Axion Inflation in String Theory

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References

— Inflation and String Theory, Baumann and McAllister (2015)
— String Cosmology – Large Field Inflation in String Theory, Westphal (2015)
— On the Cosmology of String Axions, Righi (2022)
— TASI lectures on Inflation, Baumann (2012)

1. Introduction and motivation

Inflation: – Natural place within EFT, but
– offers a unique window into Planck scale physics.

$\hookrightarrow$ $\boxed{\text{UV sensitivity}}$

Let us recall the setting of our discussions:

Hubble slow-roll parameters

$$\tilde{\epsilon} := -\frac{\dot{H}}{H^2}, \quad \tilde{\eta} := \frac{\dot{\tilde{\epsilon}}}{H\tilde{\epsilon}}$$

Inflation: $\tilde{\epsilon} < 1$, $|\tilde{\eta}| < 1$
quasi-dS

$\xrightarrow[\text{the potential energy of a scalar field}]{\text{inflation driven by}}$

Potential slow-roll parameters

$$\epsilon := \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta := M_{pl}^2 \frac{V''}{V}$$

Prolonged slow-roll inflation:
$\epsilon \ll 1$, $|\eta| \ll 1$.

<u>The η problem</u>: Take as starting point the EFT action

$$S_{eff} = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2} R + \mathcal{L}_l[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i - 4}}\right]$$

light fields: kinetic terms + renormalizable interactions

all higher-dimension operators not forbidden by the symmetries

For the EFT to be valid during the freeze-out of cosmological perturbations:

$$H \lesssim \Lambda \lesssim M_{pl}.$$

In most situations: – Planck suppressed operators make negligible contributions to the EFT as $\Lambda \to M_{pl}$.

– Inflation is sensitive even to Planck-suppressed operators.

The unknown physics that we have integrated out to construct the EFT has two effects:

1) Renormalizes the couplings of the light fields,
2) Introduces new non-renormalizable interactions.

Radiative corrections: $\Delta m^2 \sim \Lambda^2 \xRightarrow[H \lesssim \Lambda]{} \Delta\eta \sim \dfrac{\Lambda^2}{H^2} \gtrsim 1$

SUSY is not enough to prevent it:

At best SUSY is spontaneously broken at the Hubble scale.

$w \gg H$: boson-fermion loop cancellation ✓
(insensitive to curvature)

$w \lesssim H$: no cancellation.

$$\Delta m^2 \sim H^2 \Rightarrow \Delta\eta \sim 1$$

$\boxed{\eta \text{ problem}}$

Higher-dimension operators: $\mathcal{O}_\delta = c \langle V \rangle \left(\dfrac{\phi}{\Lambda}\right)^{\delta-4} \Rightarrow$

$$\Rightarrow \Delta\eta \approx c(\delta-4)(\delta-5)\left(\frac{M_{pl}}{\Lambda}\right)^2\left(\frac{\phi}{\Lambda}\right)^{\delta-4}$$

$\Lambda = M_{pl}$, $\phi < \Lambda$: operators with $\delta \gg 6$ can be neglected.

At the very least one needs to characterize Planck-suppressed interactions up to dimension 6.

<u>Natural inflation</u>: A natural bottom-up proposal to fix the aforementioned η problem.

Assume that renormalizable part of the Lagrangian respects an <u>approximate</u> shift symmetry:

$$\phi \longmapsto \phi + \text{const.}$$

Exact shift symmetry: forbids the mass term.

$\hookrightarrow$ To have interesting dynamics let the symmetry be broken by a small mass term:

$$\Delta V = \frac{1}{2} m^2 \phi^2, \quad m \ll \Lambda$$

$\hookrightarrow$ symmetry breaking parameter

$$\Delta m^2 \propto m^2$$

$\hookrightarrow |\eta| < 1$ ✓

Role of the inflaton played by a single axion ϕ with Lagrangian density:

$$\mathcal{L}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] + \cdots$$

[Freese, Frieman, Olinto '90]

f: axion decay constant, Λ: some dynamically generated scale

One can check that:

$N_* \approx 60 \wedge n_s \lesssim 0.97$

$\Downarrow$

$f \gtrsim 10\, M_{pl}$

<u>Problems</u>:
- Does QG allow for such a top-down construction of this kind?
- Is the symmetry spoilt by irrelevant operators?
- Super-Planckian decay constants have not been obtained in ST in controlled examples.

$\hookrightarrow$ WGC: $S \cdot f \lesssim M_{pl}$, $\underset{\text{perturbative control}}{S > 1}$ $\Rightarrow f < M_{pl}$

2. String Theory ingredients

Superstring Theories in 10D:

$\downarrow$

Focus on type IIB ST here

Fundamental mass (string) scale:

$M_s \sim 1/\sqrt{\alpha'}$

↳ M_{pl} is a derived quantity from M_s and g_s

Type IIB on $\mathbb{R}^{1,9}$

$\mathcal{N}=2$ theory

$\downarrow$ low energy theory

$\mathcal{N}=2$ SUGRA on $\mathbb{R}^{1,9}$ $\xrightarrow{M_{1,3}\times M_6}$ 4D $\mathcal{N}=2$ theory

$M_{1,3}$: maximally sym. spacetime

M_6: Calabi–Yau

- Kaluza–Klein replicas whose mass depends on the geometry of the internal space

4D $\mathcal{N}=1$ SUGRA $\xleftarrow{\text{Moduli Stabilization}}$

Add ingredients like D-branes, fluxes and orientifold planes

↳ backreaction → warped CY

- massless moduli without a potential parametrising the deformations of the internal geometry

- Kähler potential K
  - ↳ perturbative corrections
  - ↳ non-perturbative corrections
- holomorphic superpotential W
  - ↳ non-perturbative corrections

(ignore gauge fields here)

F-term potential:

$$V = e^{K}\left(G^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right)$$

- can give a mass to the moduli
- can provide the necessary conditions to realize slow-roll inflation

Perturbative string theory is an expansion in two parameters:

- α' - expansion: worldsheet loops
- g_s - expansion: spacetime loops

Most controlled setting to discuss inflation: SUSY only spontaneously broken during inflation

$$M_{SUSY} < H < M_{KK} < M_s < M_{pl}.$$

Performing fully explicit string constructions is in general very complicated; oftentimes one encounters string inspired constructions rather than fully fledged string theory compactifications.

Bosonic massless spectrum of type IIB:

- Graviton $G_{\mu\nu}$
- Kalb-Ramond 2-form $B_{\mu\nu}$
- Dilaton ϕ
- C_0 axion
- C_2 RR-form
- C_4 RR-form

<u>p-forms and axions:</u> 1-form case is familiar from EM.

$$S = -\frac{1}{4}\int F_2 \wedge * F_2 = -\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}, \quad F_2 := dA_1$$

$\hookrightarrow$ gauge invariant: $A_1 \longmapsto A_1 + d\Lambda.$

In string theory we have the higher dimensional analog in B_2 and C_p.

The 10D action contains terms of the form:

$$S \supset \int d^{10}x \sqrt{-G} \left[e^{-2\phi} |H_3|^2 + \sum_p |\tilde{F}_p|^2 \right]$$

Kinetic and gradient energy contributions from the p-form field strengths + they contribute as quantised fluxes in non-trivial backgrounds

Kalb-Ramond 2-form: $H_3 := dB_2$, gauge inv. $B_2 \longmapsto B_2 + d\Lambda_1$

Upon compactification decompose: $B_2(x, y) = B_2(x) + b_a(x)\, \omega^a(y)$

x: $M_{1,3}$ coordinates

y: M_6 coordinates

ω^a: basis of $H^2(M_6, \mathbb{Z})$

4D axion fields: $b_i(x) = \int_{\Sigma_2^i} B_2$

$\Sigma_2^i \rightarrow$ basis of $H_2(M_6, \mathbb{Z})$

axionic shift symmetry inherited from the gauge invariance

RR p-forms: $F_{p+1} := dC_p$, gauge inv. $C_p \longmapsto C_p + d\Lambda_{p-1}$

Upon compactification decompose:

$$C_0(x, y) = c_0(x)$$

$$C_2(x, y) = C_2(x) + c_a(x)\, \omega^a(y)$$

$$C_4(x, y) = \cdots + \tilde{c}_a(x)\, \tilde{\omega}^a(y)$$

$\tilde{\omega}^a$: basis of $H^4(M_6, \mathbb{Z})$

4D axion fields: $c_i^{(p)}(x) = \int_{\Sigma_p^i} C_p$

$\Sigma_p^i \rightarrow$ basis of $H^p(M_6, \mathbb{Z})$

axionic shift sym. from gauge inv.

<u>Note</u>: BPS D(p-1)-branes are electrically charged under C_p.

We have arrived at the conclusion that <u>classically</u> the axions described above enjoy a <u>continuous</u> shift symmetry

$$\phi \longmapsto \phi + \text{const.}$$

What happens at the quantum level?
- shift sym. present at all orders in g_s and α'.
- non-perturbatively broken to a <u>discrete</u> shift symmetry.

We follow the argument of Dine, Seiberg, Wen and Witten for B_2:

Worldsheet coupling of B_2:

$$S_\sigma \supset -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} d^2\sigma\ \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) = -\frac{1}{2\pi\alpha'}\int_{\Sigma_2} B_2 \equiv -\frac{b}{2\pi}$$

worldsheet

topological coupling

Expand $B_{\mu\nu}(X)$ around a fiducial point $X_{(0)}$:

$$B_{\mu\nu}(X) = B_{\mu\nu}(X_{(0)}) + X^\rho \partial_\rho B_{\mu\nu}(X_{(0)}) + \cdots$$

worldsheet total derivative term

$$-\frac{1}{4\pi\alpha'}\int_{\Sigma_2} d^2\sigma\ \partial_a\left(\epsilon^{ab} X^\mu \partial_b X^\nu B_{\mu\nu}(X_{(0)})\right)$$

Vanishes unless the worldsheet wraps a non-trivial cycle.

remaining terms have space-time derivatives and lead to $\partial_\mu b$ terms in the effective theory that do not break the shift symmetry.

In other words, non-perturbative effects coming from <u>worldsheet instantons</u> can and will break the continuous shift symmetry

$$b \longmapsto b + \text{const.}$$

into a discrete one

$$b \longmapsto b + (2\pi)^2.$$

The previous argument is genus agnostic: it holds at any level in g_s.

↳ Non-perturbatively in g_s the string can break open on a D-brane, acquiring a boundary that makes the worldsheet total derivative term non-vanishing $\Rightarrow$ spacetime-filling D-branes break the continuous shift symmetry.

B_2 appears in the Euclidean D-brane action and their D-brane instantons can break the shift symmetry as well.

Analogously one can argue for the rest of the axions.

Conclusion:

$$\boxed{\text{String Theory axions have a \underline{discrete} shift symmetry at the non-perturbative level.}}$$

↳ leads to periodic potentials with the periodicity set by the discrete shift symmetry.

3. KNP Alignment and N-flation

Some facts that we have learnt thus far:

- Natural inflation using an axion as the inflaton requires $f \gtrsim 10 M_{pl}$ due to phenomenological constrains.
- String theory compactifications come with a (usually abundant) number of axions $\sim h^{1,1}/2$.

__Idea:__ Use several axions, each of them with a sub-Planckian decay constant, but such that a combination of them effectively has a super-Planckian decay constant.

With 2-axions: KNP alignment [Kim, Nilles, Peloso '05]

$$V = \Lambda_a^4 \left[1 - \cos\left(c_{1a}\frac{\phi_1}{f_1} + c_{2a}\frac{\phi_2}{f_2}\right)\right] + \Lambda_b^4 \left[1 - \cos\left(c_{1b}\frac{\phi_1}{f_1} + c_{2b}\frac{\phi_2}{f_2}\right)\right]$$

$\Lambda_{a,b}^4$: some dynamical scales (e.g. the ones associated to two confining non-abelian gauge groups to which the axions couple).

Note that ϕ_1 and ϕ_2 are not yet mass eigenstates: we need to study some linear combination of them.

__Key observation:__ If $\dfrac{c_{1a}}{c_{2a}} = \dfrac{c_{1b}}{c_{2b}}$ one linear combination of the axions is unlifted. If the relation is approximately satisfied, then said combination can have a decay constant $f > M_{pl}$ while $f_1, f_2 \ll M_{pl}$.

__Example:__ $c_{1a} = c_{1b}$, $\Lambda_a^4 \gg \Lambda_b^4$:

$$\xi = \frac{\phi_2 f_2 - c_{2a}\phi_1 f_1}{c_{2a}^2 f_1^2 + f_2^2} \text{ has } f_\xi = \frac{\left(c_{2a}^2 f_1^2 + f_2^2\right)^{1/2}}{|c_{2b} - c_{2a}|}.$$

Can non-perturbative effects related to moduli stabilization spoil the flatness in the ξ-direction? It would be interesting to check this in a fully explicit stringy realization.

N-flation: Related idea with N axions that collectively excite.
[Dimopoulos, Kachru, McGreevy, Wacker '08]

N axions whose Lagrangian is N copies of that of natural inflation:

$$\mathcal{L} = \sum_{i=1}^{N} \left[-\frac{1}{2}(\partial\phi_i)^2 - \Lambda_i^4 \left[1 - \cos\left(\frac{\phi_i}{f_i}\right)\right]\right]$$

↳ We assume the cross-couplings to be negligible.

Each ϕ_i sees: – force only from its own potential V_i, but
– Hubble friction from the sum of potentials $\sum_{i=1}^{N} V_i$.

E.O.M.: $\ddot{\phi}_i + 3H\dot{\phi}_i = -\partial_i V_i$, where $3M_{pl}^2 H^2 \simeq \sum_{i=1}^{N} V_i$.
↳ Can help to achieve a friction dominated situation → slow-roll

The collective excitation $\Phi^2 := \sum_{i=1}^{N} \phi_i^2$ can have a super-Planckian displacement while the individual ϕ_i have sub-Planckian ones.

Problems: – Loops of the N light axions renormalize the Planck mass

$$\delta M_{pl}^2 \sim \frac{N}{16\pi^2} \Lambda_{UV}^2$$

Both Φ and M_{pl} grow with $\sqrt{N}$: large N is not enough to obtain a super-Planckian field displacement → UV details in concrete situations might correct this naive renormalization of M_{pl}^2 in favor of the construction.

— In stringy realizations: axions $\xleftrightarrow{\text{partners}}$ real scalar fields
(volumes of cycles)

Achieving that these have $m \gtrsim H$ such that they are frozen during inflation is challenging. If SUSY is broken at a higher scale than H this is easier, but it makes achieving control of the potential harder.

<u>Phenomenology</u>: If a quadratic approximation is valid then

$$N_* \approx 60 \Rightarrow r \approx 0.13.$$

4. Axion monodromy inflation

Idea: Make inflation persist through many cycles around the configuration space, thereby achieving an effective field range that is bigger than the fundamental period of the axion. [McAllister, Silverstein, Westphal '08]

Notice that in the action for the p-forms we wrote $|\tilde{F}_p|^2$. Here $\tilde{F}_p$ is defined as

$$\tilde{F}_p = F_p + \underbrace{B_2 \wedge F_{p-2}}_{\hookrightarrow \text{ Chern-Simons coupling}}$$

$$B_2 \mapsto B_2 + d\Lambda_1 \Rightarrow \tilde{F}_p \mapsto \tilde{F}_p + \underbrace{d\Lambda_1 \wedge F_{p-2}}_{= d(\Lambda_1 \wedge F_{p-2})}$$

Cancelled by simultaneous
$C_p \mapsto C_p - \Lambda_1 \wedge F_{p-2}$

Turning on F_{p-2} flux: generates a mass term for B_2 coming from $|\tilde{F}_p| \Rightarrow b(x)$ axions massive and with non-periodic potential

$$V \sim \int d^6y \sqrt{-g_6}\, |\tilde{F}_p|^2 \sim \int d^6y \sqrt{-g_6}\, (F_p + B_2 \wedge F_{p-2})^2 \sim$$

$$\sim (N_p + b(x) N_{p-2})^2 .$$

⇝ Parametrically extends the field range.

N_p, N_{p-2} → flux quanta (~ Dirac quantization)

Compare this with the original 4D EFT proposal of axion monodromy inflation:

$$\mathcal{L} \sim (\partial_\mu \phi)^2 + (F_{\mu\nu\rho\sigma})^2 + \frac{\mu}{\sqrt{-g}} \phi\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \rightsquigarrow V \sim (q + \mu\phi)^2$$

[Kaloper, Sorbo '08]

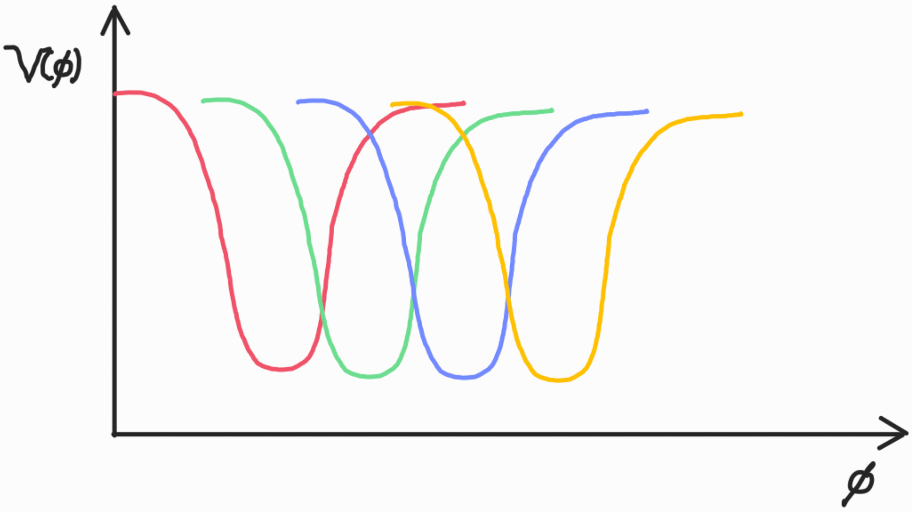
integrate out the 4-form F with q flux units

Integer shift of the axion $\xleftrightarrow[\text{compensated by}]{\text{can be}}$ Integer change in the F_{p-2} flux quanta

mediated by non-perturbative effects that are suppressed at small string coupling and large volume

We have a set of non-periodic branches of the potential labelled by the flux quanta of F_{p-2}.

→ The full theory is periodic, which we can see by summing over branches, but the branch changing effects are exponentially suppressed in the perturbative regime such that effectively we have a non-periodic potential.



The discussion above led us to a square potential, but axion monodromy can realize other variants of chaotic inflation. The generic structure of the effective Lagrangian density after picking a branch by F_{p-2} flux is

$$\mathcal{L} = f^2(\chi)(\partial_\mu b)^2 + \mu(\chi)^{4-p_0} b^{p_0} + \underbrace{\Lambda^4(\chi)\cos(b)}_{\substack{\text{periodic contribution from}\\ \text{non-perturbative effects}\\ \text{that will be exponentially suppressed}}}, \quad p_0 = 2,3,4.$$

χ: moduli of the string compactification

We have: large field monomial + tiny periodic modulations on top

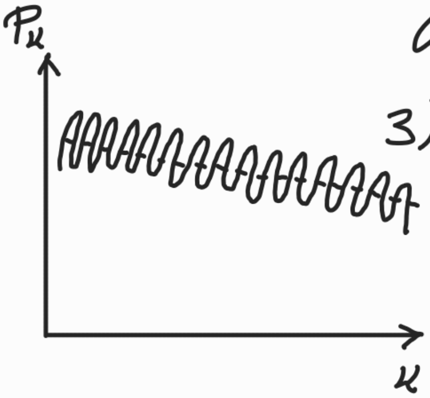
Note: 1) The backreaction of the moduli will generically lead to a flattening of the potential: $V_{eff} \sim \phi(t)^p$, $p < p_0$.

2) The same effect can arise from the coupling of the p-form gauge potentials to branes, as required by the dualities of string theory relating

compactification with branes $\xleftrightarrow[\text{same physics}]{\text{dual geometry}}$ compactification without branes, but with background fluxes

Phenomenology: 1) f is bounded from below (for perturbative control in string theory).

2) The oscillatory corrections lead to a characteristic oscillating signal in all primordial correlation functions.



3) The power spectrum will be modulated,

$$\Delta^2_R(k) = \Delta^2_R(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}\left[1 + A\cos\left(\frac{\phi_k}{f}\right)\right]$$

4) In order to reheat the axion must be coupled to other fields. Consider the dimension-five operator

$$\mathcal{L} \supset -\frac{\alpha}{4}\frac{\phi}{f}F\tilde{F}$$

$\hookrightarrow$ gauge fields are produced during inflation

Signatures: i) (Equilateral) non-Gaussianity
ii) Non-scale invariance
iii) Chiral gravitational waves

5. Harmonic hybrid inflation

Recall the idea of hybrid inflation: [Linde '94]

Two fields: χ : gives a false vacuum positive cosmological constant

ϕ : slow-rolls, triggers an instability in χ

↳ "waterfall" of χ, that rolls to zero cosmological constant

$$V(\phi,\chi) = \frac{1}{4}(\chi^2 - v^2)^2 + g\chi^2\phi^2 + \Delta V_{\substack{\text{slow}\\\text{roll}}}(\phi)$$

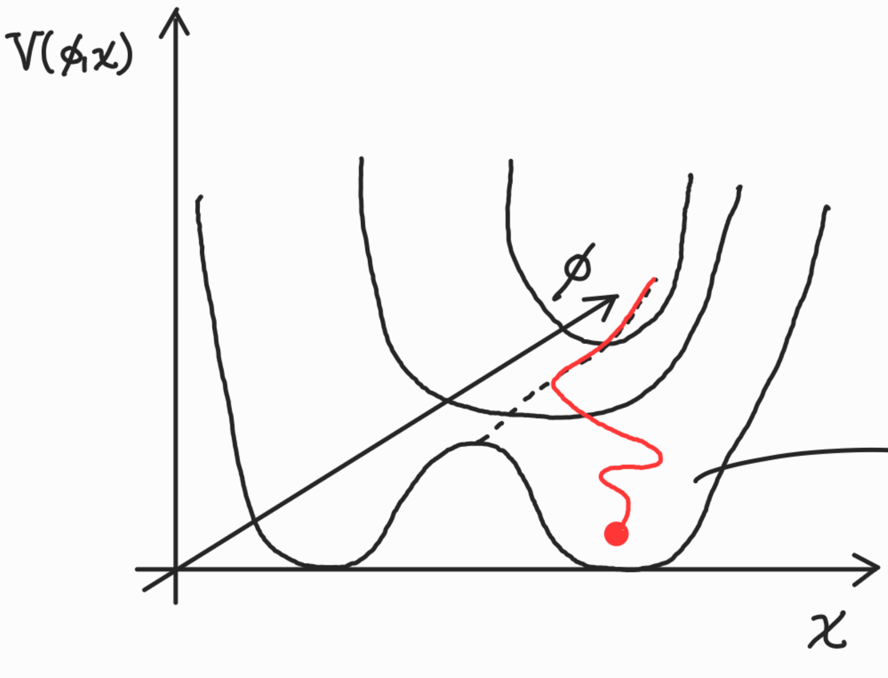
$$\hookrightarrow m_\chi^2 = -1v^2 + 2g\phi^2 \equiv -\mu^2 + 2g\phi^2 \rightsquigarrow \phi_c := \mu/\sqrt{2g}$$

$\phi > \phi_c$:

$m_\chi^2 > 0$

$\phi < \phi_c$:

$m_\chi^2 < 0$, tachyonic waterfall



tachyonic preheating leads to peculiar peaks in the spectrum of primordial gravitational waves

Realization in String Theory: Two axions with purely non-perturbative potential.

Harmonic hybrid inflation

[Carta, Righi, Welling, Westphal '20]

$$V = \Lambda^4 + \tilde{\Lambda}^4 - \left(\tilde{\Lambda}^4 + \Lambda^4 \cos(c_1\phi_1)\right)\cos(c_2\phi_2), \qquad \tilde{\Lambda} > \Lambda, \quad c_i \propto \frac{1}{f_i} \geq 1.$$

$\phi_2 \sim \phi \longrightarrow$ As ϕ_2 evolves in time the cosine flips sign

$\phi_1 \sim \chi$ and renders ϕ_1 tachyonic

$\Lambda_1^4 = \Lambda_2^4 := \Lambda/2$, $\Lambda_3^4 := \tilde{\Lambda}^4$, recast the potential as:

$$V = \Lambda_1^4\left[1-\cos(c_1\phi_1+c_2\phi_2)\right] + \Lambda_2^4\left[1-\cos(c_1\phi_1 - c_2\phi_2)\right] +$$
$$+ \Lambda_3^4\left[1-\cos(c_2\phi_2)\right]$$

Define: $\alpha := \dfrac{\tilde{\Lambda}^4 - \Lambda^4}{\tilde{\Lambda}^4 + \Lambda^4}$. We need:

1) $\tilde{\Lambda} > \Lambda$ to avoid the inflaton from being trapped in a minimum $\rightarrow$ dS saddle point

2) Dominance of vacuum energy, $\alpha \ll 1$.

3) Fast waterfall transition, $c_1 \gg 1$.

Define: $\gamma := \alpha c^2$

$$\epsilon \simeq 2\alpha\gamma \frac{e^{2\gamma\Delta N}}{(1+e^{2\gamma\Delta N})^2},$$

$$\eta \simeq \gamma \frac{1-e^{2\gamma\Delta N}}{1+e^{2\gamma\Delta N}}.$$

To avoid the domain wall problem: break the degeneracy between the two vacua such that the true vacuum domains grow or <u>avoid the tachyonic instability.</u>

→ Generalize the potential:

$$V = \Lambda_1^4\left[1-\cos\left(c_1^+\phi_1 + c_2^+\phi_2\right)\right] + \Lambda_2^4\left[1-\cos\left(c_1^-\phi_1 - c_2^-\phi_2\right)\right] +$$

$$+\Lambda_3^4\left[1-\cos\left(c_2\phi_2\right)\right]$$

Choosing $c_1^+ \neq c_2^-$ generically breaks the $\mathbb{Z}_2$ vacuum degeneracy and removes the tachyon instability so that no domain walls are generated.