

DESY Theory Seminar

QED corrections for low-energy processes

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QED calculations are not just for those who are too stupid to do QCD calculations

- they have their own motivation and justification
- they have their own issues/problems/challenges

still (for my taste) the QED world is too much separated from the QCD world

₩

McMule

Monte Carlo for MUons and other LEptons

https://mule-tools.gitlab.io

P. Banerjee, A. Coutinho, T. Engel, A. Gurgone, F. Hagelstein, S. Kollatzsch, L. Naterop, A. Proust, M. Rocco, N. Schalch, V. Sharkovska, A. Signer, Y. Ulrich



- $\bullet \ \Rightarrow \ \mathrm{McMule}, \ a \ framework$ for fully-differential higher-order QED calculations
- NNLO QED corrections available/planned for

 $\ell \to \ell' \nu \bar{\nu} \iff \text{PSI}$ $\ell p \to \ell p \quad \longleftrightarrow \quad P2 \& MUSE \quad sorry...$ $e\mu \rightarrow e\mu \quad \longleftrightarrow \quad \text{MUonE}$ $e^-e^- \rightarrow e^-e^- \iff \text{PRad}$ $e^+e^- \rightarrow e^+e^- \iff \text{luminosity}@\ell-\text{colliders}$ $e^+e^- \rightarrow \gamma\gamma \quad \longleftrightarrow \quad \text{PADME \& luminosity}@\ell\text{-colliders}$ $\ell \to \ell' \nu \bar{\nu} \gamma \quad \longleftrightarrow \quad \text{Babar, MEG}$ $\ell \to \ell' \nu \bar{\nu} (e^+ e^-) \quad \longleftrightarrow \quad \text{Mu3e}$

• planned: electroweak corrections, polarised leptons, $e^+e^-
ightarrow \ell^+\ell^-$



there are many, I pick 3 in low-energy regime ...

- Michel decay $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu \quad \Rightarrow \quad \text{looking for ALPs } \mu^+ \to e^+ X$
- Moller scattering and lepton-proton scattering \Rightarrow proton radius
- muon-electron scattering MUonE \Rightarrow HVP for $(g-2)_{\mu}$

there are also many for higher energies (not in this talk)

- Bhabha scattering and diphoton production \Rightarrow luminosity for $e^+ e^-$ colliders
- QED corrections in B decays \Rightarrow B anomalies
- . . .



- 3 teasers (ALPs, proton radius, MUonE)
- challenges of QED calculations and their solutions
- running out of time
- peeping at future results







[Plots from Bauer et al. 1908.00008, also results from Calibbi et al., Escribano et al. ...] for large diagonal couplings ! for 'normal' diagonal couplings

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 $\mu \rightarrow eX$ to look for X: a (usually very) light neutral boson, majoron

find bump in positron energy spectrum:

$$=\frac{M^2+m^2-m_X^2}{2M}$$





 $m_{\mathbf{X}}$ not too small: no theory needed: Twist

 $m_X \rightarrow 0$: theory needs to provide precise spectrum at end point! NNLO plus Logs!

 E_e

45000

40000

35000

30000

25000

20000

15000

10000 5000

0<u>`</u>

2

3



hismaig

 $(p_e - 50)$ (MeV)

0.9385

Entries 5981882 1.565

Std Dev

Mean

BCK + 10⁵ signal

mean = 1.5 MeV

events with



- proton not pointlike ightarrow electric charge distribution ho(r)
- electric form factor:

$$G_E^{(p)}(q^2) = \int d^3r \,\rho(r)e^{-iqr} = \int d^3r \,\rho(r)\left(1 - q^2\frac{r^2}{6} + \ldots\right)$$

1.5

• proton radius:
$$\langle r_p^2 \rangle = \int d^3r \, r^2 \rho(r) = -6 \frac{dG_E^{(p)}(q^2)}{dq^2} \Big|_{q^2=0}$$

- measure through scattering, slope of $G_E(q^2)$ at $q^2
 ightarrow 0$
- or measure through spectroscopy (ep) = H or better (!) (μp) lepton s wave function overlaps with nucleus, Bohr radius 1/m



confusing situation in \sim 2014, e vs μ, \rightarrow solution in sight



PRad @ JLab

measure ep and ee scattering

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{ep} = \frac{N_{\exp}(ep \to ep)}{N_{\exp}(ee \to ee)} \cdot \frac{\epsilon_{ee}}{\epsilon_{ep}} \cdot \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{ee}$$

to cancel detector effects

MUSE @ PSI

measure $e^{\pm}p$ and $\mu^{\pm}p$ scattering difference between e and μ , TPE



experimental situation will improve further

theory: 'problematic' hadronic contributions to $a_\mu^{\rm SM}$ currently largest uncertainty in $a_\mu^{\rm HLO}$

 a_{μ}^{HLO} :

 a_{μ}^{LBL} :

needs exp. or (conflicting?) lattice input or MUonE ...

Davier et al; Keshavarzi et al; Jegerlehner, 2017/18, 1001 physicists

 $a_{\mu}^{\text{HN...NLO}}$: $\sim -85(2) \times 10^{-11}$

most difficult (but smaller) $\sim 100(30) imes 10^{-11}$



μe scattering





Abbiendi et al:1609.08987

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QED and QCD calculations have many common issues, but ...

- QED matrix elements are easier due to Abelian structure [no big deal]
- The infrared structure of QED is much(!!) simpler [advantage]
- In QED we typically want to keep $m_\ell \neq 0$ since $\log(m_\ell)$ physical [problem]
- In QED we typically have to be exclusive w.r.t. hard collinear emission [problem]

still, the mules try to do QED calculations the QCD way, as much as possible QED parton shower in the plans, but not yet available





challenges

- fully differential phase-space integration
- $\Rightarrow FKS^{\ell}$
- virtual amplitudes with massive particles
- ⇒ one-loop: OpenLoops
- \Rightarrow two-loop: massification
 - numerical instabilities due to pseudo-singularities
- ⇒ next-to-soft stabilisation



only soft singularities



 \Rightarrow subtraction scheme (FKS^{ℓ})





advantage 1: $FKS^{\ell=1}$

subtraction scheme

we do not write $\sigma_n^{(1)} = \sigma_n^{(v)}(\lambda) + \sigma_n^{(s)}(\lambda, \omega) + \sigma_{n+1}^{(h)}(\omega)$ photon mass λ , resolution ω we do write $\sigma_n^{(1)} = \sigma_n^{(1)}(\xi_c) + \sigma_{n+1}^{(1)}(\xi_c)$ auxiliary unphysical parameter ξ_c

$$\sigma_n^{(1)}(\xi_c) = \int d\Phi_n^{d=4} \left(\mathcal{M}_n^{(1)} + \hat{\mathcal{E}}(\xi_c) \mathcal{M}_n^{(0)} \right) = \int d\Phi_n^{d=4} \mathcal{M}_n^{(1)f}(\xi_c)$$

$$\sigma_{n+1}^{(1)}(\xi_c) = \int d\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_1} \right)_c \left(\xi_1 \mathcal{M}_{n+1}^{(0)f} \right)$$

$$\int_0^1 \mathrm{d}\xi_1 \,\left(\frac{1}{\xi_1}\right)_c f(\xi_1) \equiv \int_0^1 \mathrm{d}\xi_1 \,\frac{f(\xi_1) - f(0)\theta(\xi_c - \xi_1)}{\xi_1}$$

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advantage 1: $FKS^{\ell \geq 2}$

 $\mathsf{FKS}^{\ell=2}$

$$\sigma_{n}^{(2)}(\xi_{c}) = \int d\Phi_{n}^{d=4} \left(\mathcal{M}_{n}^{(2)} + \hat{\mathcal{E}}(\xi_{c}) \mathcal{M}_{n}^{(1)} + \frac{1}{2!} \mathcal{M}_{n}^{(0)} \hat{\mathcal{E}}(\xi_{c})^{2} \right) = \int d\Phi_{n}^{d=4} \mathcal{M}_{n}^{(2)f}(\xi_{c})$$
$$\sigma_{n+1}^{(2)}(\xi_{c}) = \int d\Phi_{n+1}^{d=4} \left(\frac{1}{\xi_{1}} \right)_{c} \left(\xi_{1} \mathcal{M}_{n+1}^{(1)f}(\xi_{c}) \right),$$
$$\sigma_{n+2}^{(2)}(\xi_{c}) = \int d\Phi_{n+2}^{d=4} \left(\frac{1}{\xi_{1}} \right)_{c} \left(\frac{1}{\xi_{2}} \right)_{c} \left(\xi_{1} \xi_{2} \mathcal{M}_{n+2}^{(0)f} \right)$$

 FKS^ℓ

$$\begin{aligned} \text{YFS:} \quad e^{\hat{\mathcal{E}}} & \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f} \\ \sigma^{(\ell)} &= \sum_{j=0}^{\ell} \sigma_{n+j}^{(\ell)}(\xi_c) \quad \text{with} \quad \sigma_{n+j}^{(\ell)}(\xi_c) = \int \mathrm{d}\Phi_{n+j}^{d=4} \left(\prod_{i=1}^{j} \left(\frac{1}{\xi_i}\right)_c \xi_i\right) \mathcal{M}_{n+j}^{(\ell-j)f}(\xi_c) \end{aligned}$$



pheno 1: $\mu p ightarrow \mu p$ for MUSE



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- scales (e.g. masses) are the enemy of loop-integral calculators
- for one-loop amplitudes we use OpenLoops, remarkable numerical stability
- for two-loop we use massification and miss terms $\sim (lpha/\pi)^2 \, m_\ell^2/q^2$

collinear factorization



 \Rightarrow massification, i.e. obtain leading mass effects based on massless loops



simple process ($\mu
ightarrow e
u
u$ or $t
ightarrow b \ell
u$)

- $\mathcal{A}_{\mu}(m) = \mathcal{S} \times Z \times \mathcal{A}_{\mu}(0) + \mathcal{O}(m)$
- $Z \supset \log(m)$: process indep. jet fct.
- $S \supset \log(m)$: process dep. soft fct. (easy)

[Penin; Becher, Melnikov; Engel, Gnendiger, AS, Ulrich]

different process ($\mu e \rightarrow \mu e$)

• $\mathcal{A}_{\mu e}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{\mu e}(0) + \mathcal{O}(m)$

based on SCET and method of regions as calculational tool





NNLO corrections to electron energy spectrum



- full m_e effects known at NNLO analytically [Chen, McMule] and numerically [Anastasiou et al.]
- compare to logarithmic approximation [Arbuzov et al.]
- check effect of massification approximation is invisible

our result, Anastasiou et al, logarithms



real-virtual corrections trivial in principle, extremely delicate numerically



- soft limit (of collinear emission) $E_{\gamma} = \xi \sqrt{s}/2$
- Bhabha scattering (as example) [McMule, 2106.07469]
- M_{exact} Mathematica expression
- full *M* vs soft limit
- stability problem





real-virtual corrections trivial in principle, extremely delicate numerically



- soft limit (of collinear emission)
- Bhabha scattering (as example) [McMule, 2106.07469]
- M_{exact} Mathematica expression
- full M vs next-to-soft limit
- stability problem solved



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- ready for a full (Moller) NNLO calculation photonic and fermionic contributions
- compute double-real amplitudes
- massify massless two-loop amplitudes [Bern,Dixon,Ghinculov] and one-loop squared
- use OpenLoops [Buccioni, Pozzorini, Zoller] for real-virtual amplitudes
- apply next-to-soft stabilisation
- use FKS²
- open lepton production not included
- let the mule trot [McMule, 2107.12311]







pheno 3: Moller for PRad



beam energy $E_b = 1.4 \text{ GeV}$ and kinematical cuts on angles of narrow/wide electron, inelasticity $\eta = E_b + m - E_n - E_w$ and coplanarity $\zeta = |180^\circ - |\phi_n - \phi_w||$

$$\begin{array}{l} 0.5^{\circ} < \theta_n, \theta_w < 6.5^{\circ} \\ \eta < 3.5\sigma_E \\ \zeta < 3.5\sigma_{\phi} \end{array}$$

with $\sigma_E = 37.7~{
m MeV}$, $\sigma_\phi = 2.1^\circ$

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eveything that follows is **PRELIMINARY** (and rushed)

- related to ALPS and $\mu^+
 ightarrow e^+ X$
- related to measuring the HVP through $\mu\,e$ scattering



For small m_X from $\mu^+ \to e^+ X$ get shift at endpoint of Michel spectrum theory error and energy resolution of experiment are crucial



 $\begin{array}{l} \mbox{collinear logs } L_z = \log z^2 = \log m_e^2/M_\mu^2 \\ \mbox{soft logs } L_s = \log(1+z^2-x) \\ \mbox{with } x = E_e/(2M_\mu) \rightarrow 1+z^2 \mbox{ at endpoint } \\ \mbox{at } \alpha^n \mbox{ get } L_z^m, m \leq n \mbox{ and } L_s^m, m \leq n \end{array}$

include NLL L_z up to α^5 (just for fun) and resum NNLL (and partially NNNLL) L_s (YFS!!) \Rightarrow theory error at endpoint $\sim 10^{-5}$ include signal at NLO (not just a delta peak)



 $p^2 - m_W^2$

the muon Lagrangian

effective (Fermi) theory quantum field theory valid for $p^2 \ll m_W^2$

hard: $k \sim m_W$

 $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QCD}} + \frac{4G_F}{\sqrt{2}} \left(\overline{e_L} \gamma^{\mu} \mu_L\right) \left(\overline{\nu_L} \gamma_{\mu} \nu_L\right) + \text{h.c.} + \mathcal{O}(m_W^{-4})$

soft: $k \sim m_{\mu}$



 $\rightarrow \frac{1}{m_W^2} + \dots$

 $\frac{4G_F}{\sqrt{2}} = \frac{g_w^2}{2m^2} \left(1 + \mathcal{O}(\alpha)\right)$

 $SU(3) \times SU(2) \times U(1)_Y$

dim 6 operator Wilson coefficient G_F does not run ("coincidence")

 $SU(3) \times U(1)_{\text{QED}}$

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just do QED with: $\mathcal{L}_{muon} = \mathcal{L}_{QED} + \frac{4G_F}{\sqrt{2}} \left(\overline{e_L} \gamma^{\mu} \mu_L\right) \left(\overline{\nu_L} \gamma_{\mu} \nu_L\right) + h.c. + \mathcal{L}_{dirt}$

- compute LO in G_F but to any order in QED
- at some point have to deal with hadronic effects (maybe even EW)
- \Rightarrow muon is only fairly clean, not very clean

muon decay at NNLO: history calculations

- inclusive {N}NNLO for G_F Stuart, van Ritbergen 1999, Czarnecki, Pak 2008; {Fael et al. 2020}
- logarithms $\log^{\{1,2\}} rac{m_e^2}{m_\mu^2}$ of ${
 m d}\Gamma/{
 m d}E_e$ Arbuzov, Czarnecki, Gaponenko 2002, Arbuzov, Melinkov 2002
- fully inclusive w.r.t. photons, numeric energy spectrum Anastasiou, Melnikov, Petriello 2005

how?

- analytic two-loop integrals Chen 2018 and form factors Engel, Gnendiger, AS, Ulrich 2018
- fully differential Monte Carlo $_{\mbox{Engel, AS, Ulrich, 2019}}$ using FKS^2



a (not so) toy analysis for Mu3e detector response $S_e (E_e - E'_e)$ has crucial impact $\mathcal{F}_e (E_e) = \int dE'_e \left[\mathcal{E}_e (E'_e) \times \mathcal{A}_e (E'_e) \times \mathcal{S}_e (E_e - E'_e) \right] \equiv (\mathcal{E}_e \times \mathcal{A}_e) \otimes \mathcal{S}_e$





new proposal [Abbiendi et al.]: elastic scattering $\mu e \rightarrow \mu e \Rightarrow$ independent determination of HVP 150 GeV μ beam (CERN M2 beam) on e at rest (Beryllium target), \sim measure only angles

 \rightarrow nearly full coverage of integrand <code>[plots from talks by M.Passera and G.Venanzoni]</code>





 \rightarrow "theory initiative" to provide necessary computations [2004.13663, Padua group, Pavia group, McMule ...]



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MUonE



- 2-loop matrix elements known for massive muon and massless electron
- form factor contributions (emission from electron or muon line only) known with exact mass dependence
- (H)VP insertions known with exact mass dependence
- mixed 2-loop contributions through massification [McMule] or YFS approach [Pavia] quality of approximations under investigation
- fast numerical evaluation of generalised polylogarithms with [handyG]
- double-real and real-virtual (OpenLoops) known with exact mass dependence
- two completely independent Monte Carlo (subtraction vs slicing, 1/ε vs log λ)
 ⇒ found complete agreement for distributions



consistency/implementation/stability check



- $\xi_{\rm cut}$ (in)dependence
- no approximations made (subtraction not slicing)
- in principle any $\xi_{
 m cut}$ is ok
- use 'good' $\xi_{\rm cut} \sim 10^{-1}$ for actual runs (no large cancellations)



PRELIMINARY



without tweaking NNLO corrections $\sim 2\cdot 10^{-3} \Rightarrow$ need to go beyond

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QED calculations are needed for background control, very much like QCD calculations ! to tackle state-of-the-art problems, a modern approach to QED is required

future steps

- NNNLO 'form factor' contributions $\gamma^* \to \ell^+ \ell^-$
- combine fixed-order QED with (YFS?) parton shower
- combine fixed-order QED with electroweak (Bhabha, P2)
- polarised leptons
- towards higher energies (mainly numerical)



Outlook

MCMULE

Thank you