero



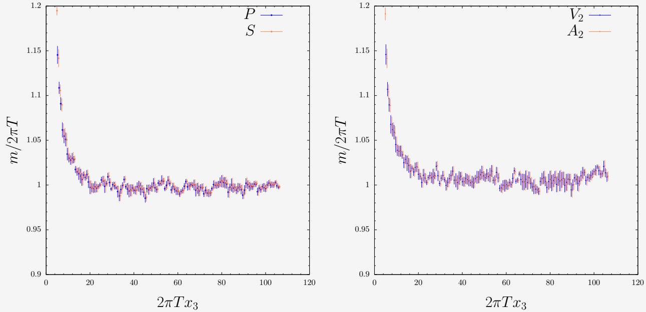
Further checks of the extrapolations

- fit excluding the coarsest lattice spacing $(L_0/a = 4)$ for $T_1 - T_8$: excellent agreement in the intercepts
- fit with additional terms $\sim (a/L_0)^2 \ln (a/L_0)$ or $\sim (a/L_0)^3$: coefficients compatible with zero

Chiral symmetry restoration

Observation of **chiral multiplets** in the entire range of temperature explored.

- Restoration of non-singlet chiral symmetry: $V_2^a \leftrightarrow A_2^a$
- Restoration of singlet axial $U(1)$: $P^a \leftrightarrow S^a$

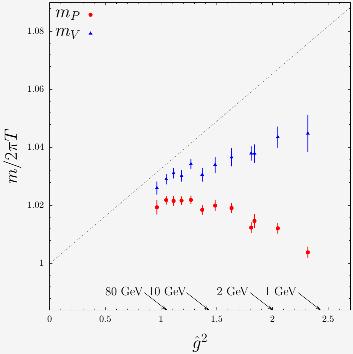


Temperature dependence

Parametrize the temperature dependence in terms of the two-loop $\overline{\mathrm{MS}}$ coupling at the scale $\mu = 2\pi T$

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} \right)$$

- Bulk of the masses given by the free theory result with a few percent positive deviation
- Mass-splitting visible in the entire range of temperature
- Masses not compatible with perturbative result (dashed line)

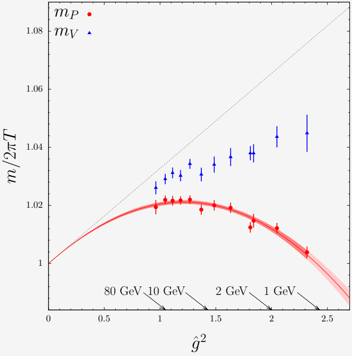


Temperature dependence: pseudoscalar mass - I

Fit the pseudoscalar mass to a quartic polynomial in $\hat{g}$

$$\frac{m_P}{2\pi T} = p_0 + p_2 \hat{g}^2 + p_3 \hat{g}^3 + p_4 \hat{g}^4$$

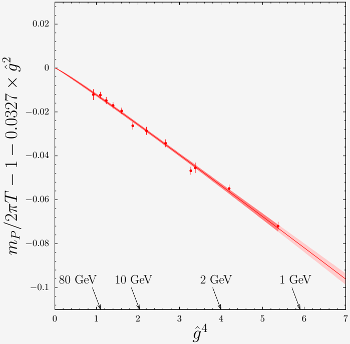
- p_0 and p_2 coefficients compatible with PT
- p_3 compatible with zero
- Negative contribution from p_4
- $|p_4| \sim |p_2|/2$. The two terms equal at $T \sim 1$ GeV



Temperature dependence: pseudoscalar mass - II

Subtract the known leading coefficients

- Almost a straight line over two orders of magnitude in the temperature
- At the electroweak scale the quartic term is half of the total contribution due to interactions

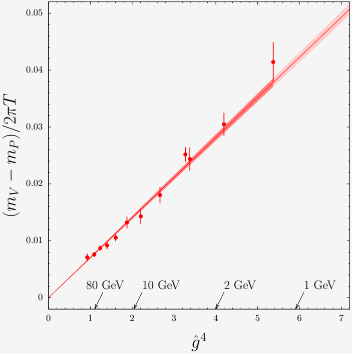


Temperature dependence: mass difference

Spin-dependent contributions are expected to be $O(g^4)$. We use the ansatz

$$\frac{m_V - m_P}{2\pi T} = s_4 \hat{g}^4$$

- $\chi^2/\mathrm{dof} = 0.79$: large spin-dependent term parameterized by a single $O(\hat{g}^4)$ term in the entire range of temperature
- mass-splitting clearly visible even at the highest temperature

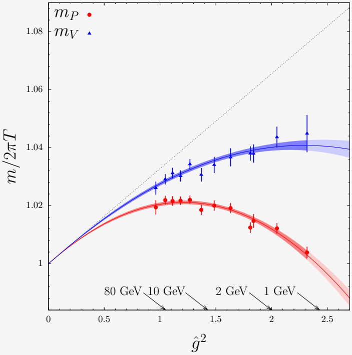


Temperature dependence: vector mass

By taking into account the best polynomial for the pseudoscalar screening mass and for the mass-splitting, the best parametrization for the vector mass is

$$\frac{m_V}{2\pi T} = p_0 + p_2\hat{g}^2 + p_3\hat{g}^3 + (p_4 + s_4)\,\hat{g}^4$$

- Positive contribution from s_4 but $|s_4| \sim |p_4|/2$
- At 1 GeV s_4 is responsible for the 4% positive deviation with respect to the free-theory result
- At the electroweak scale s_4 is responsible for $\sim 15\%$ of the total contribution due to interactions

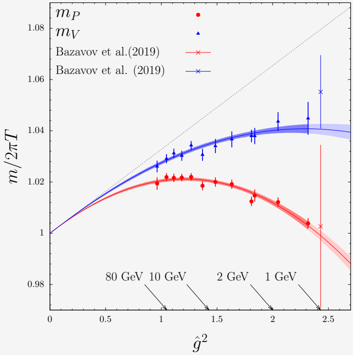


Temperature dependence: final results

Final parametrizations:

$$\begin{cases} \frac{m_P}{2\pi T} = p_0 + p_2\hat{g}^2 + p_3\hat{g}^3 + p_4\hat{g}^4 \\ \frac{m_V}{2\pi T} = \frac{m_P}{2\pi T} + s_4\hat{g}^4 \end{cases}$$

- Perturbation theory not reliable: higher order terms needed
  - $O(g^4)$ effects relevant at **low temperature**
  - mass-splitting at **high temperature**
- New perspective on past lattice results at low temperature



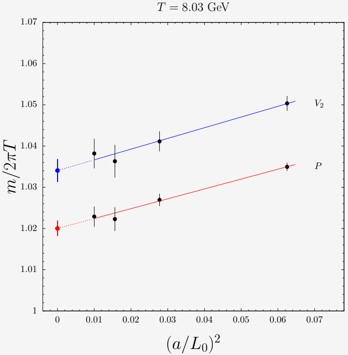
Conclusions and outlook

- Step-scaling technique provides a solid strategy to study QCD at very high temperature
- First non-perturbative results from 1 GeV up to ~ 160 GeV
- Results obtained with a few per mille-accuracy in the continuum limit
- Non-trivial pattern of different contributions explains previous lattice results at $T \lesssim 1$ GeV
- Higher order terms are still relevant at the electroweak scale

Outlook

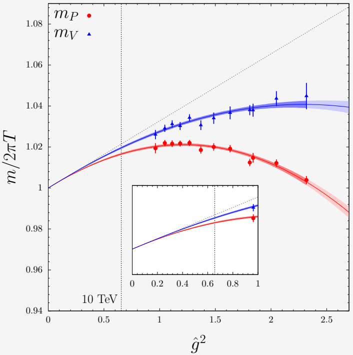
- Thermodynamic properties of thermal QCD at high temperature, baryon screening masses
- Non-perturbative matching to verify the effective field theory approach

Continuum Limit



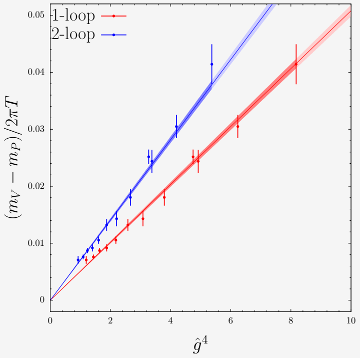
- Very good χ^2/dof
- Data at the smaller lattice spacing are within 1σ with respect to the continuum limit result

Temperature dependence



- Perturbation theory not reliable at the electroweek scale
- One-loop perturbative result reliable only for (at least) $T > 10$ TeV

Parameterization



- 1-loop:

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}}$$

- 2-loop:

$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\mathrm{MS}}}} \right)$$