Status of Electron Beam Transport Studies in Elegant, SelaV, and Impact-Z

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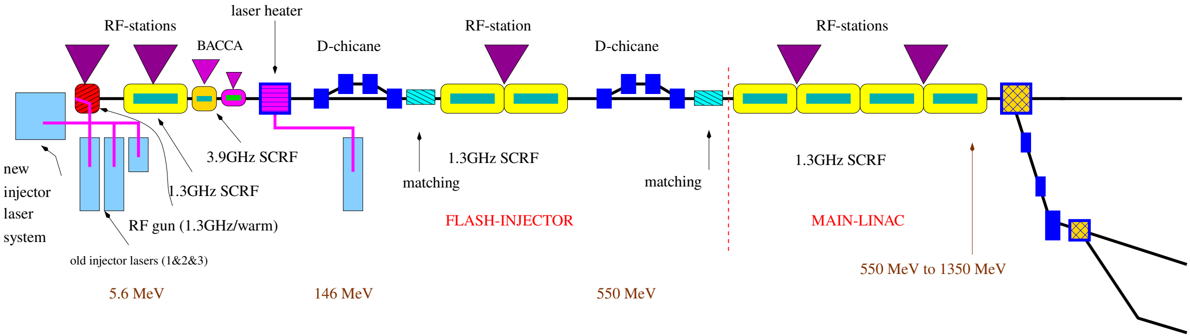
FLASH2020+ 2nd S2E Simulation Workshop
2022-04-20

v2

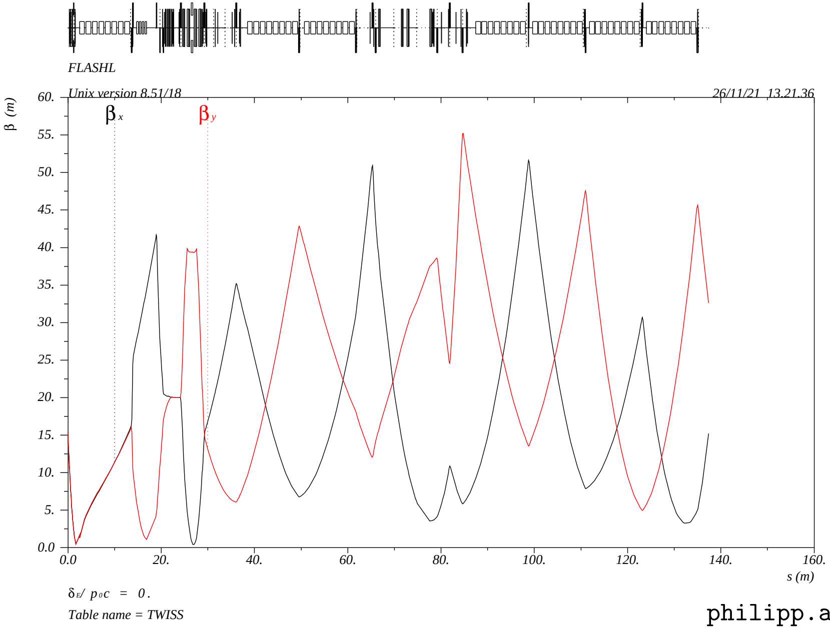
Outline

- FLASH0 Layout
- Simulation Code Overview (Elegant, IMPACT-Z, SelaV)
- RF-Calculations for "linear" bunch compression
- Comparison: Elegant & SelaV (non-collective, and quiet start)
- Calculation of the "uncorrelated slice energy spread"
- Shot-Noise in macro-particle codes and Vlasov codes
- Comparison: IMPACT-Z & SelaV (microbunching from shot-noise)

FLASH2020+ FLASH0 Layout



FLASH2020+ FLASH0 Optics



Simulation Code Overview

	Elegant	IMPACT-Z	SelaV
Type	3D macro-particle	3D macro-particle	1D semi-Lagrangian
EOM Solver	various	???	symplectic maps
Space Charge	longitudinal (charged disk)	3D PIC	longit. (ch'd. disk)
CSR	longit. (SSY'97)	longit. (SSY'97)	longit. ((modified) SSY'97)
Wakefields	possible, not used	yes	not (yet) implemented
Typ. run-time	1Mpart, 1 core: $\sim$ 20min	620Mpart, 64 cores: $\sim$ 5d	resolution=1/4096, 4 cores: $\sim$ 1min
Simulations by	D.Samoilenko & PhA	M.Dohlus	PhA

- Agreed to study a first common working point (suited for EEHG seeding):
  - $I_0 = 31.25\text{A}$, $Q = 400\text{pC}$
  - $C_1 = C_2 = 4$, hence $I_{\text{final}} = 500\text{A}$
  - $\sigma_E = 3\text{keV}$
  - start at/after first cavity of ACC1
  - well-behaved, "linear" final longitudinal phase space

SSY'97: E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov
"On the coherent radiation of an electron bunch moving in an arc of a circle", NIM-A 398 (1997)

Determining Compression Workingpoints

- Goal: Determine RF settings for "linear" bunch compression (given the compression factors and chicane settings)
- Let $\langle E\rangle(s) = f(s)$ be the mean energy deviation at bunch coordinate s.
- Consider the Taylor Expansion up to 3rd order around the bunch center $s = 0$

$$f(s) = E + h\,s + h'\,s^2 + h''\,s^3 + O(s^4)$$

- Evolution under kick maps $(s, E) \mapsto (s, E + k(s))$, with $k(s) = k_0 + \kappa\,s + \kappa'\,s^2 + \kappa''\,s^3 + O(s^4)$

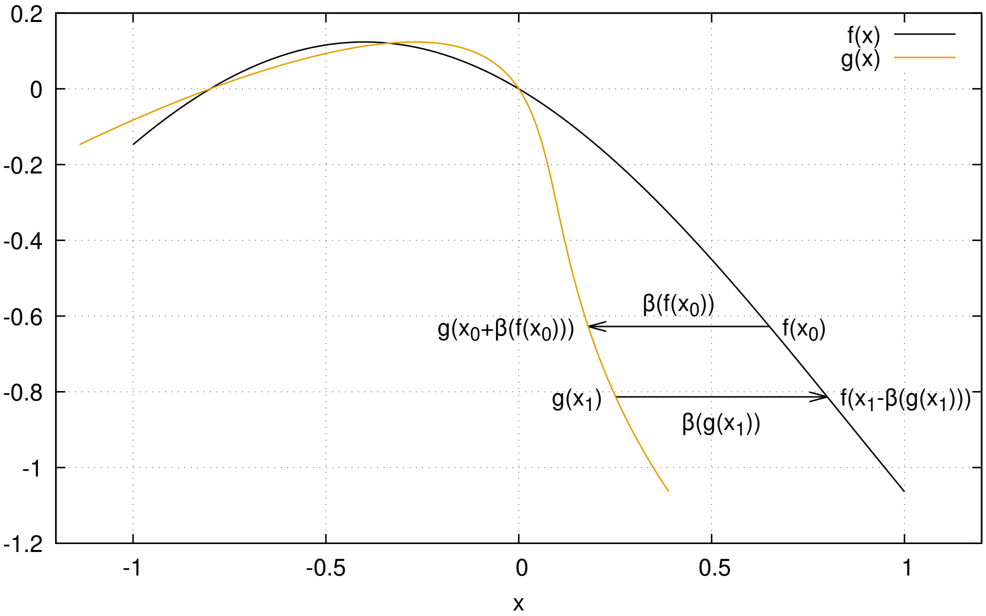
$$f(s) \mapsto f(s) + k(s) \tag{1}$$
$$(E, h, h', h'') \mapsto (E + \kappa_0, h + \kappa, h' + \kappa', h'' + \kappa'') \tag{2}$$

- Evolution under drift maps $(s, E) \mapsto (s + \beta(E), E)$, (under some assumptions)

$$f(s) \mapsto g(s), \text{with } g(s) = f(s - \beta(g(s))).$$

With $g(s)$ only implicitly defined, $(E, h, h', h'') \mapsto ???$

Implicit Function after Drift Map



Determining Compression Workingpoints, cntd.

- w.o.l.g. assume $\beta(E) = \beta(f(0)) = 0$
- Key insight: Despite $g(s)$ being implicitly defined, $g^{(n)}(0)$ can be calculated anyhow! E.g.

$$g(s) = f(s - \beta(g(s))) \implies g(0) = f(\beta(g(0))) \impliedby g(0) = f(0) \tag{3}$$

$$g'(s) = [1 - g'(s)\beta'(g(s))] f'(s - \beta(g(s))) \implies g'(0) = [1 - g'(0)\beta'(f(0))] f'(0) \tag{4}$$

$$\implies g'(0) = C\, f'(0), \text{ with } C = \frac{1}{1 + f'(0)\beta'(f(0))} \tag{5}$$

- Expression for higher derivatives become unwieldy, but manageable by defining

$$C(s) = \frac{1}{1 + f'(s)\beta'(f(s))}$$

- After some algebra

$$g(0) = f(0) \tag{6}$$

$$g'(0) = C(0)\, f'(0) \tag{7}$$

$$g''(0) = C(0)^3 \left[f''(0) - \beta''(f(0))\, f'(0)^3\right] \tag{8}$$

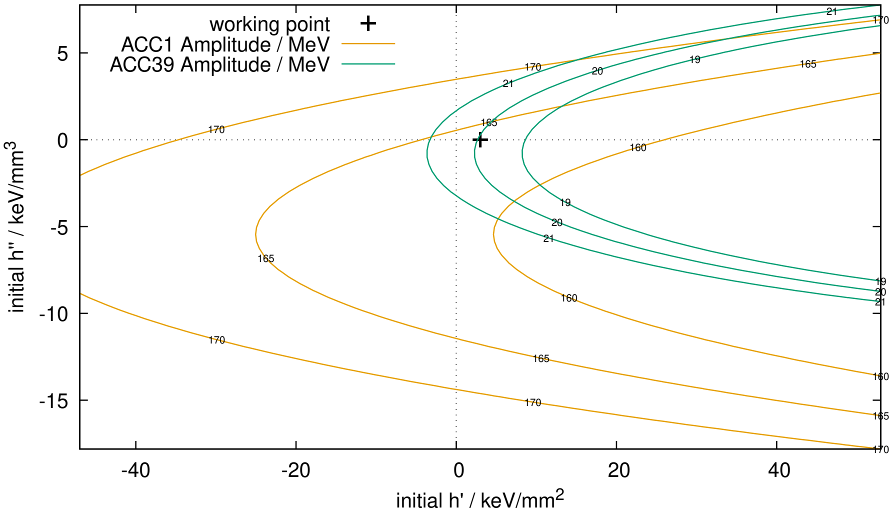
$$g'''(0) = C''(0)\, f'(0) + 3C(0)C'(0) f''(0) + C(0)^3 f'''(0) \tag{9}$$

Determining Compression Workingpoints, cntd.[2]

$$\overbrace{\begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_0 \xleftarrow{K[A_1,\phi_1]} \begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_{\mathrm{ACC1}} \xleftarrow{K[A_{39},\phi_{39}]} \begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_{\mathrm{ACC39}} \xleftarrow{D[\alpha_1]} \begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_{\mathrm{BC1}}}^{C_1} \overbrace{\xleftarrow{K[A_{23},\phi_{23}]} \begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_{\mathrm{ACC23}} \xleftarrow{D[\alpha_2]} \begin{pmatrix}E\\h\\h'\\h''\end{pmatrix}_{\mathrm{BC2}}}^{C_2}$$

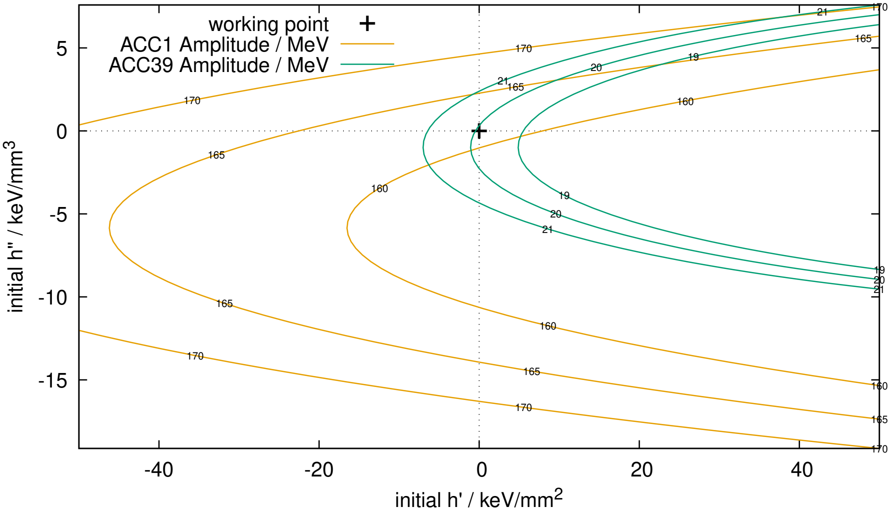
- Specify
  - compression factors C_1, C_2; energy profile $E_{\mathrm{ACC39}}, E_{\mathrm{ACC23}}$
  - initial condition $\vec{b}_0 = \left(E, h, h', h''\right)_0$; final non-linearities $h'_{\mathrm{BC2}}, h''_{\mathrm{BC2}}$
- Calculate
  - $E_{\mathrm{BC1}} = E_{\mathrm{ACC39}}$, $E_{\mathrm{BC2}} = E_{\mathrm{ACC23}}$
  - $h_{\mathrm{BC1}} = \frac{C_1 - 1}{R_{56,\mathrm{BC1}}(\alpha_1)}$, $h_{\mathrm{BC2}} = \frac{C_2 - 1}{R_{56,\mathrm{BC2}}(\alpha_2)}$
  - $\vec{b}_{\mathrm{ACC23}} = D[\alpha_2](\vec{b}_{\mathrm{BC2}})$
  - Find A_{23}, ϕ_{23} so that with $(\hat{E}, \hat{h}, \hat{h}', \hat{h}'') = K[A_{23}, \phi_{23}](\vec{b}_{\mathrm{ACC23}})$, it is $\hat{E} = E_{\mathrm{BC1}}$, $\hat{h} = h_{\mathrm{BC1}}$
  - $\vec{b}_{\mathrm{ACC39}} = D[\alpha_1] \circ K[A_{23}, \phi_{23}](\vec{b}_{\mathrm{ACC23}})$
  - Find $A_1, \phi_1, A_{39}, \phi_{39}$ so that $\vec{b}_0 = K[A_1, \phi_1] \circ K[A_{39}, \phi_{39}](\vec{b}_{\mathrm{ACC39}})$
- Not covered today: Calculating drift-coefficients of chicanes; solving for RF parameters

ACC39 Limits Achievable Linearity



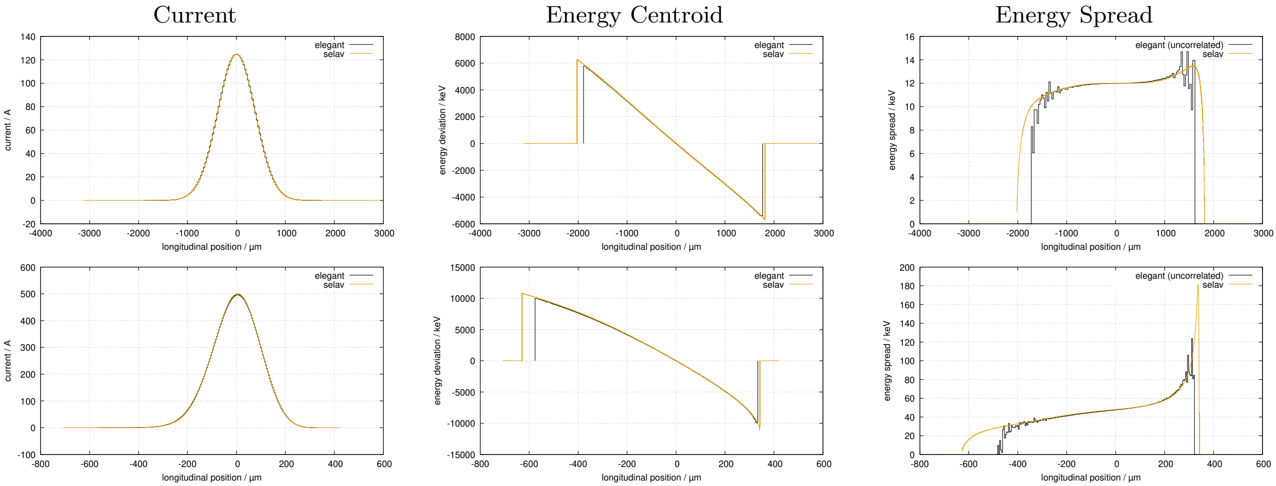
- Amplitude of ACC39 is limited to $\leq$ 20MeV
- Working point found by allowing for initial (or final) non-linearities

Identified Improved Working Point for Future Studies



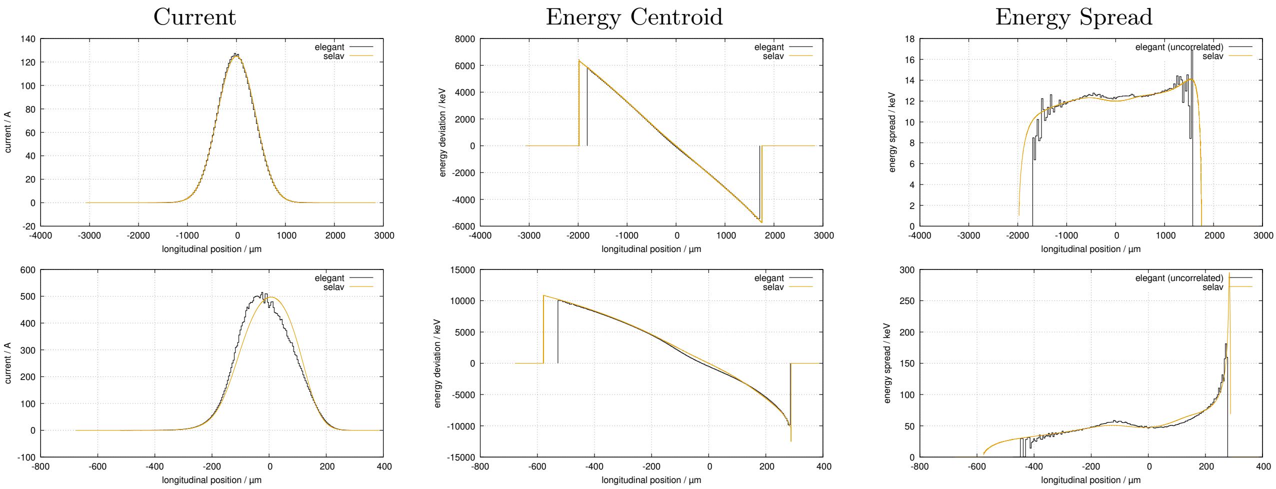
- Reducing energy in BC1 from 146MeV to 143MeV relaxes ACC39 requirements
- $Q = 400\text{pC}$, $I_0 = 20\text{A}$, $C_1 = C_2 = 5$

Comparison: Elegant & SelaV — collective effects: None



- top (bottom) row: after BC1 (BC2)
- nearly perfect agreement

Comparison: Elegant & SelaV — collective effects: LSC (quiet start)



- top (bottom) row: after BC1 (BC2)
- slight differences probably due to differences in transverse beam-size (under investigation)

Excursion: Uncorrelated Slice Energy Spread

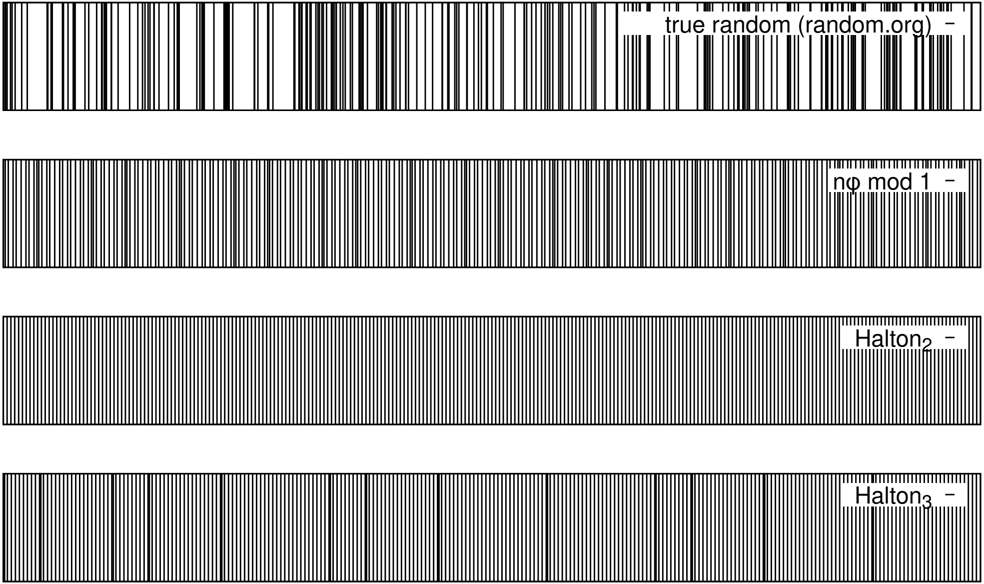
- Macro-particle codes rely on slicing to determine "local" beam properties
- (Vlasov codes can integrate/project the PSD directly and do not require slicing)
- Problem with slicing in the longitudinal phase-space:
  Chirp induces artificial increase of the slice energy spread
- Therefore we define the uncorrelated energy spread:
  - Let $\{z_1, \ldots, z_n\}$ with $z_i = (s_i, E_i)$ be an ensemble of phase-space coordinates.
  - Define $\langle f \rangle \equiv \sum_{i=1}^{n} f(z_i)/n$.
  - Assume $\langle s\, E \rangle = 0$, i.e. s and E are "uncorrelated".
  - Let $E'_i = E_i + h\, s_i$. Then $\langle s\, E' \rangle = \langle s\, E \rangle + h \langle s^2 \rangle$, therefore $h = \langle s\, E' \rangle / \langle s^2 \rangle$.
  - Further $\langle E'^2 \rangle = \langle E^2 \rangle + 2\, h\, \langle s\, E \rangle + h^2 \langle s^2 \rangle$, therefore $\langle E^2 \rangle = \langle E'^2 \rangle - h^2 \langle s^2 \rangle$.
  - Finally, we get the "uncorrelated energy variance" $\langle E^2 \rangle = \langle E'^2 \rangle - \langle s\, E' \rangle^2 / \langle s^2 \rangle$.
- Works also for in case of 2nd-order chirps: $E'_i = E_i + h\, s_i + g\, s_i^2$

$$\langle E^2 \rangle = \langle E' \rangle - h^2 \langle s^2 \rangle - g^2 \langle s^4 \rangle - 2hg \langle s^3 \rangle, \text{with } \begin{pmatrix} h \\ g \end{pmatrix} = \begin{pmatrix} \langle s^2 \rangle & \langle s^3 \rangle \\ \langle s^3 \rangle & \langle s^4 \rangle \end{pmatrix}^{-1} \begin{pmatrix} \langle sE' \rangle \\ \langle s^2 E' \rangle \end{pmatrix}$$

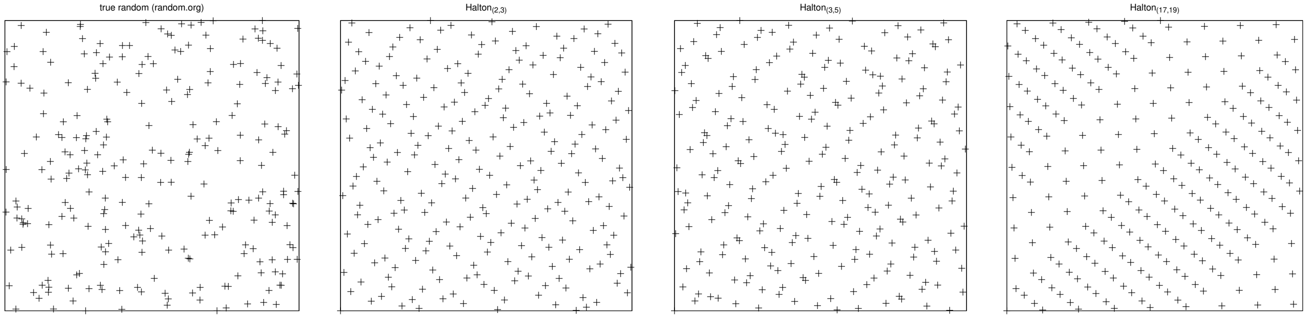
Phase-Space Statistics in Macro-Particle Simulations

- Shot-noise plays a major role in the formation of the microbunching instability
- Typical number of electrons in a bunch: $N = 400\text{pC}/\text{e} \approx 2.5 \times 10^9$
  $\implies$ Computationally, it is attractive to represent multiple electrons by one macro-particle $N = \nu N_{\text{sim}}$
- Base assumption: Phase-space coordinates of electrons are initially independent and identically distributed (i.i.d.)
  - bunch is described by a phase-space density $\Psi(z)$, so that $\langle n \rangle(V) = N \int_V \Psi(z)\mathrm{d}z$ is the expected number of electrons in V
  - phase-space population of one bunch realization (ensemble) *locally* follows a Poisson distribution with parameter $\langle n \rangle(V)$, hence $\sigma(V) = \sqrt{\langle n \rangle(V)}$
  - $\sigma_{\text{sim}}(V) = \sigma(V)/\sqrt{\nu}$, hence the current variation $\sigma_{I,\text{sim}}(V) \propto \nu\sigma_{\text{sim}}(V) = \sigma(V)\sqrt{\nu}$
- Two possible solutions:
  - take $N_{\text{sim}} = N$
  - or use quasirandom sequence to generate low-discrepancy phase-space coordinates and artificially introduce correct statistics

Quasirandom Sequences



Higher-Dimensional Quasirandom Sequences



- In higher dimensions quasirandom sequences can show correlations, if not chosen carefully
- Typically, the unit-(hyper)cube is mapped to final distribution via inverse cumulative distribution function (ICDF)
  - Is low-discrepancy property preserved?

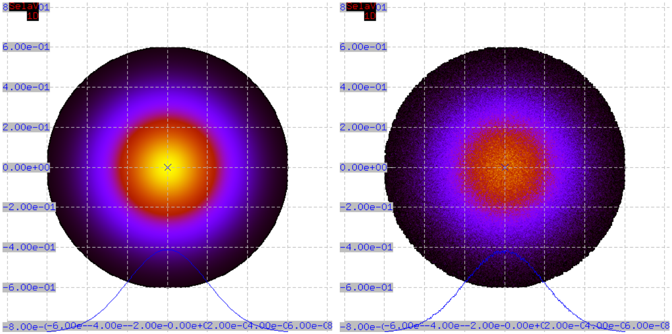
Shot-Noise in Vlasov Codes

- Generate "smooth" initial PSD as usual
- Then modify PSD values according to Poisson distribution with local particle number

$$\Psi(z_{ij}) \to X(z_{ij})/\mathrm{d}^2 z$$

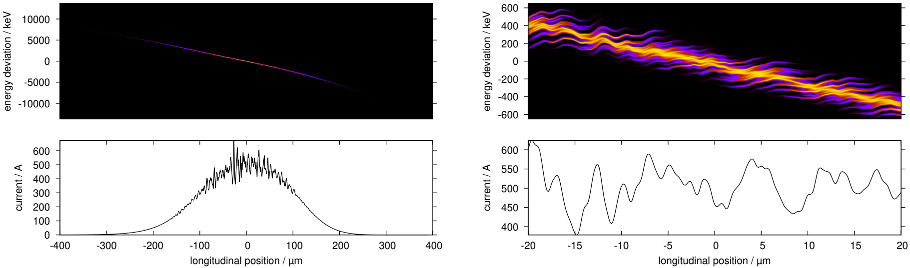
$$P(X(z)) = \lambda(z)^k \mathrm{e}^{-\lambda(z)}/k!$$

$$\lambda(z) = \lceil Q/e\,\Psi(z)\,\mathrm{d}^2 z \rceil$$

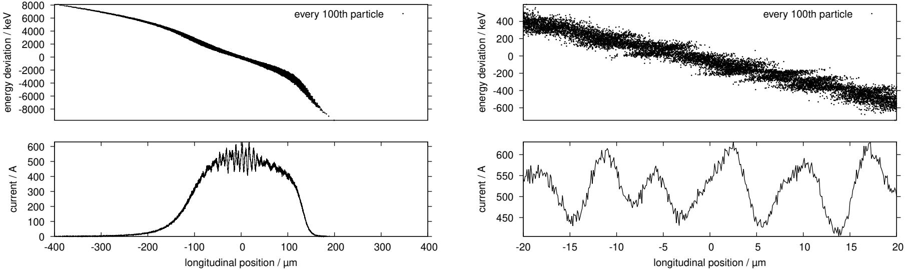


IMPACT-Z & SelaV: Longitudinal Phase-Space after BC2

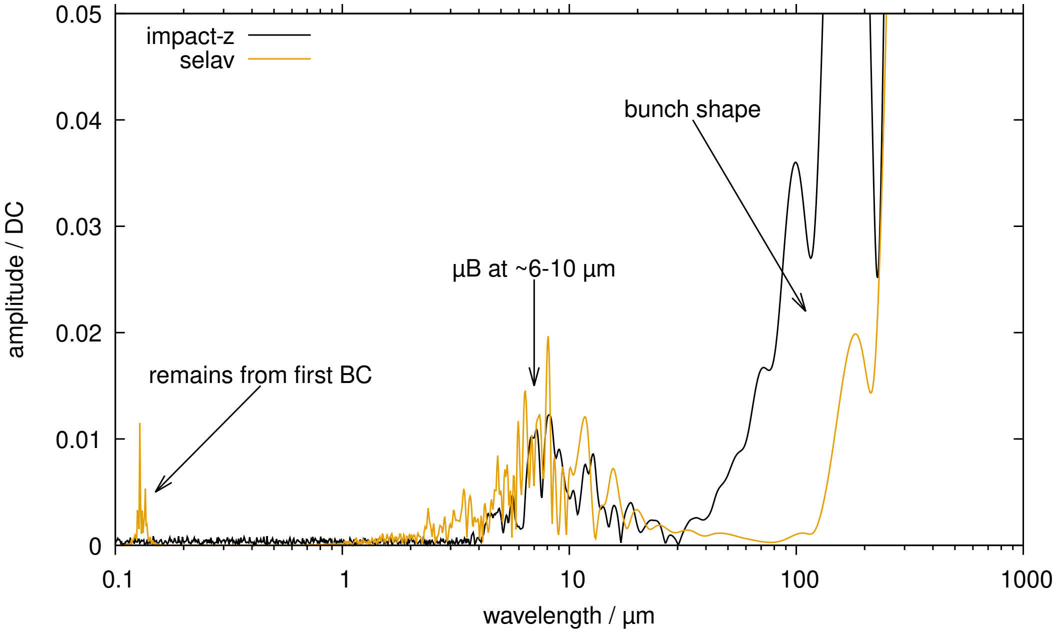
SelaV (LSC only):



IMPACT-Z:



Microbunching Spectra: IMPACT-Z & SelaV after BC2



Summary & Outlook

- First comparison studies between Elegant, IMPACT-Z, SelaV show
  - Excellent agreement between Elegant & SelaV in the non-collective case (quiet-start collective case is improvable)
  - Good agreement between IMPACT-Z & SelaV regarding microbunching spectra
- Future studies:
  - Elegant simulations with increased macro-particle numbers
  - Compare different working points: EEHG seeding, SASE, ...
  - Study impact of laser heater