Blind Test for Unfolding

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Outline

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Basic conventions

We measure probability density functions or cross sections. Probability density function (PDF) $P(x')$ of the reconstructed characteristics x' of events obtained by detectors with finite resolution and limited acceptance can be represented as:

$$P(x') = \frac{\int_\Omega p(x)A(x)R(x,x')dx}{\int_{\Omega'}\int_\Omega p(x)A(x)R(x,x')dxdx'},$$

where $p(x)$ is true PDF, $A(x)$ is the setup acceptance, i.e. a probability to record an event with characteristic x; $R(x,x')$ is the experimental resolution, i.e. a probability to obtain x' instead of x after the event reconstruction. Integration in (3) is carried out over the domain Ω of variable x and domain Ω' of variable x'.

Basic conventions

An events with the reconstructed PDF $P(x')$ can be obtained as result of experiment or simulation that has three steps:

1. random value x is choosing according PDF $p(x)$;
2. with probability $1 - A(x)$ go to step 1 again;
3. random value x' is choosing according PDF $R(x, x')$.

The quantity $\hat{P}_i = n_i/n$ where n_i is number of events belonging to the ith bin for histogram with total number of events n is an estimator of P_i:

$$P_i = \int_{S'_i} P(x')dx', \; i = 1, \dots, m.$$

With expectation value of the estimator equal to :

$$\mathrm{E}\,\hat{P}_i = P_i.$$

Basic conventions

Results of unfolding must be:

- vector $\hat{\boldsymbol{p}} = (\hat{p}_1, \hat{p}_2, ..., \hat{p}_m)^t$ of estimators of quantities
$$p_i = \int_{S_i} p(x)dx, \; i = 1, \dots, m, \quad (1)$$
integration in (1) is carried out over the bin S_i and $\sum_1^m p_i = 1$;
- estimator of covariance matrix $\mathrm{Cov}\,\hat{\boldsymbol{p}}$ of vector $\hat{\boldsymbol{p}}$.

Question we must answer:

- What is bias of estimator $\hat{\boldsymbol{p}}$?
- What happened with bias if statistics of experimental data increase?
- What is average covariance matrix $\mathrm{Cov}\,\hat{\boldsymbol{p}}$?
- What happened with average covariance matrix if statistics of experimental data increase?

Testing of a method

For testing should be done numerical experiment it is usual minimum 10000 runs. Bias of unfolded vector and average covariance matrix can be calculated after that.

Another test that is not complete test of method is "blind test"

- take some parametrization of true distribution $p(x, a_1, a_2, ..., a_l)$;
- simulate randomly parameters and create sample of "experimental data" according procedure mentioned above.

There is person called tester. The tester takes well documented unfolding program and uses this program according manual.

Testing of the method

Input for tester:

- type of parameterizations $p(x, a_1, a_2, ..., a_l)$, without values of parameters;
- acceptance function $A(x)$;
- resolution function R(x,x');
- number of bins m for the unfolded vector;
- sample of "experimental data".

Input for tester is prepared by analyst as well as analyst is analyzed output from tester

Output from tester:

- unfolded vector $\hat{\boldsymbol{p}}$;
- covariance matrix $\mathrm{Cov}\,\hat{\boldsymbol{p}}$;
- estimators of parameters $\hat{a}_1, \hat{a}_2, ..., \hat{a}_l$;
- covariance matrix $\mathrm{Cov}\,\hat{a}$.

The test for one dimensional case

- type of parametrization $p(x, a_1, a_2, ..., a_l)$:

$$p(x, ...) \propto A_1 \frac{C_1^2}{(x - B_1)^2 + C_1^2} + A_2 \frac{C_2^2}{(x - B_2)^2 + C_2^2}$$

  Ω: $x \in [4, 16]$ and Ω': $x' \in [...]$ defined by tester;

- acceptance function $A(x)$:

$$A(x) = 1 - \frac{(x - 10)^2}{36};$$

- resolution function $R(x, x')$:

$$R(x, x') = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x - x')^2}{2\sigma^2}), \ \sigma = 1.5;$$

- number of bins m for the unfolded vector: $m = 30$.

The test for one dimensional case

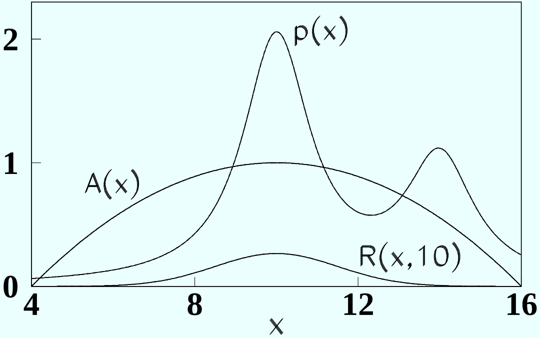

Figure: An example of the true distribution $p(x)$, the acceptance function $A(x)$, the resolution function $R(x, 10)$

The test for one dimensional case

- sample of "experimental data":
  Parameters defined randomly according to uniform distributions on the intervals :
  $[1, 3]$ for A_1; $[5, 10]$ for B_1; $[0.5, 1.5]$ for C_1;
  $[0.0, 1.5]$ for A_2; $[10, 13]$ for B_2; $[0.5, 1.5]$ for C_2.

Number of events in sample is 10000.
Number of events in Monte Carlo is not more then 100000.
Must be used well documented, with manuals, published and popular programs.

The test for one dimensional case

Programs (methods) could be used:

- RUN program
  V. Blobel, Unfolding Methods in High Energy Physics Experiments, DESY 84-118 (1984) and Proc. 8th CERN School of Computing (Aiguablava, Spain, 1984), CERN 85-09.
- Bayes unfolding program
  G. D'Agostini, A multidimensional unfolding method based on Bayes' theorem. Nucl. Instrum. Meth. A 362 (1995) 487-498.
- SVD program
  A. Höcker, V.Kartvelishvili, SVD approach to data unfolding. Nucl. Instrum. Meth. A 372 (1996) 469-481.

The test for one dimensional case

Results of unfolding procedure must be presented as

- vector $\hat{\boldsymbol{p}} = (\hat{p}_1, \hat{p}_2, ..., \hat{p}_{30})^t$;
- covariance matrix 30×30 $\mathrm{Cov}\,\hat{\boldsymbol{p}}$;
- errors of vector $\hat{\boldsymbol{p}}$ if they are asymmetric;
- parameters $A_1, A_2, B_1, B_2, C_1, C_2$;
- covariance matrix 6×6 of the parameters;
- errors of parameters if they are asymmetric.

Each algorithm should be tested 10-100 times by 3-5 testers. Each tester must work independent according manual to program, use program as some kind “black box”.

The test for one dimensional case

For comparison different algorithms can be calculated:

- deviation of unfolded vector from true vector that is:

$$X^2 = (\hat{\boldsymbol{p}} - \boldsymbol{p})^t\,(\mathrm{Cov}\,\hat{\boldsymbol{p}})^{-1}(\hat{\boldsymbol{p}} - \boldsymbol{p});$$

- volume of dispersion ellipsoid that is $\det(\mathrm{Cov}\,\hat{\boldsymbol{p}})$;
- trace of covariance matrix $\mathrm{Tr}(\mathrm{Cov}\,\hat{\boldsymbol{p}})$;
- deviation of parameters vector from true vector of parameters;
- volume of dispersion ellipsoid for parameters;
- trace of covariance matrix for parameters.

If test will be done many times (more then 100) then can be calculated some average characteristics of algorithms.

The test for two dimensional case

- type of parameterizations $p(x, y, a_1, a_2, ..., a_l)$:

  $$A_1 \frac{C_1^2}{(x-B_1)^2+(y-B_2)^2+C_1^2} + A_2 \frac{C_2^2}{(x-B_3)^2+(y-B_4)^2+C_2^2}$$

  Ω: $(x-10)^2+(y-10)^2 \leq 36$ and Ω': $x' \in [...]$ defined by tester
- acceptance function $A(x)$

  $$A(x) = 1 - \frac{(x-10)^2+(y-10)^2}{36};$$
- resolution function $R(x, x')$

  $$R(x, x') = \frac{1}{2\pi\sigma^2}\exp(-\frac{(x-x')^2+(y-y')^2}{2\sigma^2}),\ \sigma = 1.5;$$
- number of bins m for the unfolded vector: $m = 15 \times 15$.

Conclusions

- Formulation of problem has given.
- Main characteristics of unfolding have presented.
- Examples of tests for one dimensional and two dimensional cases are proposed.

Possible outcome of this work is:
Useful and interesting publication.