Running of the top quark mass at NNLO in QCD

DESY Top Mass mini-workshop - 28.06.2022

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The most famous “running”: $\alpha_S(Q)$

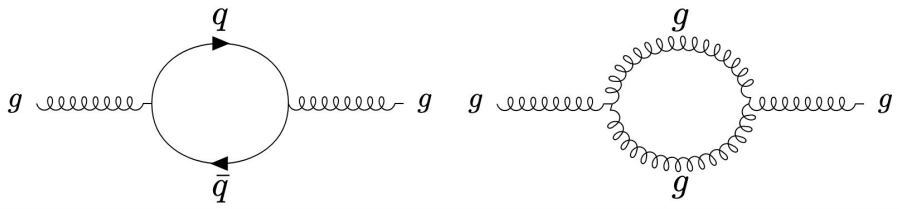
Self-energy corrections to gluon propagators lead to running of α_S

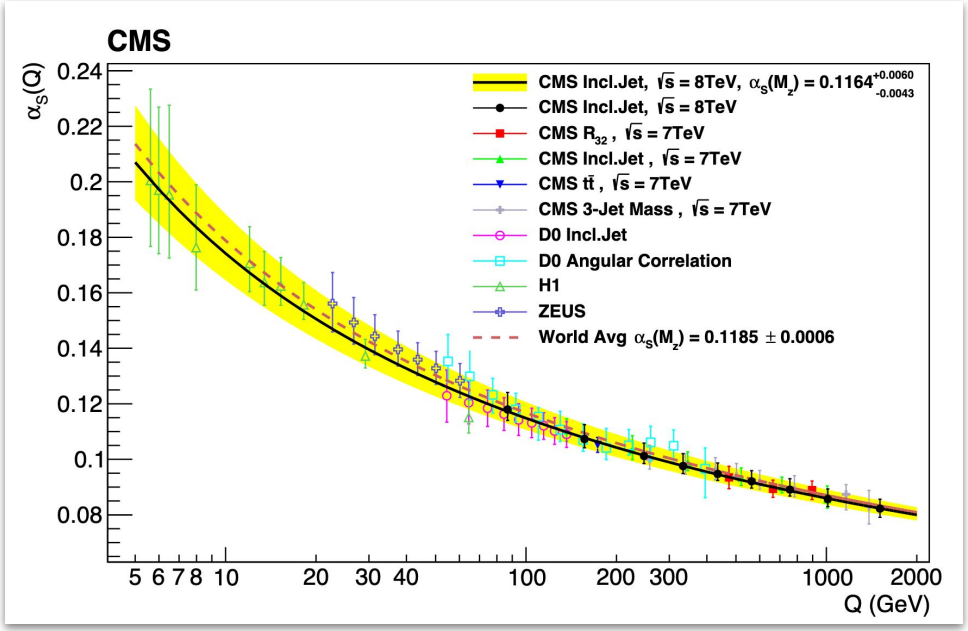
- described by renormalisation group equations (RGE)
- Tested experimentally by measuring $\alpha_S(Q)$ as a function of energy scale Q

@1 loop

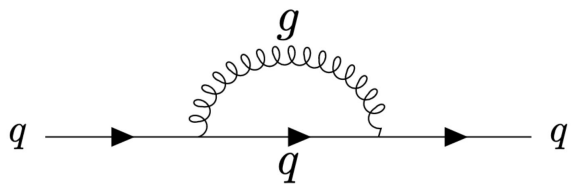
$$\alpha_S(\mu^2) = \frac{\alpha_S(\mu_0^2)}{1 + \beta_0 \alpha_S(\mu_0^2) \ln(\mu^2/\mu_0^2)}$$

Can be modified by BSM physics at high scales





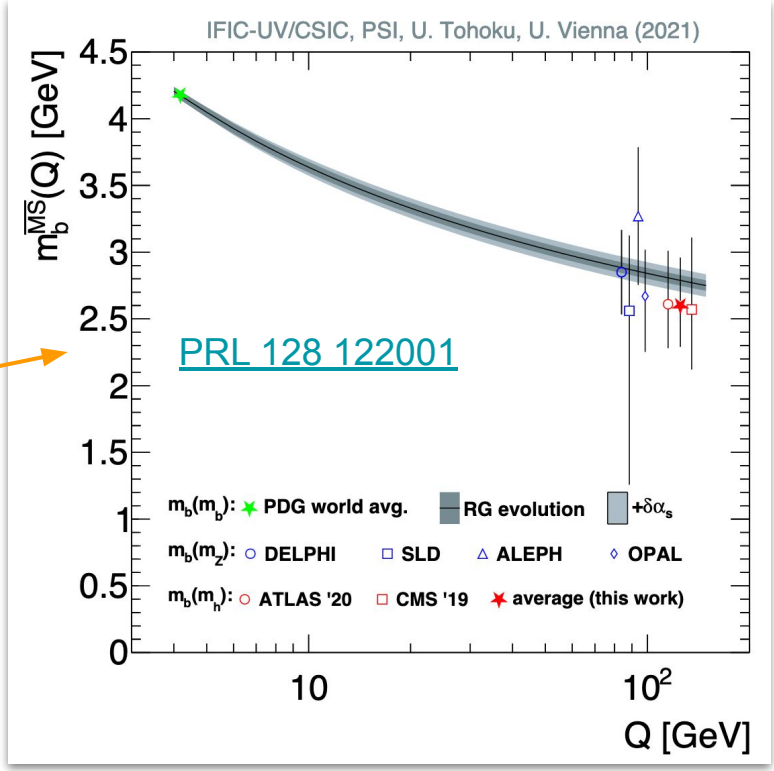
Running of the quark masses



Similarly, in the MS renormalisation scheme, the values of the quark masses depend on an additional scale μ_m

$$m(\mu) = m(\mu_0)\left[1 - c_0\alpha_S(\mu)\ln\left(\frac{\mu^2}{\mu_0^2}\right)\right]$$ @1 loop

- Running of m_c studied at HERA
- Running of m_b recently studied up to the m_H scale for the first time
- Running of m_t investigated by CMS for the first time in 2019 (at NLO in QCD)



How to extract m_t in the MS scheme

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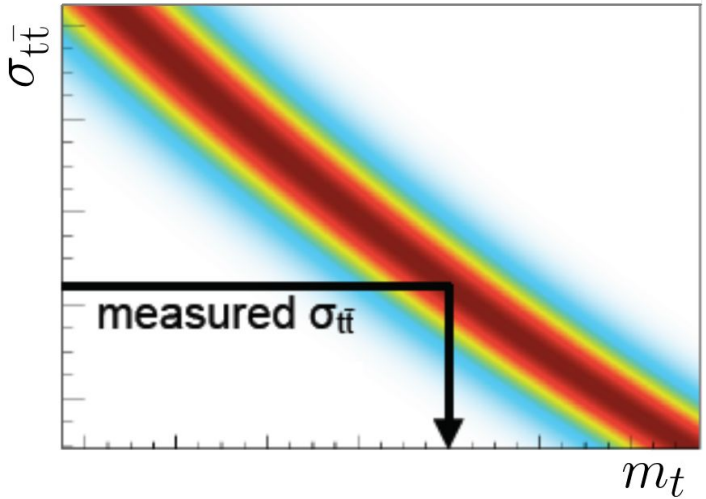
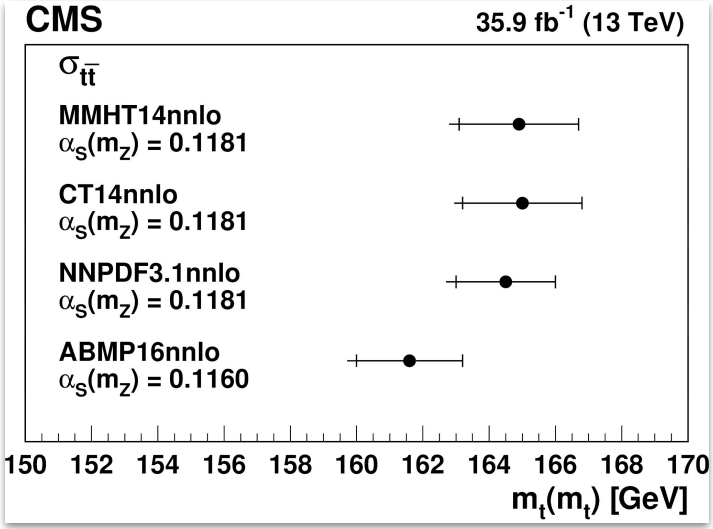




- Compare measurement of inclusive σ_{tt} to theoretical prediction in the MS scheme
  - σ_{tt} measured by likelihood fit to multi-differential distributions
  - Dependence of σ_{tt} on m_t^{MC} mitigated in the fit (J. Kieseler et. al. PRL 116 (2016) 162001)

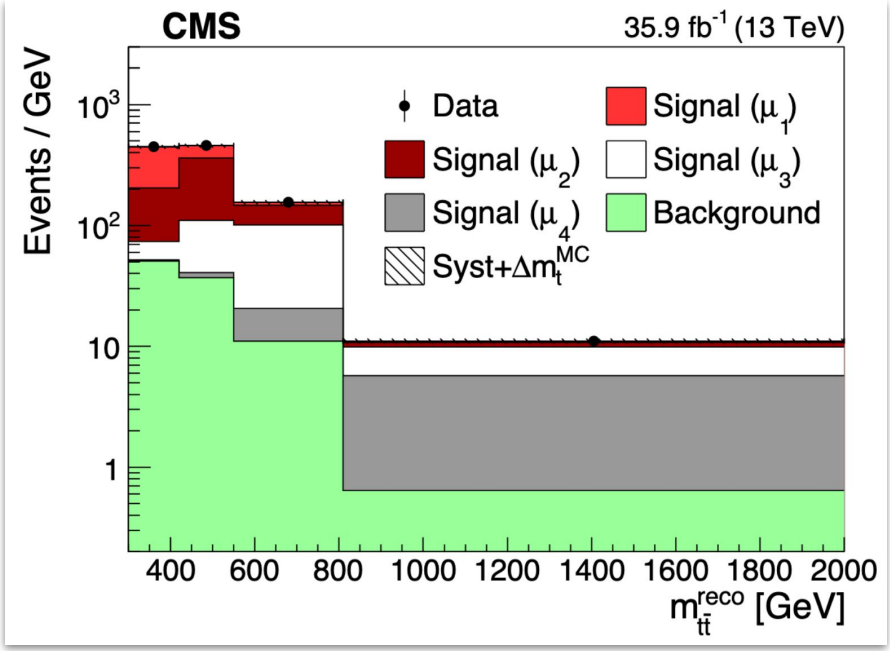
Running of m_t can be obtained by extending this method to a differential measurement

The CMS analysis at NLO in QCD

- Measure $m_t(\mu_m)$ as a function of $\mu_m = m_{tt}$ using a differential measurement of the tt production cross section

- Cross section measured by means of maximum-likelihood unfolding to multi-differential distributions
  - Reduce the impact of systematic uncertainties
  - Simultaneous fit of signal and background contributions

PLB 803 (2020) 135263



Extraction of the running of m_t @ NLO

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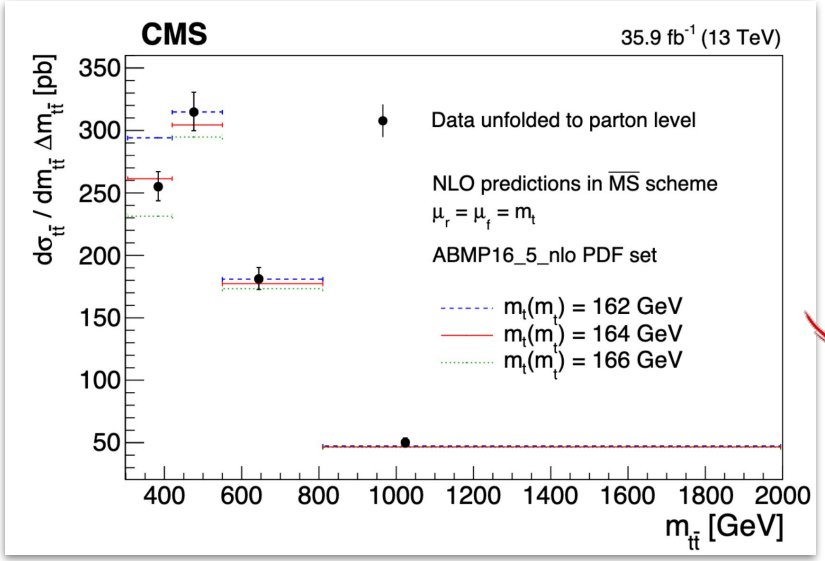
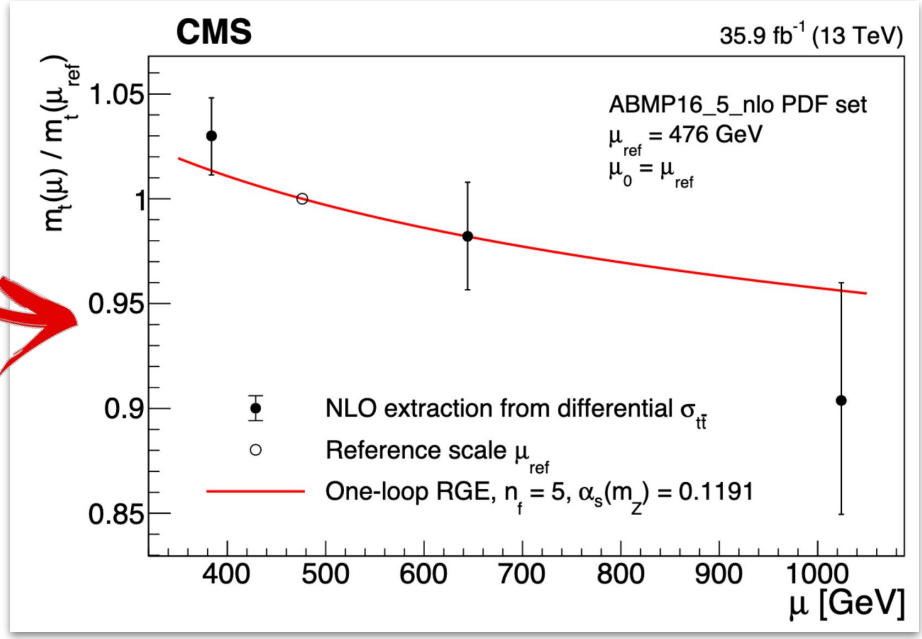




Result compared to theoretical predictions in the MS scheme at NLO (MCFM) with fixed QCD scales ($\mu_r = \mu_f = \mu_m = m_t$)

-> $m_t(m_t)$ converted to $m_t(\mu)$ after extraction

Good agreement with QCD running at one loop, within uncertainties

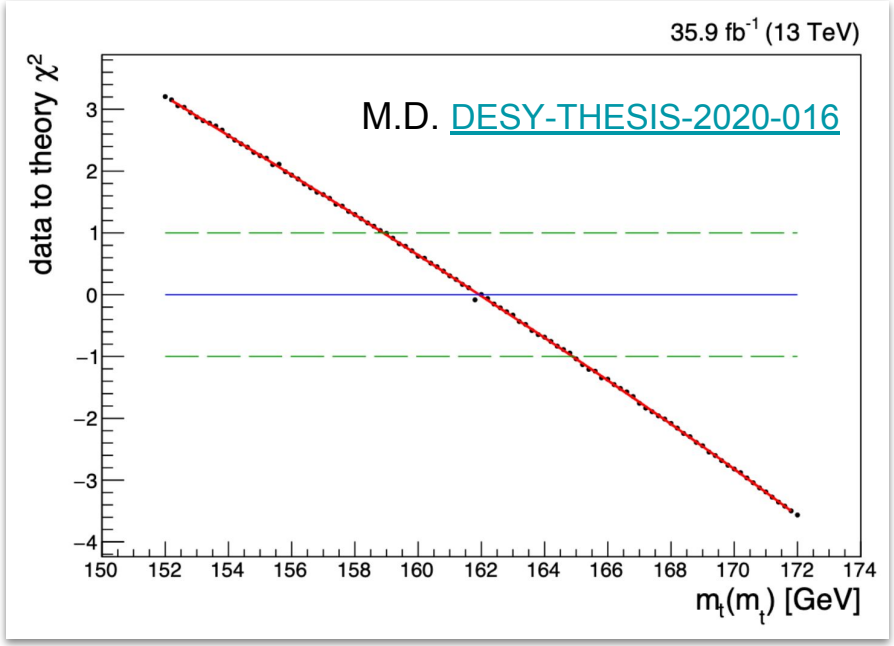
Details of the NLO fit

□^{2} fit of theoretical prediction as a function of m_t in each bin of m_{tt} separately

$$\sqrt{\chi_k^2}(m_t) = \frac{\delta_k}{\Delta\sigma_k}\sqrt{1 - 2A_k\frac{\delta_k}{\Delta\sigma_k} + 5A_k^2\left(\frac{\delta_k}{\Delta\sigma_k}\right)^2}.$$

Relatively short computing time:

- Large number of mass points can be calculated
- Numerical uncertainty of the calculation can be made negligible
- Calculation can be repeated using different PDF eigenvectors for all the mass points



Effect of PDF uncertainties estimated by repeating the □^{2} fit (*externalised*)

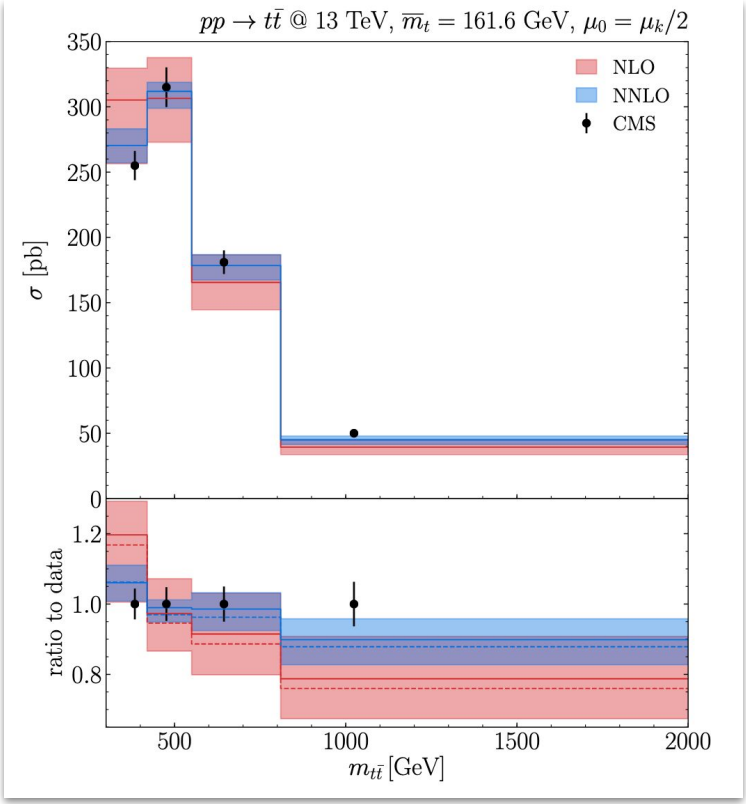
Matrix NNLO prediction in MS scheme

S. Catani et. al. JHEP 08 (2020) 027

- First differential prediction of this kind, implemented in *Matrix*
- Significant reduction of QCD scale uncertainties
- Possibility to set scale dynamically bin-by-bin -> extract directly $m_t(\mu_m)$ (instead of $m_t(m_t)$ ->$m_t(\mu_m)$ conversion)

Also, it is argued that a better choice for the dynamic scale is $\mu_m = m_{tt}/2$ (instead of m_{tt}), since $m_{tt}/2$->m_t near the production threshold

-> **first step**: repeat CMS analysis at NLO with dynamic scale and $\mu_m = m_{tt}/2$ (which was not possible at the time of analysis)

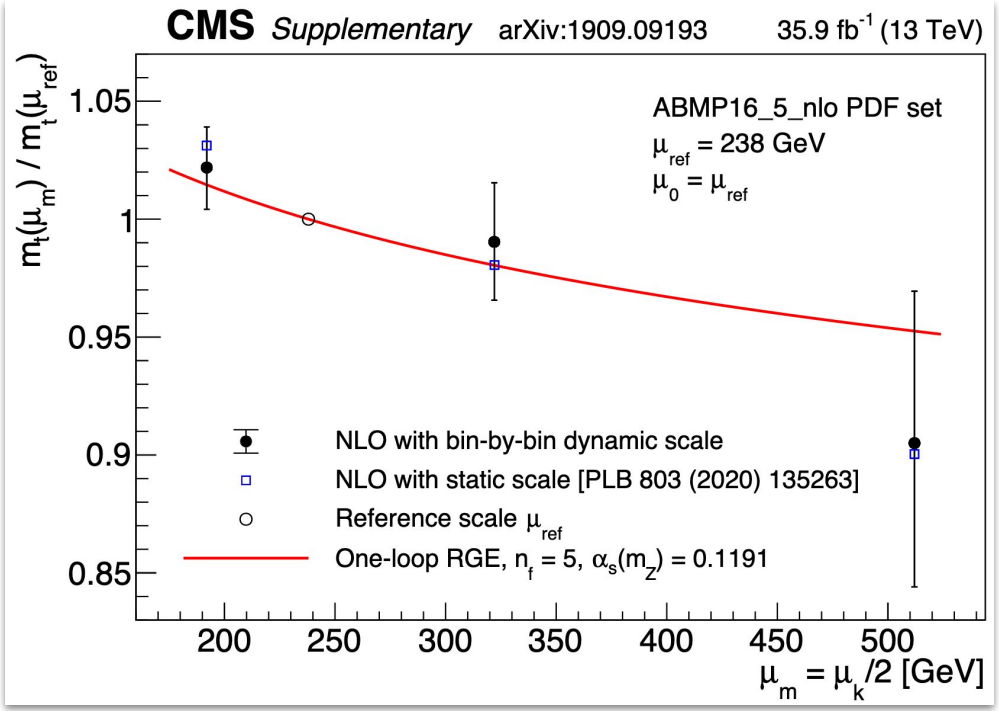


m_t running @NLO with bin-by-bin dynamic scales

- Version of MCFM with bin-by-bin dynamic scale (as in *Matrix*)
- Results well compatible within systematic uncertainties
- Overall conclusions of the analysis are not changed

Optimal result can be achieved by:

- Making use of the new NNLO theoretical prediction in *Matrix*
- Using improved estimate of CMS integrated luminosity (2.5% -> 1.2%)

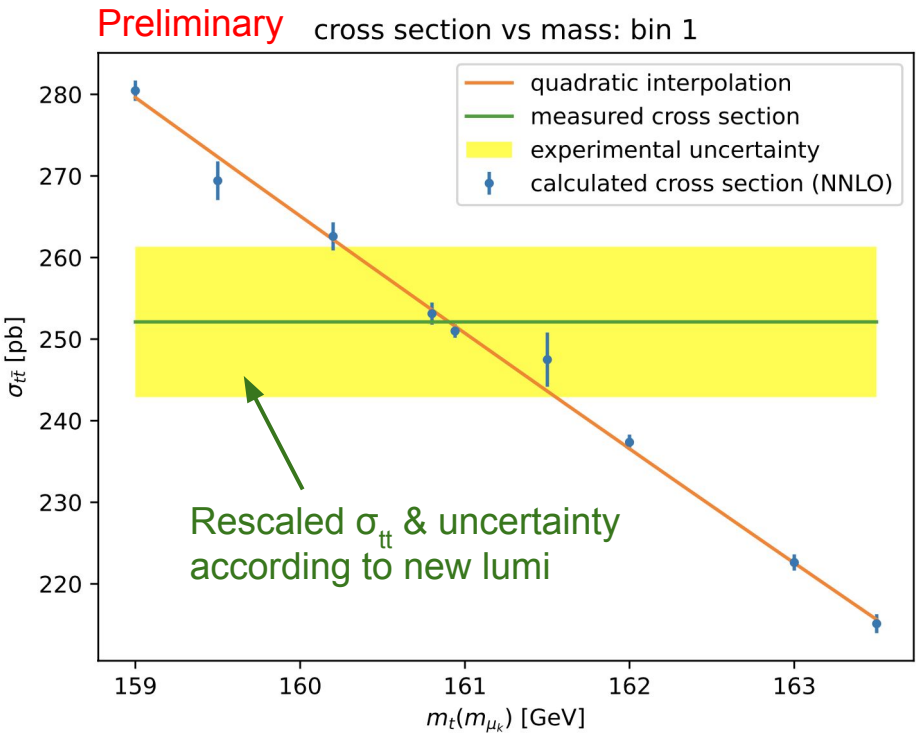


Theoretical inputs to the NNLO fit

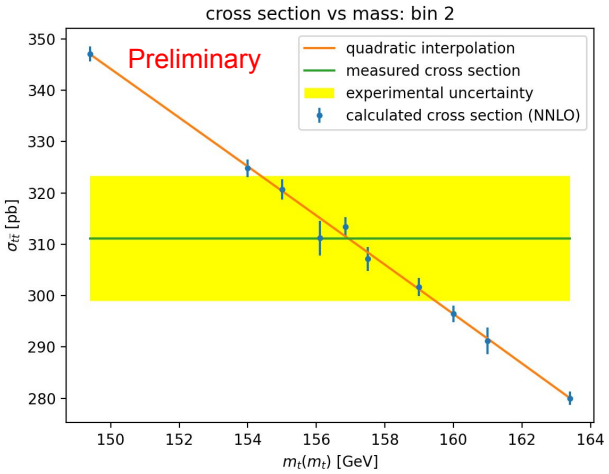
Theoretical prediction at NNLO in MS scheme obtained with ***Matrix*** (using ABMP16 NNLO PDF set)

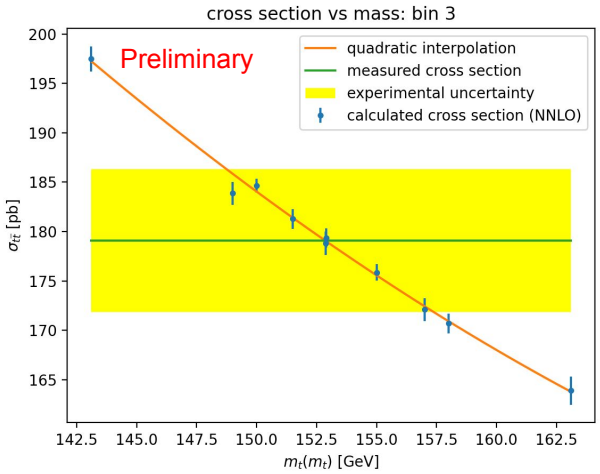
Much more computationally expensive than NLO -> not possible to reach the same level of numerical precision due to resource limitations

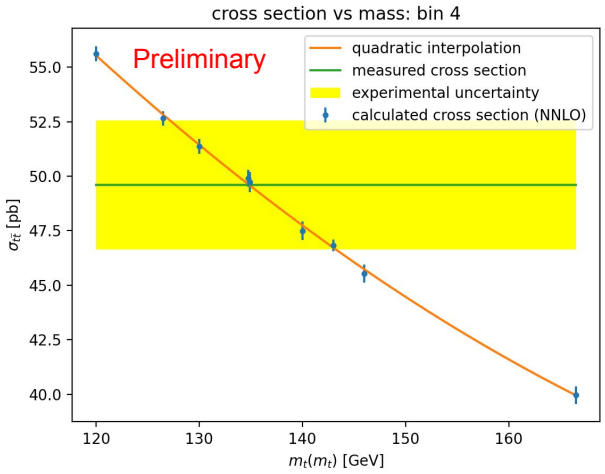
- Theoretical dependence of σ_{tt} on m_t modelled with quadratic function
- Effectively smooths the numerical uncertainty (-> mitigates impact)



Inputs to the NNLO fit & PDF uncertainties







Due to resource constraints, not possible to re-derive the full dependence on m_t for each PDF eigenvectors (as in NLO analysis) -> approximations

- Effect of PDF variations estimated on a single mass point, and assumed to be constant in relative terms -> larger impact of numerical uncertainty (no smoothing!)
- Relative variation obtained using calculation in the on-shell scheme (TBU)

Improved $m_t(\mu)$ fit for NNLO analysis

Goal: consistently take into account effect of numerical uncertainties, especially those related to PDF variations (cannot be smoothed)

$$\chi^2(\vec{m},\vec{j},\vec{\eta}) = [\vec{\sigma}_{\rm exp} - \vec{\sigma}_{\rm th}(\vec{m},\vec{j},\vec{\eta})]^T C_{\rm exp}^{-1}[\vec{\sigma}_{\rm exp} - \vec{\sigma}_{\rm th}(\vec{m},\vec{j},\vec{\eta})] + \sum_{i=0}^{\rm nPDF} j_i^2 + \sum_{i=0}^{\rm nPred} \eta_i^2$$

PDFs

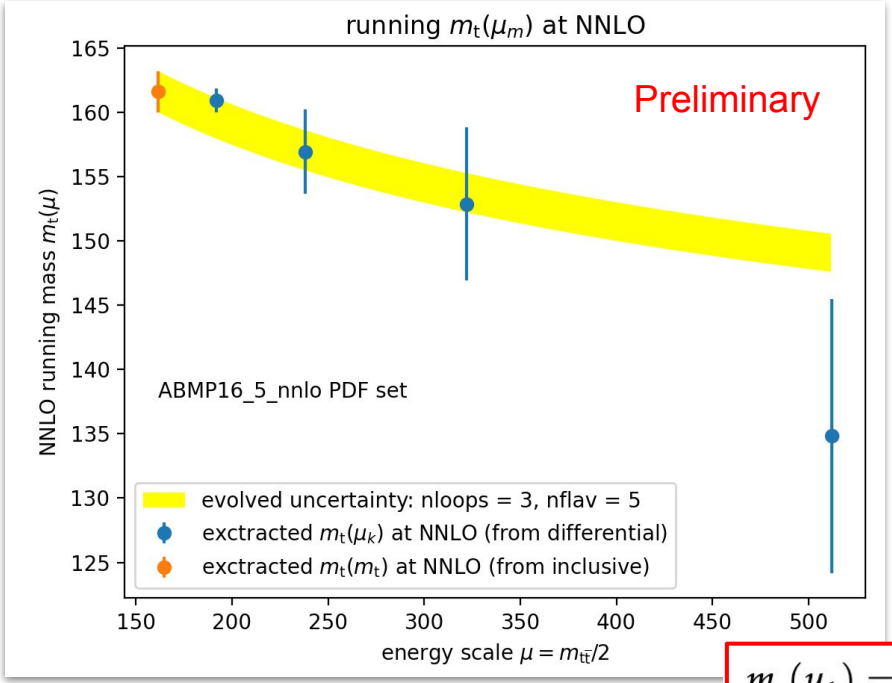
numerical

priors

For each set of values j_i and η_i, the function $\sigma^k_{th}(m_k)$ is obtained by interpolation (as shown in previous slides for $j_i = \eta_i = 0$), where k indicates the bin in m_{tt}

In this way, also the correlations between the numerical uncertainties and the PDF variations are fully taken into account

Extracted $m_t(\mu_m)$ at NNLO



$m_t(\mu_1)$ = 160.9 ± 0.7 (exp) ± 0.7 (PDF+α_S+num) GeV
$m_t(\mu_2)$ = 156.9 ± 2.6 (exp) ± 2.0 (PDF+α_S+num) GeV
$m_t(\mu_3)$ = 152.9 ± 4.5 (exp) ± 3.9 (PDF+α_S+num) GeV
$m_t(\mu_4)$ = 134.8 ± 8.6 (exp) ± 6.4 (PDF+α_S+num) GeV

- Scale uncertainties not yet included
- **Improved experimental precision** (CMS lumi)
- Larger impact from PDF uncertainties
  -> being investigated (conversion to MS?)

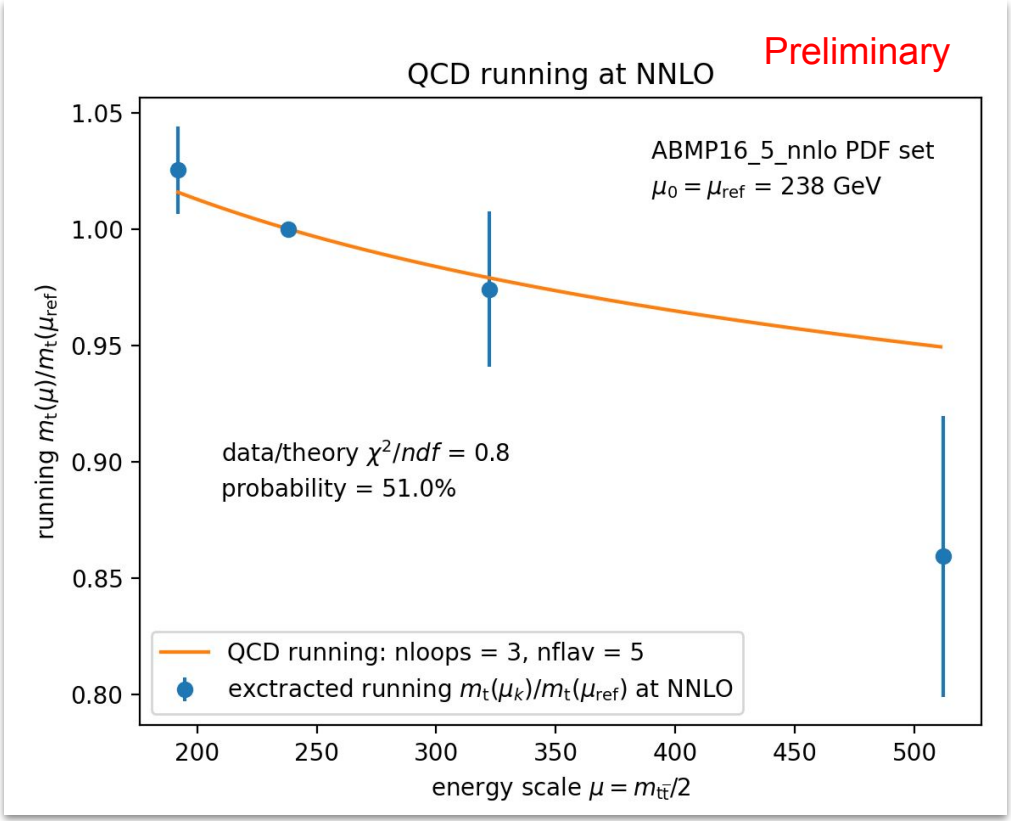
At NNLO: (exp) = (fit+extr)

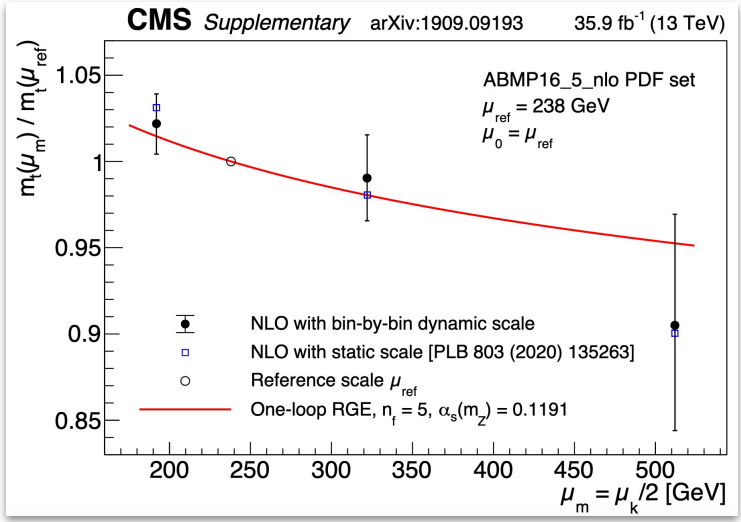
NLO results

$$m_t(\mu_1) = 155.4 \pm 0.8\ (\text{fit}) \pm 0.2\ (\text{PDF}+\alpha_S) \pm 0.1\ (\text{extr})\ ^{+0.9}_{-0.6}\ (\text{scale}),$$
$$m_t(\mu_2) = 150.9 \pm 3.0\ (\text{fit})\ ^{+1.1}_{-0.7}\ (\text{PDF}+\alpha_S)\ ^{+0.4}_{-0.5}\ (\text{extr})\ ^{+3.9}_{-4.3}\ (\text{scale}),$$
$$m_t(\mu_3) = 148.2 \pm 4.6\ (\text{fit})\ ^{+2.0}_{-1.4}\ (\text{PDF}+\alpha_S)\ ^{+0.9}_{-1.0}\ (\text{extr})\ ^{+7.3}_{-9.5}\ (\text{scale}),$$
$$m_t(\mu_4) = 136.4 \pm 9.0\ (\text{fit})\ ^{+3.8}_{-3.0}\ (\text{PDF}+\alpha_S)\ ^{+2.8}_{-2.3}\ (\text{extr})\ ^{+9.6}_{-16.1}\ (\text{scale}).$$

N.B. central values cannot be compared directly (obtained with different scale choices & CMS lumi)

QCD running at 3 loops (NNLO)





Similar trend as in NLO analysis

- Small discrepancy observed at NNLO will likely be covered by scale uncertainties

New physics in the m_t running?

It is possible to investigate scenarios in which lepton masses are generated dynamically (e.g. PRL 94 (2005) 241801)

For μ<<Λ:

$$m_t(\mu) = m_t(m_t)\frac{1-(m_t/\Lambda)^2}{1-(\mu/\Lambda)^2}$$

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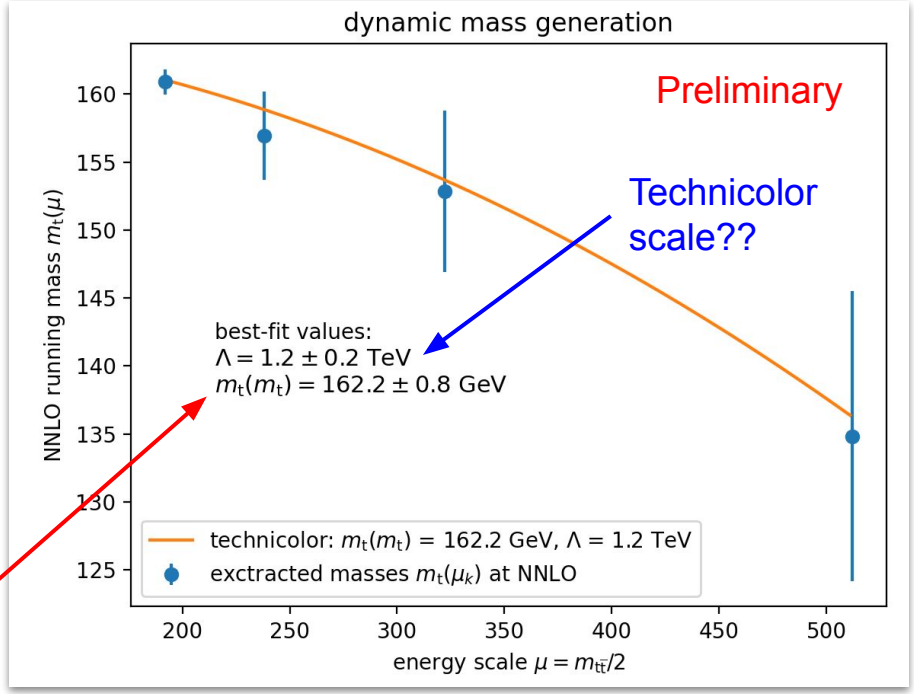
PDF set	$m_t(m_t)$ [GeV]
ABMP16	$161.6 \pm 1.6\,(\text{fit} + \text{PDF} + \alpha_S)\,^{+0.1}_{-1.0}\,(\text{scale})$
NNPDF3.1	$164.5 \pm 1.6\,(\text{fit} + \text{PDF} + \alpha_S)\,^{+0.1}_{-1.0}\,(\text{scale})$
CT14	$165.0 \pm 1.8\,(\text{fit} + \text{PDF} + \alpha_S)\,^{+0.1}_{-1.0}\,(\text{scale})$
MMHT14	$164.9 \pm 1.8\,(\text{fit} + \text{PDF} + \alpha_S)\,^{+0.1}_{-1.1}\,(\text{scale})$



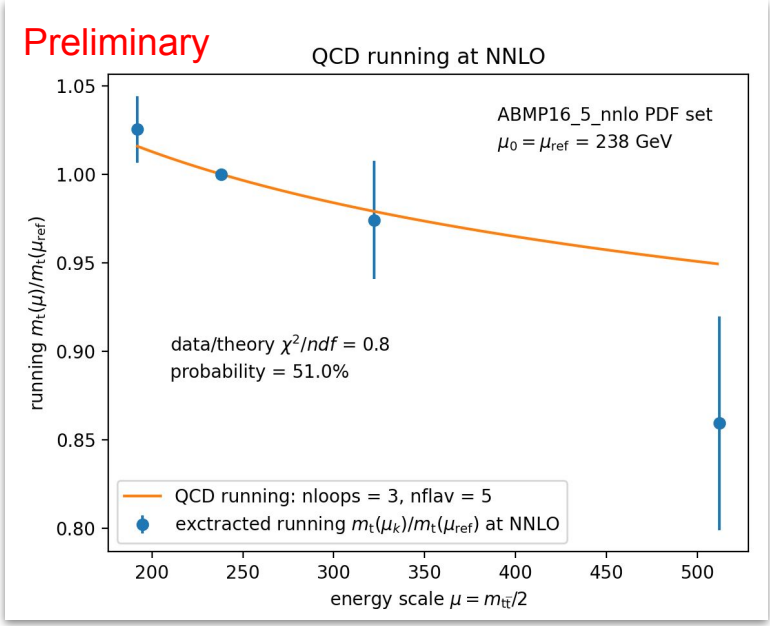
Just a toy example showing how this type of measurements can be used to probe BSM physics at high energy scales

Summary and outlook

We presented preliminary results for the running of m_t at NNLO in QCD for the first time

- *Matrix* NNLO calculation is MS scheme, with bin-by-bin dynamic scale choice
- **Improved method** of extraction of $m_t(\mu_m)$ which takes into account the numerical precision in the calculation

- PDF uncertainties estimated from calculation in the pole scheme -> to be updated
- Uncertainties related to choice of μ_r and μ_f to be included -> big improvement expected wrt NLO



Publication to appear…