Glueball Molecules

Table of Contents:

Alexey A. Petrov

Wayne State University

Quantum Chromodynamics is simple!

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

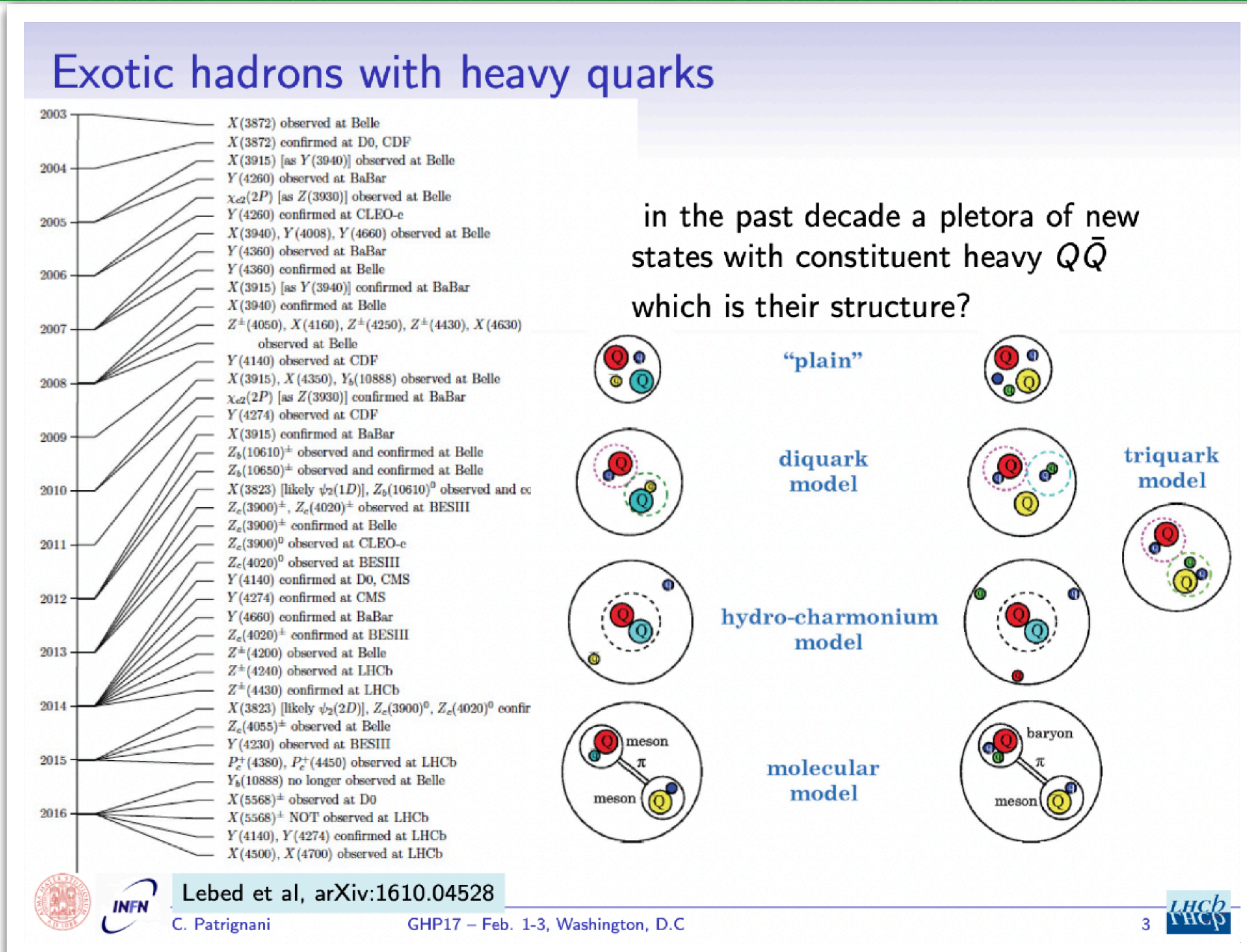
where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

<u>That's it!</u>

F. Wilczek, "QCD made simple", Physics Today, August 2000

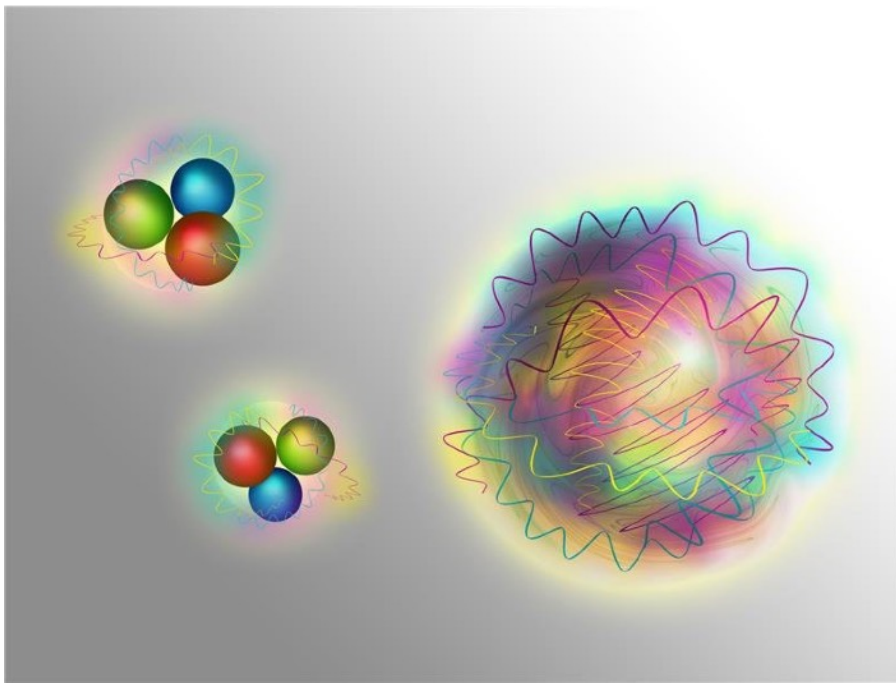
Introduction



- Lingo: what do we mean by "exotic" (quark model-driven)?
  - exotic states:
    - quantum numbers are not allowed in $q\bar{q}'$ or $qq'q''$
    - states require more than 2 or 3 quarks
  - cryptoexotic states:
    - mass/width do not fit in meson or baryon spectra
    - production or decay properties incompatible with ordinary states

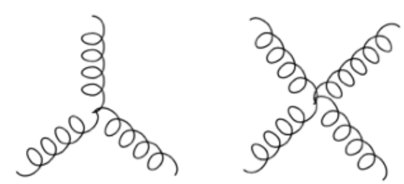
We often do not follow our own definitions. This talk included.

Ok, maybe Quantum Chromodynamics is not so simple…



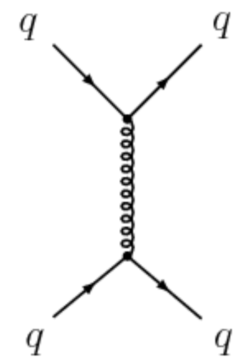
- QCD Lagrangian is written in terms of the ``wrong” degrees of freedom: we see mesons and baryons, not quarks/gluons!
- Since gluons carry color charge, they can self-interact

Can there be bound states of pure glue?

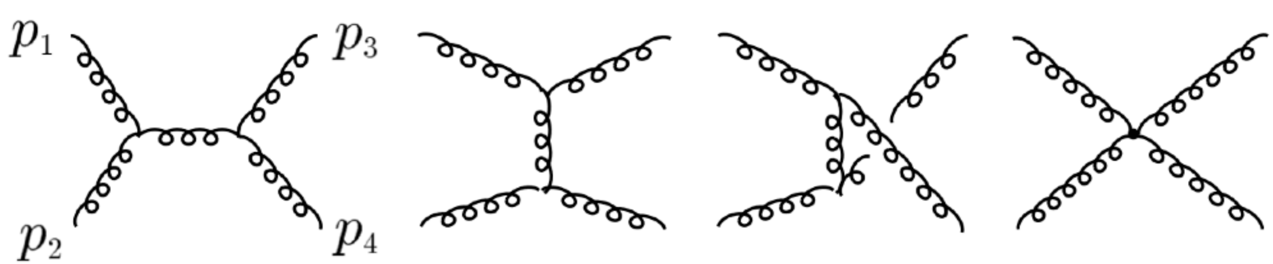


Curious: Higgs field has nothing to do with mass!

- Can we predict glueball spectrum?
  - quark models: quark-antiquark potential
  - not so easy for gluons: gauge invariance



  - quark models (constituent, flux tube, bag, etc.)

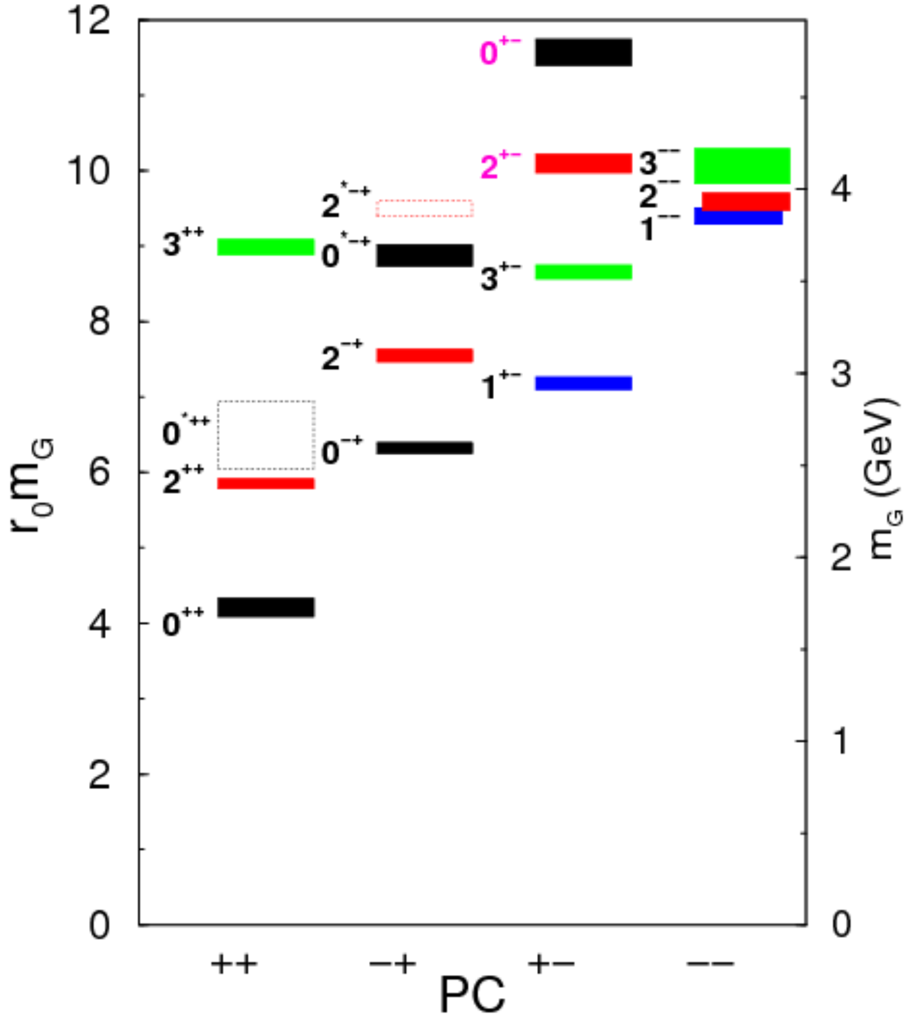


Cornwall and Soni, PLB120 (1983) 431
Hou and Wong, PRD67, 034003 (2003)

  - since gluons have spin-one, all glueballs are bosons
- Lattice QCD, QCD Sum Rules, bag models, ADS/QCD, …

Glueball spectrum: masses

Old: lattice and model-dependent



Morningstar and Peardon, PRD60, 034509 (1999)

J^{PC}	Constituent	Lattice	Experiment
	{1730} (0.72)	1730 (0.72)	1500[b] (0.76)
0^{++}	2685 (1.12)	2670 (1.11)	2105[b] (1.06)
	2710 (1.13)		2320[c] (1.17)
	2790 (1.16)		
	{2400} (1.00)	2400 (1.00)	1980[b] (1.00)
	2693 (1.12)	3290 (1.37)	2020[d] (1.02)
2^{++}	2700 (1.13)		2240[d] (1.13)
	2730 (1.14)		2370[d] (1.20)
	2810 (1.17)		
0^{-+}	2570 (1.07)	2590 (1.08)	2140[d] (1.08)
	2765 (1.15)		2190[b] (1.11)
1^{-+}	2605 (1.09)		
	2770 (1.15)		
2^{-+}	2615 (1.09)	3100 (1.29)	2040[d] (1.03)
	2775 (1.16)	3890 (1.62)	2300[d] (1.16)
1^{++}	2690 (1.12)		2340[d] (1.18)
3^{++}	2694 (1.12)	3690 (1.54)	2000[d] (1.01)
			2280[d] (1.15)
4^{++}	2695 (1.12)	3650[a] (1.52)	2044[d] (1.03)
			2320[d] (1.17)
$3g(0^{-+})$	3780 (1.58)	3640 (1.52)	
$3g(1^{--})$	3680 (1.53)	3850 (1.60)	
$3g(3^{--})$	3690 (1.54)	4130 (1.72)	

Hou and Wong, PRD67, 034003 (2003)

Glueball spectrum: masses

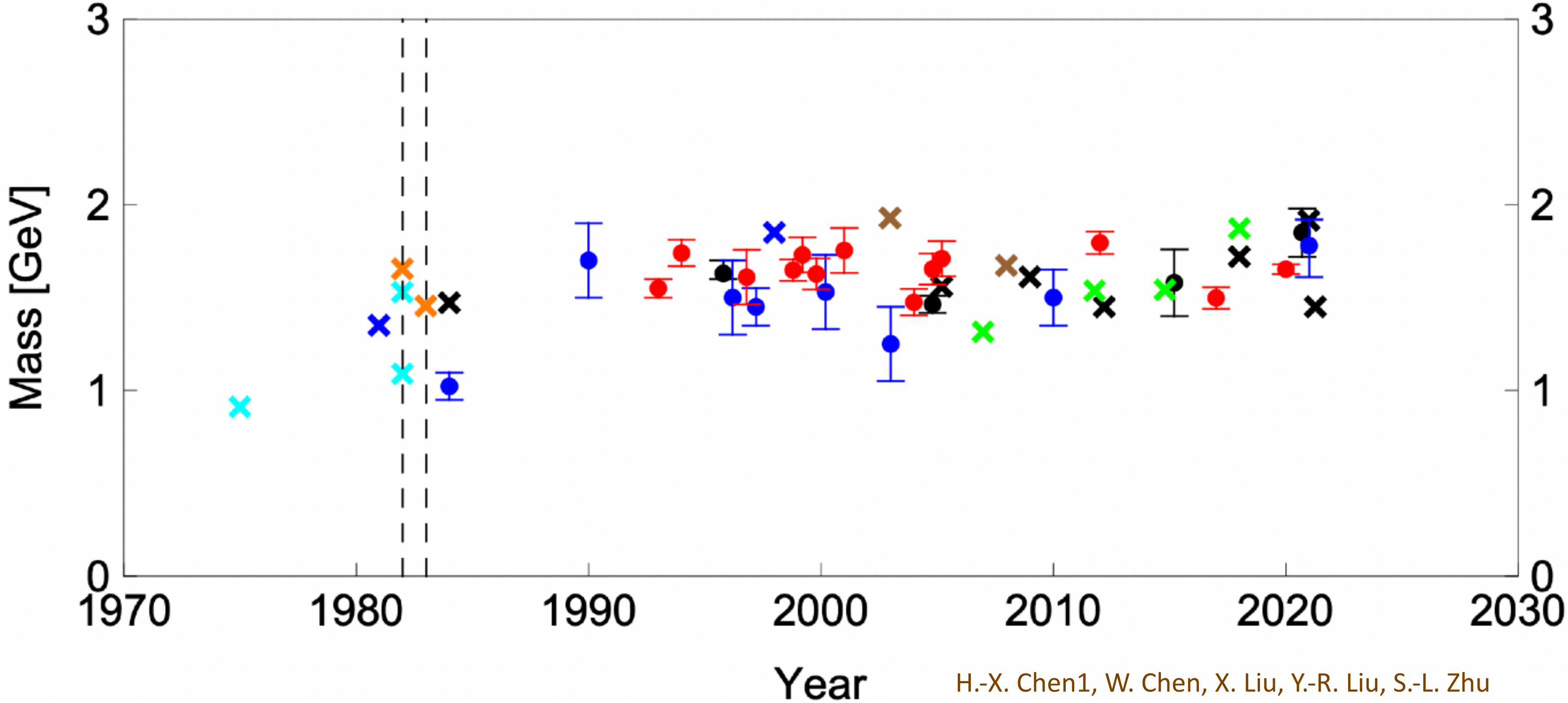
- The predictions for the glueball masses “stabilized”…

Glueball	Ref. [795]	Ref. [796]	Ref. [797]	Ref. [798]	Ref. [799]	QSR [807]
$\|GG;0^{++}\rangle$	$1730 \pm 50 \pm 80$	$1710 \pm 50 \pm 80$	$1475 \pm 30 \pm 65$	1795 ± 60	1653 ± 26	1780^{+140}_{-170}
$\|GG;2^{++}\rangle$	$2400 \pm 25 \pm 120$	$2390 \pm 30 \pm 120$	$2150 \pm 30 \pm 100$	2620 ± 50	2376 ± 32	1860^{+140}_{-170}
$\|GG;0^{-+}\rangle$	$2590 \pm 40 \pm 130$	$2560 \pm 35 \pm 120$	$2250 \pm 60 \pm 100$	–	2561 ± 40	2170^{+110}_{-110}
$\|GG;2^{-+}\rangle$	$3100 \pm 30 \pm 150$	$3040 \pm 40 \pm 150$	$2780 \pm 50 \pm 130$	3460 ± 320	3070 ± 60	2240^{+110}_{-110}
$\|GGG;0^{++}\rangle$	$2670 \pm 180 \pm 130$	–	$2755 \pm 70 \pm 120$	3760 ± 240	2842 ± 40	4460^{+170}_{-190}
$\|GGG;2^{++}\rangle$	–	–	$2880 \pm 100 \pm 130$	–	3300 ± 50	4180^{+190}_{-420}
$\|GGG;0^{-+}\rangle$	$3640 \pm 60 \pm 180$	–	$3370 \pm 150 \pm 150$	4490 ± 590	3540 ± 80	4130^{+180}_{-360}
$\|GGG;2^{-+}\rangle$	–	–	$3480 \pm 140 \pm 160$	–	3970 ± 70	4290^{+200}_{-220}
$\|GGG;1^{+-}\rangle$	$2940 \pm 30 \pm 140$	$2980 \pm 30 \pm 140$	$2670 \pm 65 \pm 120$	3270 ± 340	2944 ± 42	4010^{+260}_{-950}
$\|GGG;2^{+-}\rangle$	$4140 \pm 50 \pm 200$	$4230 \pm 50 \pm 200$	–	–	4240 ± 80	4420^{+240}_{-290}
$\|GGG;3^{+-}\rangle$	$3550 \pm 40 \pm 170$	$3600 \pm 40 \pm 170$	$3270 \pm 90 \pm 150$	3850 ± 350	3530 ± 80	4300^{+230}_{-260}
$\|GGG;1^{--}\rangle$	$3850 \pm 50 \pm 190$	$3830 \pm 40 \pm 190$	$3240 \pm 330 \pm 150$	–	4030 ± 70	4910^{+200}_{-180}
$\|GGG;2^{--}\rangle$	$3930 \pm 40 \pm 190$	$4010 \pm 45 \pm 200$	$3660 \pm 130 \pm 170$	4590 ± 740	3920 ± 90	4250^{+220}_{-330}
$\|GGG;3^{--}\rangle$	$4130 \pm 90 \pm 200$	$4200 \pm 45 \pm 200$	$4330 \pm 260 \pm 200$	–	–	5590^{+330}_{-220}

H.-X. Chen1, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu
arXiv:2204.02649 [hep-ph]

Glueball spectrum: 0^{++} masses

- … but the accuracy seem not to improve much over time

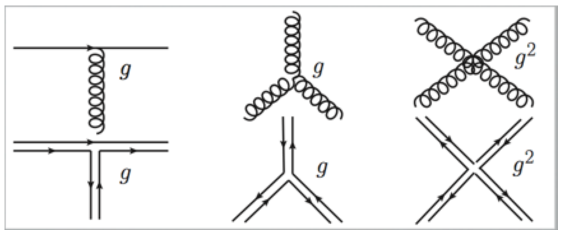


H.-X. Chen1, W. Chen, X. Liu, Y.-R. Liu, S.-L. Zhu
arXiv:2204.02649 [hep-ph]

What do we know about glueballs' widths?

Glueball spectrum: widths

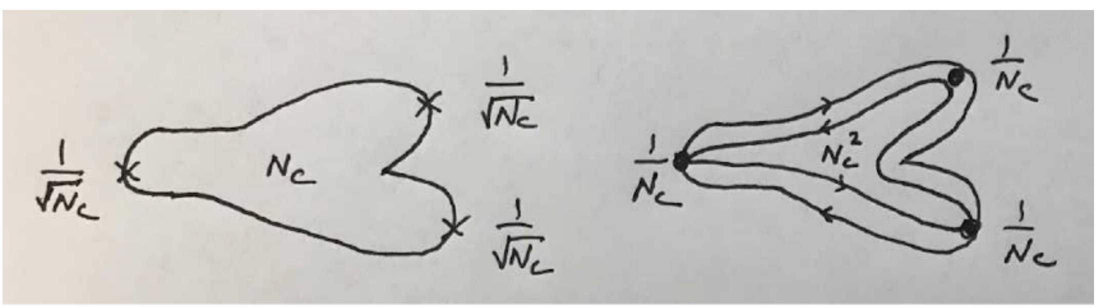
- Should we expect wide or narrow glueball states?
  - difficult to say model-independently; lots of model-dependent results
  - large N_c counting rules can provide guidance ('t Hooft limit)



Each coupling: $g \sim \frac{1}{\sqrt{N_c}}$

Each quark loop: N_c

  - meson and glueball decay amplitudes



$\sim N_c^{-\frac{1}{2}}$ $\qquad$ $\sim N_c^{-1}$

$$A_{n(q\bar{q})} \sim N_c^{-\frac{n-2}{2}}$$

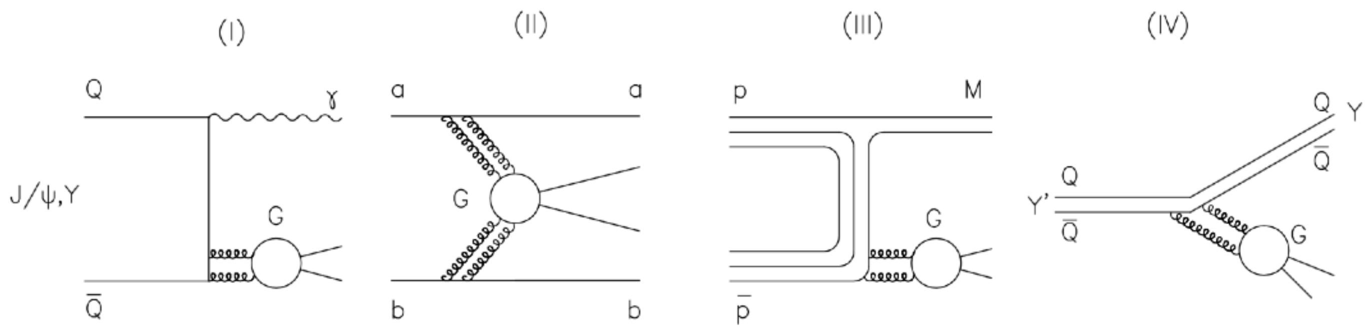
$$A_{n(G)} \sim N_c^{-(n-2)}$$

$$A_{n(G),m(q\bar{q})} \sim N_c^{-\frac{n}{2}+m-1}$$

- Glueballs are narrow in the large N_c limit, expect smaller widths

Experimental searches for glueballs

- It appears that 0^{++} glueball is the lightest glueball state
  - it must be produced copiously in the glue-rich environment and couples strongly to the color-singlet di-gluon (radiative J/ψ decays)
  - its production in gamma-gamma collisions must be suppressed
  - the decay/production amplitude for the glueballs is flavor symmetric



  - it must be narrow (at least in the large N_c limit; also chiral)

Chanowitz, PRL95, 172001 (2005)

- All of this is generally true for other glueball states as well

“Experimental” searches for glueballs

1/6

automatic sweet dumpling **machine/rice glue balls making machine**

$852.00-$2,738.00/ Set

1 Set (Min. Order)

Verified 9 YRS CN Supplier >

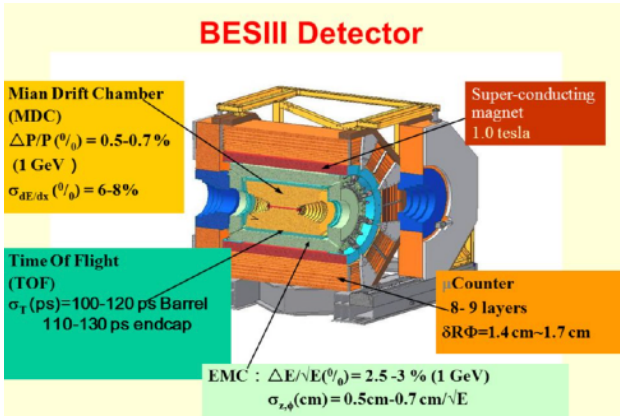
3.9 ★ (5)

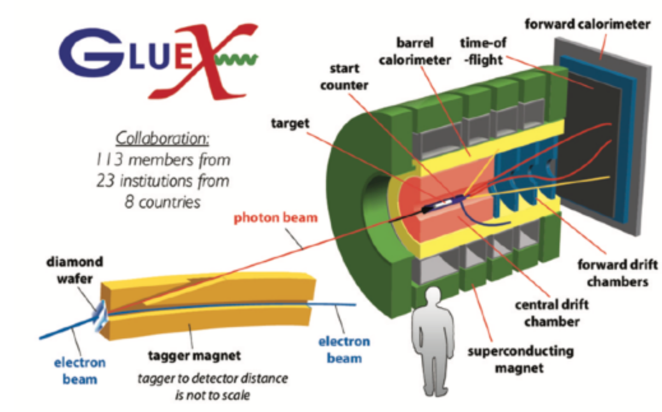
Contact Supplier

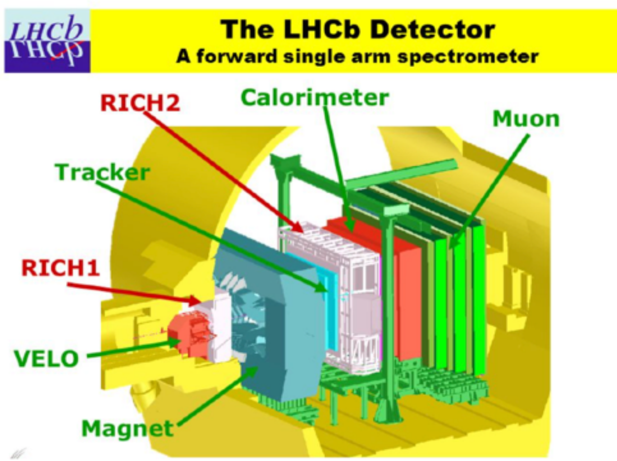
Compare

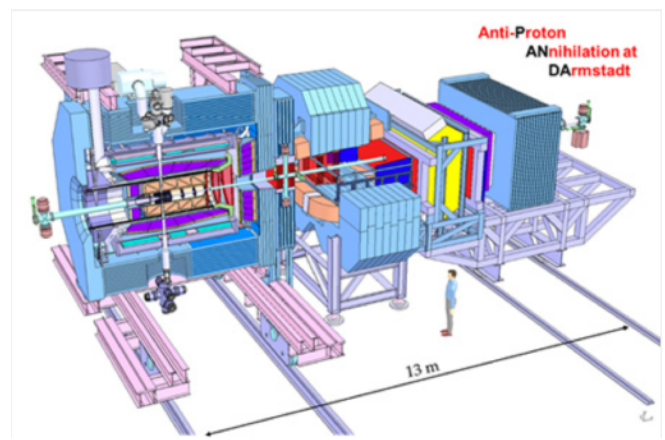
Experimental searches for glueballs

- Searches at dedicated and general-purpose detectors









- No convincing observation of a pure glueball state yet. Why?

Problems with finding glueballs?

- Glueballs and some $q\bar{q}$ states have the same quantum numbers
  - quantum mechanics requires mixing of those states

    … which means that "pure glueballs" do not exist!
  - let us still concentrate on scalar 0^{++} states

$$f_0(500),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

  - these states are admixtures $|f_{0i}\rangle = \alpha_i|N\rangle + \beta_i|S\rangle + \gamma_i|G\rangle$

$$N \equiv n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$$
$$S \equiv s\bar{s}$$

  - fit to experiment (decays $f_0 \to \pi\pi, KK, \ldots\ J/\psi \to \gamma f_0$, …)
  - various fits exist for the relative coefficients, here is an example

$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}$$

Cheng, Chua, and Liu, PRD92, 094006 (2015)

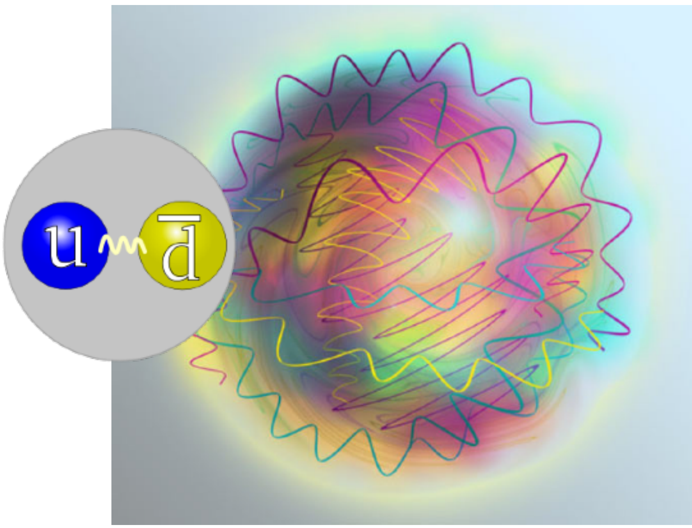
3. Molecular states with glueballs

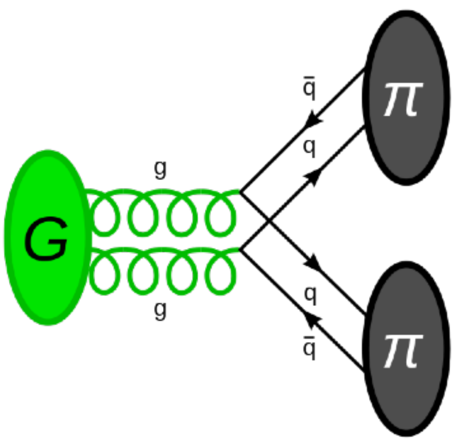
- Are there any other mechanisms for "glueball hadronization"?
  - meson-meson and meson-baryon molecular states:
    - why not glueball-meson or glueball-baryon molecular states?
    - glueballs have smaller widths than mesons in the large N_c, which might have implications for some observed highly excited states
  - some hints from Nature from observations of a few unusual states?
    - for small binding energy: $m_{G(0^{++})} + m_\pi \approx m_{\pi(1800)}$

$$m_{G(1^{--})} + m_\pi \approx m_{X(3872)}$$

  - need non-relativistic description of components to build molecular states (consider lightest glueball and lightest octet of pseudoscalars)

Molecular states with glueballs





- Lifetime of the state is expected to be governed by a lifetime of the glueball component
  - smaller widths, at least from the large N_c arguments
  - possible large mixing with highly excited $q\bar{q}$ states
  - expect unusually long-lived "highly excited states"
- Alternatively can be viewed as a "glueball excitation of a state"
- The lightest state (πG): 0^{-+} or a "pseudo-glueball" $\mathscr{P}$

Components: non-relativistic pions

- For a weakly-bound system need non-relativistic pions
  - not an unusual situation for pionic atoms!

Kong and Ravndal, PRD61, 077506 (2000)

- kinetic part

$$\mathcal{L}_0(\pi_i) = \pi_i^* \left(i \frac{\partial}{\partial t} + \frac{1}{2m_i} \nabla^2 \right) \pi_i$$

- interaction part

$$\mathcal{L}_{int}(\boldsymbol{\pi}) = \frac{1}{4} A_0 (\pi_0^* \pi_0^* \pi_0 \pi_0) + B_0 (\pi_+^* \pi_-^* \pi_+ \pi_-)$$

$$+ \frac{1}{2} C_0 (\pi_+^* \pi_-^* \pi_0 \pi_0 + \pi_0^* \pi_0^* \pi_+ \pi_-)$$

$$+ \frac{1}{4} D_0 (\pi_+^* \pi_+^* \pi_+ \pi_+ + \pi_-^* \pi_-^* \pi_- \pi_-$$

$$+ 2 \pi_+^* \pi_0^* \pi_+ \pi_0 + 2 \pi_-^* \pi_0^* \pi_- \pi_0)$$

- NR pion propagator

$$G(E, \mathbf{k}) = \frac{1}{E - \mathbf{k}^2/2m_\pi + i\epsilon}$$

Components: scalar glueball in EFT

- It is sufficient to have an effective description of a 0^{++} glueball
  - consider massless QCD $\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + i\bar{q} \not{D} q$
  - use the fact that QCD is classically invariant under dilatations

$$x^\mu \to \lambda x^\mu \ , \quad \psi_q(x) \to \lambda^{3/2} \psi_q(\lambda x) \ , \quad A^a_\mu(x) \to \lambda A^a_\mu(\lambda x)$$

  - this symmetry is broken at quantum level

$$(T_{\rm YM})^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0,$$

  - can introduce a scalar dilaton field G describing the trace anomaly

$$\mathcal{L}_{\rm dilaton} = \frac{1}{2}\left(\partial_\mu \tilde{G}\right)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left[\tilde{G}^4 \log\left|\frac{\tilde{G}}{\Lambda}\right| - \frac{1}{4}\tilde{G}^4\right]$$

Salomone, Schechter, Tudron
Migdal and Shifman

- To calculate the binding energy need to couple pions and glueballs
  - use extended linear sigma model $\mathcal{L} = \mathcal{L}_{\rm LSM} + \mathcal{L}_{\rm dilaton} + \mathcal{L}_{\rm int}$

Jankowski et al, PRD84, 054007 (2011)

$$\begin{aligned}\mathcal{L}_{\rm LSM} = & \ \mathrm{Tr}\left[\left(\partial^{\mu}\Phi\right)^{\dagger}\left(\partial_{\mu}\Phi\right)\right] - \lambda_1\left(\mathrm{Tr}\left[\Phi^{\dagger}\Phi\right]\right)^2 \\ & - \lambda_2\ \mathrm{Tr}\left[\left(\Phi^{\dagger}\Phi\right)^2\right] + \mathrm{Tr}\left[H\left(\Phi^{\dagger}+\Phi\right)\right] \\ & + c\left(\det(\Phi^{\dagger}) + \det(\Phi)\right),\end{aligned}$$

with $\Phi = \frac{1}{2}\left(\sigma + i\eta_N\right)\sigma^0 + \frac{1}{2}\left(\vec{a}_0 + i\vec{\pi}\right)\cdot\vec{\sigma}$

  - … with the interaction term

$$\mathcal{L}_{\rm int} = -m_0^2\ \mathrm{Tr}\left[\left(\frac{\tilde{G}}{\Lambda}\right)^2\Phi^{\dagger}\Phi\right]$$

- Small momentum transfer: match to determine πG coupling

- Matching to NR EFT for pions and glueballs
  - expand G and σ about the minimum (G $\rightarrow \Lambda$ + G, $\sigma \rightarrow \sigma + \langle\sigma\rangle$)…

$$\mathcal{L}_{\sigma G} = -\frac{m_0^2 \langle\sigma\rangle}{\Lambda^2} G^2 \sigma + \ldots$$

$$\mathcal{L}_{\pi\pi\sigma} = -\lambda_1 \left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 - \lambda_2 \ \mathrm{Tr}\left[\left(\Phi^\dagger \Phi\right)^2\right]$$

$$\mathcal{L}_{\pi \mathrm{G}} = -\lambda \pi^2 G^2$$

$$\rightarrow \quad \mathcal{L}_{\pi \mathrm{G}} = -\lambda \pi^2 G^2$$

  - … resulting in

$$\mathcal{L}_{\pi \mathrm{G}} = -\lambda \pi^2 G^2 \quad \text{with} \quad \lambda = \frac{m_0^2}{2\Lambda^2}\left[1 - \frac{\langle\sigma\rangle^2}{m_\sigma^2}\left(2\lambda_1 + \lambda_2\right)\right]$$

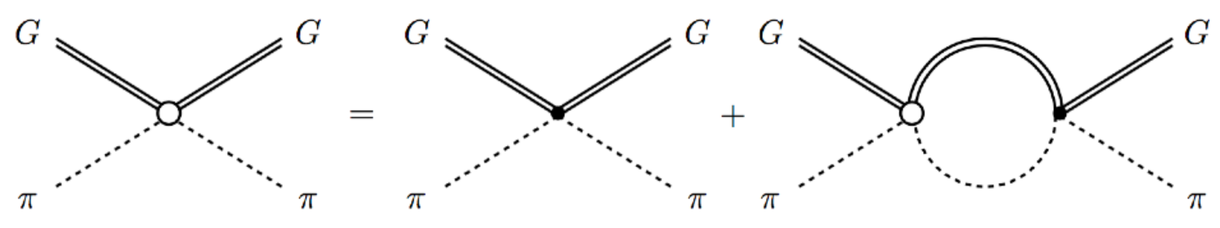
- Now we can calculate the low energy π-G scattering amplitude

Glueball molecules: binding energy

- Calculate binding energy from the pole of transition amplitude
  - in quantum mechanics

$$T_{\pi G} = \frac{4\pi}{\mu_{\pi G}} \frac{1}{p \cot \delta_s - ip} = -\frac{4\pi}{\mu_{\pi G}} \frac{a}{1 + ipa}$$

  - QFT: solve Lippmann-Schwinger equation to find the transition amplitude



$$iT_{\pi \mathrm{G}} = -i\lambda + \int \frac{d^4 q}{(2\pi)^4} \left(iT_{\pi \mathrm{G}}\right) G_{\pi \mathrm{G}} \left(-i\lambda\right)$$

- Need to evaluate one loop integral: divergence?

Glueball molecules: binding energy

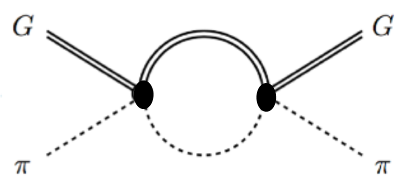
- Calculate binding energy from the pole of transition amplitude

S. Weinberg

  - resuming the "bubbles"…

$$T_{\pi \mathrm{G}} = \frac{\lambda}{1 + i\lambda \widetilde{A}}$$

  - …need to calculate (expect a divergence, move to d-1 dim), $\lambda \to \lambda_R$



$$\widetilde{A} = -\frac{i}{2}\frac{\mu_{\pi G}}{m_G m_\pi}\int \frac{d^3 q}{(2\pi)^3}\frac{1}{\vec{q}^2 - 2\mu_{\pi G}E - i\epsilon}$$

  - We find a scattering amplitude with a pole corresponding to

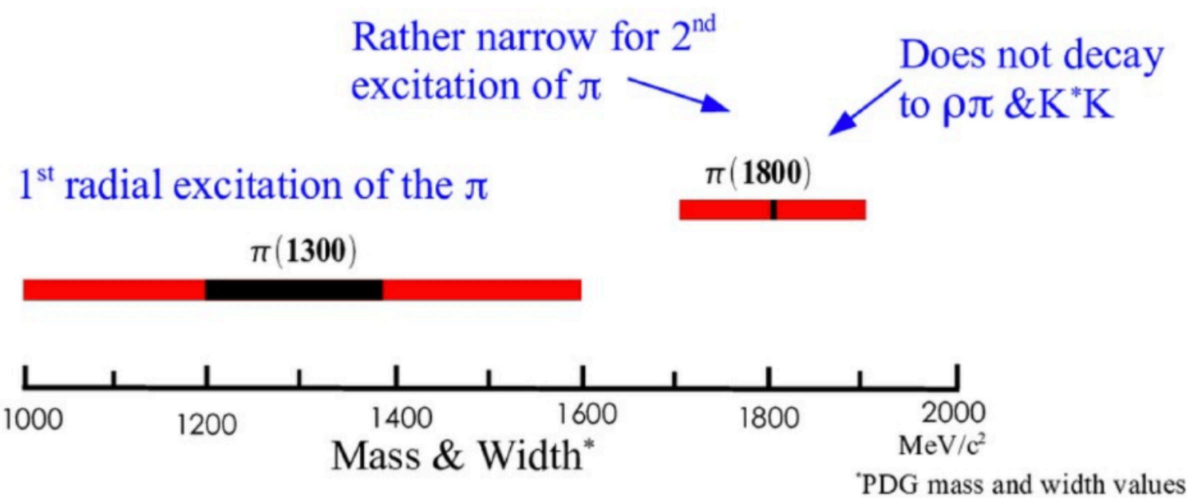
$$E_{\mathrm{bound}} = E_{pole} = \frac{32\pi^2}{\lambda_R^2}\frac{m_\pi^2 m_G^2}{\mu_{\pi G}^3}$$

  - NR bound state: small binding energy. Observed state of $\pi(1800)$?

The $\pi(1800)$ puzzle?

$\pi(1800)$ 0^{-+} Hybrid

$$\pi(1800) \to f_0(980)\pi,\ f_0(500)\pi,\ a_0(980)\eta,\ \omega\rho,\ \eta\eta'\pi,\ K_0^*(1430)K$$



Many† have suggested that the $\pi(1800)$ is a 0^{-+} hybrid meson

†See for example T. Barnes, F. E. Close, P. R. Page, & E. S. Swanson Phys. Rev. D55 4157 (1997)

P. Eugenio, talk at 2016 APS April meeting

Maybe it is a molecular state $\mathscr{P}$?

$\pi(1800)$ as a glueball molecule

- It appears that most issues with understanding of $\pi(1800)$ would go away if a dominant part of the $\pi(1800)$ wave function is built up from a glueball-π molecule
  - lifetime of a glueball-pi molecule is driven by a glueball lifetime
    - expect smaller width than usual $q\bar{q}$ excitations
  - $\pi(1800)$ mass is tantalizingly close to that of a G(0^{++})-π molecule
    - for small binding energy, as considered before,

$$m_{G(0^{++})} + m_\pi \approx m_{\pi(1800)}$$

If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.

Wikipedia's definitions of a "Duck test"

4. Phenomenology of glueball molecules

- Phenomenology of glueball molecules: a word of caution
  - note: quantum mechanics requires that the states of different nature but the same quantum numbers mix
  - we can only make definite statements if molecular component dominates!
  - assume: $\pi(1800)$ is mostly a glueball molecular state

- Phenomenology of glueball molecules: production
  - the molecular state $\mathscr{P}$ can be produced where the glueballs can be produced
    - heavy ion collisions
    - decays of the heavy quark states such as $J/\psi \to \gamma\pi\mathscr{P}$

- Phenomenology of glueball molecules: decay patterns
  - decays of the molecular state $\mathscr{P}$ are driven by the glueball decay
    - decays $\mathscr{P} \to 3\pi$, $\mathscr{P} \to \pi KK$, etc. can be related
    - decays in the f_0 states can be related

Glueball molecules: decays into f_0 states

- Study decay patterns into the f_0 states:
  - assume: $\pi(1800)$ is mostly a glueball molecular state
    - decays $\pi(1800) \to \pi f_0(1500)$ and $\pi(1800) \to \pi f_0(1370)$ can be related

- Recall: the f_0 states seem to contain varying amounts of glue

$$\begin{pmatrix} |f_0(1370)\rangle \\ |f_0(1500)\rangle \\ |f_0(1710)\rangle \end{pmatrix} = \begin{pmatrix} 0.819(89) & 0.290(91) & -0.495(118) \\ -0.399(113) & 0.908(37) & -0.128(52) \\ 0.413(87) & 0.302(52) & 0.859(54) \end{pmatrix} \begin{pmatrix} |N\rangle \\ |S\rangle \\ |G\rangle \end{pmatrix}$$

- … then the decay amplitude for a decay into an f_0 state can be written as

$$\mathcal{A}(\pi(1800) \to \pi f_0) = \langle f_0 | G \rangle \langle \pi G | \mathcal{H} | \pi(1800) \rangle,$$

- … where for different f_0 states we can write (must invert the matrix above)

$$\begin{aligned} |G\rangle &= \langle f_0(1370|G\rangle \, |f_0(1370)\rangle \\ &+ \langle f_0(1500)|G\rangle \, |f_0(1500)\rangle \\ &+ \langle f_0(1710)|G\rangle \, |f_0(1710)\rangle \end{aligned}$$

Glueball molecules: decays into f_0 states

- Recall: the f_0 states seem to contain varying amounts of glue

$$\mathbb{F} = \mathbb{M}\,\mathbb{Q},$$

$$\mathbb{M}_1 = \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & -0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \quad \mathbb{M}_2 = \begin{pmatrix} 0.79 & -0.54 & 0.29 \\ 0.49 & 0.84 & 0.22 \\ -0.37 & 0.023 & 0.93 \end{pmatrix}$$

- … then the ratios of the branching ratios can be written as

$$\frac{\mathcal{B}(\pi(1800) \to \pi f_0(1500))}{\mathcal{B}(\pi(1800) \to \pi f_0(1370))} = \left| \frac{\langle f_0(1500)|G\rangle}{\langle f_0(1370)|G\rangle} \right|^2 r_p \quad \text{with} \quad r_p = p_{f_0(1500)}/p_{f_0(1500)}$$

- … then numerically

$$\frac{\mathcal{B}(\pi(1800) \to \pi f_0(1500))}{\mathcal{B}(\pi(1800) \to \pi f_0(1370))} = (4 \div 7) \times 10^{-3}$$

Things to take home

- Glueballs are expected to be there from QCD
  - smaller widths, at least from the large N_c arguments
  - possible large mixing with highly excited $q\bar{q}$ states
  - expect unusually long-lived highly excited states
- Proposed a new mechanism for "glueball hadronization"
- Alternatively can be viewed as a "glueball excitation of a qq-bar or a qqq state"
  - has direct implications for the N* program at JLab
  - opens up new opportunities in identifying gluon degrees of freedom of ordinary hadrons
- How do you know that X(3872) and other molecules/tetraquarks contain charmed quarks? What about new pentaquark states?


EXTRA!

$\pi(1800)$ as a glueball molecule