Beyond single-stage inflation & New GW signatures.

Recap of Axion Monodromy Inflation

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \mu^{4-p}\phi^p - \Lambda^4 \cos(\phi/f)$$

· arXiv:0803.3085
Silverstein & Westphal

Where does this come from?

· Φ = dilaton

Ex. Wrapped D4-branes

· X^M = coordinates of D4-brane

$$S_{D4} = -\int \frac{d^5\xi}{(2\pi)^4(\alpha')^{5/2}} e^{-\Phi} \sqrt{\det\left[(G_{MN}+B_{MN})\partial_\alpha X^M \partial_\beta X^N\right]} + S_{CS}$$

· $\alpha' = l_s^2$

$$S_{CS} = \frac{1}{(2\pi)^4(\alpha')^{5/2}} \int \left(\sum_p C^{(p)}\right) e^{-B} e^{2\pi\alpha' F}$$

· F = worldvolume gauge flux

· compactify IIA on product of Nil manifolds $N_3 \times \tilde{N}_3$ w/ coordinates $\{u_1, u_2, x\}$ & $\{\tilde{u}_1, \tilde{u}_2, \tilde{x}\}$

· $\frac{ds^2_{N_3}}{\alpha'} = L_{u_1}^2 du_1^2 + L_{u_2}^2 du_2^2 + L_x^2(dx' + Mu_1 du_2)^2$

· wrap D4 on u_2 & let it move along u_1

· $\beta = \frac{L_{u_2}}{L_{u_1}} = \frac{L_u^2}{L_{u_1}^2}$

$$S_{D4}[u_1] = \frac{1}{(2\pi)^4 g_s \alpha'^2} \int d^4x \sqrt{-g_4} \left(\beta^{-1} L_u^2 \sqrt{\beta L_u^2 + L_x^2 M^2 u_1^2}\, \frac{\alpha' \dot{u}_1^2}{2} - \sqrt{\beta^2 L_u^2 + L_x^2 M^2 u_1^2} \right)$$

· canonical normalization

$$\dot{\phi} = \phi'(u_1)\dot{u}_1$$

$$\phi = \frac{L_u^{3/2}\beta^{-1/4}}{3(2\pi)^2\sqrt{g_s\alpha'}} u_1 \cdot \left[F^2_{1,\frac{1}{2},\frac{3}{4},\frac{3}{2}}\left(\frac{-M^2L_x^2}{\beta L_u^2}u_1^2\right) + 2\left(1 + \frac{M^2 L_x^2}{\beta L_u^2} u_1^2\right)^{1/4} \right]$$

· $S_{D4} = \int d^4x \sqrt{-g_4} \left[\frac{1}{2}\dot{\phi}^2 - V_R(\phi) \right]$

$$V_R = \frac{\sqrt{\beta} L_u}{(2\pi)^4 g_s \alpha'^2} \sqrt{1 + \frac{M^2 L_x^2}{\beta L_u^2} u_1^2(\phi)} = \begin{cases} \frac{m^2}{2}\phi^2 & \text{for small } \phi \\ \mu^{10/3}\phi^{2/3} & \text{for large } \phi \end{cases}$$

- Flux Monodromy EFT
  - at 2-derivative level, all known string axion & brane monodromy inflationary models can be dualized into 4D massive 4-form description.

$$\mathcal{L} = -\frac{1}{48} F_{\mu\nu\lambda\sigma}^2 - \frac{m^2}{12}\left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^2 - \sum_{n \geq 2} \frac{a_n'}{M^{4n-4}} \tilde{F}^n$$

$$- \sum_{n \geq 1} \frac{a_n''}{M^{4n-4}} m^{2n} \left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^{2n}$$

$$- \sum_{\substack{k \geq 1 \\ l \geq 1}} \frac{a_{k,l}'''}{M^{4k+2l-4}} m^{2k} \left(A_{\mu\nu\lambda} - h_{\mu\nu\lambda}\right)^{2k} \tilde{F}^l$$

- $F = dA$
- $\tilde{F} = *F$
- $h = db$

  - dual frame: F replaced by compact scalar

$$F \leftrightarrow \varepsilon(m\phi + Q)$$

$$mA \leftrightarrow \varepsilon \partial\phi$$

- $Q = Nq$
  q = 4-form charge & $N \in \mathbb{Z}$

- NDA
  #1. $\phi \to \frac{4\pi\phi}{M}$
  #2. overall $\frac{M^4}{(4\pi)^2}$
  #3. factorials for symmetry factors in S-matrix

- strong coupling scale

$$M_S = M/\sqrt{4\pi}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(m\phi+Q)^2 - \sum_{n\geq 2} \frac{c_n'(m\phi+Q)^n}{n!\left(\frac{M^2}{4\pi}\right)^{n-2}}$$

$$-\sum_{n\geq 1} c_n'' \frac{(\partial_\mu\phi)^{2n}}{2^n n!\left(\frac{M^2}{4\pi}\right)^{2n-2}} - \sum_{\substack{k\geq 1\\ l\geq 1}} c_{k,l} \frac{(m\phi+Q)^l}{2^k k!\, l!\left(\frac{M^2}{4\pi}\right)^{2k+l-2}} (\partial_\mu\phi)^{2k}$$

- $c_n \sim \mathcal{O}(1)$
- define inflaton $\varphi = \phi + Q/m$
- φ can be $\gg M_{Pl}$, but all parameters below M_{Pl}

$\Rightarrow$ rewrite

$$\mathcal{L} = K(\varphi, X) - V_{eff}(\varphi)$$

- $X = -(\partial_\mu\varphi)^2$

$$= \frac{M^4}{16\pi^2} K\left(\frac{4\pi m\varphi}{M^2}, \frac{16\pi^2 X}{M^4}\right) - \frac{M^4}{16\pi^2} V_{eff}\left(\frac{4\pi m\varphi}{M^2}\right)$$

- two distinct phases $\begin{cases} \text{nonlinear in } X \text{ suppressed} \rightarrow \text{flattened potential} \\ \text{nonlinear in } X \text{ present} \rightarrow \text{k-flation.} \end{cases}$

- in first phase, if $m\varphi < \frac{M^2}{4\pi}$, theory in weak coupling regime $\Rightarrow V_{eff}$ reduces to quadratic + corrections.

$\Rightarrow r = 0.16$

- strong coupling needed: $\frac{M^2}{4\pi} < m\varphi$

· but EFT valid at least to $m\varphi < M^2$

⇒ window of validity w/ strong coupling w/o exciting UV d.o.f.

⇒ here potential leaves quadratic regime & flattens out.

Details depend on UV completion that fixes Z_{eff} & V_{eff}

• Rock & Roll(ercoaster) Cosmology

• arXiv: 2011.09489
D'Amico & Kaloper

- Why should inflation be continuous?
  Not necessary – one could imagine bursts of accelerated expansion w/ breaks of different cosmological evolution (i.e. radiation domination)
- Why consider this?
  - String theory & field displacements
    $\Delta\phi > M_{Pl}$ may be problematic $\Rightarrow$ Swampland Distance Conjecture, infinite # of Kaluza-Klein modes become light & destroy EFT
  - Enrich model building & signals

- How does it work?
  - Horizon problem
    - correlations seen on scales $\ell \sim 1/H_{now}$ $\Rightarrow \ell(t) = \frac{1}{H_{now}} \frac{a(t)}{a_{now}}$

    for normal matter, $\ell(t) \sim t^{2/3(1+w)} \sim \left(\frac{1}{H}\right)^{2/3(1+w)}$
    so particle horizon $L_H = a(t) \int_{t_i}^{t} \frac{dt'}{a(t')} \sim t \sim 1/H$
    and

    $$\ell/L_H \sim H^{(3w+1)/(3w+3)}$$

    $\Rightarrow$ ℓ slower than L_H when $w > -1/3$

    for ratio ~ 1 now, $\ell_{in} \ggg L_{H_{in}}$
  - in expanding universe, L_H integral dominated by early times. After 1^{st} stage of inflation

    $$\int_{t_0}^{t_e} \frac{dt'}{a(t')} \simeq \frac{1}{\sqrt{H_1 H}}$$

    • $H_1 < H$

· stringing together expansion epochs,

$$a(t) = \prod_j \frac{a(t_{j+1})}{a(t_j)} \Rightarrow L_H \sim \frac{a(t)}{\sqrt{H H_1}} \lesssim \frac{a(t)}{H_1}$$

so $\ell / L_H \gtrsim \ell_{in} H_1$

· flatness problem

· Rollercoasters & Axion Monodromy

•arXiv: 2101.05861 & 2112.13861
D'Amico, Kaloper, Westphal

- apply rollercoaster cosmo. to axion monodromy

$\Downarrow$

double* axion monodromy

*or more!

- Why?
  - fix n_s problem
  - obtain interesting GW signal
- Overall Idea
  - Inflation driven by 2 stages of monodromy inflation

    $\Rightarrow$ 1st stage: 30-40 efolds, $V \sim \phi_1^p$ w/ $p \lesssim 1/2$ $\Rightarrow$ $n_s \sim 0.965$, $r \lesssim 0.06$

    $\Rightarrow$ phase where ϕ_1 oscillates, matter domination

    $\Rightarrow$ 2nd stage: remaining e-folds, 20-30
  - axion-U(1) coupling $\mathcal{L} \supset \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$ · $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$

    causing tachyonic instability at end of 1st stage $\Rightarrow$ nonperturbative generation of F

    $\Rightarrow$ F sources chiral GWs with amplitude $\gg$ non-chiral GW from δg_{ij}

    $\Rightarrow$ GWs superhorizon just before end of 1st stage, wavelengths stretched to $\sim 10^8$ km in 2nd stage $\Rightarrow$ LISA detection.

· The model

· $V(\phi_1,\phi_2) = M_1^4\left[\left(1+\frac{\phi_1^2}{\mu_1^2}\right)^{p_1/2}-1\right] + M_2^4\left[\left(1+\frac{\phi_2^2}{\mu_2^2}\right)^{p_2/2}-1\right]$

· $M_1, M_2 \sim \mathcal{O}(0.1 M_{Pl})$

· $M_2 \lesssim M_1$

· take ϕ_i's as arising from p-forms $\Rightarrow \mu_i \sim f_i \Rightarrow 10^{-2} M_{Pl} \lesssim f_i \lesssim M_{Pl}$

· during 1^{st} stage, ignore ϕ_2

$$V_{eff}(\phi_1) \simeq M_1^4\left[\left(\frac{\phi_1}{\mu_1}\right)^{p_1}-1\right]$$

· $\phi_1(\text{end}) \simeq \mu_1$

$$n_s - 1 = 2\eta_V - 6\epsilon_V$$

$$r = 16\epsilon_V$$

$$N_e \simeq \left(\frac{\phi_1^2}{2p_1 M_{Pl}^2}\right)\left[1-\frac{2}{2-p_1}\left(\frac{\mu_1}{\phi_1}\right)^{p_1}\right] \Rightarrow$$

if $\phi_{max}/M_{Pl} \sim 2\text{-}3$

$p \sim 0.1 \Rightarrow N_{max} \sim 20\text{-}45$

larger p, smaller N_{max}

· Predictions

· $r \gtrsim 0.02 \Rightarrow$ in reach of B-mode searches

$r \lesssim 0.06$

· GWs

· first, production of $U(1)$ bosons.

$$\mathcal{L}_{int} \supset -\sqrt{-g}\,\frac{\phi_1}{4f_1} F_{\mu\nu}\tilde{F}_{\mu\nu}$$

dynamics:

$$\begin{cases} \ddot{\phi}_1 + 3H\dot{\phi}_1 + \partial_1 V - \frac{1}{f_a}\langle \vec{E}\cdot\vec{B}\rangle = 0 \\ 3H^2 = \dot{\phi}_1^2/2 + V(\phi_1) + \frac{1}{2}\rho_{EB} \\ A''_\pm + \left[k^2 \pm 2\lambda\xi k a H\right]A_\pm = 0 \end{cases}$$

· $\rho_{EB} = \frac{\vec{E}^2 + \vec{B}^2}{2}$

· $\lambda = \mathrm{sgn}[\dot{\phi}_1]$

· $\xi = \frac{1}{2f_1}\frac{d\phi_1}{dN_e}$

· if $\dot{\phi}_1 \neq 0$ & $2\xi > \frac{k}{aH} \Rightarrow -\lambda$ helicity is tachyonic

• approx. w/ dS patch & $\xi \simeq$ constant

$$A_{-\lambda}(\tau,\vec{k}) = \frac{e^{\pi\xi/2}}{\sqrt{2k}} W_{-i\xi,1/2}(2ik\tau)$$

• $W_{k,m}(z)$ = Whittaker function

and

$$\rho_{EB} \simeq 1.3\times10^{-4} H^4 \frac{e^{2\pi\xi}}{\xi^3}$$

$$\langle \vec{E}\cdot\vec{B} \rangle \simeq -2.4\times10^{-4} \lambda H^4 \frac{e^{2\pi\xi}}{\xi^4}$$

• feeding into Ω_{GW},

• $\Omega_{r,0} = 8.6\times10^{-5}$

$$\Omega_{GW} = \frac{\Omega_{r,0}}{24}\Delta_T^2$$

$$\simeq \frac{\Omega_{r,0}}{12}\left(\frac{H}{\pi M_{Pl}}\right)^2 \left\{1 + 4.3\times10^{-7}\frac{H^2}{M_{Pl}^2\xi^6}e^{4\pi\xi}\right\}$$

↪ secondary production from U(1)

$\cdot$ arXiv:2112.13861

· Hairy Inflation

- rollercoaster + axion monodromy + strong coupling corrections.

$$\mathcal{L}_i = K(\phi_i, X_i) - V_{eff}(\phi_i)$$

$\cdot X_i = -(\partial_\mu \phi_i)^2$

k-flation

$$= \frac{M^4}{16\pi^2} K\left(\frac{4\pi m_i \phi_i}{M^2}, \frac{16\pi^2 X_i}{M^4}\right) - \frac{M^4}{16\pi^2} V_{eff}\left(\frac{4\pi m_i \phi_i}{M^2}\right)$$

- reduction in r if $\begin{cases} V_{eff} \text{ plateaus at large } \phi \\ \text{higher derivative operators contribute.} \end{cases}$

- keep only

$$K = Z X_1 + \tilde{Z} \frac{16\pi^2}{M^4} X_1^2 + \text{sometimes sextic}$$

and consider DBI:

$$K(X) = M^4\left(1 - \sqrt{1 - 2X/M^4}\right)$$

· Lower bound on r

- bound on non-Gaussianities $f_{NL} \lesssim \mathcal{O}(10)$

$\Rightarrow$ lower bound on r

- inflaton EFT

$$S = -\int dt\, d^3x\, a^3 M_{Pl}^2 \dot{H}\left[\frac{1}{c_s^2}\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} + \left(\frac{1}{c_s^2} - 1\right)\left(\dot{\pi}^3 + \frac{2}{3} c_3 \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right)\right]$$

$\cdot\, c_s = \partial_X K / (\partial_X K + 2X\partial_X^2 K)$

$\cdot\, c_3\left(\frac{1}{c_s^2} - 1\right) = 2X^2 \partial_X^3 K / \partial_X K$

- $$\langle \Phi_{k_1} \Phi_{k_2} \Phi_{k_3} \rangle = (2\pi)^3 \delta_D(k_1 + k_2 + k_3) \frac{G\Delta_{\Phi}^2}{(k_1+k_2+k_3)^3} \left[f_{NL}^{(1)} F_1(k_1, k_2, k_3) + f_{NL}^{(2)} F_2(k_1, k_2, k_3) \right]$$

- $$f_{NL}^{(1)} = \frac{-85}{324}\left(\frac{1}{c_s^2} - 1\right)$$

- $$f_{NL}^{(2)} = \frac{10}{243}\left(1 - c_s^2\right)\left(\frac{3}{2} + c_3\right).$$

$\Rightarrow$ see DBI plot $\Rightarrow 0.006 \leq r$