The Tadpole Problem

Mariana Graña
CEA / Saclay
France

Work in collaboration with

Iosif Bena, Johan Blåbäck and Severin Lüst	arXiv: 2010.10519
	arXiv: 2103.03250
Iosif Bena, Callum Brodie	arXiv: 2112.00013
Thomas Grimm, Damian van de Heisteeg, Alvaro Herraez, Erik Plauschinn	arXiv: 2204.05331
Andreas Braun, Bernardo Fraiman, Severin Lust, Hector Parra De Freitas	arXiv: 2205.xxxxx

Hamburg, May 2022

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Ashok, Denef, Douglas 03

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  $\longrightarrow$ Can be done with fluxes

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- Fluxes induce charges

- How large is the charge induced by fluxes needed to stabilize a given number of moduli?

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The tadpole conjecture

Bena, Blåbäck, M.G., Lüst 20

• For a large number N of moduli

$$\boxed{Q_{\text{flux s.t. all mod stabilized at a generic point in mod space}} > \alpha \, \mathrm{N}}$$

with $\alpha > \frac{1}{3}$

Here: IIB flux Compactifications on Calabi-Yau

$$M_{10} = M_4 \times_w CY_3$$

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- Add 3-form fluxes

$$\int_{\alpha_I} F_3 = \underbrace{M^I}_{\in \mathbb{Z}} \qquad \int_{\alpha_I} H_3 = K^I$$

basis of 3-cycles

$I = 1, ..., 2h^{2,1} + 2$

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$$S \sim \int F_3 \wedge \star F_3 + e^{-2\phi} H_3 \wedge \star H_3$$

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Tadpole cancelation condition

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(maximum charge from O3-planes $100(-\frac{1}{4})$)

Carta, Moritz, Westphal 20

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- Unified description in F-theory

F-theory on CY_4

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$$\left.\begin{array}{l}h^{2,1} \text{ complex structure moduli} \\ \text{D7-brane moduli}\end{array}\right\} \quad h^{3,1} \text{ complex structure moduli of } CY_4$$

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D7-brane moduli

$\Big\}$ $h^{3,1}$ complex structure moduli of CY_4

3-form fluxes H_3, F_3

2-form fluxes F_2 on D7

$\Big\}$ 4-form flux G_4

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Tadpole conjecture

$$\boxed{\frac{1}{2} \int G_4 \wedge G_4 \Big|_{\text{all moduli are stabilized}}} > \boxed{\tfrac{1}{3} N}$$

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Tadpole conjecture

$$\frac{1}{2}\int G_4 \wedge G_4 \Big|_{\text{all moduli are stabilized}} > \frac{1}{3}N$$

If true, cannot stabilize
a large number of moduli!!

Supporting arguments

Tadpole conjecture $\alpha > \frac{1}{3}$

Description	N	Q_{flux}	$\alpha = Q_{flux}/N$	Ref
IIB at highly symm pt in mod space	$h^{2,1} = 128$	48	0.38	Giryavets, Kachru, Tripathy, Trivedi 03
	$h^{2,1} = 272$	124	0.46	Demirtas, Kim, Mc Allister, Morritz 19
F-theory on sextic CY	$h^{3,1} = 426$	775/4	0.45	Braun, Valandro 20
F-theory on $\mathbb{CP}^3$ base	$n_7 = 3728$	1638	0.44	Collinucci, Denef Esole 08
F-theory on any weak-Fano base	$n_7 = 58c_1^3(B) + 16$	$\frac{7}{16}(58c_1^3(B) + 15)$	0.44	Bena, Brodie, M.G. 21
M-theory on K3xK3	57	25	0.44	Bena, Blåbäck, M.G., Lust 20
F-theory close to boundaries of mod. space	N	> 0.5 N	> 0.5	M.G., Grimm, van de Heisteeg, Herraez, Plauschinn 22

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lattice of signature (3,19)

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- Fixing moduli on K3: choosing 3-plane Σ of self-dual 2-forms

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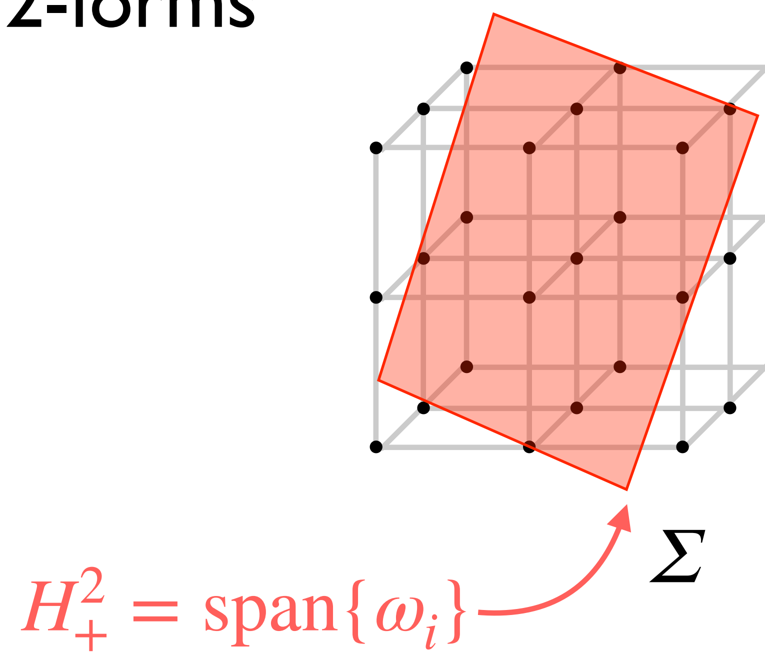
$$\Omega = \omega_1 + i\omega_2 \qquad J \sim \omega_3$$



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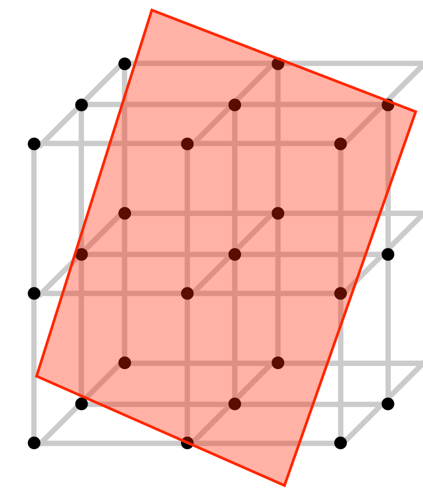


Σ

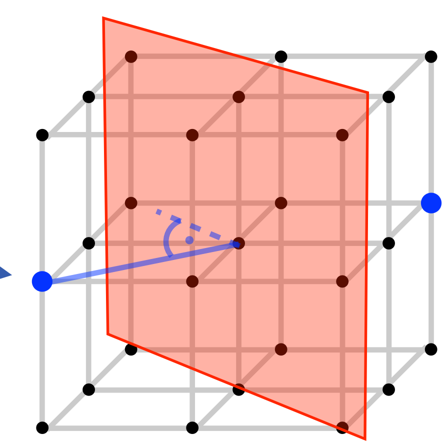
$H^2_+ = \mathrm{span}\{\omega_i\}$

- We require smooth compactification (no orbifold singularity)

orbifold singularity if ∃

root $\alpha \in H^2(K3,\mathbb{Z})$ such that $\alpha \perp \Sigma$

$(\alpha,\alpha) = -2$



α

- K3 x K3' with 4-form flux

$G_4 \in H^2(K3,\mathbb{Z}) \times H^2(K3',\mathbb{Z})$

basis of $H^2(K3,\mathbb{Z})$ $\quad I = 1, ..., 22$

$$G_4 = N^{I\tilde{J}} \alpha_I \wedge \alpha'_{\tilde{J}}$$

22x22 integer matrix

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Braun, Hebecker, Ludeling, Valandro 08

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-Define a map $M : H^2(K3) \to H^2(K3)$

$$d_{IJ} = \int_{K3} \alpha_I \wedge \alpha_J$$

$$M^I{}_J = N^{I\tilde{K}} d_{\tilde{K}\tilde{L}} N^{M\tilde{L}} d_{MJ}$$

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(i) M is diagonalizable with non-negative eigenvalues

$$\{\underbrace{a_1^2, a_2^2, a_3^2}_{\substack{\text{eigenvectors with}\\ \text{positive norm}\\ \Sigma}}, \underbrace{b_1^2, ..., b_{19}^2}_{\substack{\text{eigenvectors with}\\ \text{negative norm}}}\}$$

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(iii) No root in the lattice $\perp \ \Sigma$

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- Used evolutionary algorithm

Evolutionary algorithm

- Optimization inspired by biological evolution (population, mutation, selection)
- Random initial population: P={ $N \in \mathbb{R}^{484}$ } (rounded to $\mathbb{Z}$)

  22×22
- For each N, mutate some entries using other elements of population
- From original and mutated, select the one that minimizes a fitness function

weights (determined empirically)

$$f = \sum_{k=1}^{3} w^k p_k(N) + w^Q Q_{\rm flux}(N)$$

penalty if (i)-(iii) is violated

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- Perform local search (brute force) around minima

Results

Bena, Blåbäck, M.G., Lüst 20

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- This behavior confirmed by looking at smaller dimensional lattices

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lattice Λ	$D = \dim(\Lambda)$	$Q_{\min}(\Lambda)$
$3\,U$	6	5
$A_4 \oplus U$	6	6
$D_4 \oplus U$	6	6
$A_4 \oplus 2\,U$	8	7
$D_4 \oplus 2\,U$	8	6
$E_6 \oplus U$	8	9
$A_4 \oplus 3\,U$	10	9
$D_4 \oplus 3\,U$	10	9
$E_8 \oplus U$	10	10
$E_8 \oplus 2\,U$	12	12
$E_8 \oplus 3\,U$	14	13
$2\,E_6 \oplus 2\,U$	16	14
$2\,E_8 \oplus U$	18	20
$2\,E_8 \oplus 2\,U$	20	21
$2\,E_8 \oplus 3\,U$	22	25

- Minimum charge ≥ D-1

Analytic supporting evidence on K3xK3

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- Using properties of lattice embeddings we proved: has no roots

no roots in the lattice $\perp$ Σ $\Leftrightarrow$ rank 6 lattice $T_S^{\perp} \subset E_8$

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- One could think of this as a positive result, but non-Abelian gauge groups come with extra moduli (brane moduli) that need to be stabilised

Analytic supporting evidence: D7 moduli stabilization

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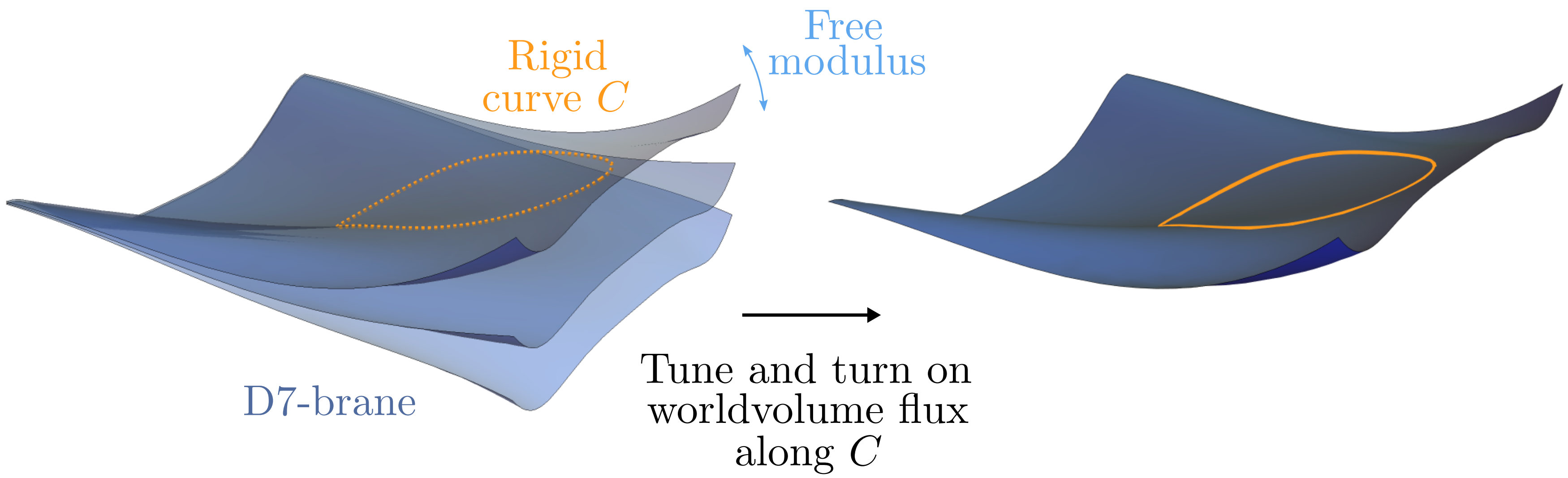

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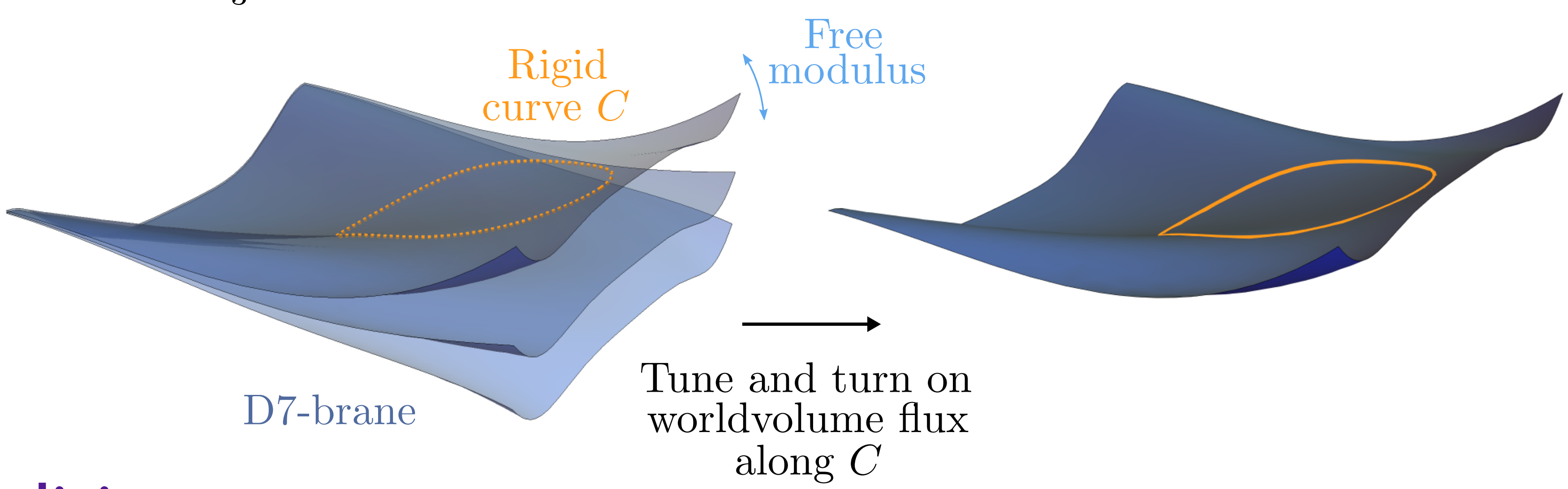

Tadpole cancelation condition

$$\boxed{Q_{\text{flux}} = \frac{1}{2}\int_S F_2 \wedge F_2} \leq \boxed{15 \int_{B_3} c_1(B_3)^3 + 12}$$

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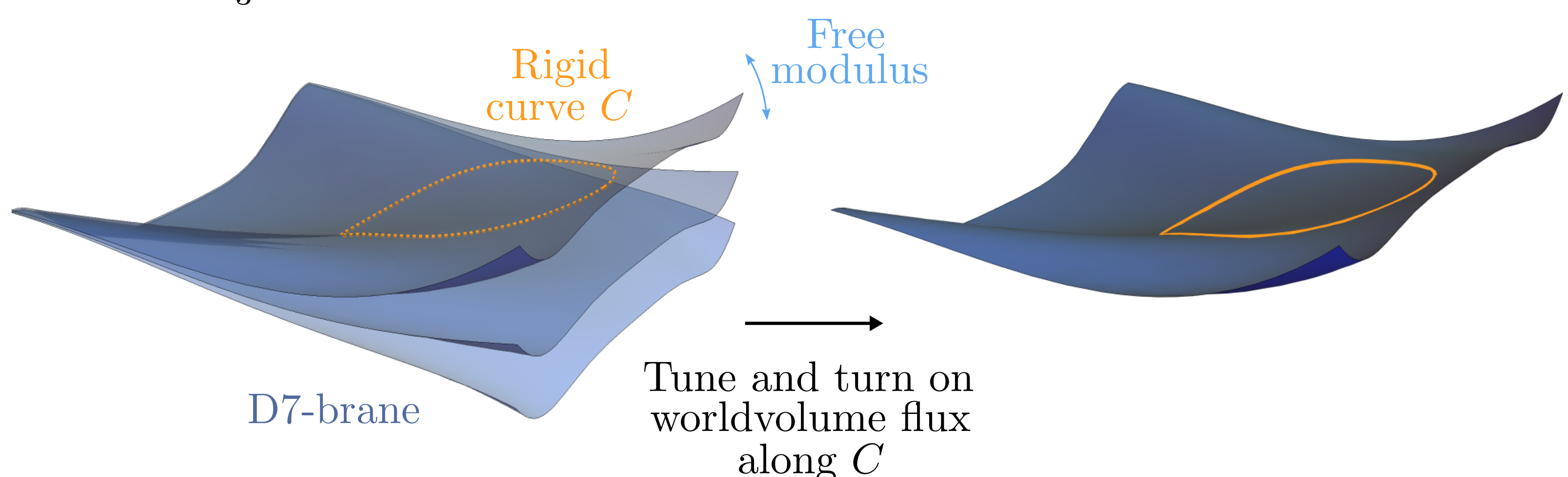

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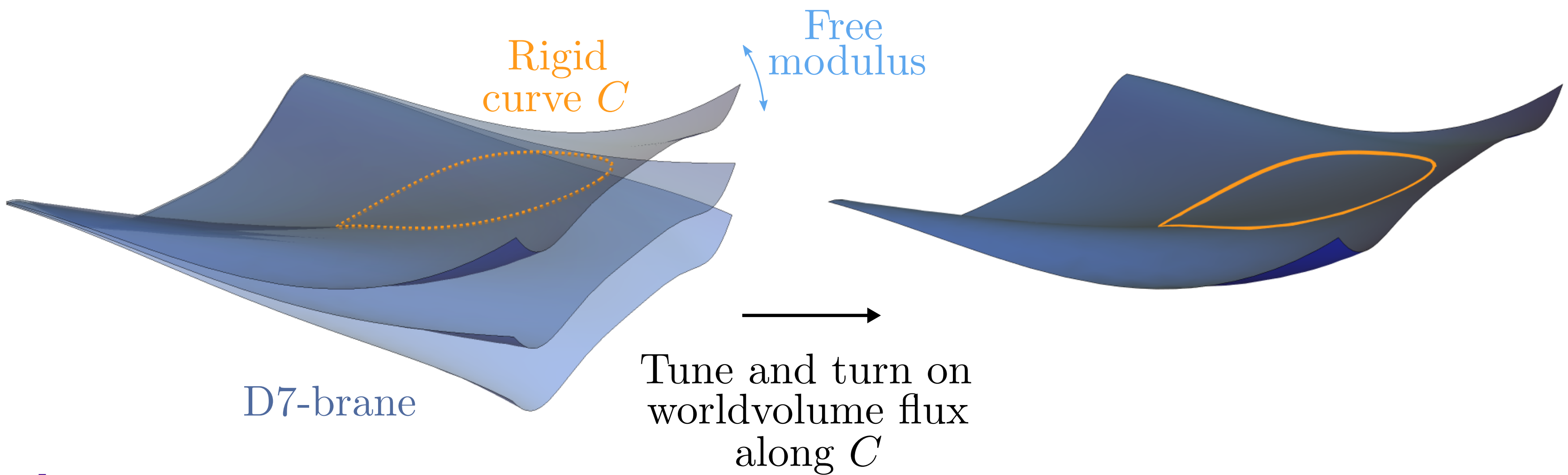

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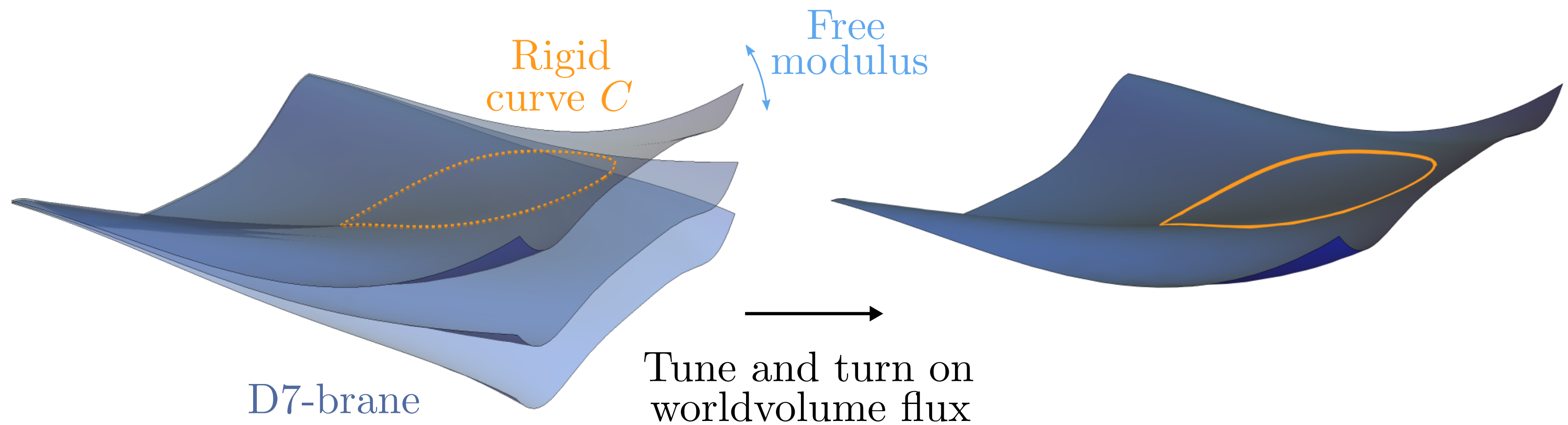

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If true, cannot stabilize a large number of moduli

D7-moduli stabilization

I.Bena, C. Brodie, M.G. 21

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$$C \cdot (-K) = 4d \text{ for } \mathbb{CP}^3$$

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- This reduces to the result for B3= $\mathbb{CP}^3$, genus 0

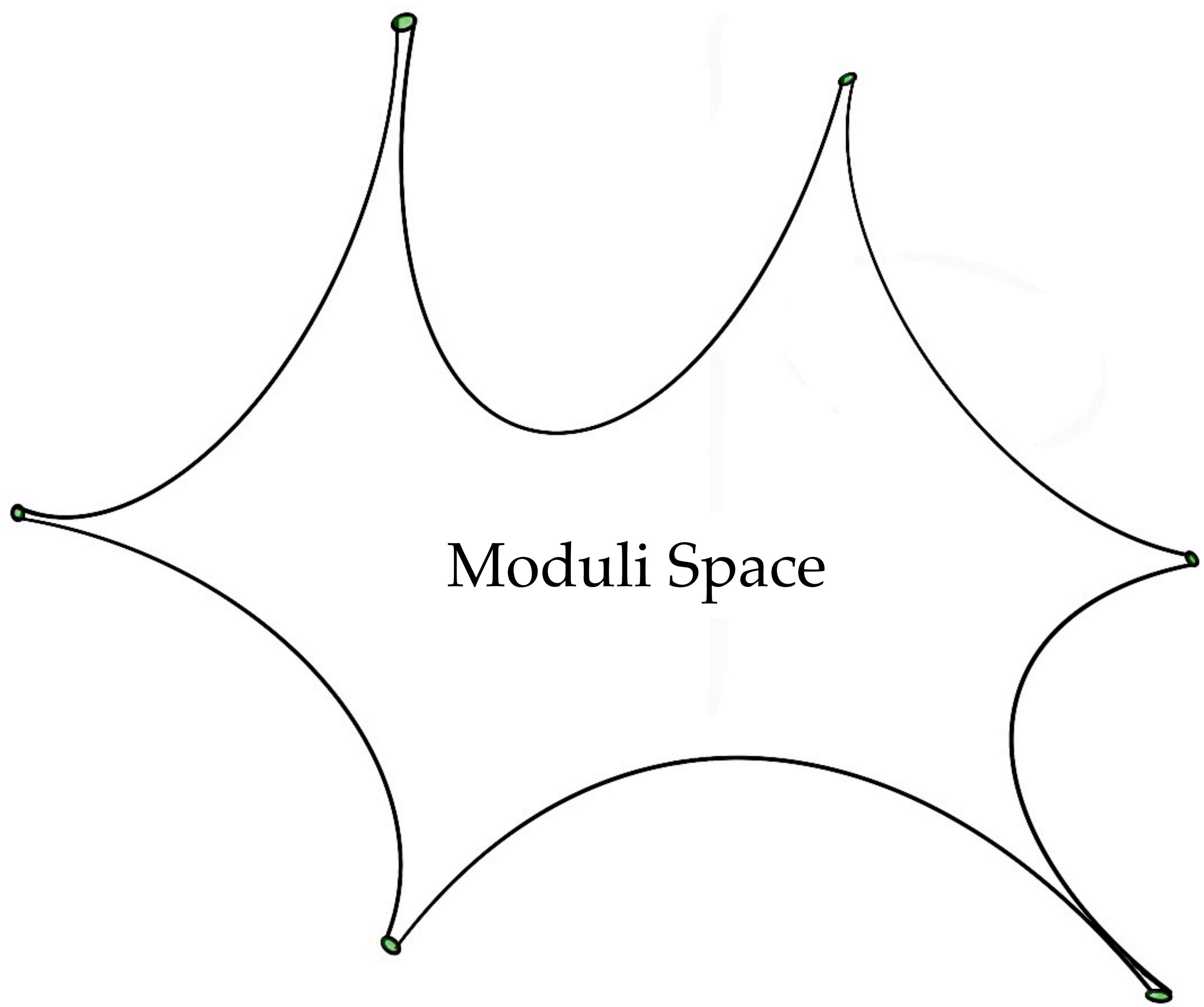
Collinucci, Denef, Esole 08

$$n_{\text{stab.moduli}} = 32d + 1 = 3728 \qquad Q_{\text{flux}} \geq 14d + 1 \geq 1640 \qquad |Q_{\text{neg}}| = 972 \qquad \alpha \geq \frac{14}{32} \simeq 0.44$$

Analytic supporting evidence: asymptotic limits

M.G., Grimm, van de Heisteeg, Herraez, Plauschinn 22

- Asymptotic limits of moduli space (here complex structure)



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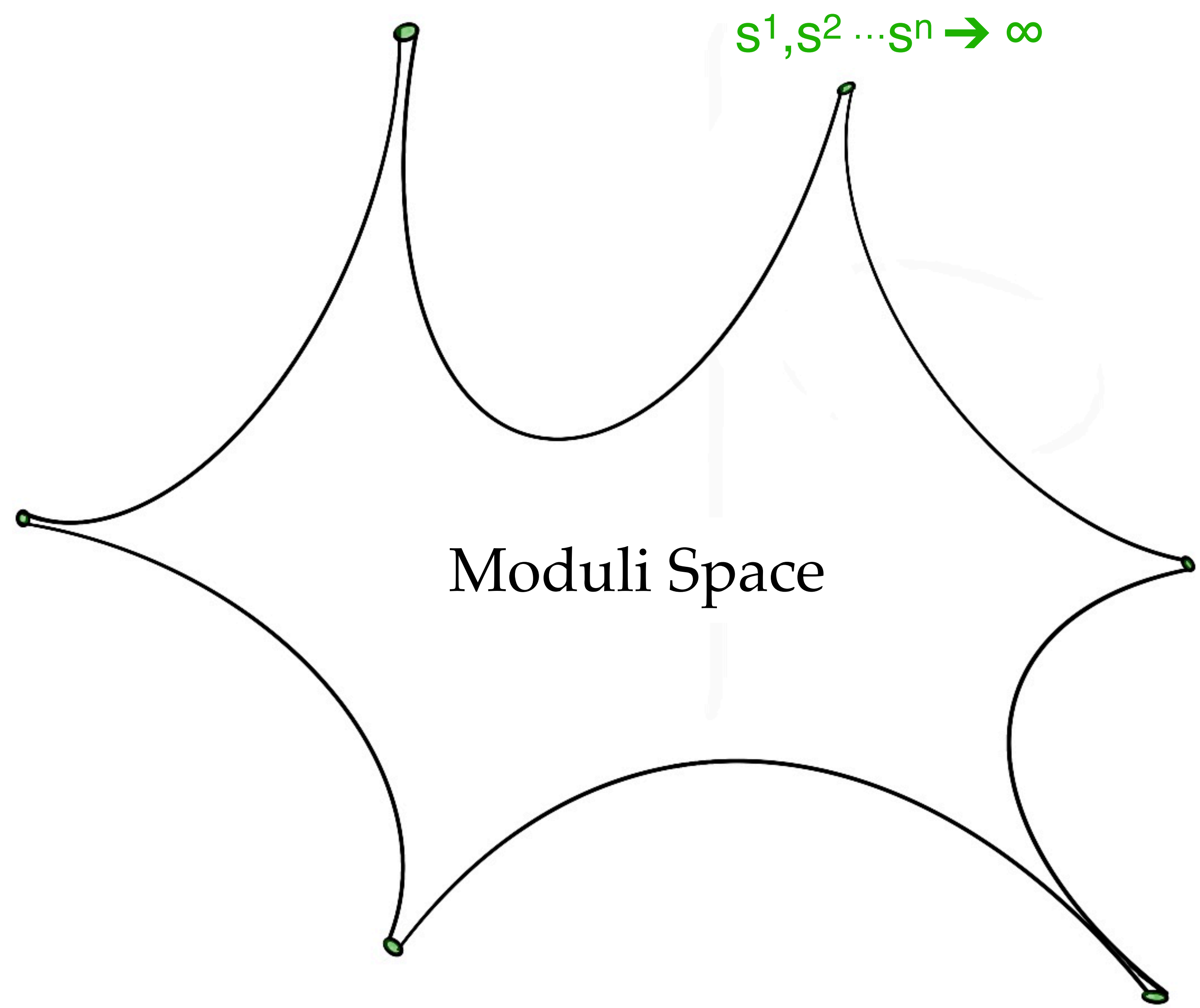
$$t^i = \phi^i + i\, s^i \qquad\qquad i = 1, \ldots, n \;\leq h^{3,1}$$

$$\gg 1$$



Approaching a boundary

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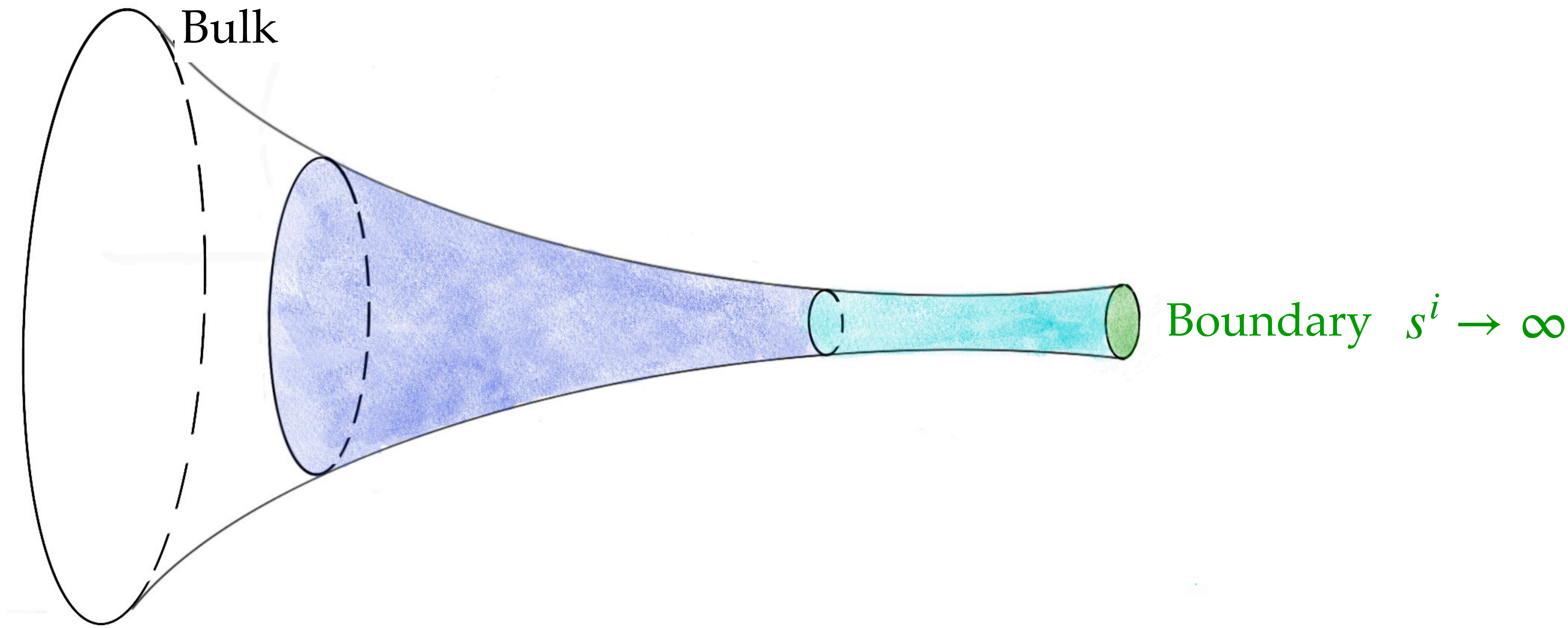

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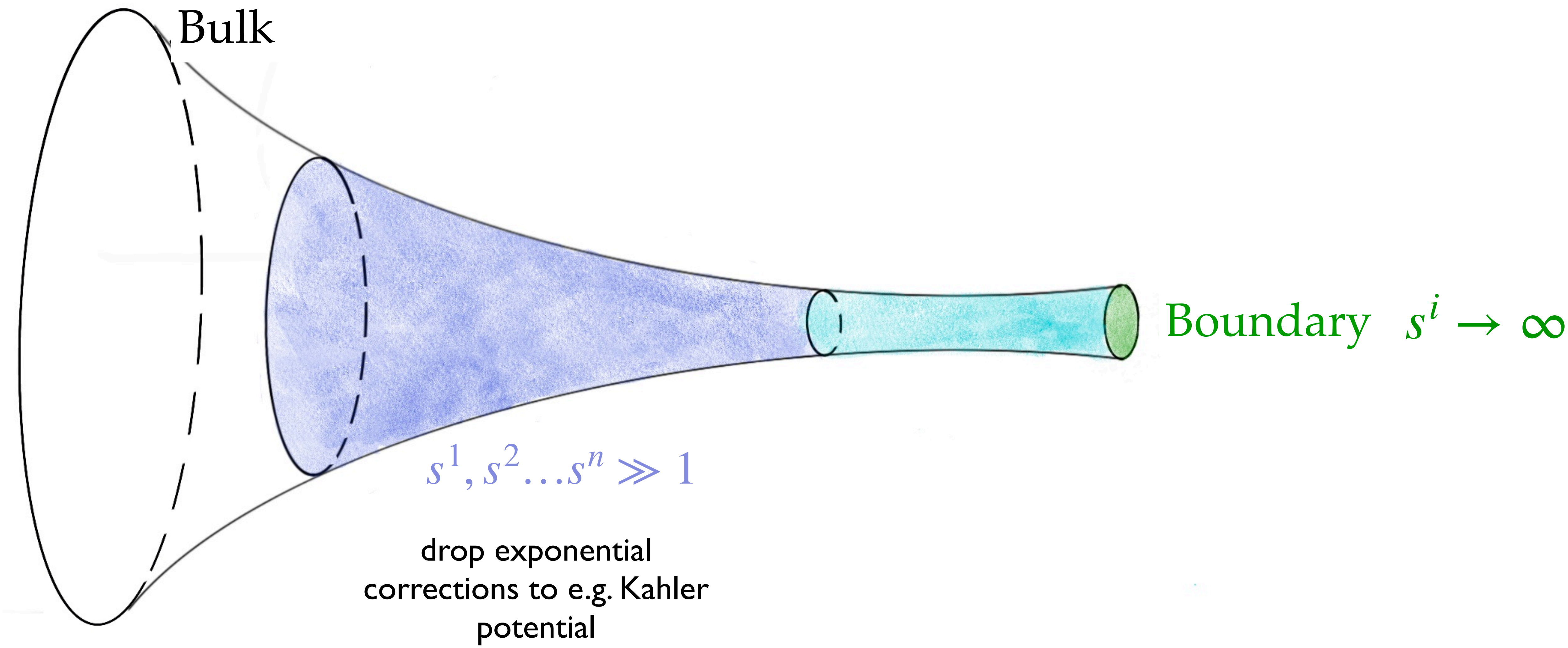

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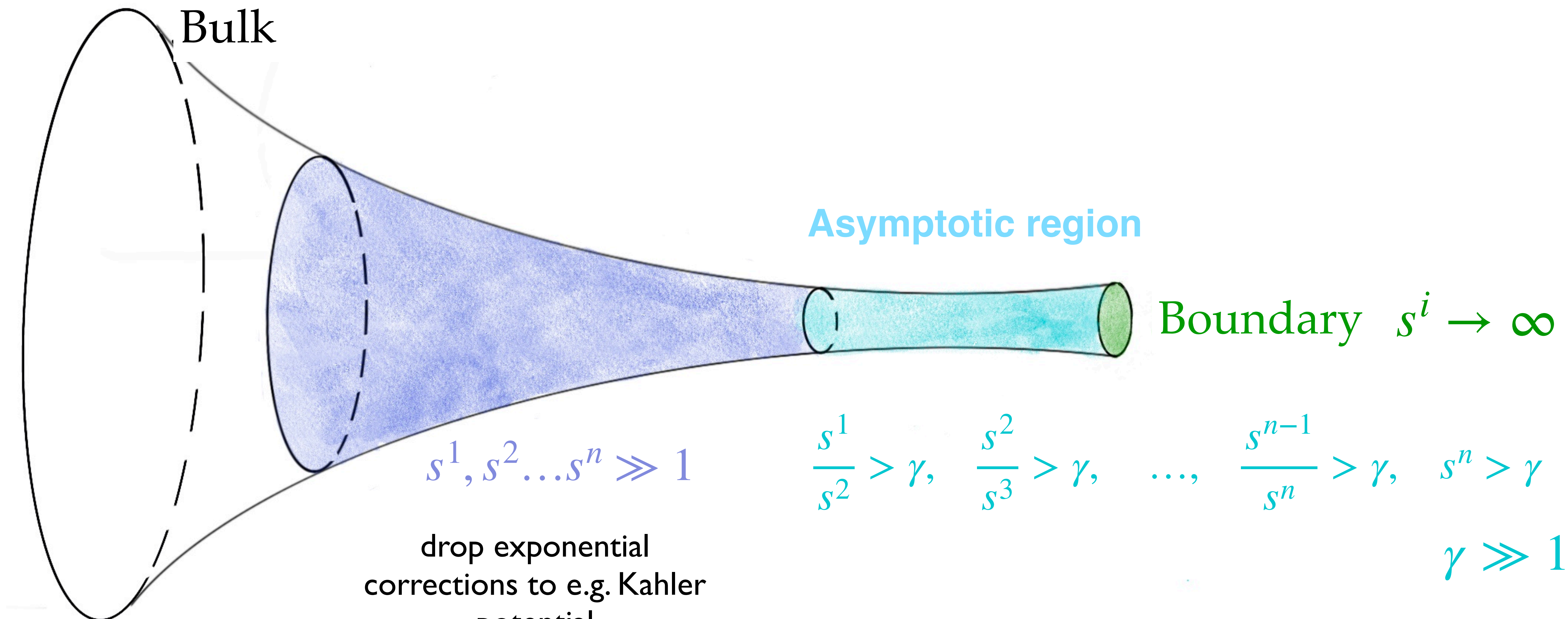

drop exponential corrections to e.g. Kahler potential

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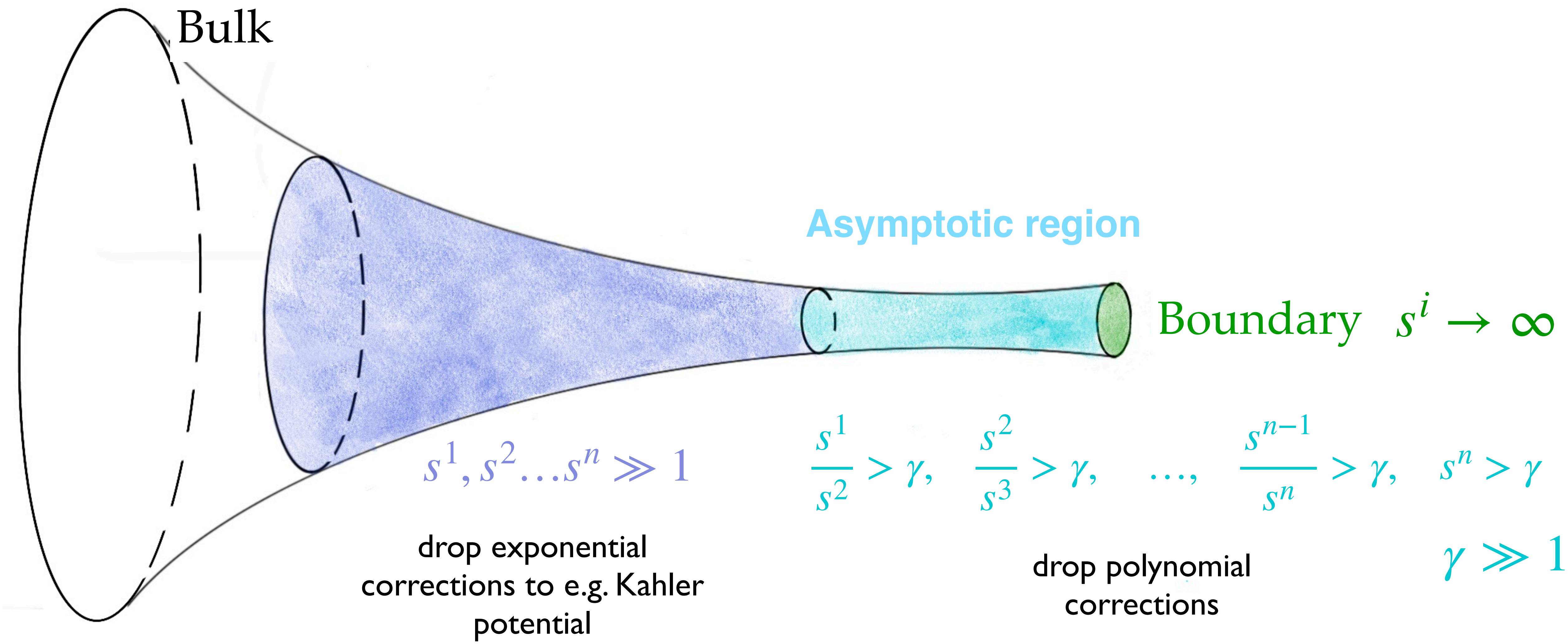

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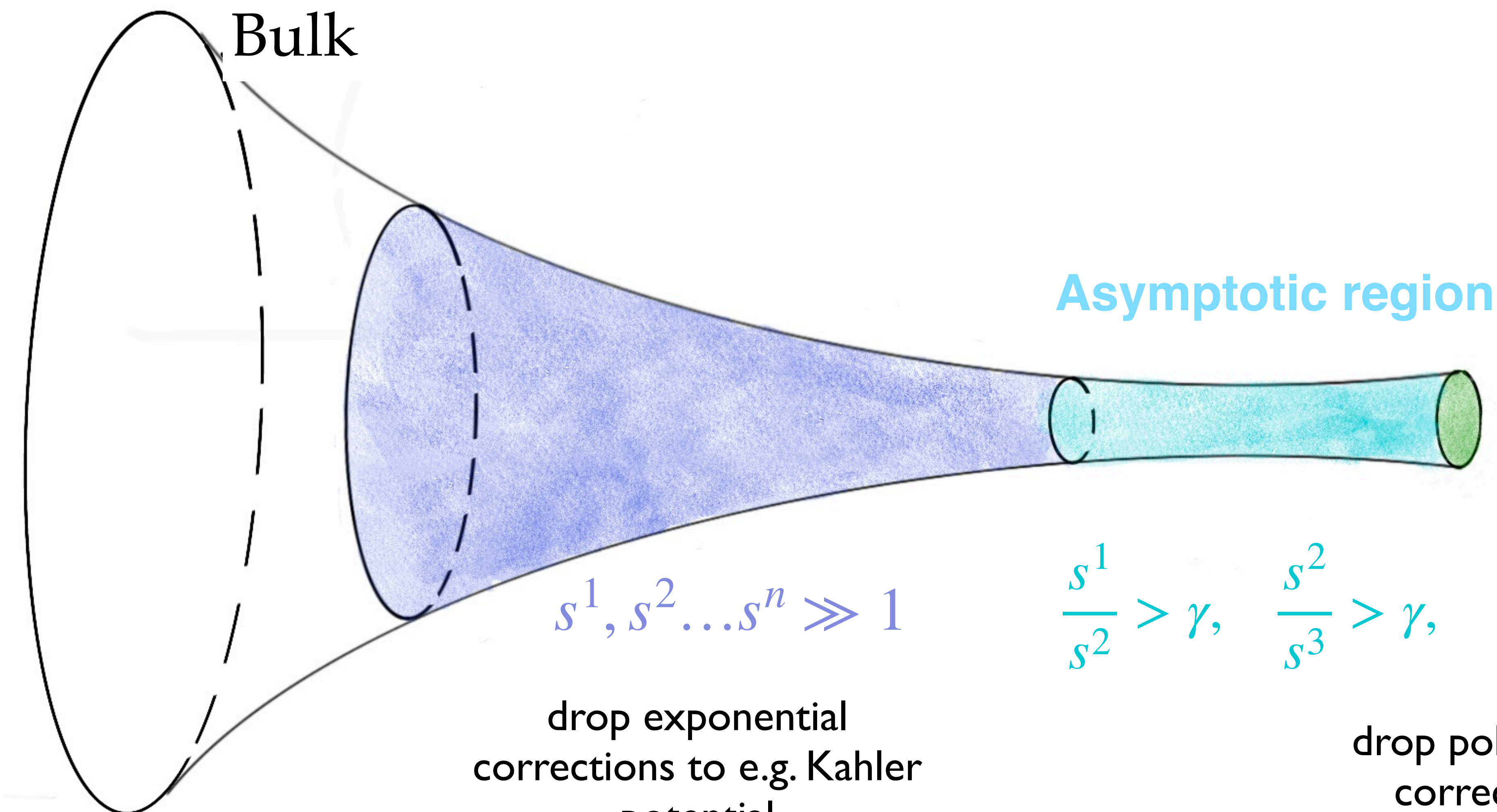

Boundary $s^i \to \infty$

$s^1, s^2 \ldots s^n \gg 1$

drop exponential corrections to e.g. Kahler potential

$$\frac{s^1}{s^2} > \gamma, \quad \frac{s^2}{s^3} > \gamma, \quad \ldots, \quad \frac{s^{n-1}}{s^n} > \gamma, \quad s^n > \gamma$$

drop polynomial corrections

$\gamma \gg 1$

$\gamma \gtrsim 4$

gives good approximation

Asymptotic Hodge theory

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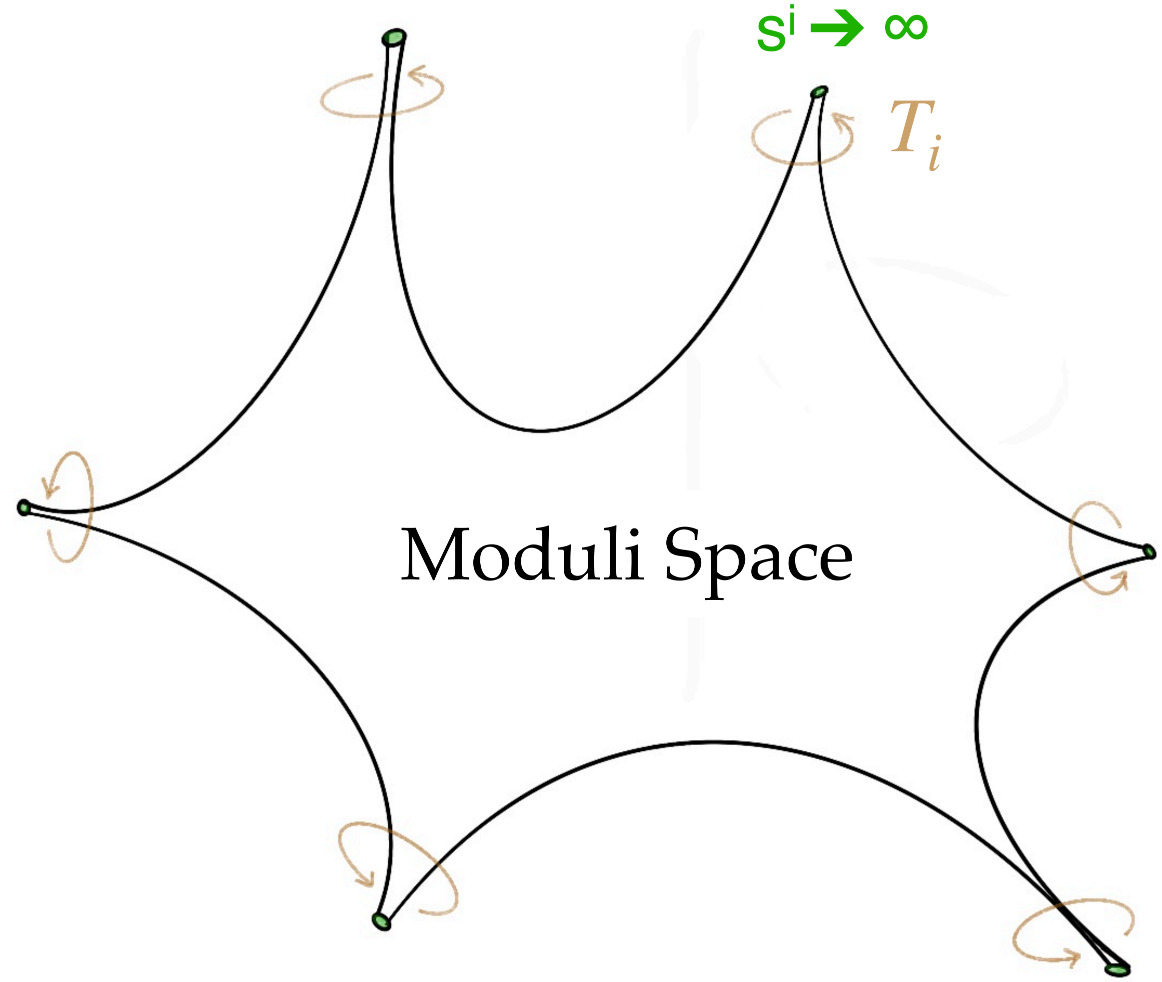
$$i = 1, \ldots, n \;\leq h^{3,1}$$



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- Shift symmetry $\phi^i \to \phi^i + 1$

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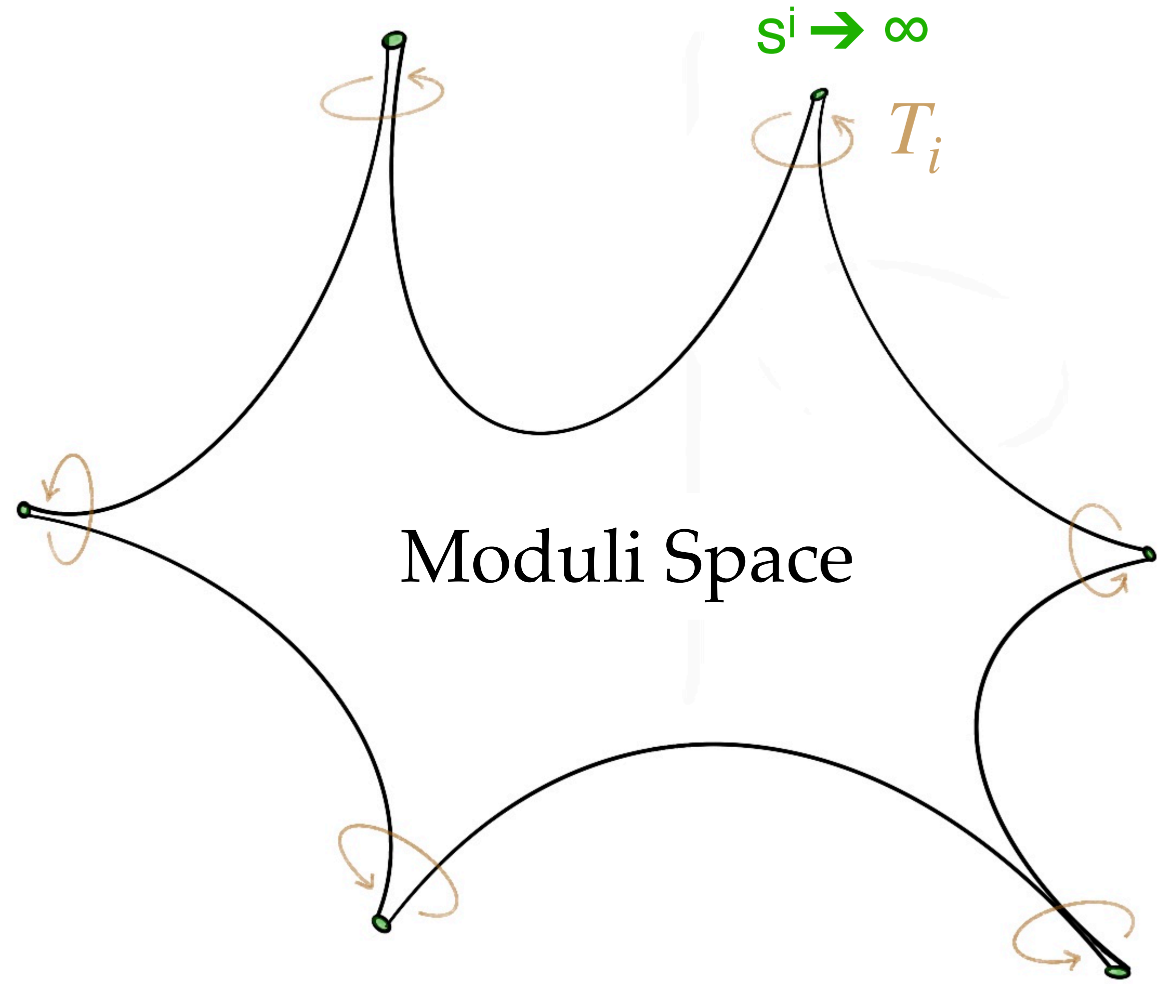
monodromy matrices



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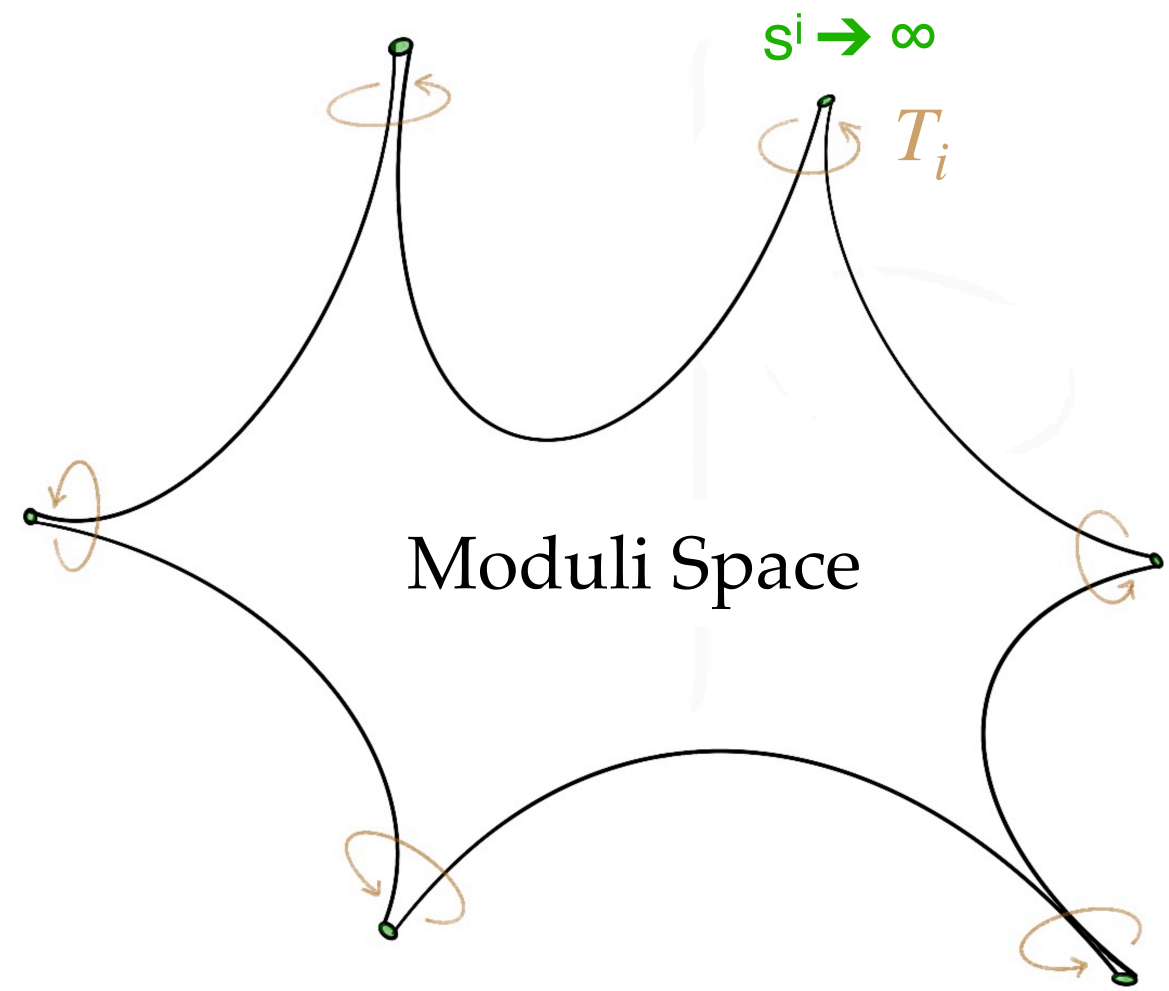

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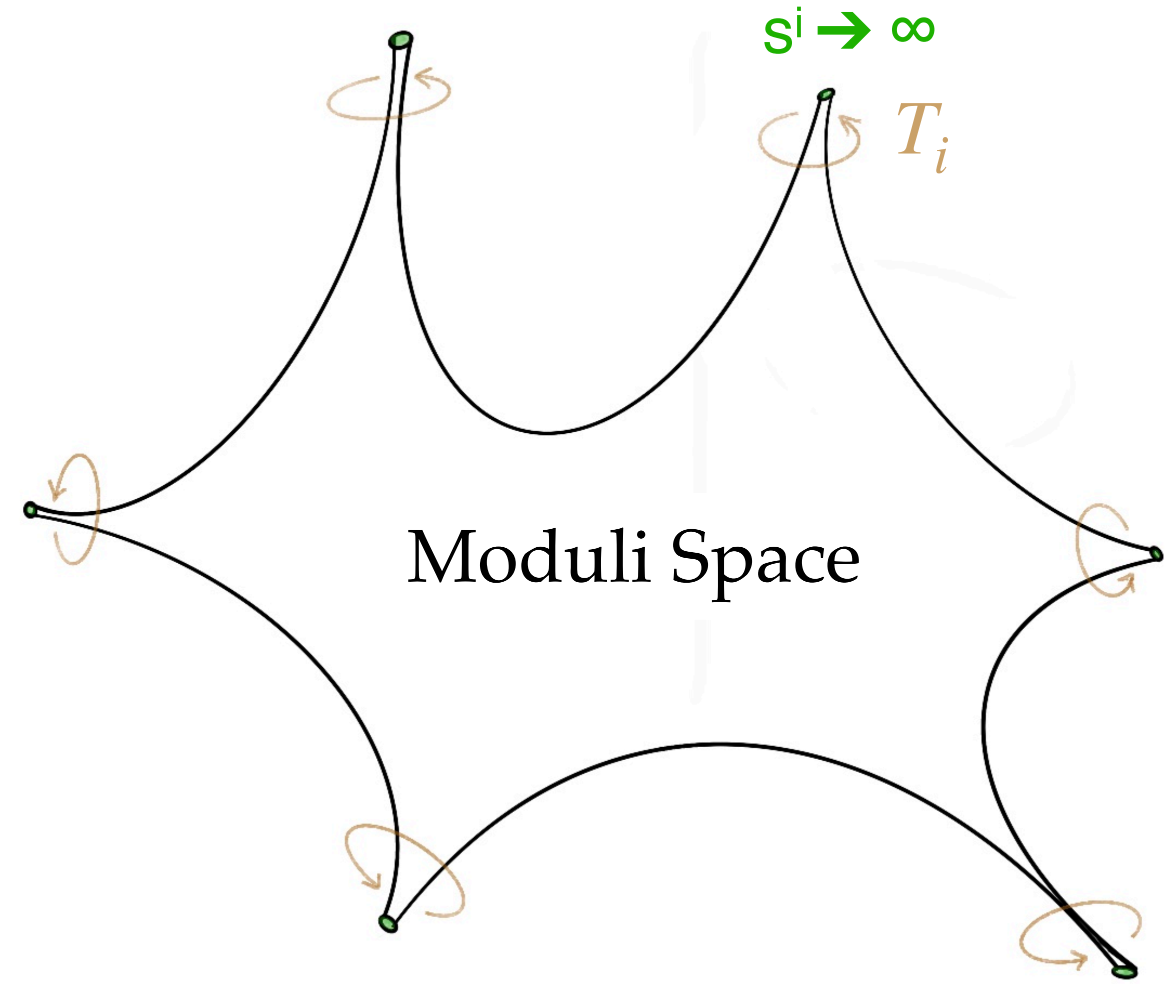

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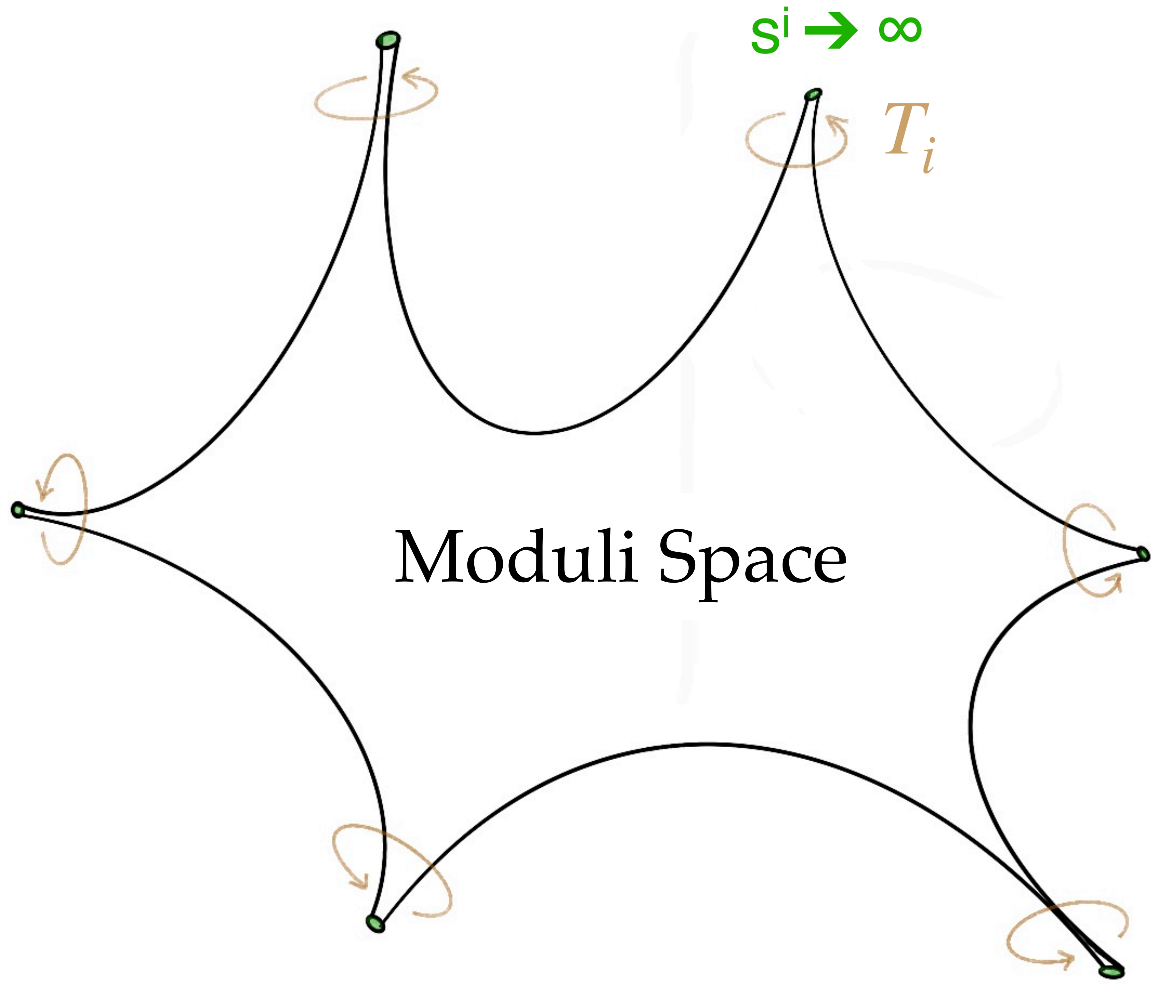

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- n **commuting** sl(2) triplets: $\{N_i^-, N_i^+, N_i^0\}$

$$\boxed{H^4_{\text{prim}}\left(Y_4, \mathbb{R}\right) = \bigoplus V_\ell} \qquad \ell = (\ell_1, \ldots, \ell_n)$$

$$N_i^0\, v_\ell = (\ell_i - \ell_{i-1})\, v_\ell \qquad v_\ell \in V_\ell \qquad \text{For a 4-fold} \qquad -4 \leq \ell_i \leq 4$$

Moduli stabilisation

$$G_4 = \star\, G_4$$

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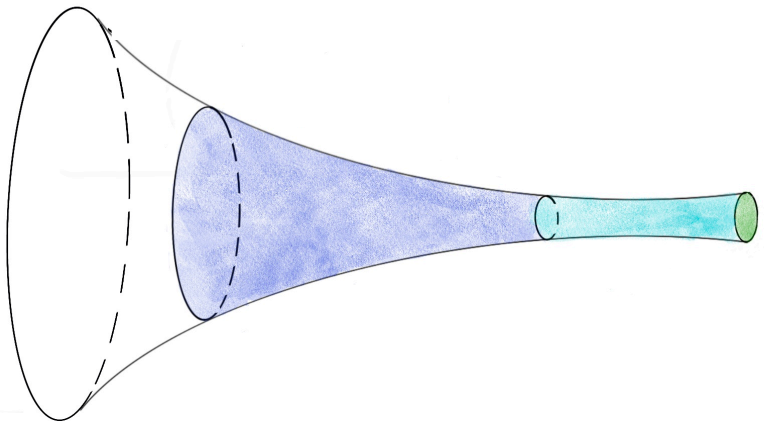
- Not hard to see that one (pair of) G_ℓ flux stabilises one modulus

Tadpole



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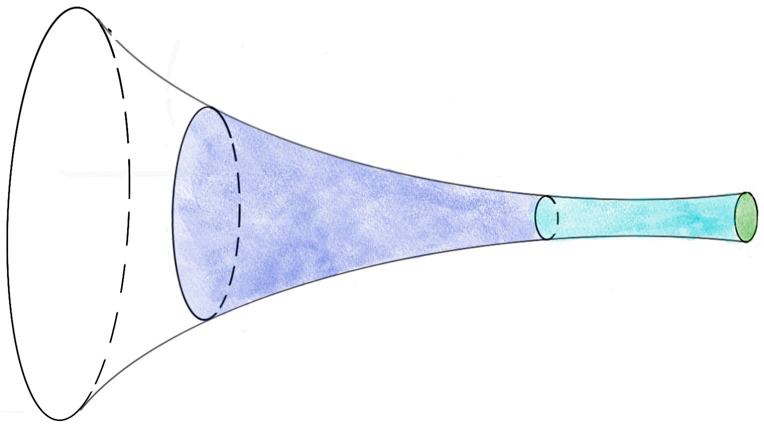

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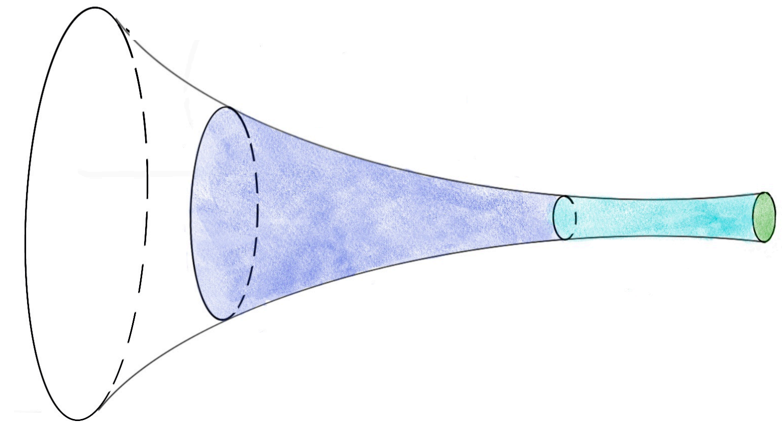

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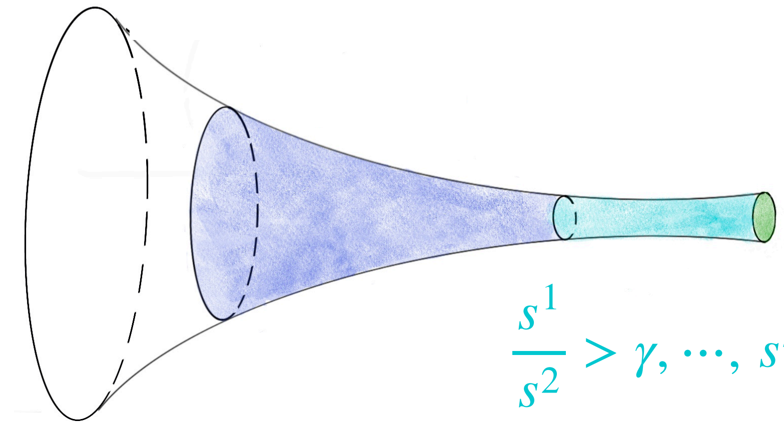
note: each individual term is positive!

Tadpole



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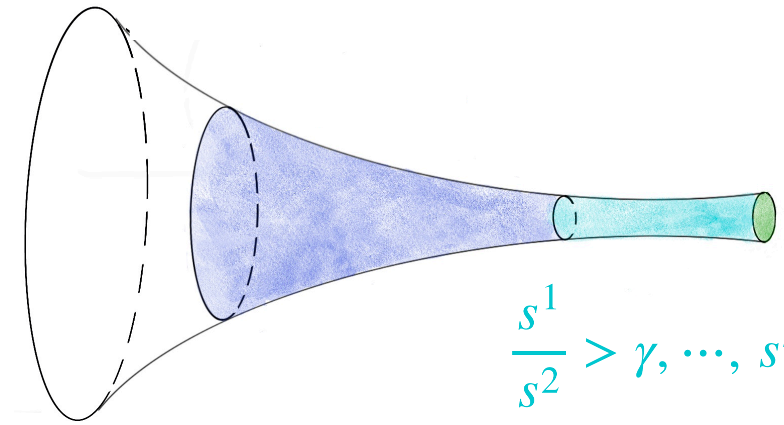

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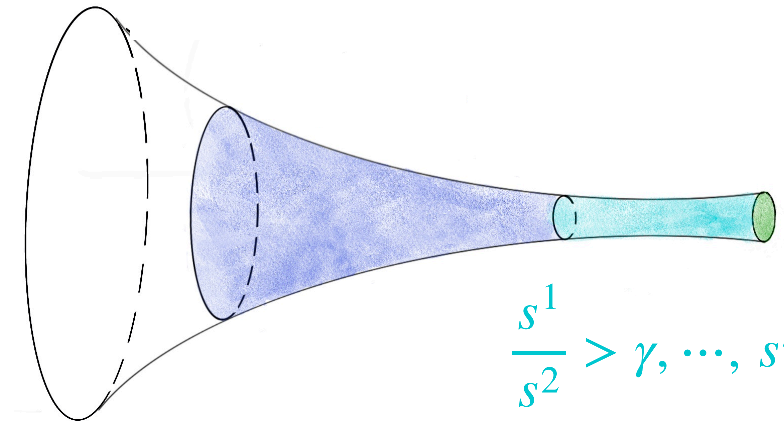
Tadpole



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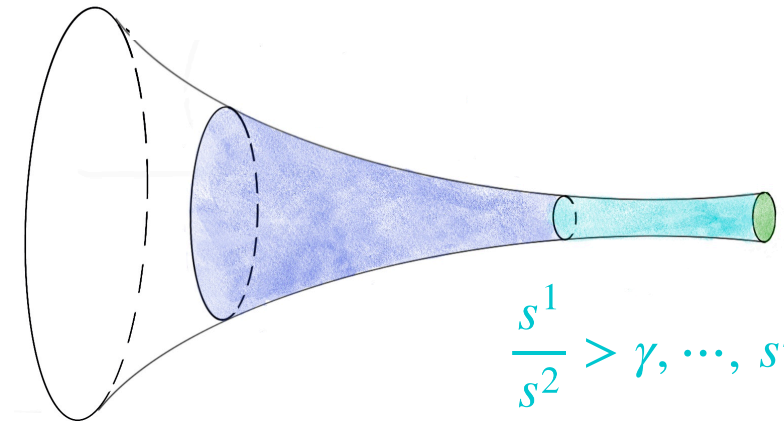
as many terms as moduli

Tadpole



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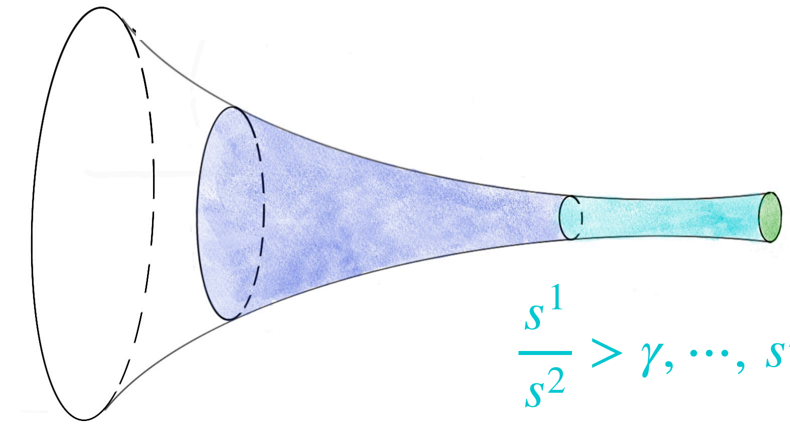
Tadpole



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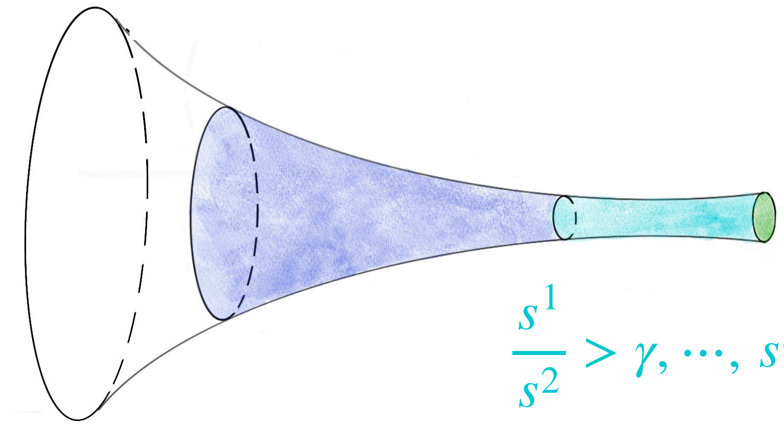
"experimental" evidence

$$\sum_{\ell \geq 0} \|G_\ell\|_\infty^2 > \frac{1}{8}$$

Tadpole



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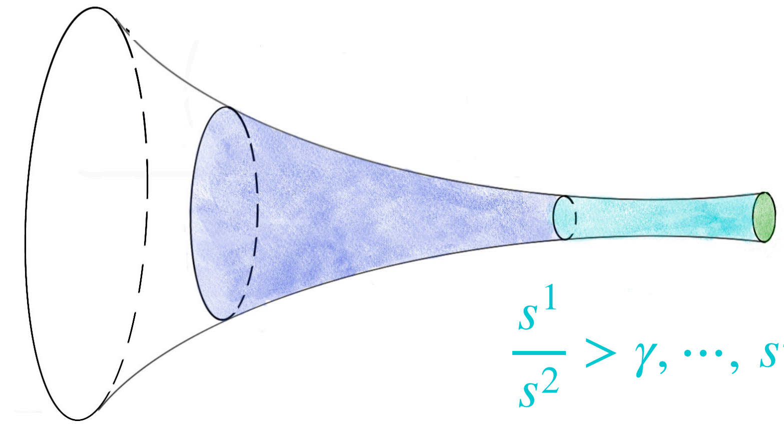
Tadpole



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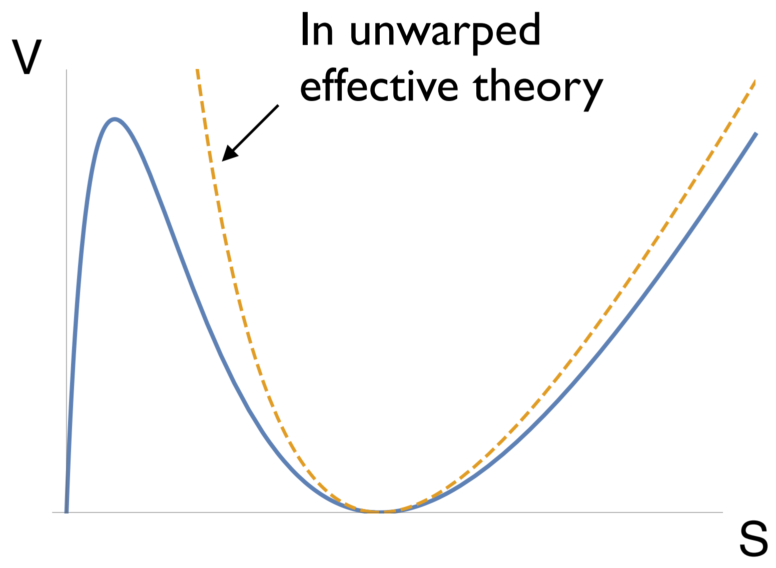
Implications for de Sitter with anti-brane uplift

Implications for de Sitter with anti-brane uplift

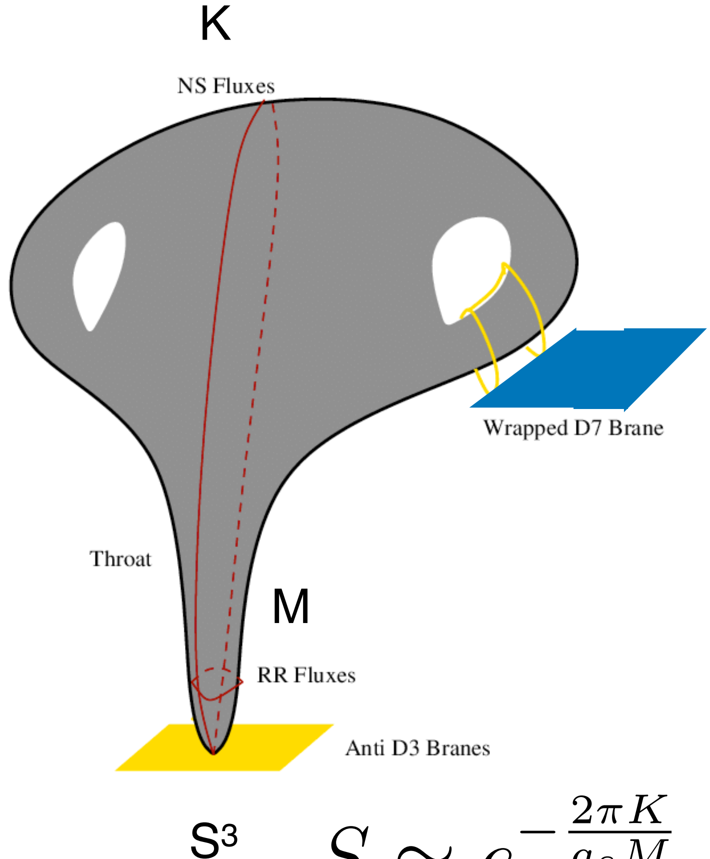
Bena, Dudas, M.G., Lust 18

Moduli stabilization using warped effective field theory for conifold modulus

Douglas, Torroba 08



$$ds^2 = e^{2A} ds_4^2 + e^{-2A} ds_{\rm CY}^2$$



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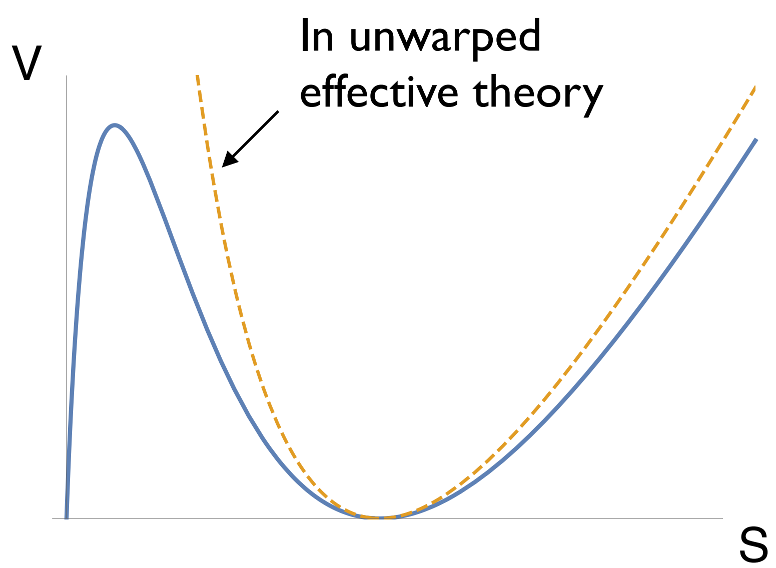

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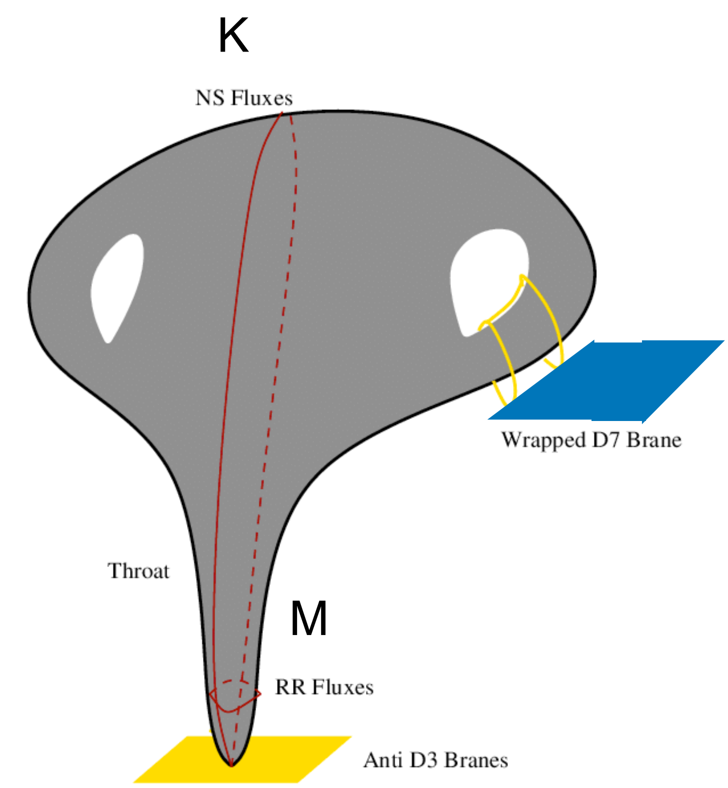

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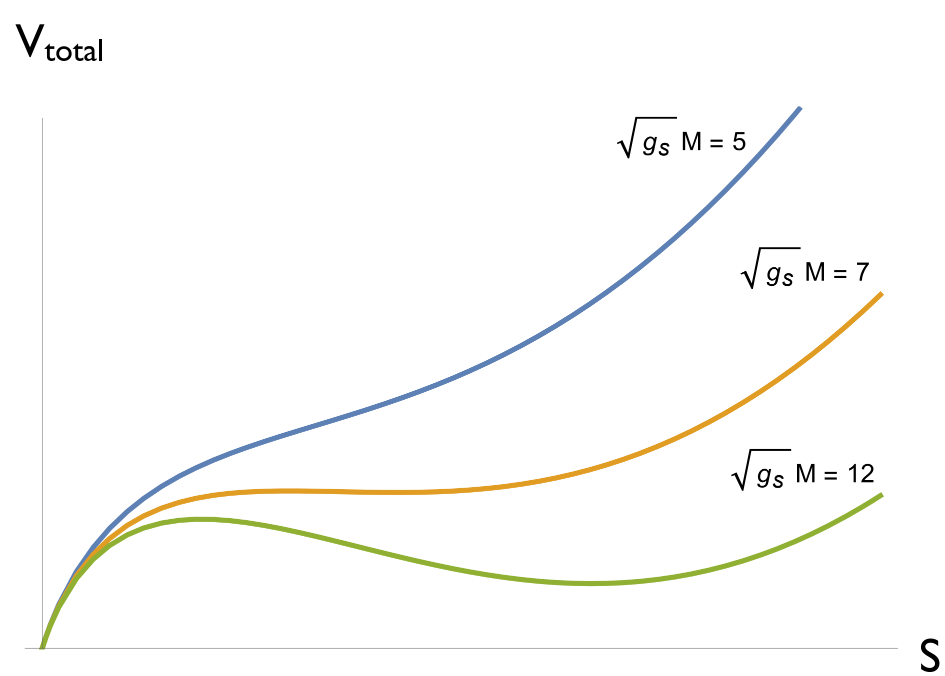
- Add $\overline{\text{D3}}$ wants to collapse the S^3!

Full flux + $\overline{\text{D3}}$ warped potential for size of S^3



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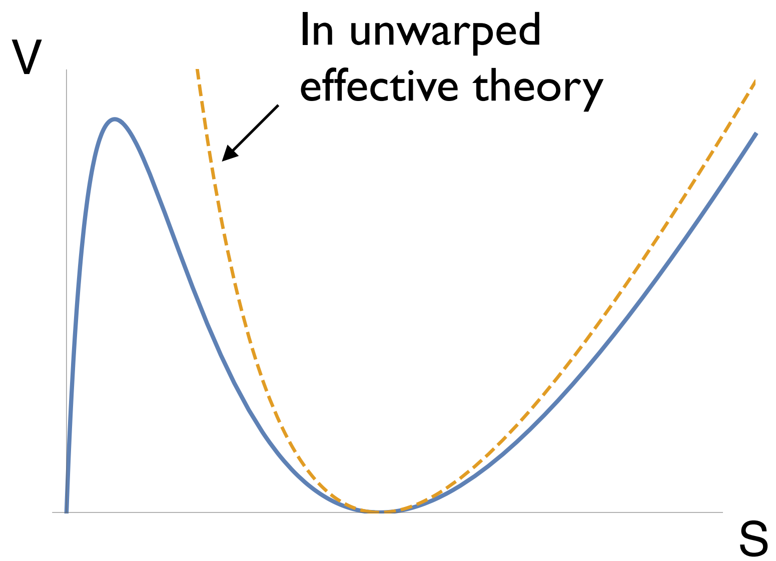

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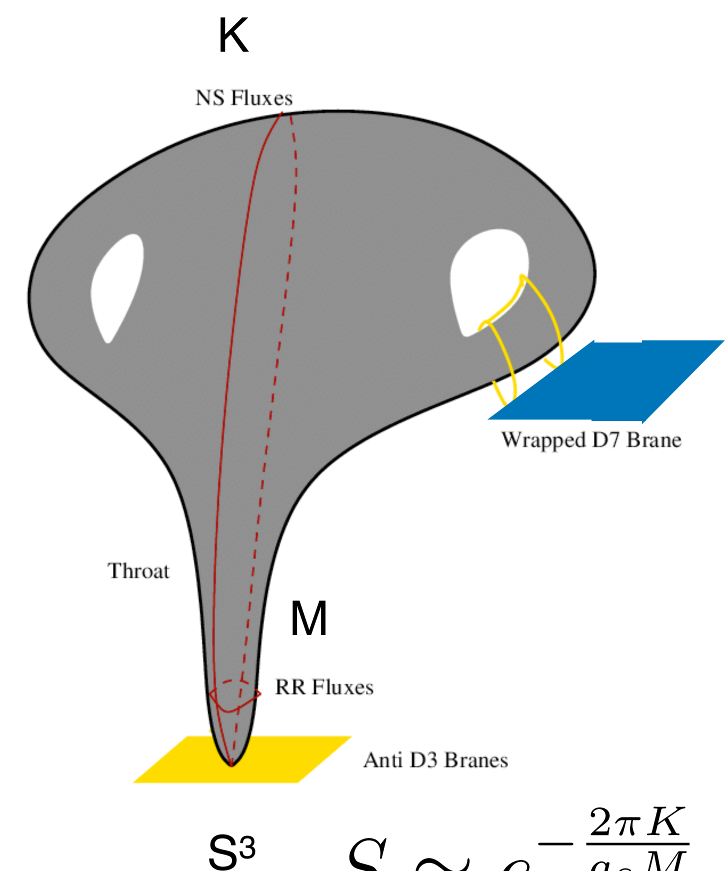

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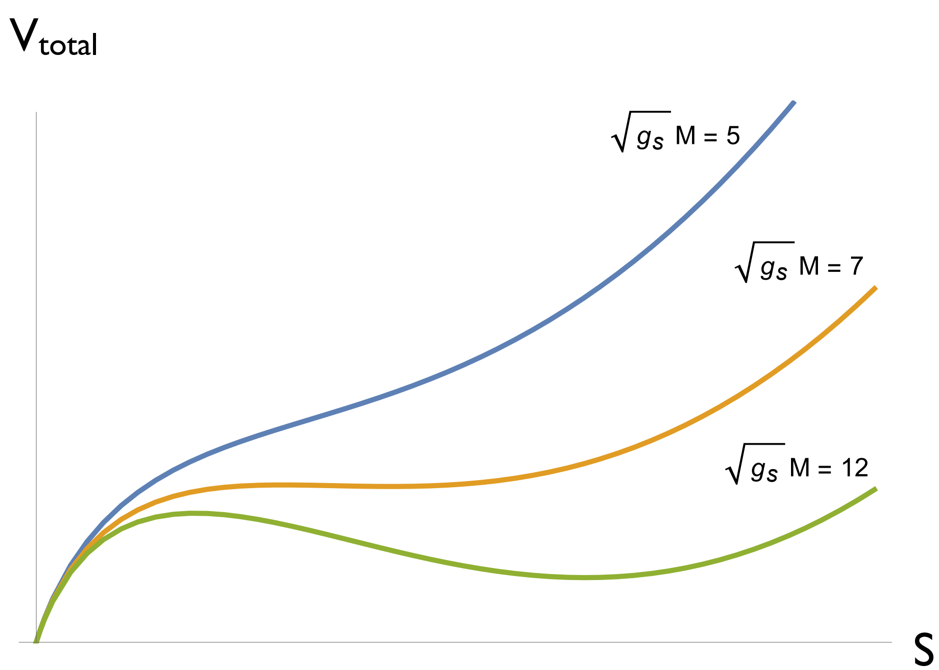
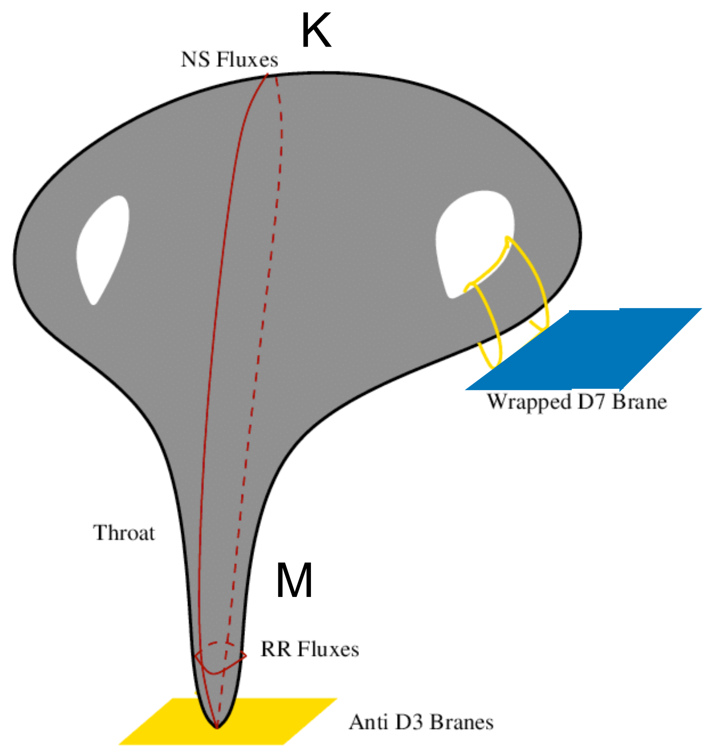


Need $\sqrt{g_s} M \geq 6.7$ to avoid collapse

• But then hierarchy:

$$e^{A}|_{\text{bottom}} = \frac{\Lambda_{IR}}{\Lambda_{UV}} = \exp\left(-\frac{2\pi}{3}\frac{K}{g_s M} \right)$$



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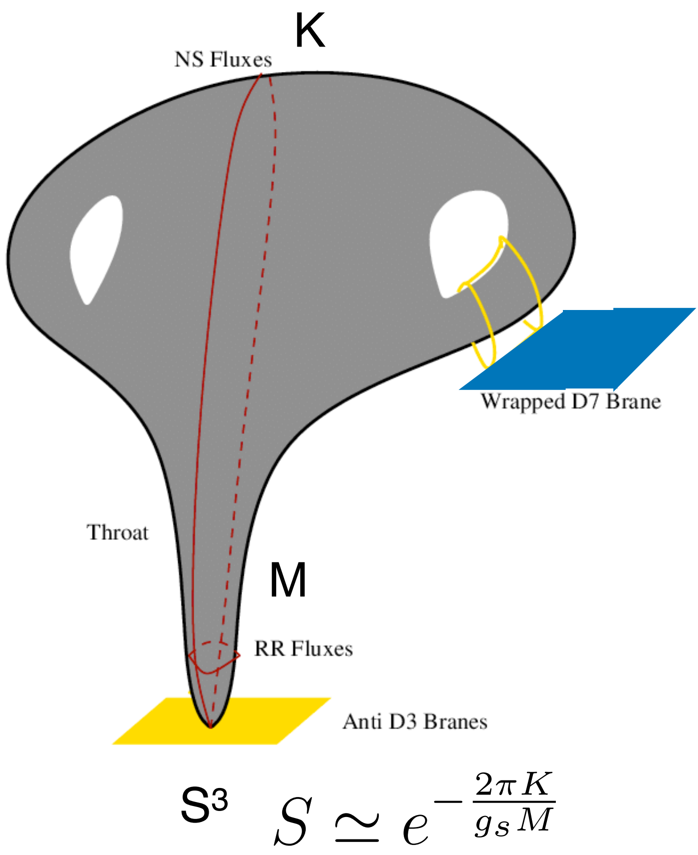

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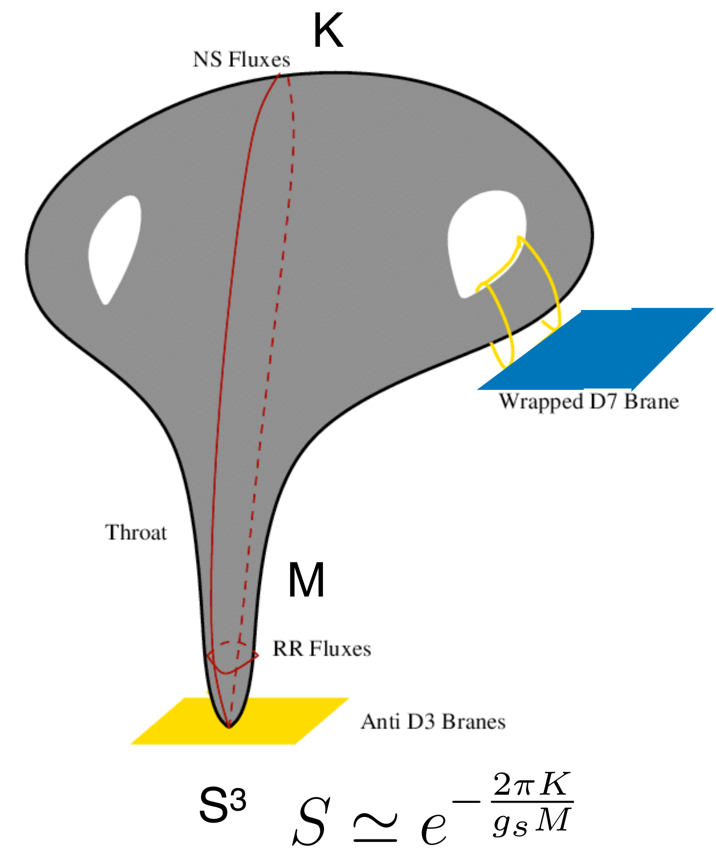

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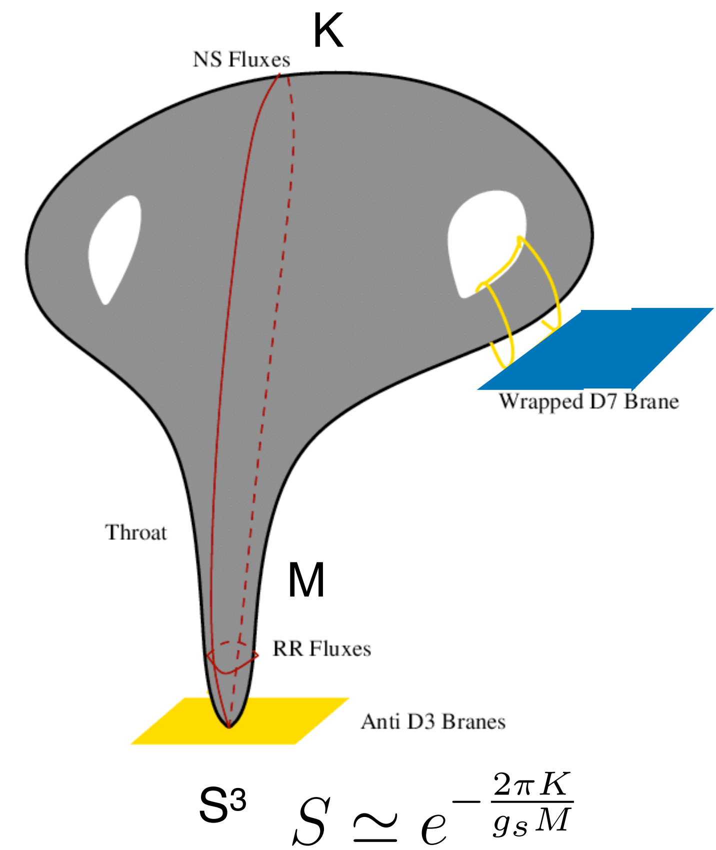

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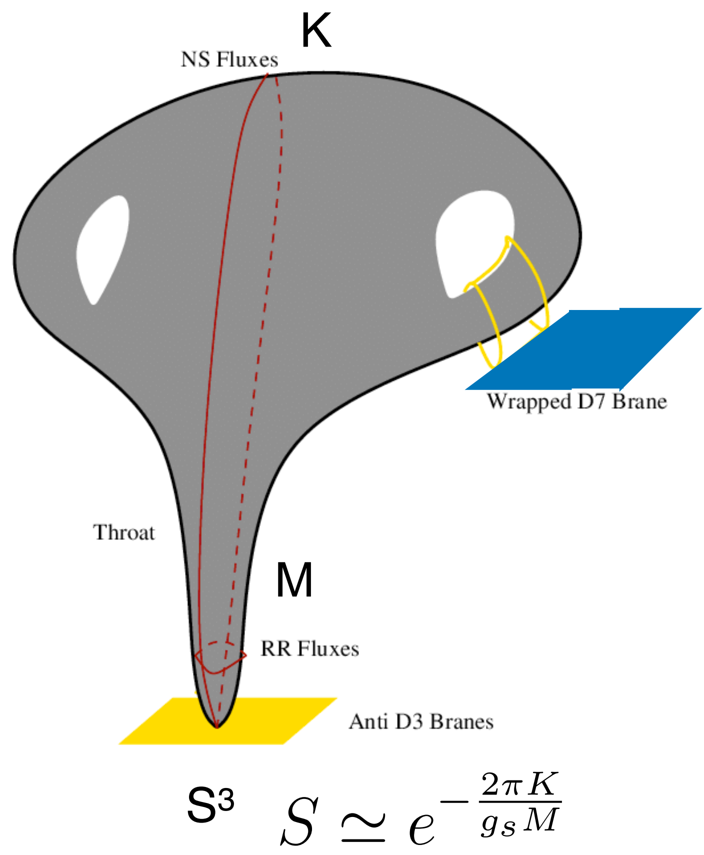

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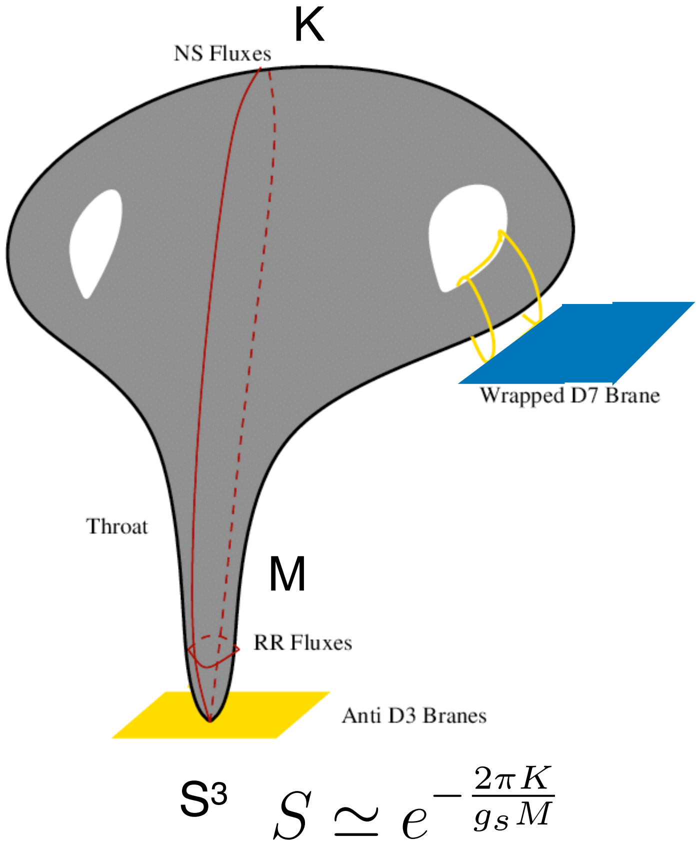

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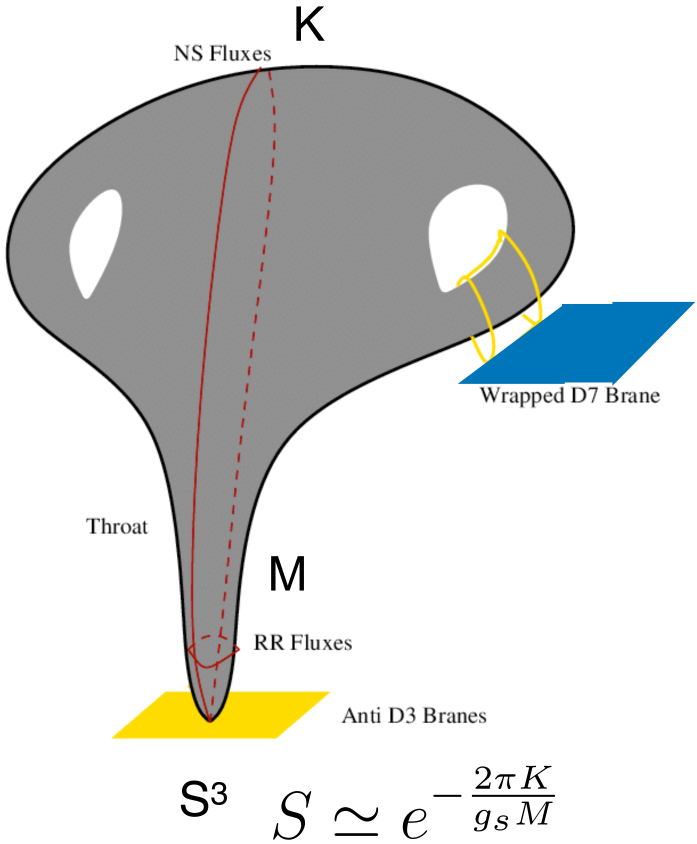

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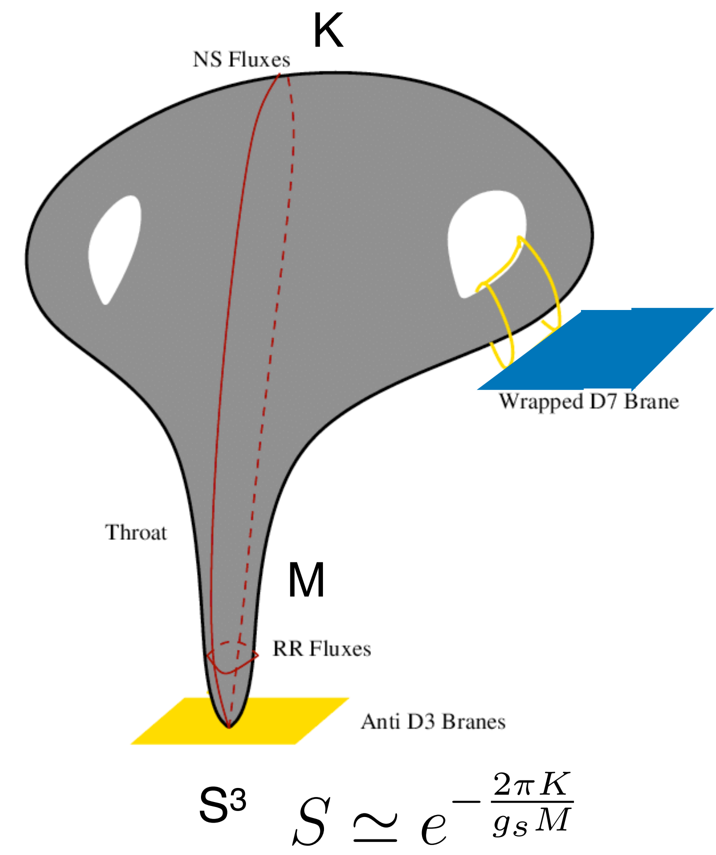

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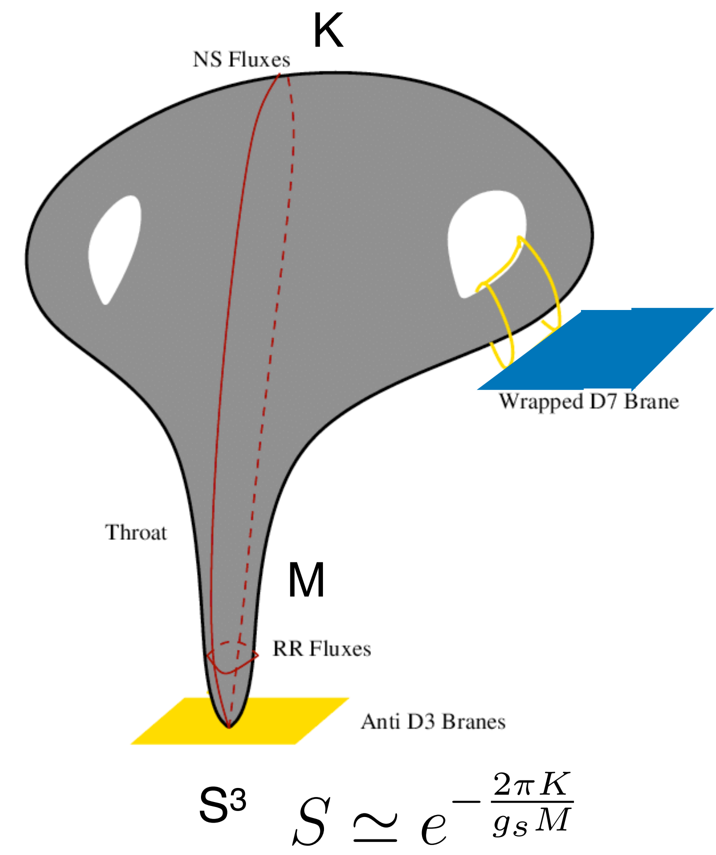

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