Discussion: Progress in theory

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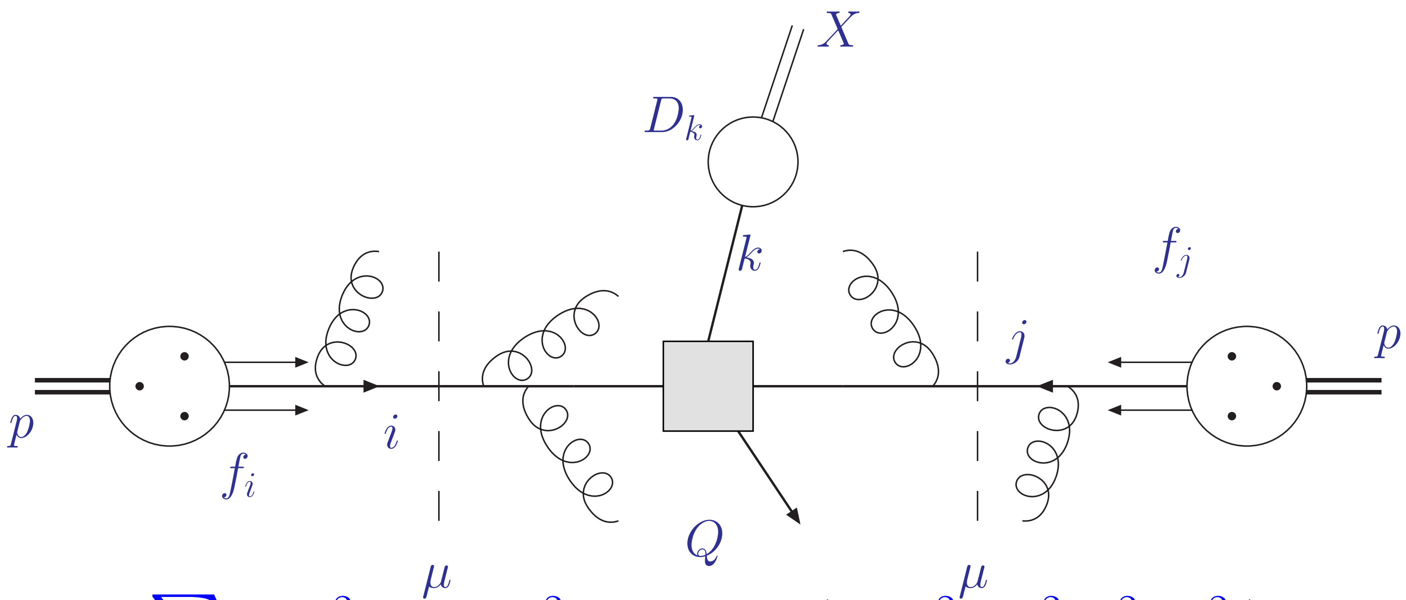
Theory-Experimental Top Quark Mass Workshop, DESY, Jun 28, 2022

Based on work done in collaboration with:

- *One-loop soft anomalous dimension matrices for $t\bar{t}j$ hadroproduction*
  B. Chargeishvili, M. V. Garzelli, and S. M. arXiv:2206.10977
- *Phenomenology of $t\bar{t}j + X$ production at the LHC*
  S. Alioli, J. Fuster, M. V. Garzelli, A. Gavardi, A. Irles, D. Melini, S. M. P. Uwer and K. Voß arXiv:2202.07975
- *Cross-sections for $t\bar{t}H$ production with the top quark $\overline{MS}$ mass*
  A. Saibel, S. M. and M. Aldaya Martin arXiv:2111.12505
- *Heavy-flavor hadro-production with heavy-quark masses renormalized in the $\overline{MS}$, MSR and on-shell schemes*
  M. V. Garzelli, L. Kemmler S. M. and O. Zenaiev arXiv:2009.07763
- [. . .]
- *Parton distribution functions, α_s, and heavy-quark masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1701.05838
- [. . .]
- *Top-quark pair production near threshold at LHC*
  Y. Kiyo, J. H. Kühn, S. M. M. Steinhauser and P. Uwer arXiv:0812.0919
- [. . .]

QCD factorization

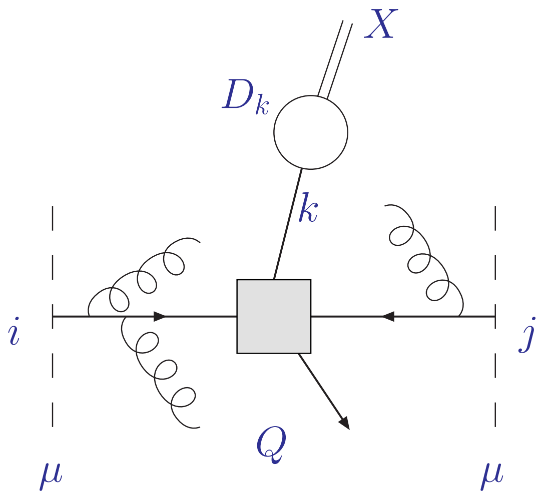


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Hard scattering cross section

- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^3LO (next-to-next-to-next-to-leading order)
  - ...

Parton luminosity

- Long distance dynamics due to proton structure

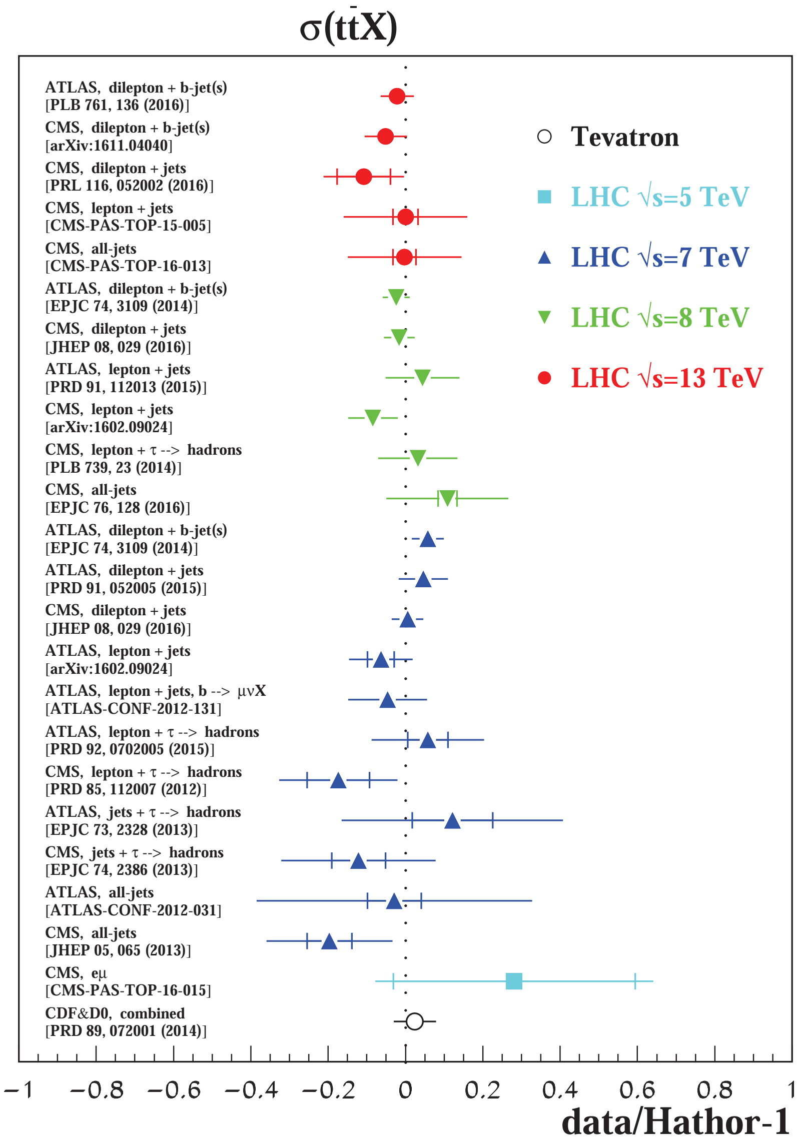
- Cross section depends on parton distributions f_i

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[\ldots \right]$$

- Parton distributions known from global fits to exp. data
    - available fits accurate to NNLO
    - information on proton structure depends on kinematic coverage

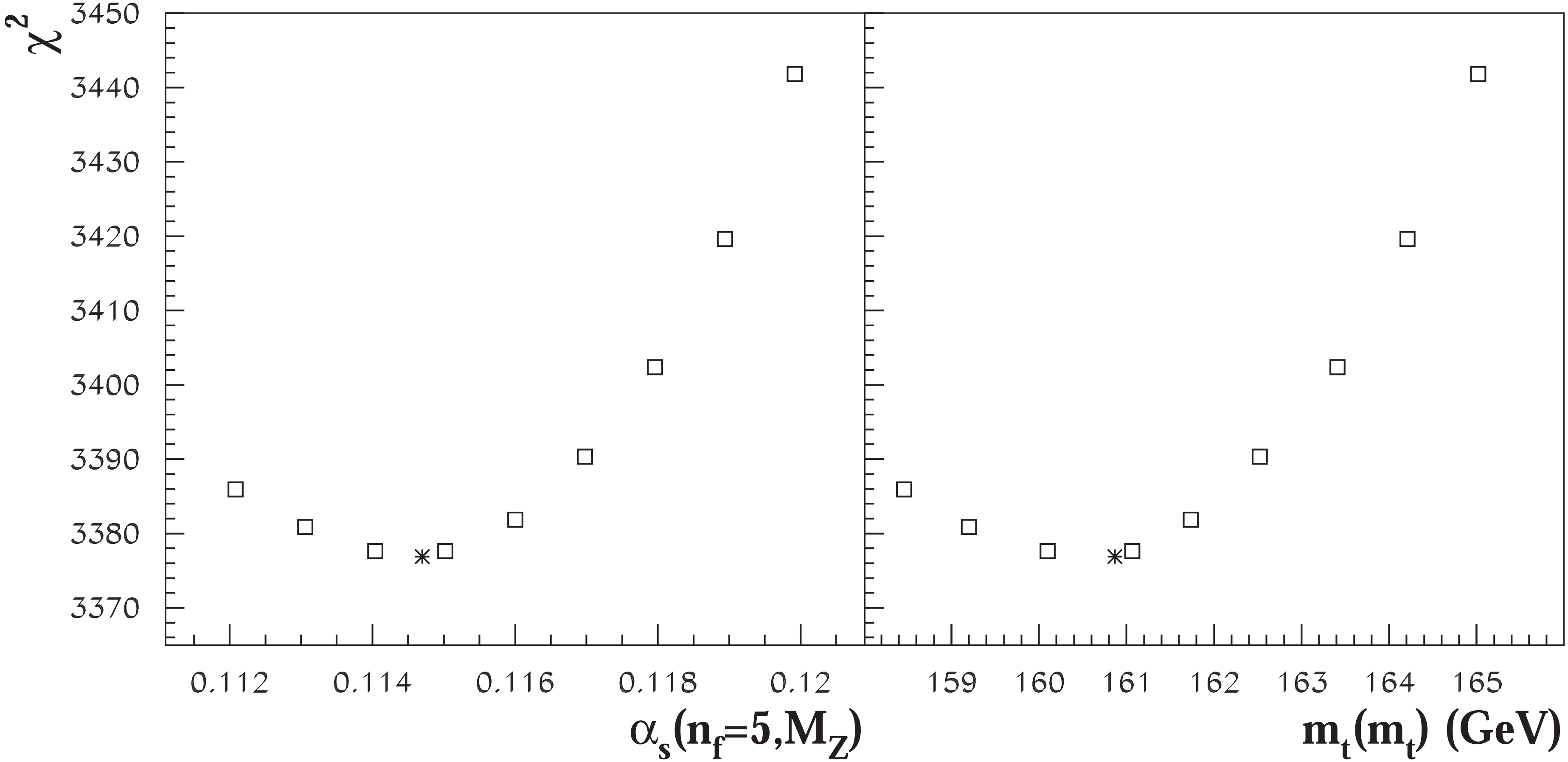
Data on top-quark cross sections

- Pulls for $t\bar{t}$-inclusive cross sections in ABMP16



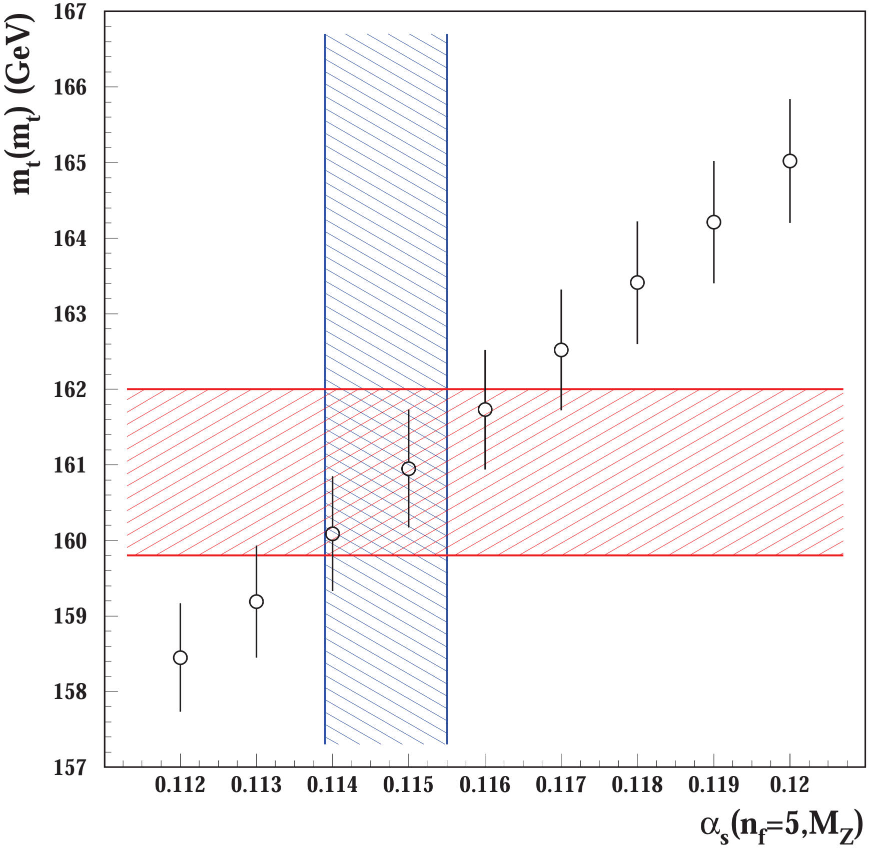
Fit quality

- Goodness-of-fit estimator χ^2 for extracted $\alpha_s(M_Z)$ and $m_t(m_t)$ values
  - χ^2 of global fit with $NDP = 2834$
  - data on top-quark production with $NDP = 36$ D0, ATLAS, CMS, LHCb



Correlations

- Correlations between gluon PDF $g(x)$, $\alpha_s(M_Z)$ and $m_t(m_t)$



- Fits with fixed values of m_t and $\alpha_S(M_Z)$ carry significant bias

Data on top-quark cross sections (I)

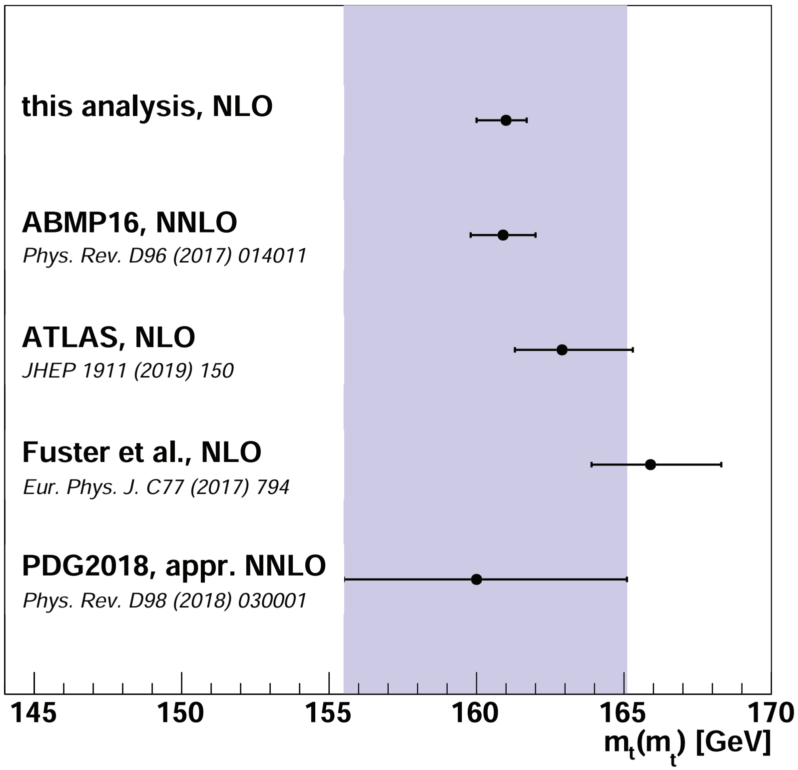
- Correlated determination of PDFs , $\alpha_s(M_Z)$ and $m_t(m_t)$ using HERA DIS data and $t\bar{t}$ cross sections by CMS collaboration
  Garzelli, Kemmler, S. M., Zenaiev '20
  - ansatz from HERAPDF for PDFs

Settings	Fit results
pole mass	$\chi^2/\mathrm{dof} = 1364/1151,\ \chi^2_{t\bar{t}}/\mathrm{dof} = 20/23$
$\mu_R = \mu_F = H'$	$m_t^{\mathrm{pole}} = 170.5 \pm 0.7(\mathrm{fit}) \pm 0.1(\mathrm{mod})^{+0.0}_{-0.1}(\mathrm{par}) \pm 0.3(\mu)$ GeV
CMS, arXiv:1904.05237	$\alpha_S(M_Z) = 0.1135 \pm 0.0016(\mathrm{fit})^{+0.0002}_{-0.0004}(\mathrm{mod})^{+0.0008}_{-0.0001}(\mathrm{par})^{+0.0011}_{-0.0005}(\mu)$
pole mass	$\chi^2/\mathrm{dof} = 1363/1151,\ \chi^2_{t\bar{t}}/\mathrm{dof} = 19/23$
$\mu_R = \mu_F = m_t^{\mathrm{pole}}$	$m_t^{\mathrm{pole}} = 169.9 \pm 0.7(\mathrm{fit}) \pm 0.1(\mathrm{mod})^{+0.0}_{-0.0}(\mathrm{par})^{+0.3}_{-0.9}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1132 \pm 0.0016(\mathrm{fit})^{+0.0003}_{-0.0004}(\mathrm{mod})^{+0.0003}_{-0.0000}(\mathrm{par})^{+0.0016}_{-0.0008}(\mu)$
$\overline{\mathrm{MS}}$ mass	$\chi^2/\mathrm{dof} = 1363/1151,\ \chi^2_{t\bar{t}}/\mathrm{dof} = 19/23$
$\mu_R = \mu_F = m_t(m_t)$	$m_t(m_t) = 161.0 \pm 0.6(\mathrm{fit}) \pm 0.1(\mathrm{mod})^{+0.0}_{-0.0}(\mathrm{par})^{+0.4}_{-0.8}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1136 \pm 0.0016(\mathrm{fit})^{+0.0002}_{-0.0005}(\mathrm{mod})^{+0.0002}_{-0.0001}(\mathrm{par})^{+0.0015}_{-0.0009}(\mu)$
MSR mass, $R = 3$ GeV	$\chi^2/\mathrm{dof} = 1363/1151,\ \chi^2_{t\bar{t}}/\mathrm{dof} = 19/23$
$\mu_R = \mu_F = m_t^{\mathrm{MSR}}$	$m_t^{\mathrm{MSR}} = 169.6 \pm 0.7(\mathrm{fit}) \pm 0.1(\mathrm{mod})^{+0.0}_{-0.0}(\mathrm{par})^{+0.3}_{-0.9}(\mu)$ GeV
this work	$\alpha_S(M_Z) = 0.1132 \pm 0.0016(\mathrm{fit})^{+0.0003}_{-0.0004}(\mathrm{mod})^{+0.0002}_{-0.0000}(\mathrm{par})^{+0.0016}_{-0.0008}(\mu)$

Table 1. The values for $\alpha_S(M_Z)$ and the top-quark mass in different mass schemes obtained in CMS, arXiv:1904.05237 and in this work by fitting the CMS data on $t\bar{t}$ production and the HERA DIS data arXiv:1506.06042 to theoretical predictions. The fit, model (mod), parametrisation (par) and scale variation (μ) uncertainties are reported. Also the values of χ^2 are reported, as well as the partial χ^2 values per number of degrees of freedom (dof) for the $t\bar{t}$ data ($\chi^2_{t\bar{t}}$) for 23 $t\bar{t}$ cross-section bins in the fit. The scale H' is defined in the text.

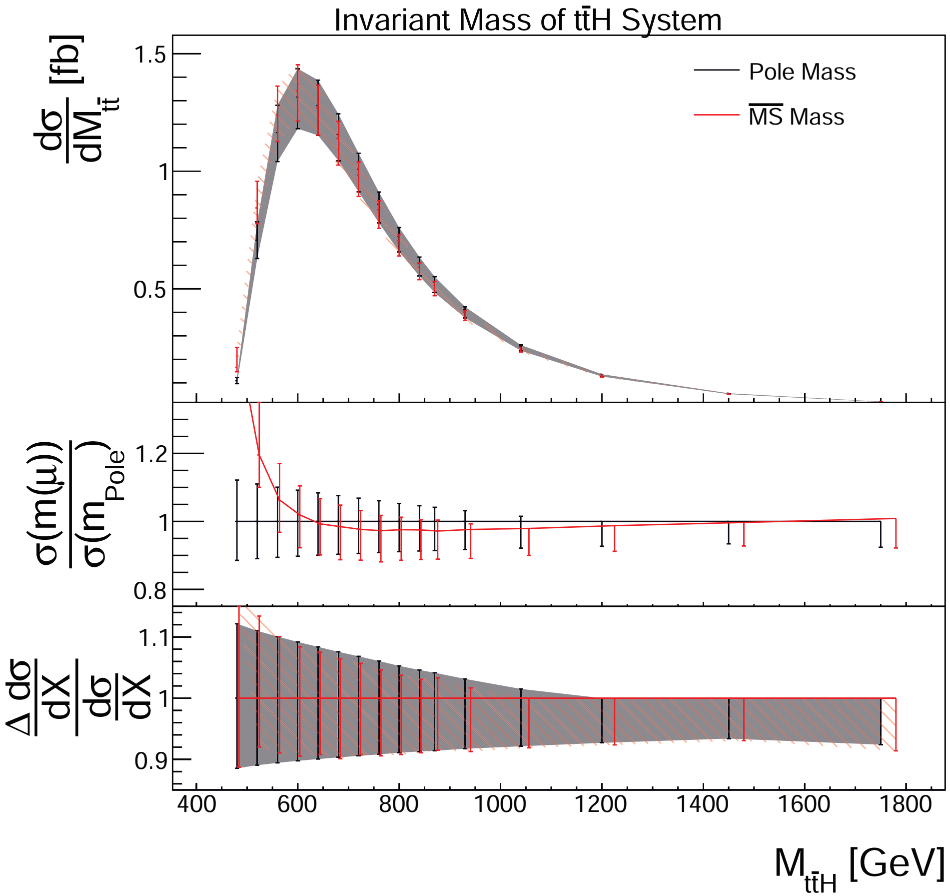
Data on top-quark cross sections (II)

- Extraction of $m_t(m_t)$ at NLO from differential $t\bar{t}$ cross-sections using data of CMS collaboration
  Garzelli, Kemmler, S. M., Zenaiev '20
  - value of $m_t(m_t)$ compared to other determinations
  - world average labelled as PDG2018, appr. NNLO is based on a single determination of D0 collaboration



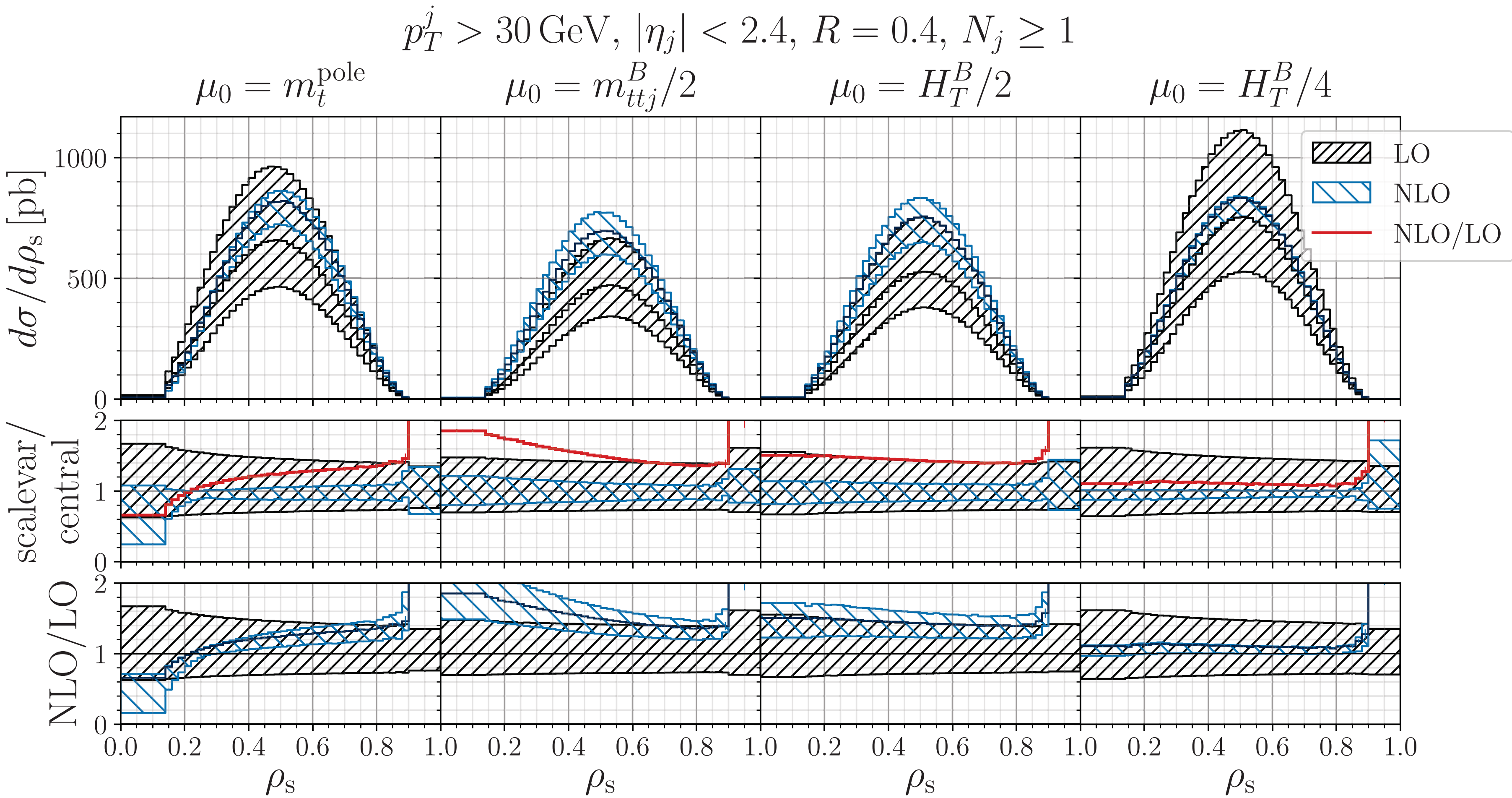
Theory predictions with running mass

- Theory predictions for $t\bar{t}H$ production with top quark $\overline{MS}$ mass at NLO
  Saibel, S. M., Aldaya Martin '21
  - shape of differential distributions largely unaffected



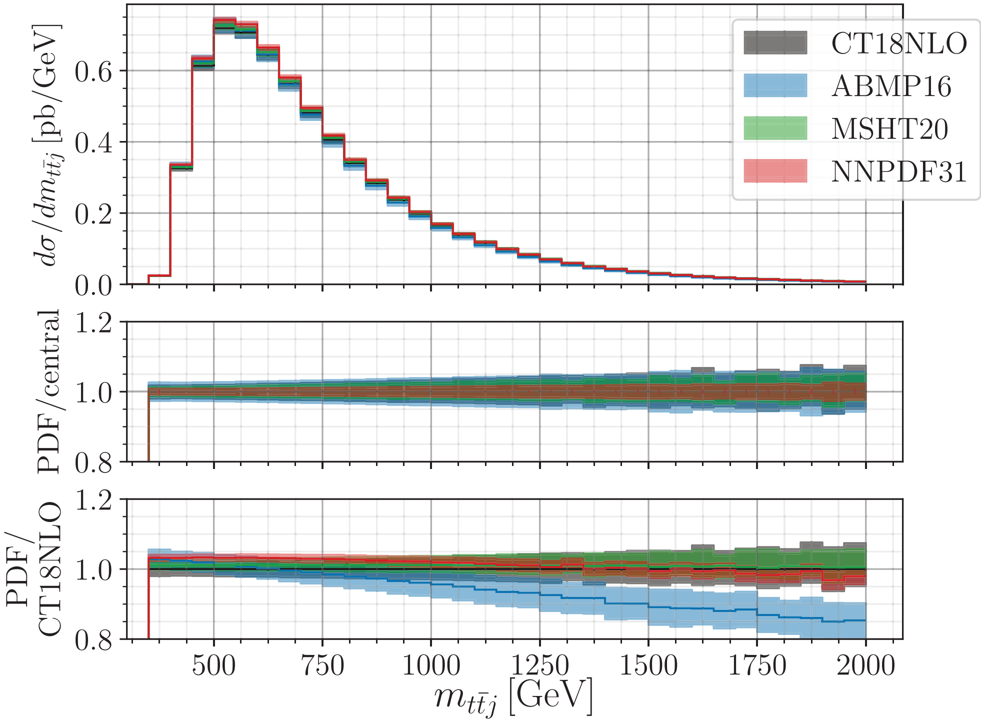
Phenomenology of $t\bar{t}j$ production at LHC

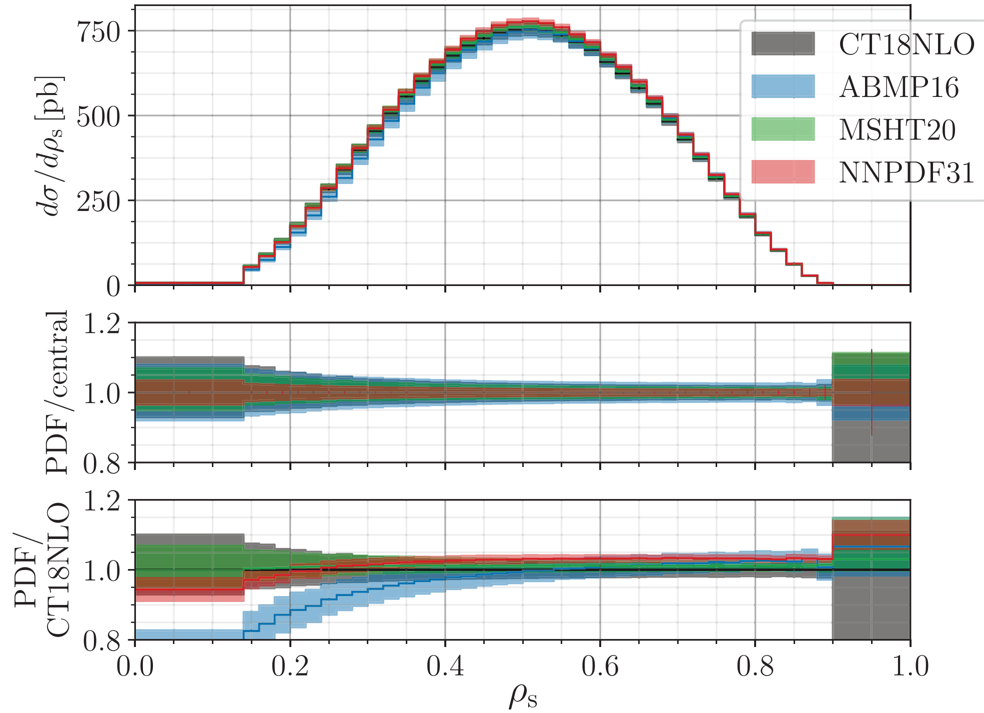
- Theory predictions for $t\bar{t}j$ production with different scale choices
  Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, S. M., Uwer, Voß '22
  - dynamical scale with better apparent perturbative convergence



PDF dependence of $t\bar{t}j$ production

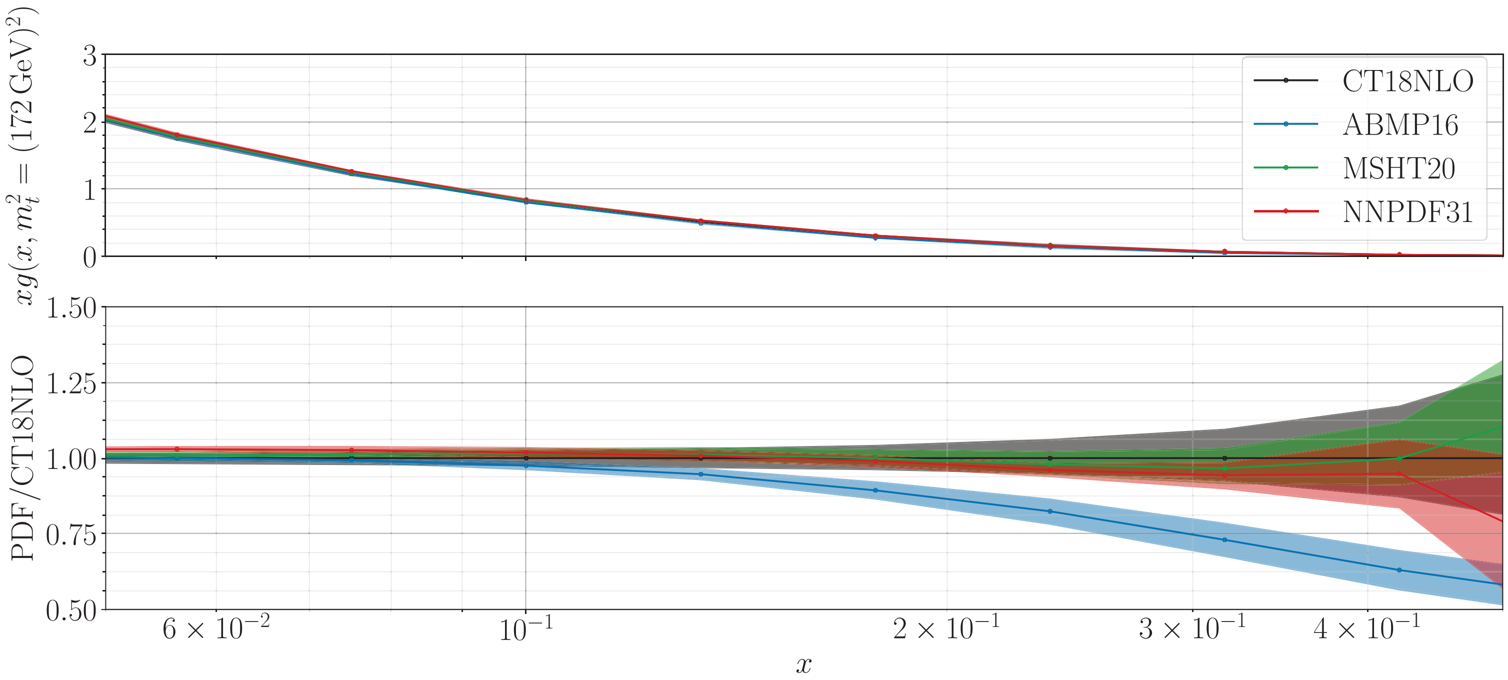
- Predictions for $m_{t\bar{t}j}$ (left) and ρ_s (right) distributions at LO computation with $\mu_0 = H_T^B/4$
  Alioli, Fuster, Garzelli, Gavardi, Irles, Melini, S. M., Uwer, Voß '22
  - PDF uncertainties of ABMP16, CT18, MSHT20 and NNPDF3.1 NLO sets





Gluon PDF

- PDFs sets ABMP16, CT18, MSHT20 and NNPDF3.1 at NLO for $Q^2 = m_t^2 = (172\,\text{GeV})^2$
  - effective parton $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 5 \cdot 10^{-2} \ldots 10^{-1}$ for $m_{t\bar{t}j}$



Progress in theory

Challenge

- Improve theory predictions and reduce theoretical uncertainty
  - hard scattering cross section $\hat{\sigma}_{ij \to X}$

Beyond NLO

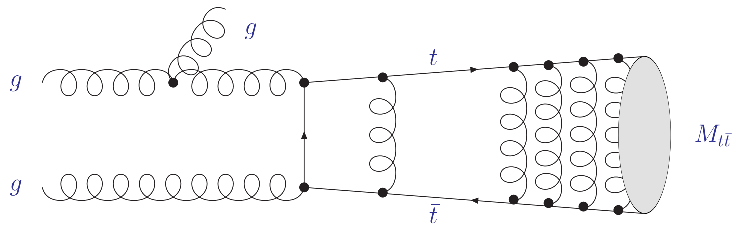
- Scale uncertainties dominate theoretical uncertainties; need NNLO computations (very difficult)
- Focus on kinematical limits
  - soft and collinear kinematics
  - high energy (boosted) regime
  - Coulomb corrections

Threshold logarithms

- Sudakov logarithms in velocity $\beta_{t\bar{t}} = \sqrt{1 - 4m^2/s}$ of heavy quarks or $\beta_{t\bar{t}j} = \sqrt{1 - m^2_{t\bar{t}j}/s}$ for $t\bar{t}j$-system
  - all order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$ in Mellin space (renormalization group equation) Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; ...

Coulomb corrections

- Heavy quark production very close to threshold
  - resummation of Coulomb corrections $\sim 1/\beta$ to all orders
  - factorization in non-relativistic QCD Bodwin, Braaten, Lepage '95
- **ILC**: much work (theory and phenomenology)
  - fixed center-of-mass energy S allows threshold scan at $\sqrt{S} \sim 2m_t$
  - dominant color-singlet production $\rightarrow t\bar{t}\left({}^3S_1^{[1]}\right)$
- **LHC**: effects on top-mass measurement Hagiwara, Sumino, Yokoya '08
  - complete NLO NRQCD result Petrelli, Cacciari, Greco, Maltoni, Mangano '97 (corrections by Hagiwara, Sumino, Yokoya '08)
  - NLL resummation Cacciari '99
  - detailed study in NRQCD assembling existing knowledge at NLO/NLL Kiyo, Kühn, S.M., Steinhauser, Uwer '08
  - Recent phenomenological update Wan-Li Ju, Guoxing Wang, Xing Wang, Xiaofeng Xu, Yongqi Xu,Li Lin Yang '20

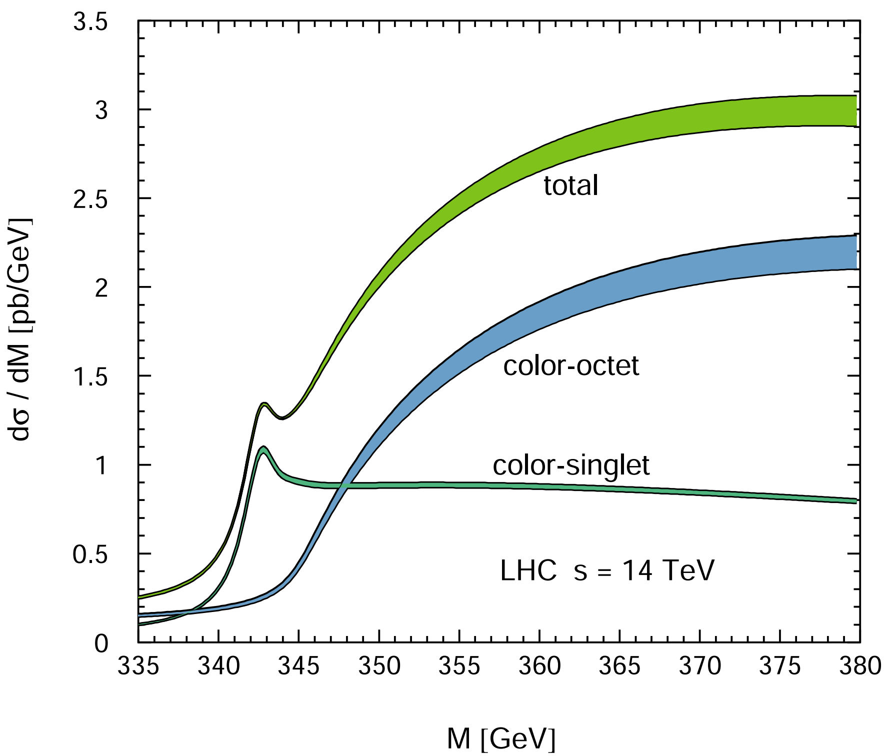


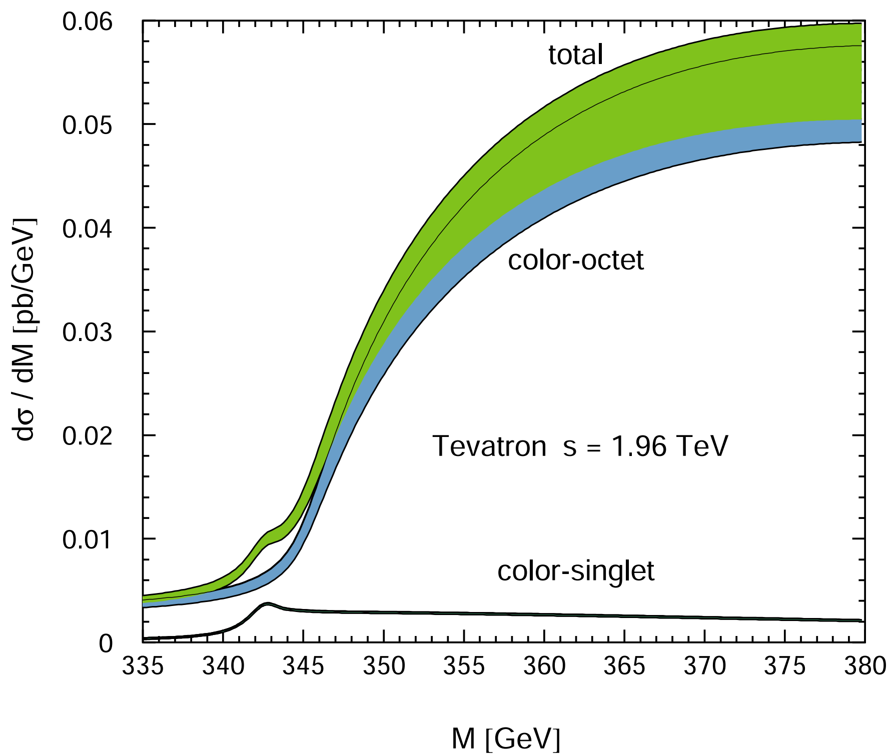
Coulomb corrections

- Recall master equation $$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$
- Convolution with PDFs $f_i \otimes f_j$
  - top-quark pairs produced as color-singlets and color-octets $\to t\bar{t}\left({}^{2s+1}S_J^{[1,8]}\right)$
  - threshold at $M_{t\bar{t}} \sim 2m_t$ with $M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$
- NRQCD factorization of partonic cross section into $\hat{\sigma}_{ij\to t\bar{t}} = F_{ij\to T} \otimes G(M_{t\bar{t}})$
  - free $t\bar{t}$ production rate F
  - evolution factor into “boundstate” (Green’s function) G
- Differential kinematics $$\frac{d\hat{\sigma}_{ij\to t\bar{t}}}{dM_{t\bar{t}}^2} = F_{ij\to T} \times \Im G^{[1,8]}(M_{t\bar{t}})$$
  - factorization of soft-collinear dynamics (real emission radiation)
  - matching at NLO and NLL resummation
- Effective theory formulation Beneke, Falgari, Schwinn ‘09; Beneke, Kiyo, Schuller ‘13

Invariant mass distristribution

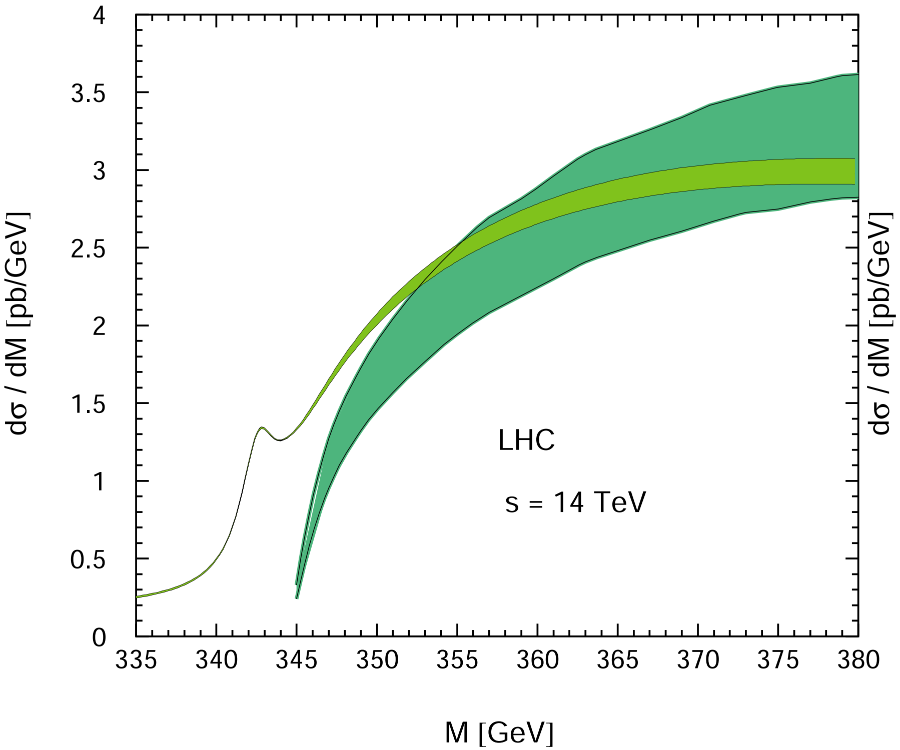
- $d\sigma/dM_{t\bar{t}}$ at LHC driven by large gluon luminosity
  - $gg \to t\bar{t}\left({}^1S_0^{[1]}\right)$ dominates
- $d\sigma/dM_{t\bar{t}}$ at Tevatron with small bound state effects
  - $q\bar{q}$-channel large with only color-octet configurations only
- Validity of Coulomb resummation restricted to $dM_{t\bar{t}} \geq 335$ GeV

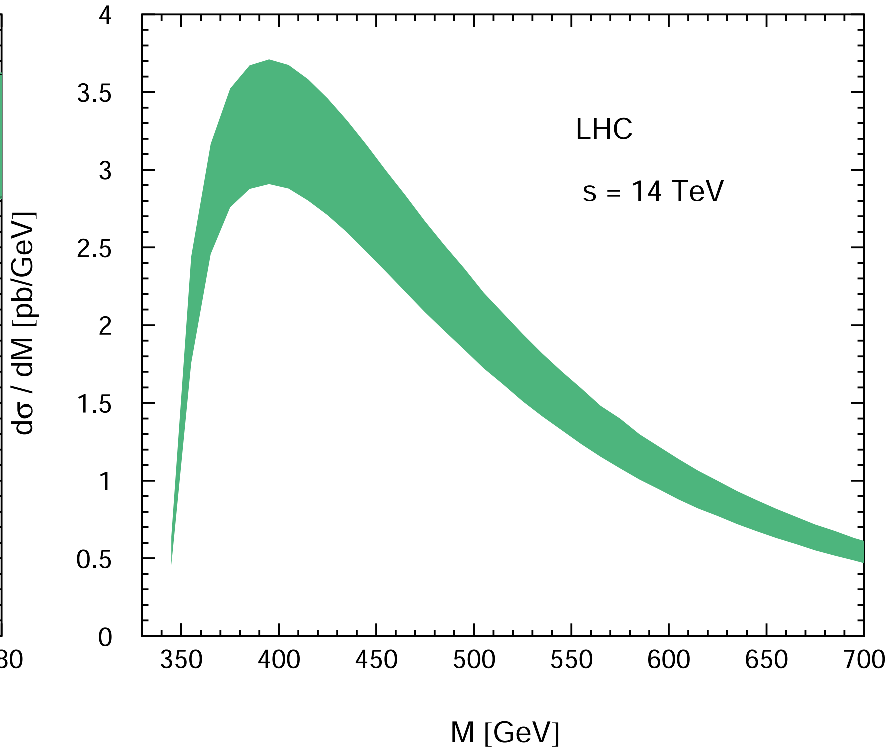




Matching to fixed order

- $d\sigma/dM_{t\bar{t}}$ with at LHC
  - compare NLL resummed result in NRQCD with (plain vanilla) NLO (use HVQMNR Mangano, Nason, Ridolfi '92 for matching)
  - consistency check OK
- Resolution of bound state effects in $d\sigma/dM_{t\bar{t}}$ at LHC difficult (requires rather fine binning)



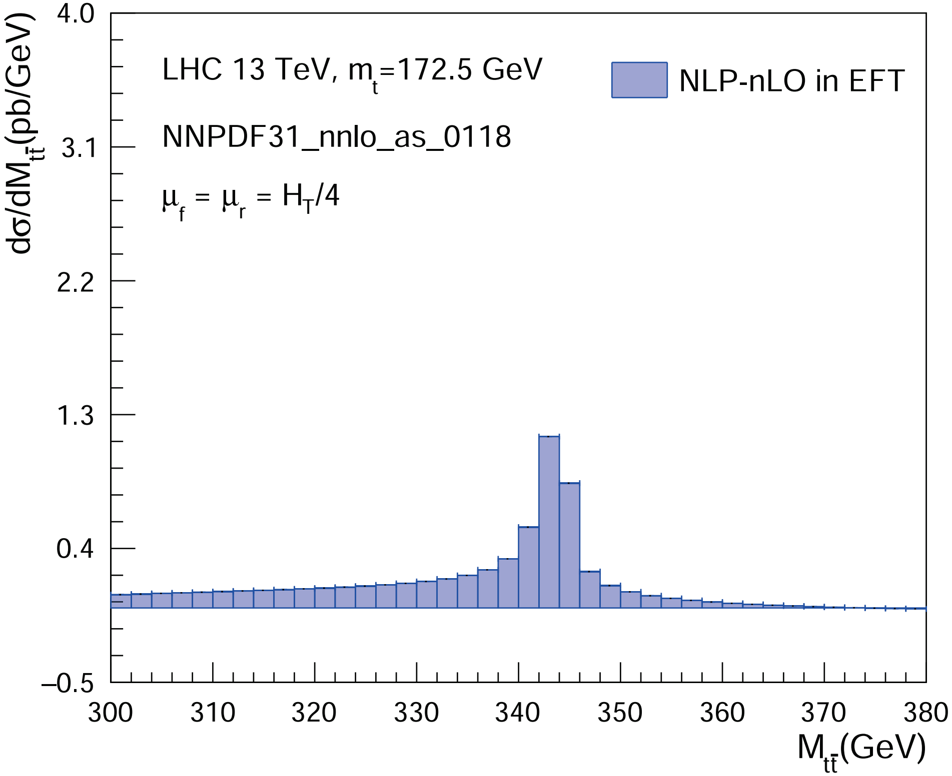


Comment on 2004.03088

- Resummation effects in top quark mass determination with claimed shift of $m_t \sim 1.4$ GeV

  Wan-Li Ju, Guoxing Wang, Xing Wang, Xiaofeng Xu, Yongqi Xu,Li Lin Yang '20

- Contributions from resummation in region 300 GeV $\leq M_{t\bar{t}} \leq 380$ GeV at the 13 TeV LHC



Summary

- Top mass extractions need to keep correlations with $\alpha_s(M_Z)$ and with PDFs
  - fixing of gluon PDF $g(x)$ and $\alpha_s(M_Z)$ may lead to bias
- Use of different mass schemes: $\overline{MS}$, MSR and on-shell schemes
  - preference for short distance mass schemes $\overline{MS}$, MSR
- Push QCD perturbation theory to NNLO for $t\bar{t}H$ or $t\bar{t}j + X$ production
  - generally very difficult: $2 \to 3$ processes with masses are beyond current state-of-the-art
  - progress in controlled kinematic limits (threshold, high-energy, . . .) feasible
- Coulomb corrections and their resummation
  - update of resummation studies at NLL for upcoming analyses