

EFTs: the on-shell way

Yael Shadmi, Technion



Baron Münchhausen
bootstrapping out of the swamp

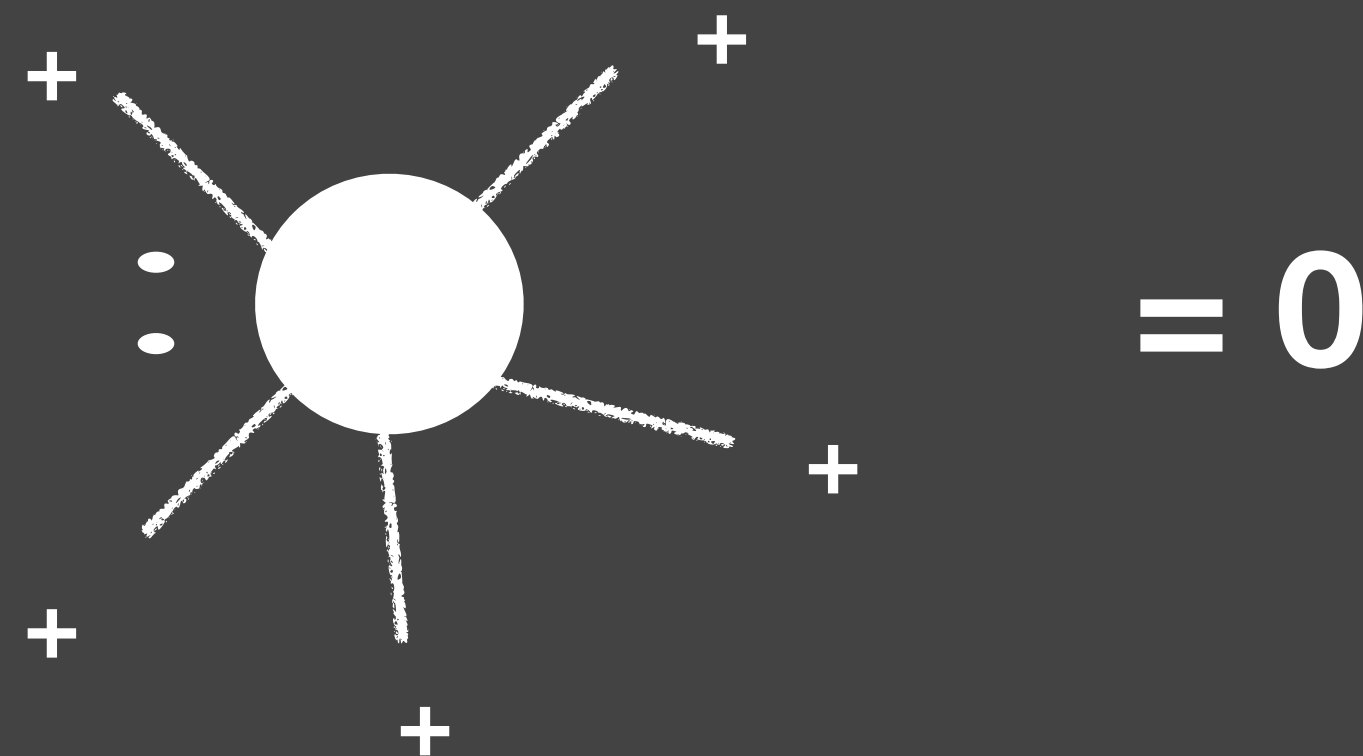
Münchhausen zieht sich am
Zopf aus dem Sump, Distelli

Why on-shell?

0) 1st clue: amplitudes: the whole is SMALLER than the sum of its parts:

gauge boson amplitudes: many Feynman diagrams (~10 million for tree 10-gluon):

Mangano Parke review



Why on-shell?

describe massless *spin-1 particle* (2 dof's) via *vector field* (4 dof's)

➡ more efficient: focus on physical dof's only

Why on-shell?

1) various ways developed for expressing amplitudes: make various properties/symmetries transparent

here:

massless & massive amplitudes in terms of **2-component spinor** products

- uniform description of amplitudes of different spins
- properties of amplitudes under Lorentz manifest: Little Group
 - > selection rules
- simple relations between massive \longleftrightarrow massless

Why on-shell?



2) bootstrapping amplitudes:

construct amplitudes based on their properties: little group; poles, cuts

$$\rightarrow \mathcal{A}_{SM} + \mathcal{A}_{EFT}$$

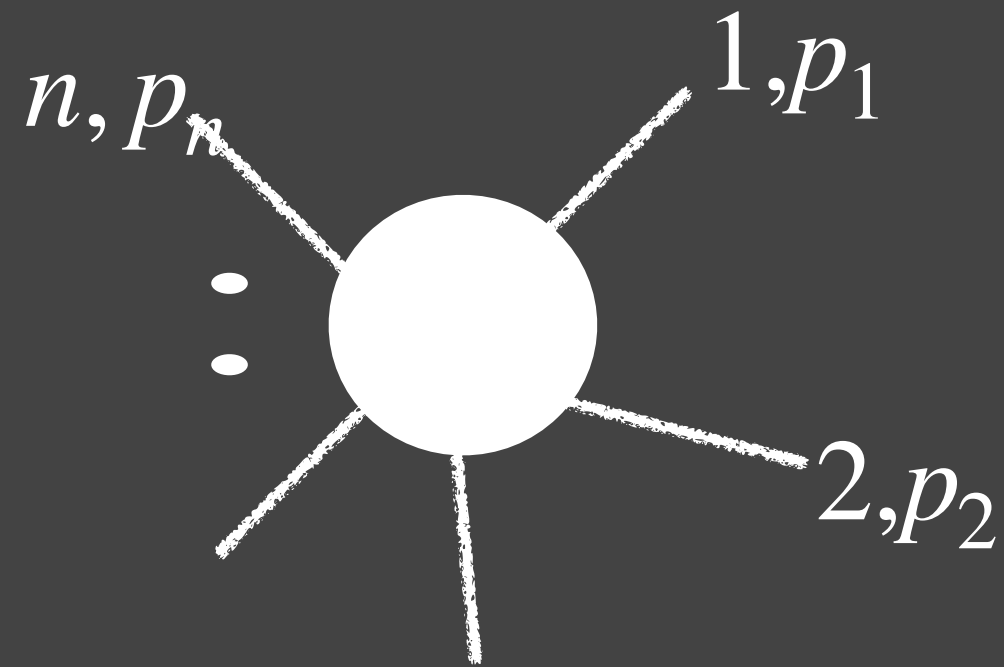
rediscover SM
(more generally, gauge theory,
Higgs mechanism)

- most general EFT amplitude
- model independent
- no issues of field redefinitions, basis dependence
- natural approach as we try to go beyond SM

Plan

- amplitude basics
- sketchy overview of EFT applications
- bootstrapping amplitudes
 - rediscovering the SM
 - going BSM: on-shell EFTs

amplitude basics: spinor variables: massless



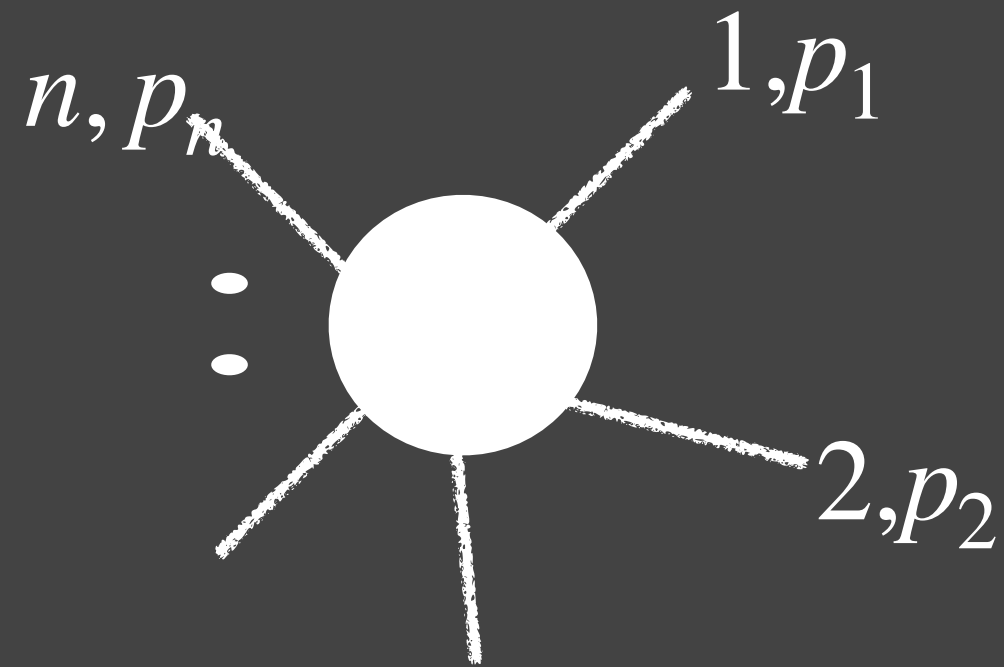
function of momenta, polarizations ($s = 1/2, s = 1$)

all can be written in terms of 2-component spinors:

external $s = 1/2$ fermion $h = \pm$: $p]$ or $p\rangle$

leg i : $i]$ or $i\rangle$

amplitude basics: spinor variables: massless



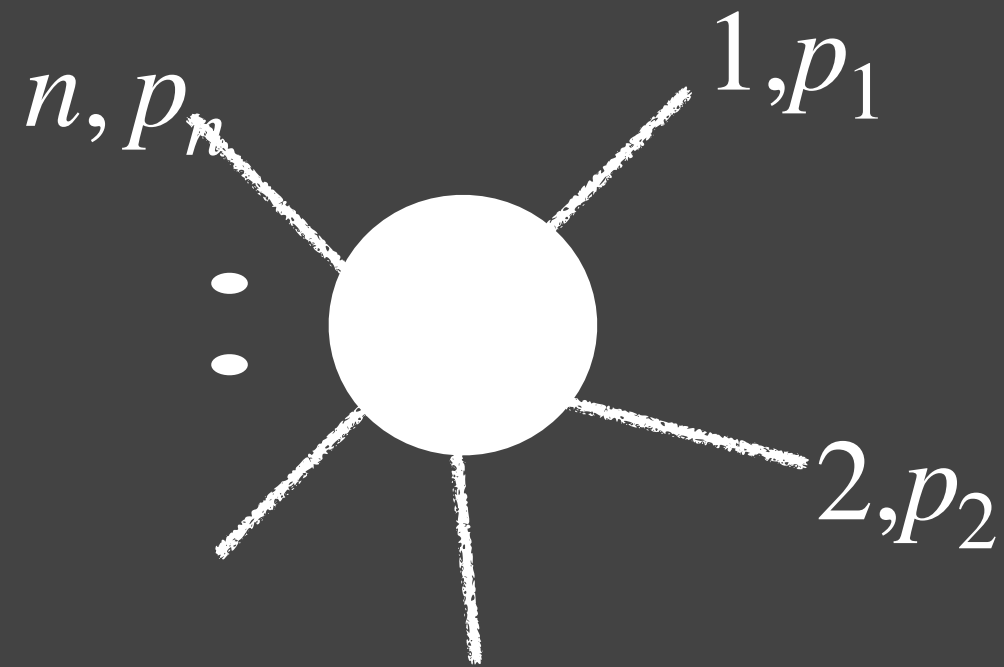
function of momenta, polarizations ($s = 1/2, s = 1$)

all can be written in terms of 2-component spinors:

external momentum p_i : $p_i^\mu \rightarrow p_{i,\alpha\dot{\alpha}} \equiv p_i^\mu (\sigma_\mu)_{\alpha\dot{\alpha}}$

$$\det(p_i) = 0 \quad \rightarrow \quad p_i = i\rangle[i$$

amplitude basics: spinor variables: massless



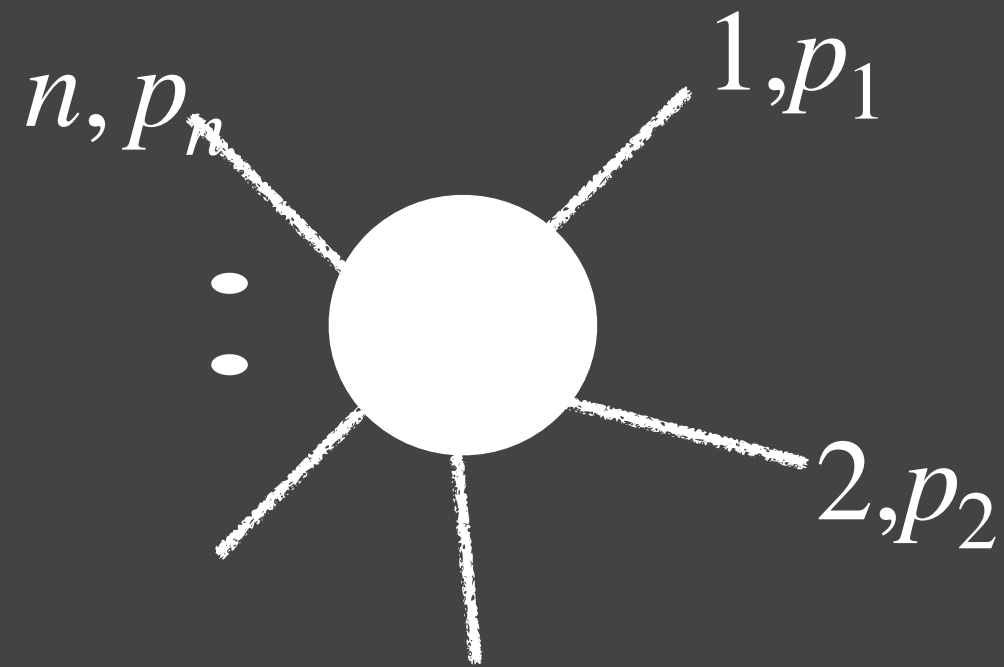
function of momenta, polarizations ($s = 1/2, s = 1$)

all can be written in terms of 2-component spinors:

external spin-1 ε_i : standard polarization vectors can be written as

$$\varepsilon_i(p_i; +) \sim \frac{r\rangle[i}{\langle ir\rangle} \quad \leftarrow \quad \varepsilon_{i,\alpha\dot{\alpha}}$$

amplitude basics: spinor variables: massless



function of momenta, polarizations ($s = 1/2, s = 1$)

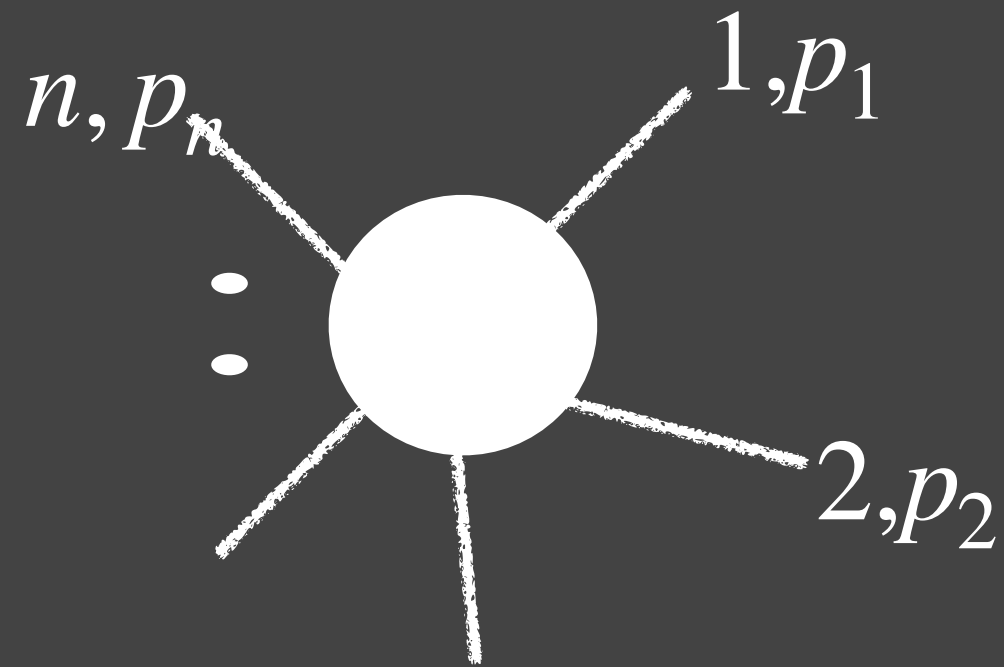
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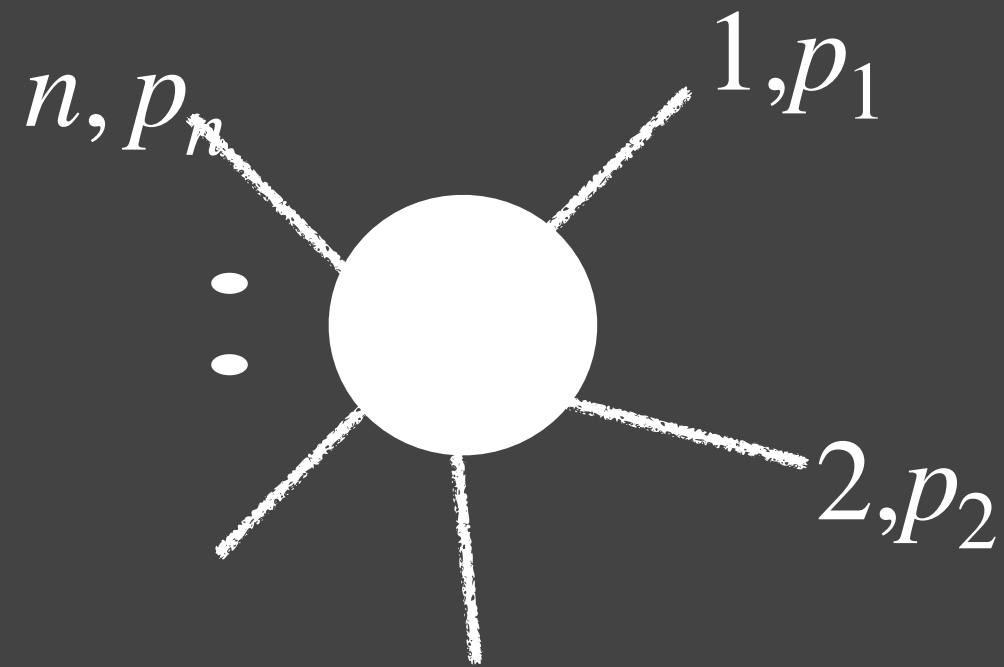
arbitrary “reference” spinor
 changing $r \longleftrightarrow$ changing $\varepsilon_\mu(p) \rightarrow \varepsilon_\mu(p) + \#p_\mu$

amplitude basics: spinor variables: massless



amplitude = function of spinor products $\langle ij \rangle, [ij]$
& Lorentz invariants $s_{ij} = (p_i + p_j)^2$ ($= 2[ij]\langle ji \rangle$)

amplitude basics: spinor variables: massless



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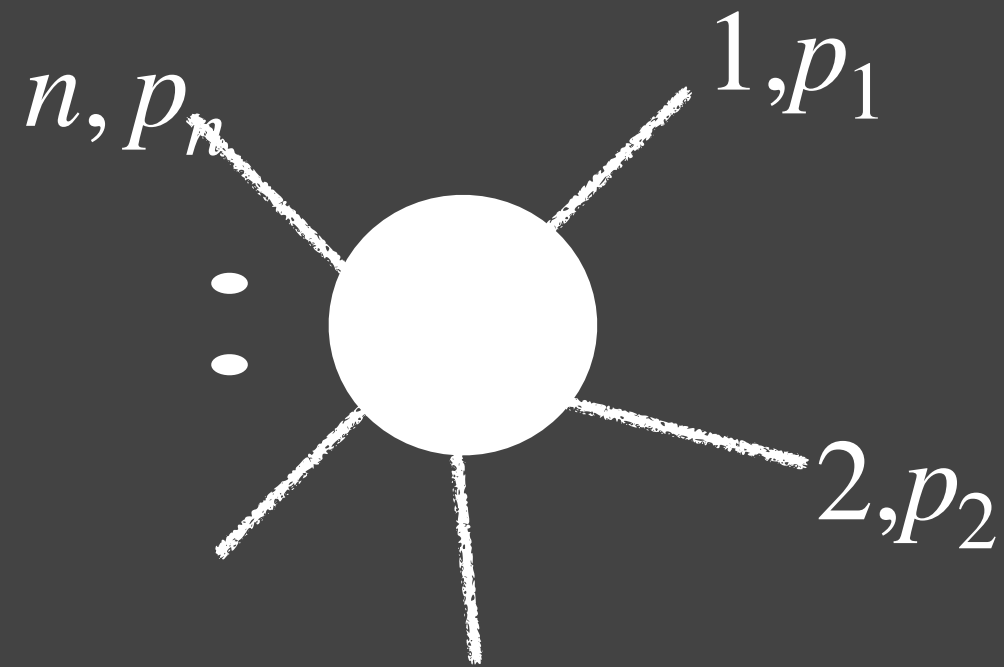
Little Group (LG) transformations transparent:

$p_i = i\rangle[i$: Lorentz transformations keeping p_i invariant:

$$i] \rightarrow e^{i\phi} i] : \text{charge} + 1$$

$$i\rangle \rightarrow e^{-i\phi} i\rangle : \text{charge} - 1$$

amplitude basics: spinor variables: massless



$$i, h = 1/2 \quad i]$$

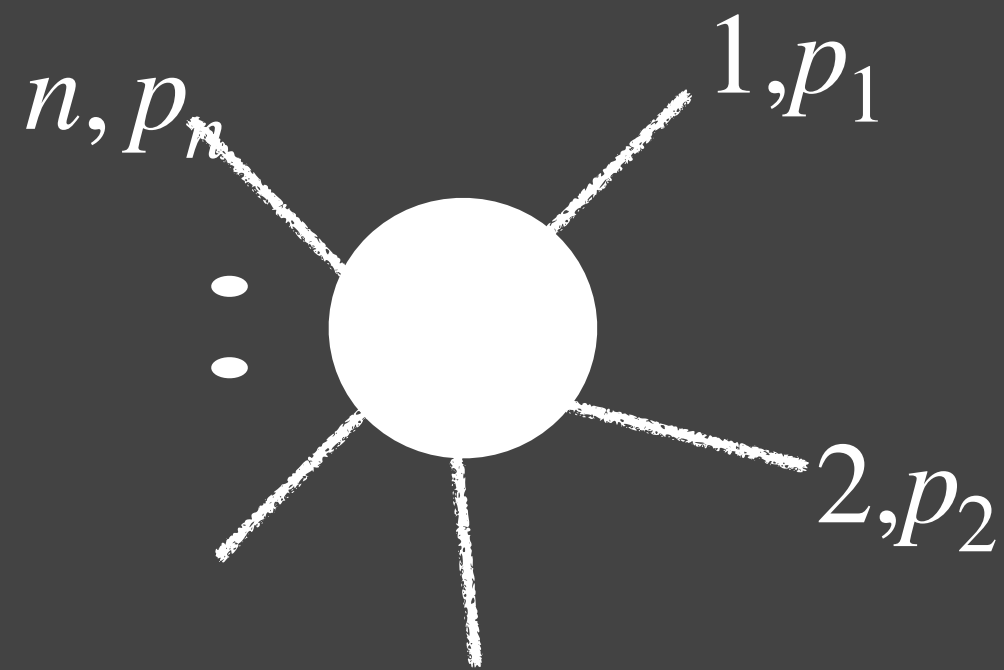
$$i, h = -1/2 \quad i\rangle$$

$$i, h = +1 \quad i]i]$$

$$i, h = -1 \quad i\rangle i\rangle$$

selection rules:
dictate allowed form of amplitudes

amplitude basics: spinor variables: **massive**



$$\det(p) \neq 0 \quad \rightarrow \quad p = p^{I=1} + p^{I=2} \quad \text{lightlike vectors}$$

massless particle of definite helicity: single direction

massive particle of definite spin polarization:

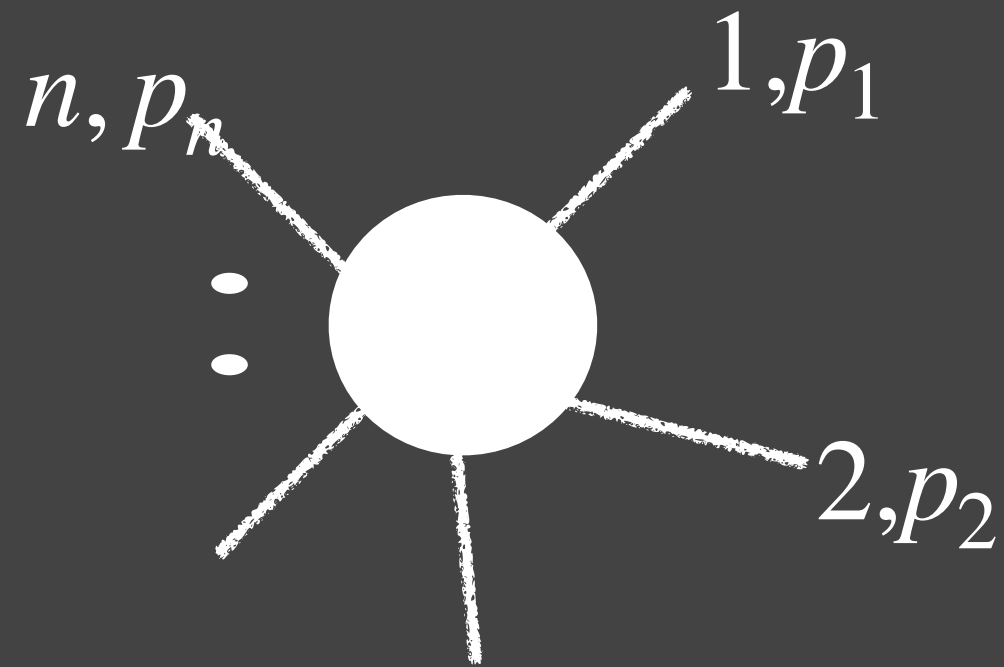
\rightarrow 2 directions: momentum + spin axis

$$\det(p) \neq 0 \quad \rightarrow \quad p = p^{I=1} + p^{I=2}$$

$$p = p \rangle^I [p_I$$

amplitude basics: spinor variables: **massive**

Arkani-Hamed Huang Huang '17



$$p = p^{I=1} + p^{I=2} \quad \text{lightlike vectors}$$

$$p = p^I \rangle [p_I$$

external spin-1/2 fermion: $p^I \rangle \quad I = 1, 2 \quad h = \pm 1/2$

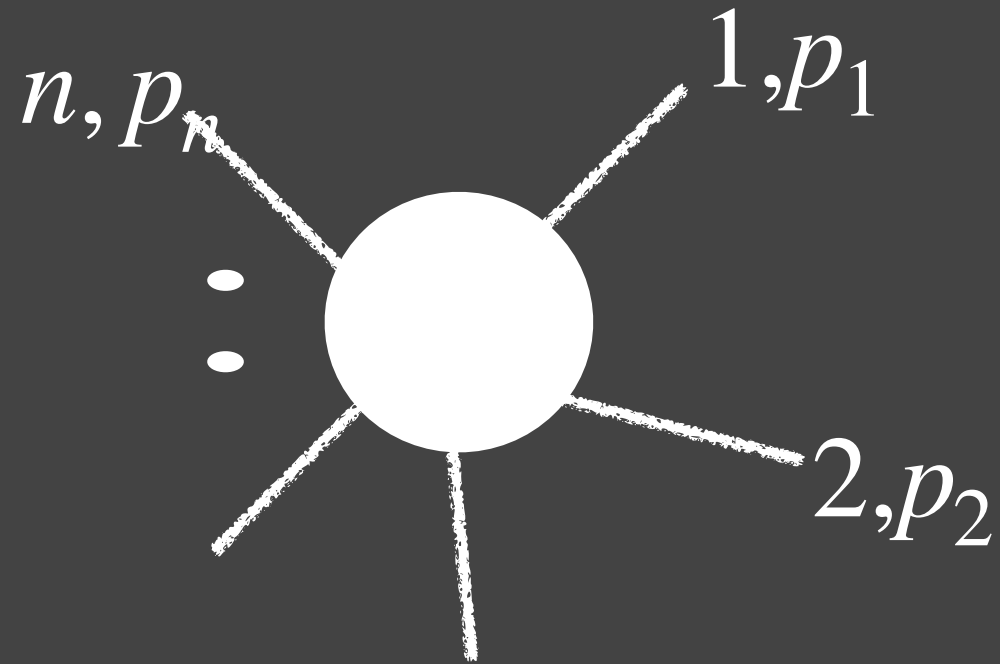
external spin-1: $\varepsilon(p) \sim \frac{p^I \rangle \{p^J \}}{m} \quad I = J : h = \pm 1, \quad I \neq J : h = 0$

no gauge freedom

$$\frac{\mathbf{p} \rangle [\mathbf{p}}{m}$$

BOLD notation
Arkani-Hamed Huang Huang

amplitude basics: spinor variables: **massive**



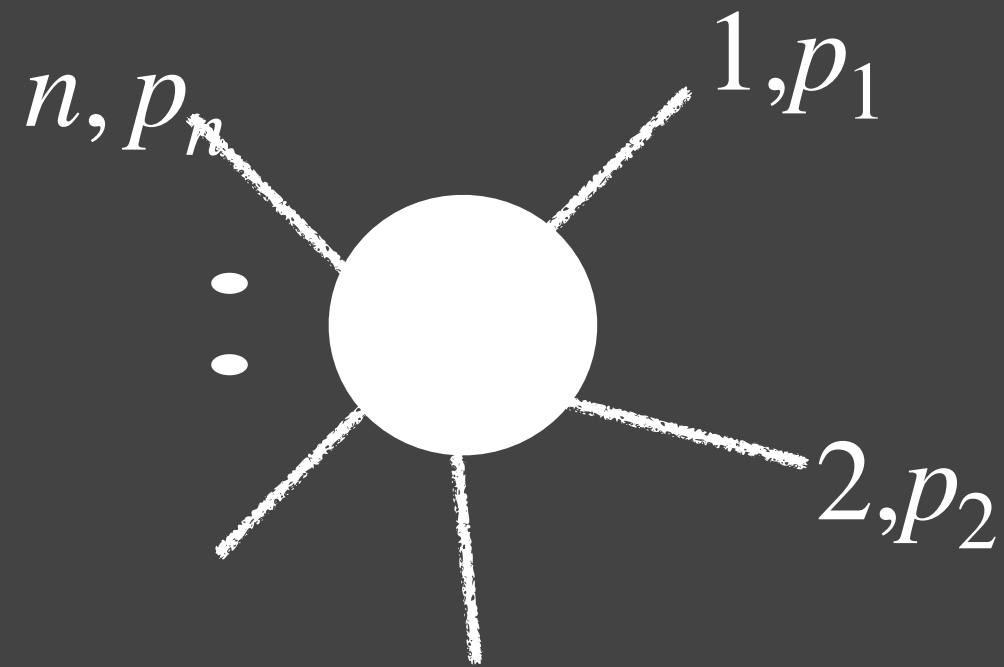
$$\mathbf{p} = \mathbf{p}\rangle^I[\mathbf{p}_I$$

LG transformation $p\rangle^I \rightarrow W_J^I p\rangle^J \quad [p_I \rightarrow (W^{-1})_I^J [p_J$

massive LG SU(2) transformations

little-group covariant massive spinor formalism

amplitude basics: spinor variables: **massive**



$i, s = 1/2$ $i]$ or $i\rangle$

selection rules:
dictate allowed form of amplitudes

$i, s = +1$ $i]i]$ or $i\rangle i\rangle$ or $i\rangle i]$

amplitude basics: spinor variables:

- selection rules: dictate allowed form of amplitudes
- determine all 3-point amplitudes *(complex momenta)*

massless: easy

massive: in some cases: constructing a basis of independent spinor structures

requires some work: EOM $\mathbf{p} \mathbf{p}] = m \mathbf{p} \rangle$

explicit bases for spins ≤ 3

Durieux Kitahara YS Weiss '19
Durieux Kitahara Machado YS Weiss '20

amplitude basics: more on LG covariant massive spinors

high-energy limit:

$$p = p^{I=1} + p^{I=2} \quad \equiv k + q$$

$$\text{HE:} \quad k = \mathcal{O}(E) \sim p \quad q = \mathcal{O}(m^2/E)$$

eg, only $\mathbf{p}]^{I=1} \sim p]$ survives; $\mathbf{p}]^{I=2} = q]$ subleading

—> HE limit: simply unbold spinor structures

use extensively in constructing massive EFT amplitudes:

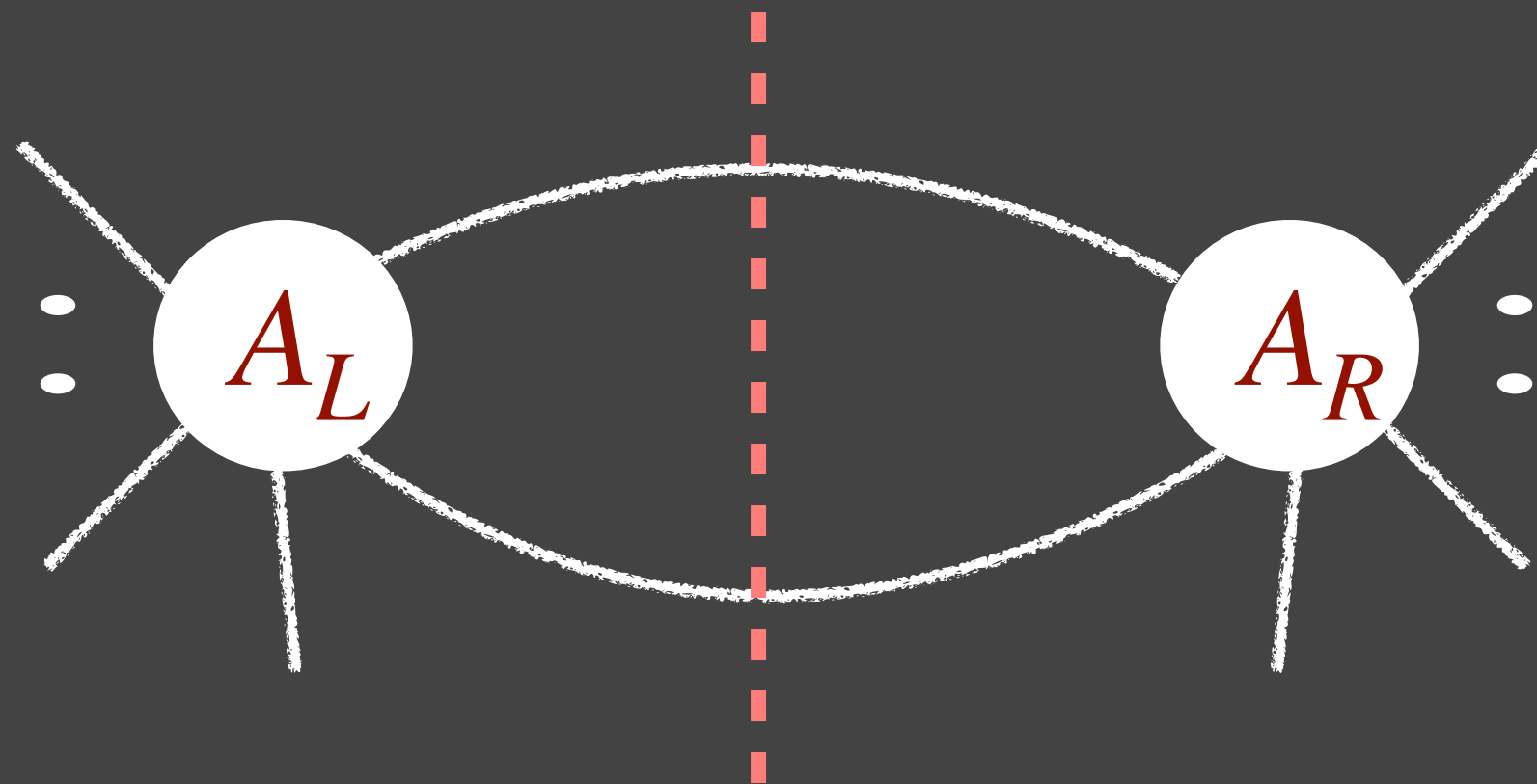
massless \longleftrightarrow massive amplitudes from (un)bolding

amplitude basics: factorization

tree: simple poles: residue =



loop: branch cuts:



+ generalized cuts with more propagators on-shell

“generalized unitarity”

amplitude basics: bootstrap

construct amplitudes recursively from the bottom up: w/out Lagrangian

LG: determines all 3-point amplitude

factorization \rightarrow higher point amplitudes

+ n-point contact-terms:
determined by LG + locality (no poles)

rediscover QFT: gauge theory massless + **massive**

massless SM: eg Accettulli-Huber De Angelis '21



example: massive degenerate spin-1 particles

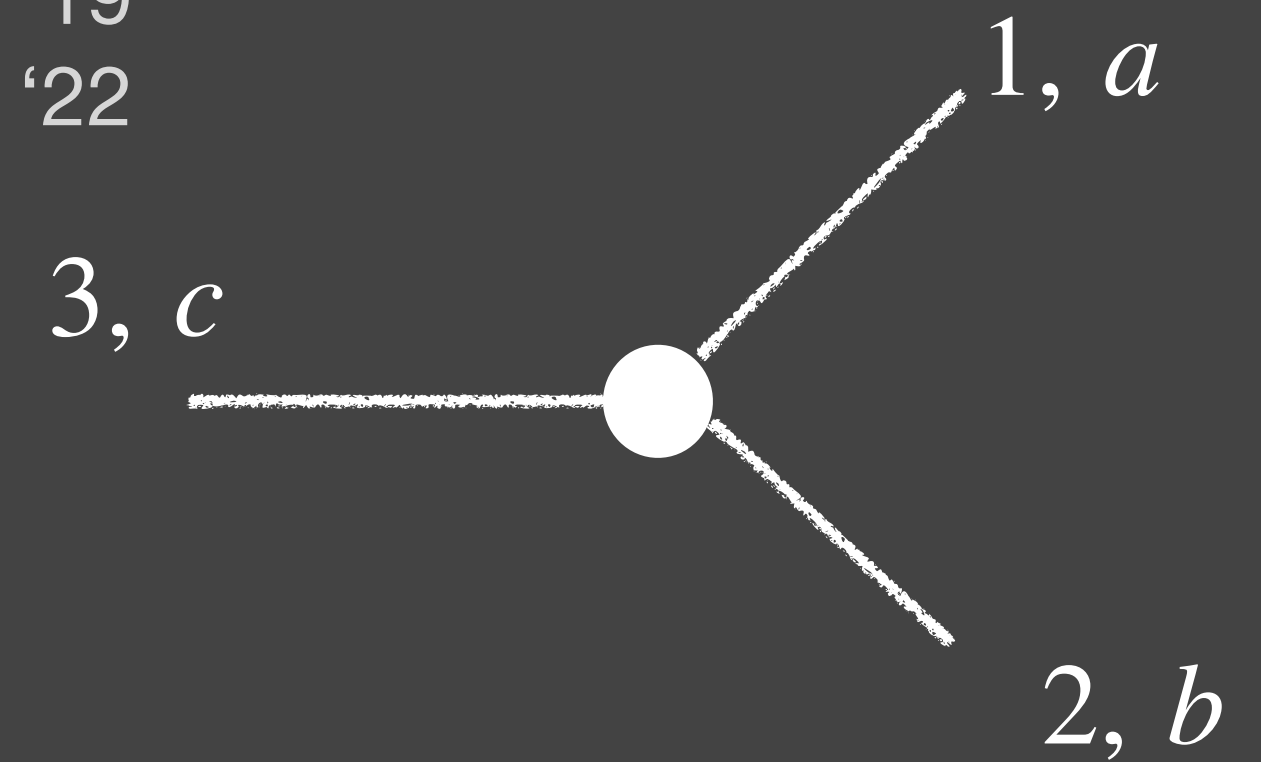
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Liu Yin '22

LG: most general amplitude:

$$C^{abc} (\langle 12 \rangle [23] \langle 31 \rangle + [12] \langle 23 \rangle [31] + \text{perm}) / M^2$$

$$+ C'^{abc} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle / \Lambda^2 + C''^{abc} [12] [23] [31] / \Lambda^2$$



example: massive degenerate spin-1 particles

Durieux Kitahara YS Weiss '19

Liu Yin '22

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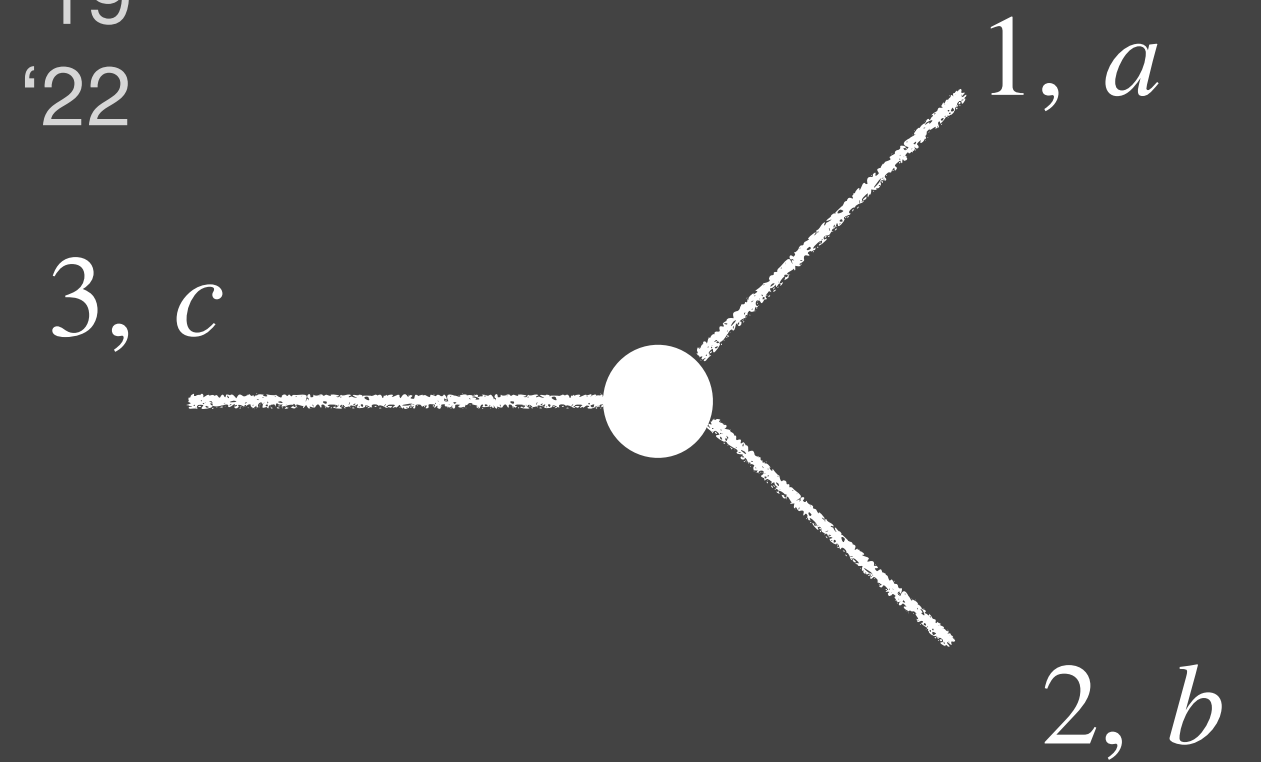
completely antisymmetric

$$+ C'^{abc} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle / \Lambda^2 + C''^{abc} [12] [23] [31] / \Lambda^2$$

$\rightarrow C^{abc}$ completely antisymmetric

structure constants!

+ factorization of 4-points on 3-points: Jacobi identity



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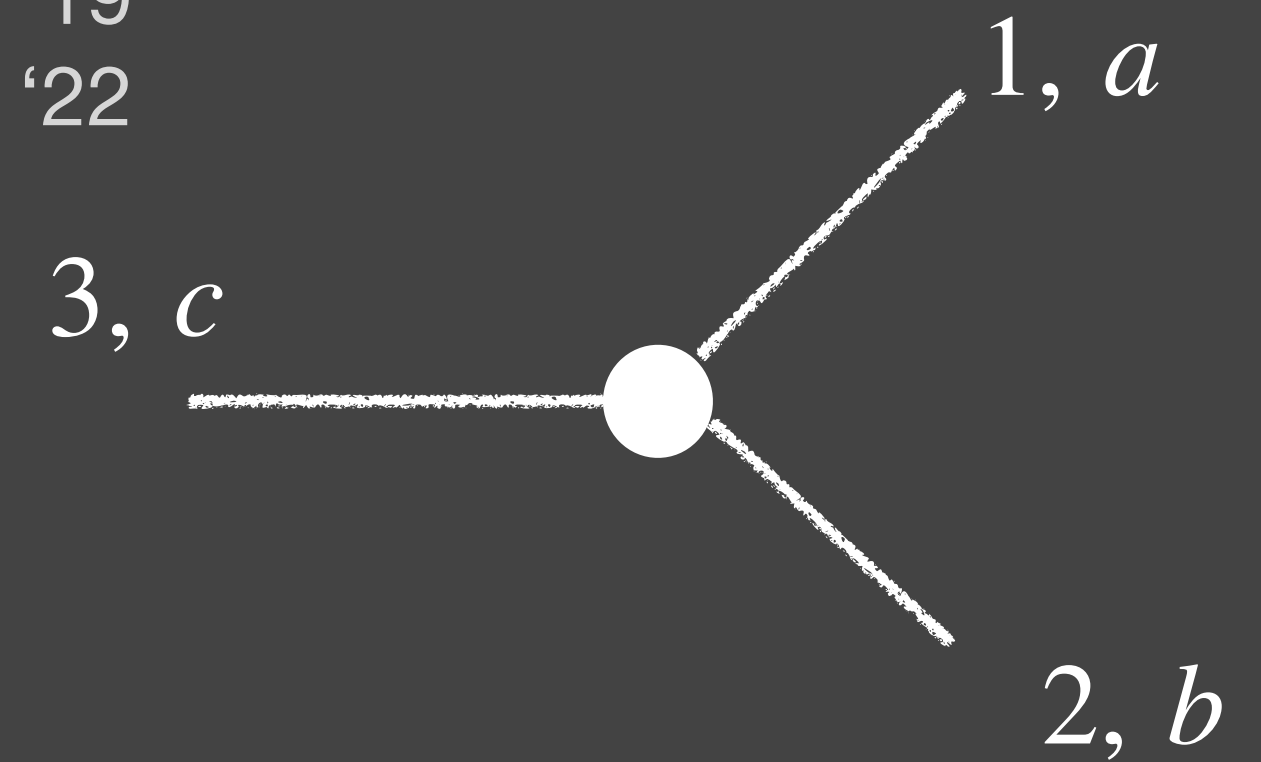
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vector self-coupling

Lie algebras

EFT applications

(massless) EFT applications (1)

- selection rules: explain zeros in

- matrix of anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Cheung Shen '15

Bern Parra-Martinez Sawyer '20

- interference of SM x EFT amplitudes (tree)

Azatov Contino Machado Riva '16

- derive anomalous dimensions of EFT operators (loop cuts & generalized cuts)

Barratella Fernandez von Harling Pomarol '20

Bern Parra-Martinez Sawyer '20

Jiang Ma Shu '20

De Angelis Accettulli-Huber '21

Barratella '22

...

(massless+ massive) EFT applications (2)

- count & construct bases of EFT operators:

operator (n-fields) \longleftrightarrow n-point contact term

determine from LG scaling + locality (no poles)

YS Weiss '18

Ma Shu Xiao '19

Remmen Rodd '19

Li Ren Shu Xiao Yu Zheng '20

Durieux Machado '20

...

also used in Henning Melia Murayama '15

will come back to this

amplitude



\mathcal{L}



amplitude



LHC

EFT applications (3)



work directly with amplitudes

bottom-up EFTs: parametrize our ignorance about the UV

bottom-up construction of amplitudes does just that

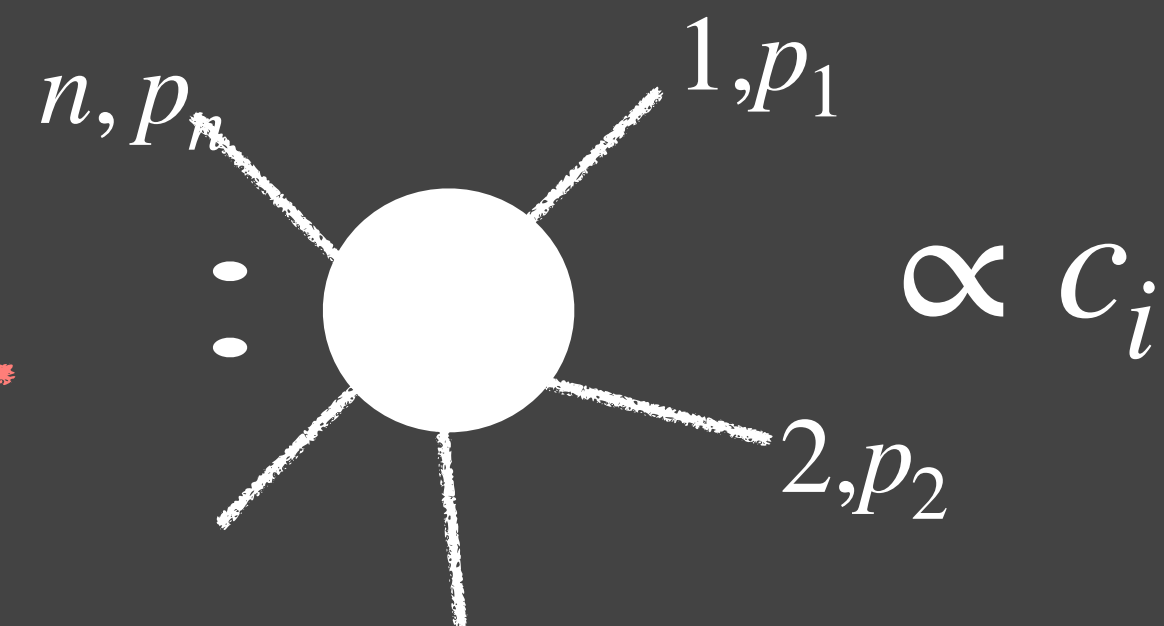
EFT via on-shell bootstrap

usually: start with SM fields: most general \mathcal{L}
consistent with symmetries (global, gauge)

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i(\phi_1, \dots, \phi_n)$$

on-shell: start with SM particles: most general \mathcal{A}
consistent with symmetries (global, gauge)

1-1 correspondence



EFT via on-shell bootstrap

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start with the massive (and massless) particles we know:
construct most general amplitudes



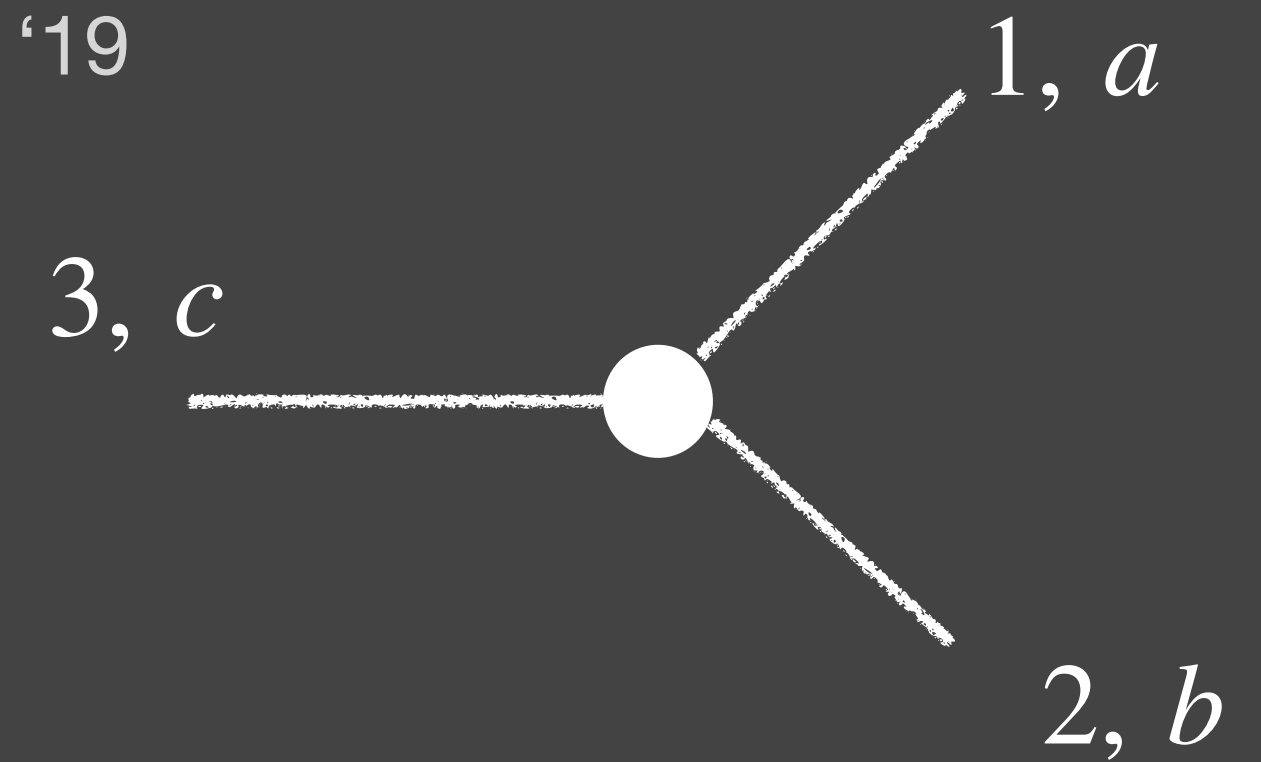
already saw an example:

bootstrap 3-pt of degenerate massive spin-1 particles

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LG: most general amplitude:

$$C^{abc} (\langle 12 \rangle [23] \langle 31 \rangle + [12] \langle 23 \rangle [31] + \text{perm}) / M^2 \\ + C'^{abc} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle / \Lambda^2 + C''^{abc} [12] [23] [31] / \Lambda^2$$



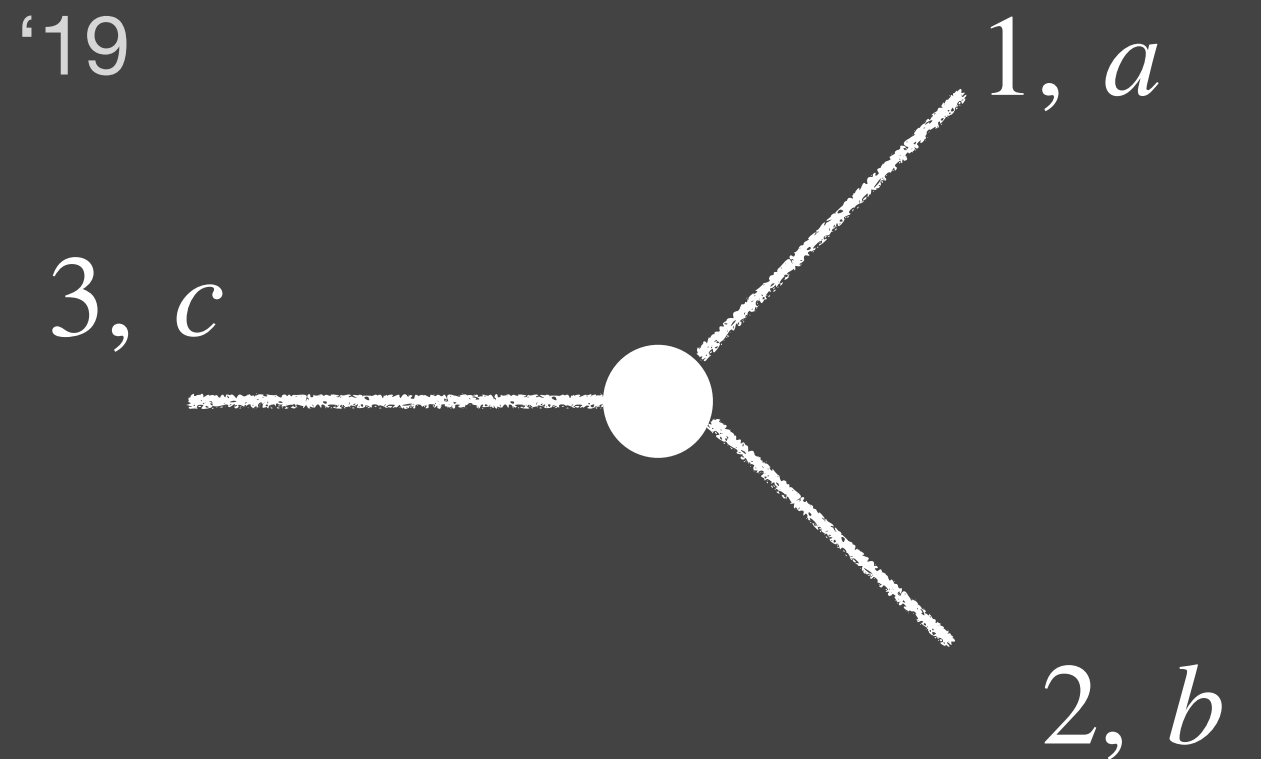
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Durieux Kitahara YS Weiss '19

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$$\text{EFT contributions} \longleftrightarrow \text{tr}(G^3) / \Lambda^2 \quad \text{tr}(G^2 \tilde{G}) / \Lambda^2$$

Λ^2 vs M^2 normalization based on high-energy behavior

EFT via on-shell bootstrap

YS Weiss '18

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...

$$\mathcal{A} = \frac{[\dots] \cdots \langle \dots \rangle}{\Lambda^\#} P \left(\frac{s_{ij}}{\Lambda^2} \right)$$

local: no poles

EFT via on-shell bootstrap

$$\mathcal{A} = \frac{[\dots] \cdots \langle \dots \rangle}{\Lambda^\#} P \left(\frac{s_{ij}}{\Lambda^2} \right)$$

carries LG weight; “stripped” of
all Lorentz invariants s_{ij}
“stripped contact term” SCT

EFT via on-shell bootstrap

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polynomial in Lorentz
invariants s_{ij}

subject to kinematical constraints,
eg, $s_{12} + s_{13} + s_{23} = \sum m^2$

derivative expansion

EFT via on-shell bootstrap

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derivative expansion

finding operator basis:

1. finding SCT basis (exploit massless limit; massless basis)
2. finding basis of polynomials in invariants *very simple!*

only briefly
today

EFT via on-shell bootstrap

YS Weiss '18

example: higgs + 3 gluons:

- factorizable + EFT (most general)

$$\mathcal{M}(h; g^{a+}(p_1) g^{b+}(p_2) g^{c+}(p_3)) = \frac{[12][13][23]}{\Lambda} \left[f^{abc} \left(-i \frac{m^4 g_s c_5^{hgg}}{s_{12}s_{13}s_{23}} + \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6} (s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \right) \right. \\ \left. \text{derivative expansion} + d^{abc} \frac{c'_{13}}{\Lambda^8} (s_{12} - s_{13})(s_{12} - s_{23})(s_{13} - s_{23}) \right] + \dots$$

- full kinematic behavior of amplitude
- going to dim-13: academic exercise: here see that nothing important beyond dim-7
- by-product: counting & classifying basis of EFT operators

EFT via on-shell bootstrap:

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example: most general fermion-fermion-vector-scalar amplitude (contact-term part):

$$\begin{aligned}\mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) &= \frac{C_6^{(1)}}{\bar{\Lambda}^2} [\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\bar{\Lambda}^2} [\mathbf{13}]\langle \mathbf{23} \rangle \\ &+ \frac{C_7^{(1)}}{\bar{\Lambda}^3} [\mathbf{312}]\langle \mathbf{13} \rangle + \frac{C_7^{(2)}}{\bar{\Lambda}^3} [\mathbf{321}]\langle \mathbf{23} \rangle \\ &+ \text{angle} \leftrightarrow \text{square}\end{aligned}$$

each coefficient: derivative expansion:

$$C_6^{(1)} = c_6^{(1)} + c_8^{(1,1)} \frac{\tilde{s}_{12}}{\bar{\Lambda}^2} + c_8^{(1,2)} \frac{\tilde{s}_{13}}{\bar{\Lambda}^2} + \dots \quad [\tilde{s}_{ij} \equiv 2p_i \cdot p_j]$$

massive EFT amplitude bases:

finding operator basis:

1. finding SCT basis (exploit massless limit; massless basis)
2. finding basis of polynomials in invariants *very simple!*

massive EFT amplitude bases:

systemized for massless EFTs + derivation of bases for SMEFT, GRSMEFT Durieux Machado '20

massive EFTs + 4-points for SM particle content: spins 0, 1/2, 1
heavily exploit massless bases & bolding-unbolding

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diagrammatic representation + general algorithm for n-points massless

Accettulli-Huber De Angelis '21

+ massive: using bolding
automated, some publicly available code

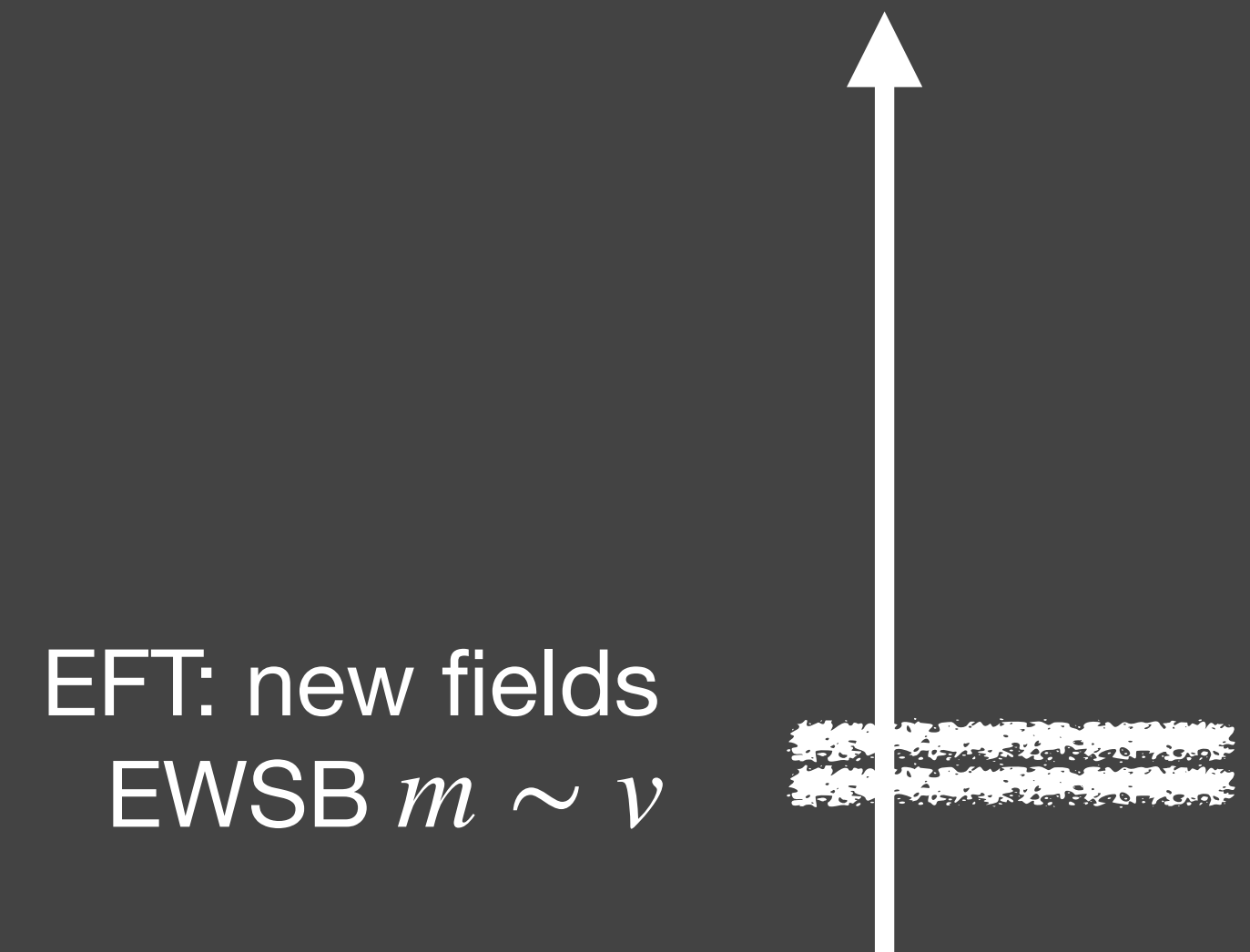
De Angelis '22

different approach: use EOM to work with just \mathbf{p} -spinors
obscures mapping to operators of given dim

Dong Ma Shu '21

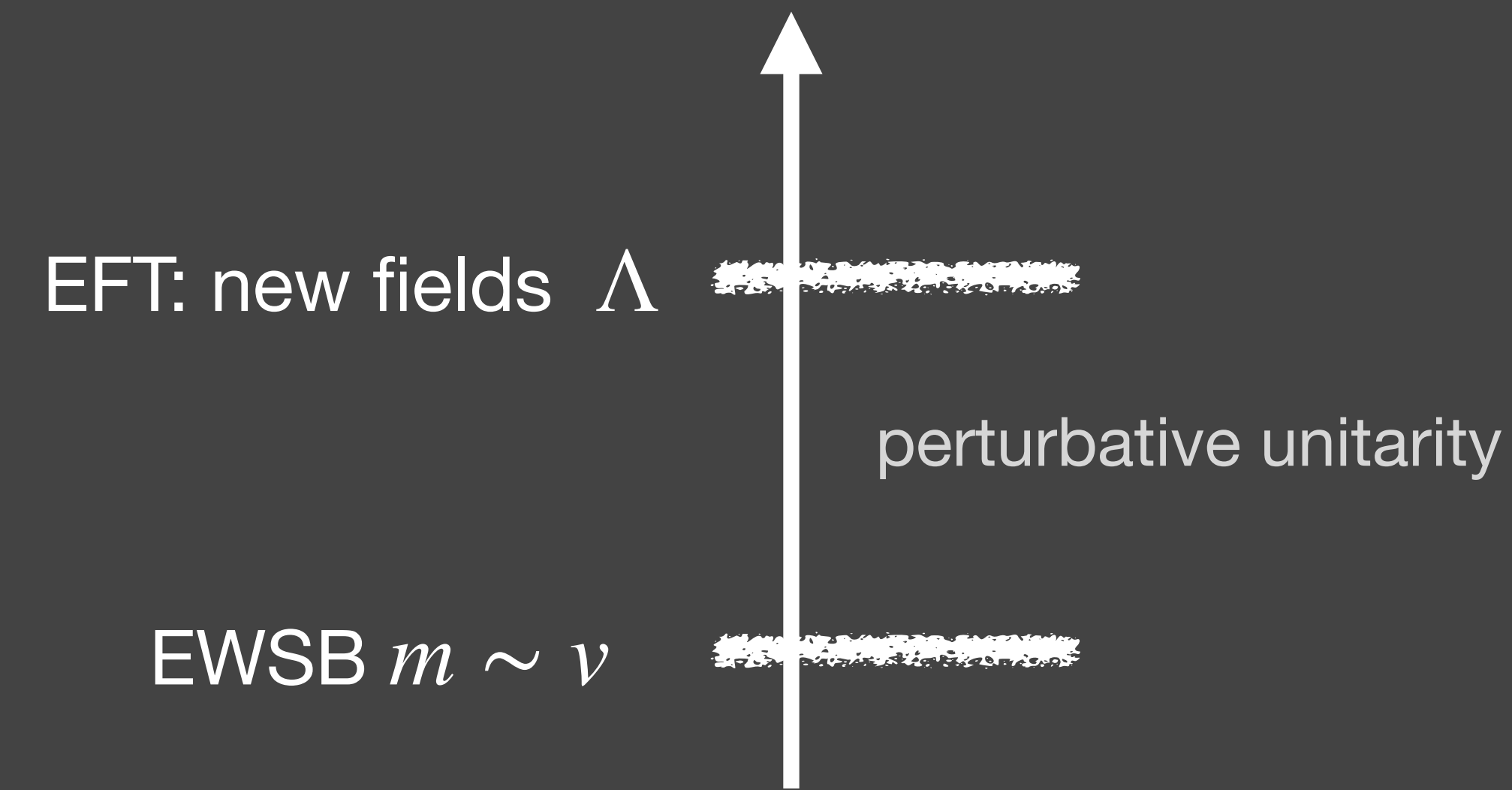
so: have bottom-up construction of most general amplitude:

but different possible bottom-up EFTs: which amplitude gives which EFT?

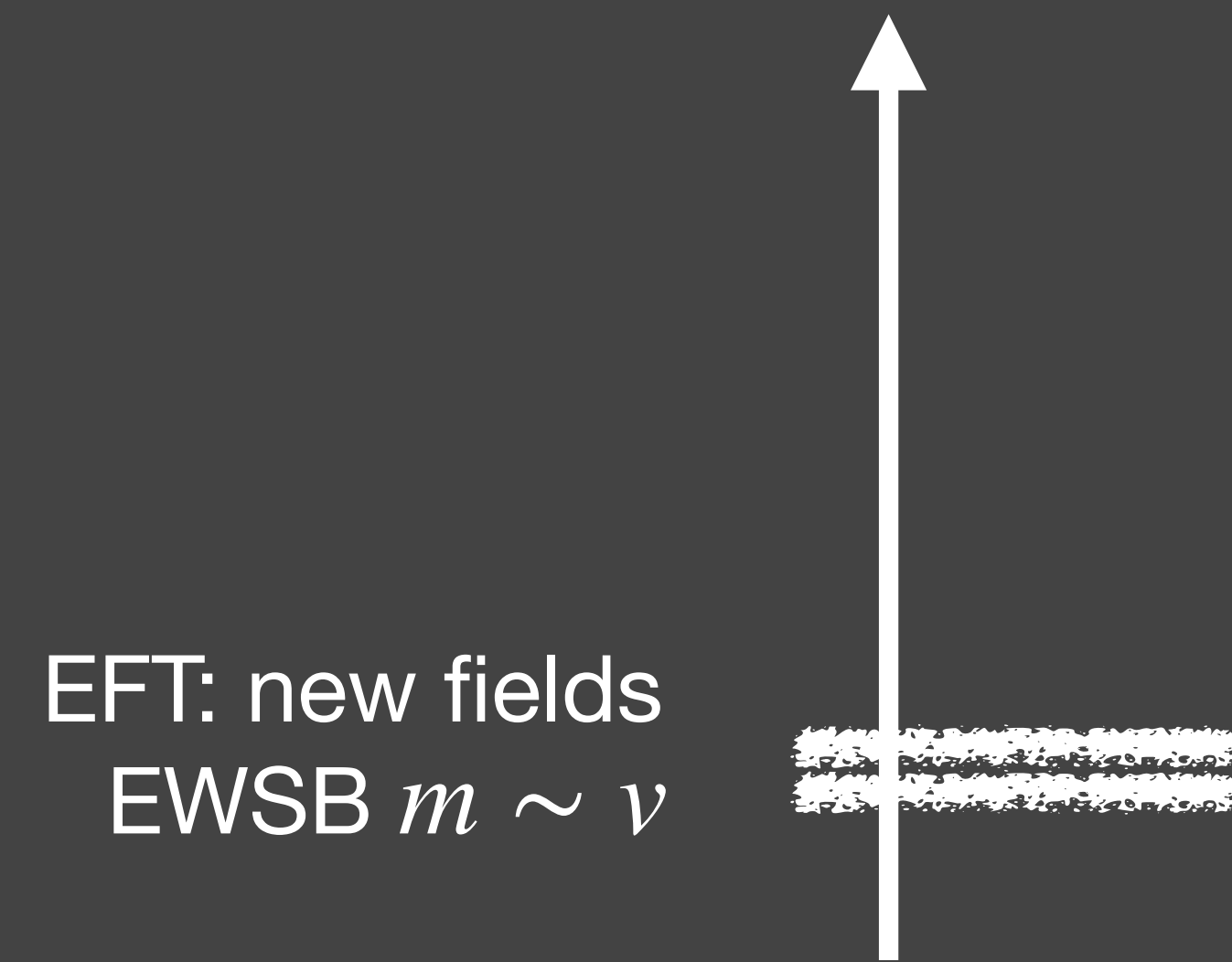


bootstrap: different possible EFTs

most general amplitude: but different possible bottom-up EFTs:



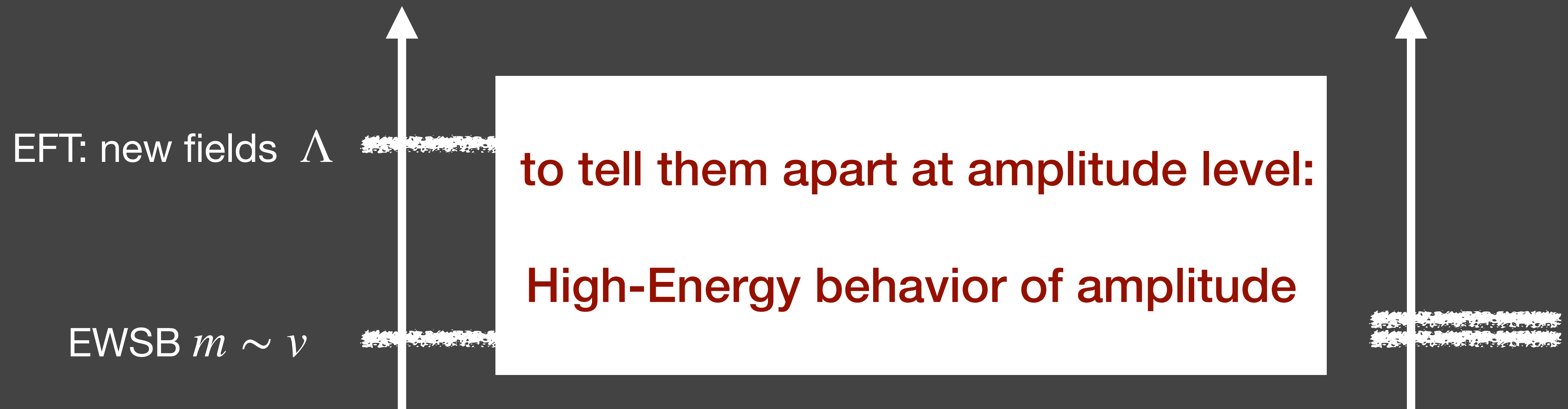
large hierarchy of scales possible
SU(2)xU(1) linearly realized = "SMEFT"



cutoff $\sim v$

bootstrap: different possible EFTs

most general amplitude: but different possible bottom-up EFTs:



$O(E^n)$ growth suppressed by powers of Λ (not mass)
heavy states decouple as Λ

bootstrap: different possible EFTs

2 approaches to getting massive SMEFT amplitudes:

- 1) **purely bottom-up**: construct amplitudes and require perturbative unitarity
guaranteed to recover SM above EW scale

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(imposing $SU(2) \times U(1)$ relations) Aoude Machado '19

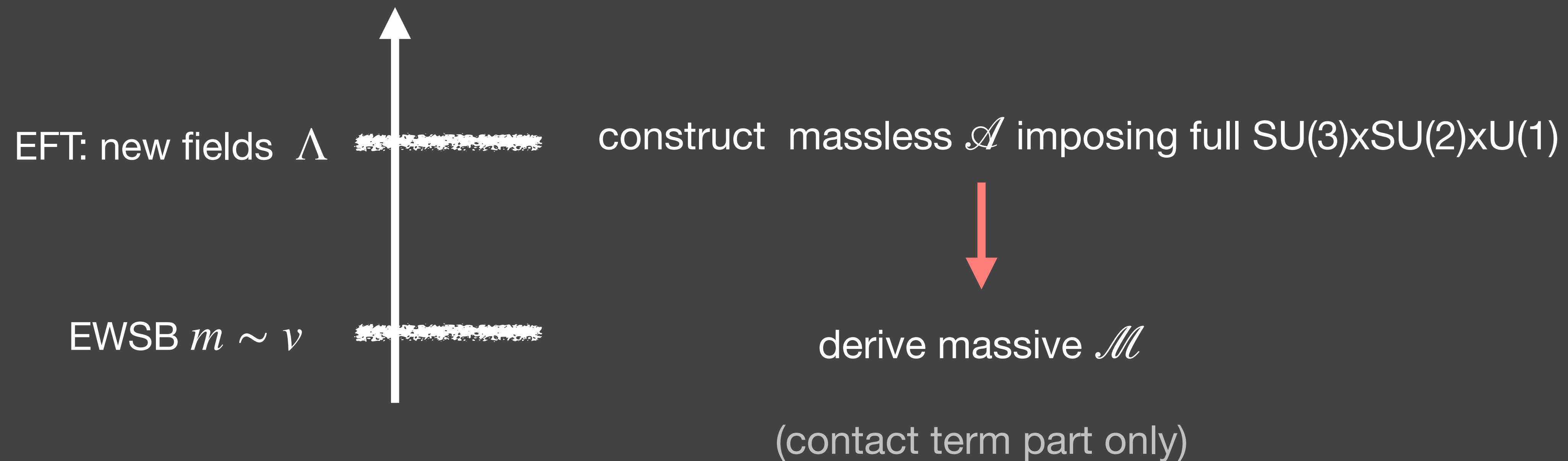
- 2) **top-down (on-shell)**: construct amplitudes of unbroken theory & “Higgs” them to get massive amplitudes \leftarrow *here*

Balkin Durieux Kitahara YS Weiss '21

bootstrap: different possible EFTs

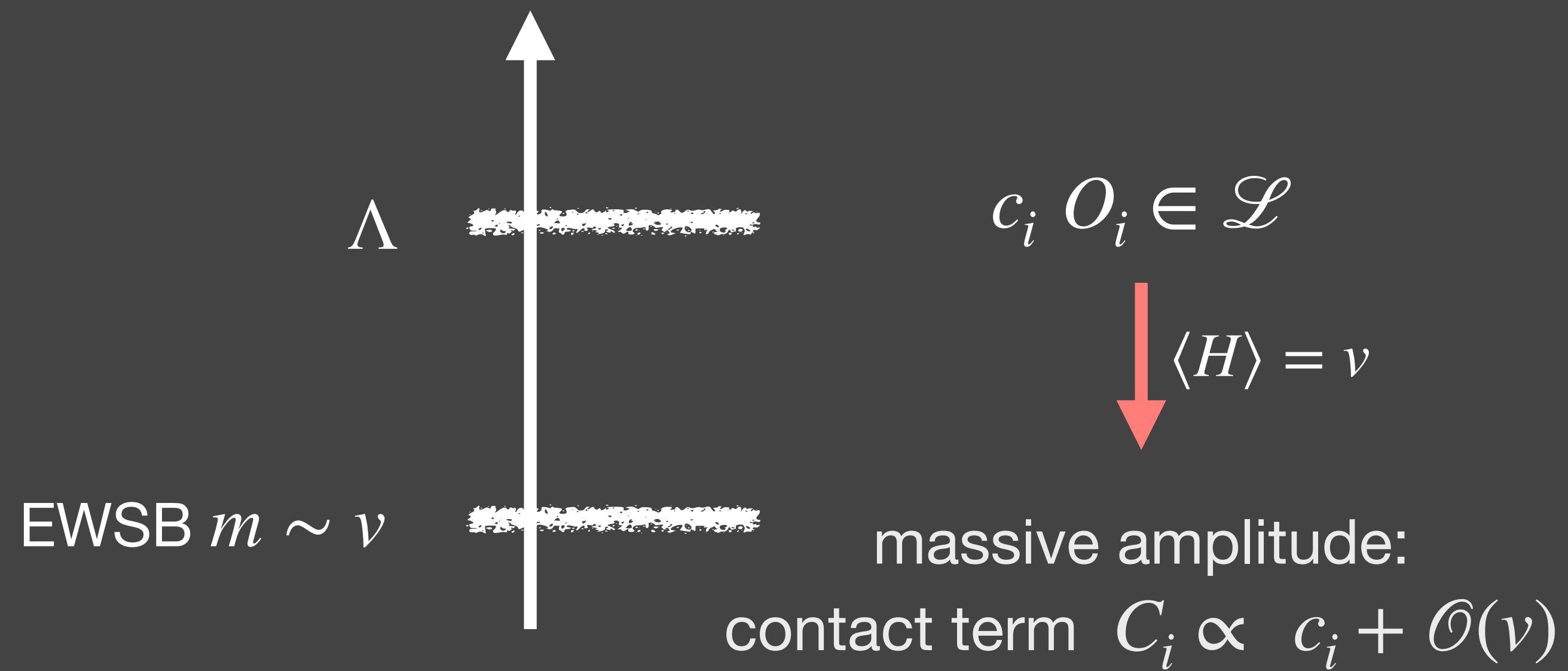
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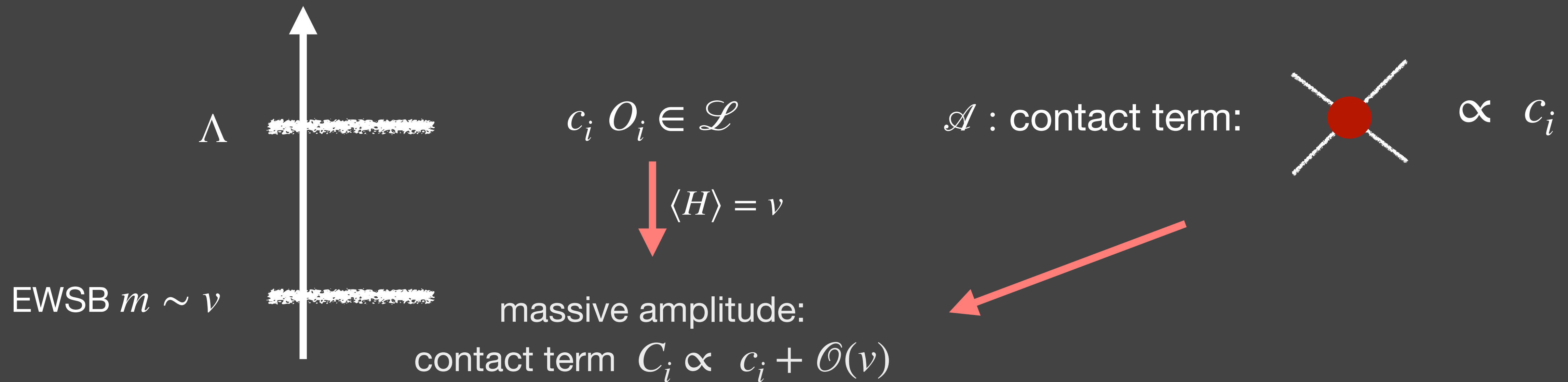
SMEFT amplitudes from on-shell Higgsing

Balkin Durieux Kitahara YS Weiss '21



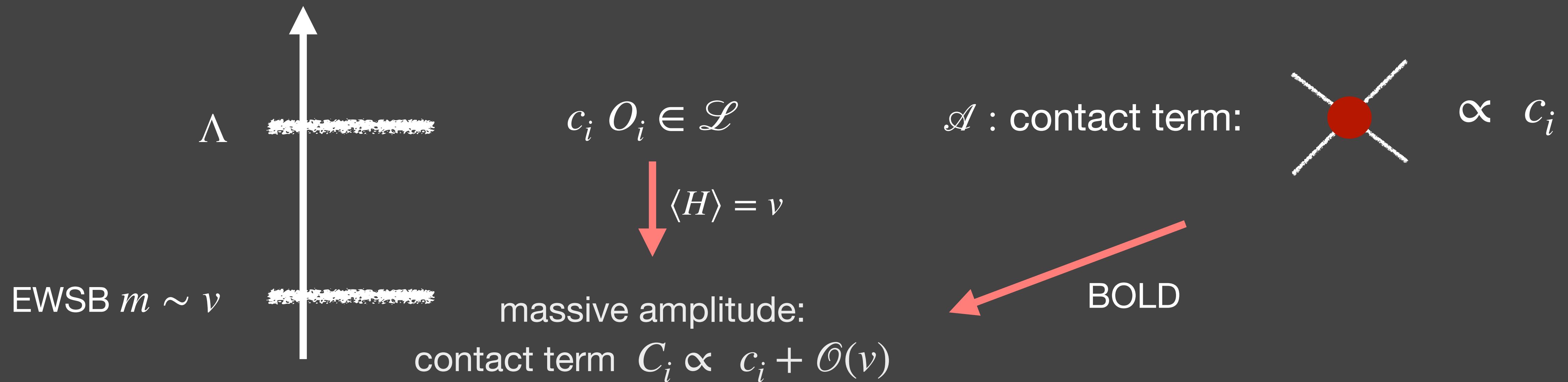
SMEFT amplitudes from on-shell Higgsing

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SMEFT amplitudes from on-shell Higgsing

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exploit simple relation between massive and massless amplitudes written in terms of massive spinor formalism:

take massless contact term: BOLD spinor structure \rightarrow massive contact term

SMEFT amplitudes from on-shell Higgsing

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let's make contact with the Higgs mechanism:

low-energy n -point contact term $\propto c_i$ gets contributions from:

massless n -point contact term $\propto c_i$

massless $n + n_H$ amplitudes with n_H **soft** Higgs legs $A_n \sim v^{n_H} A_{n+n_H}$

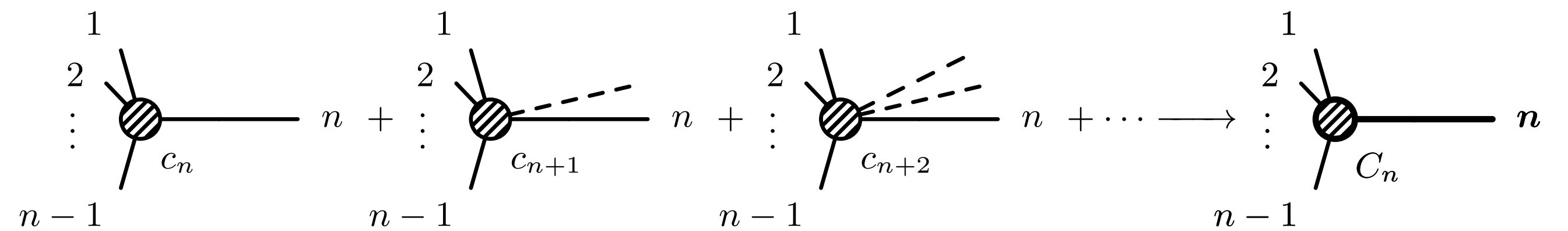
2 types of contributions to massive contact terms:

SMEFT amplitudes from on-shell Higgsing

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2 types of contributions to massive contact terms:

1) high-energy contact terms:
give $\mathcal{O}(v)$ corrections to C_i

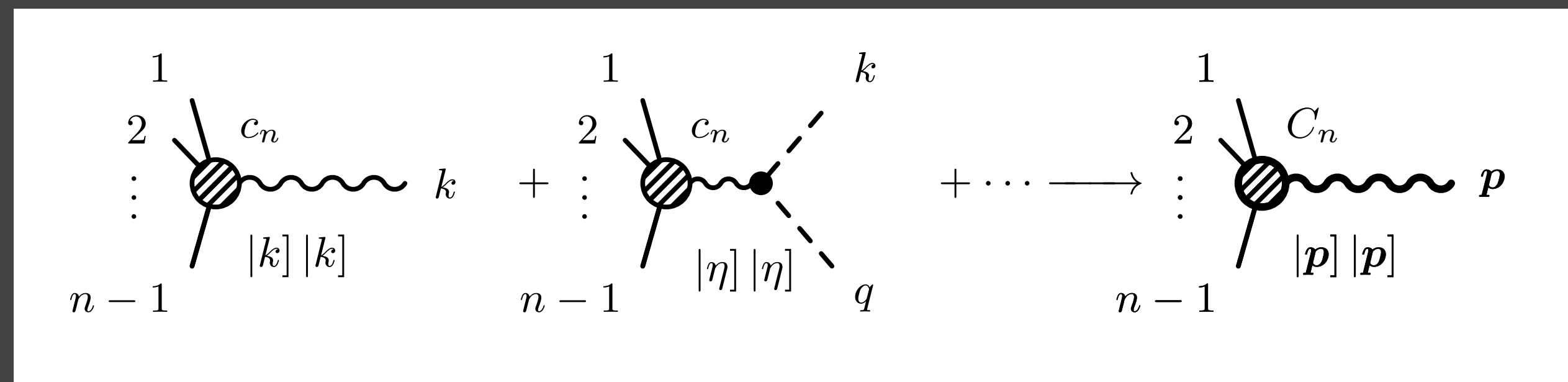


SMEFT amplitudes from on-shell Higgsing

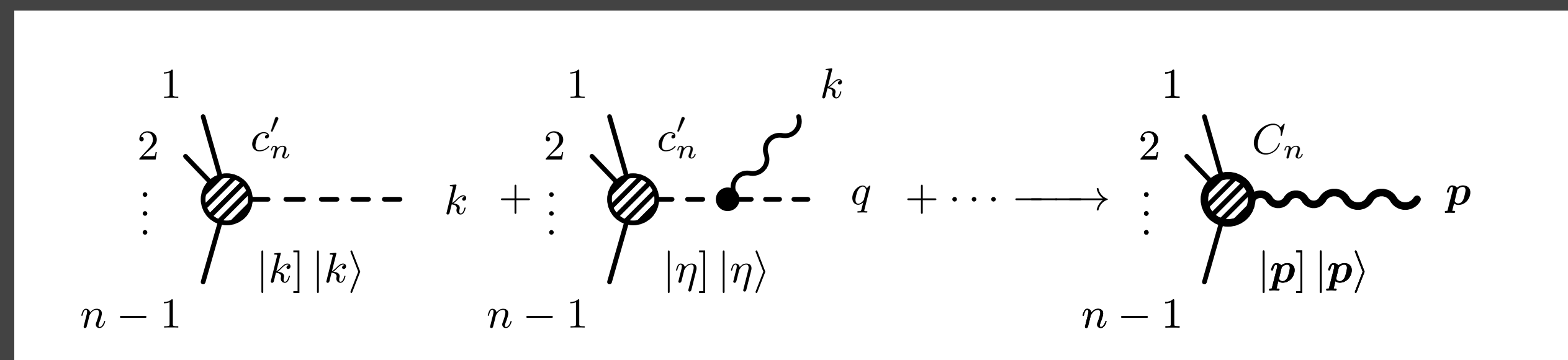
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2) high-energy factorizable pieces: give subleading pieces of massive (bold) spinor structures

$\mathbf{p}] \mathbf{p}] :$



$\mathbf{p}] \mathbf{p}\rangle :$

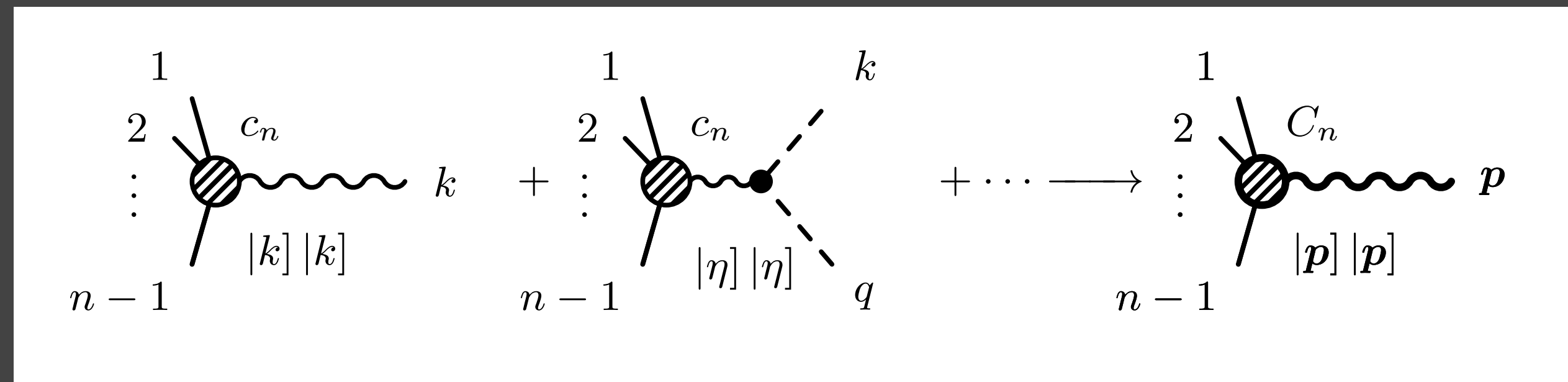


SMEFT amplitudes from on-shell Higgsing

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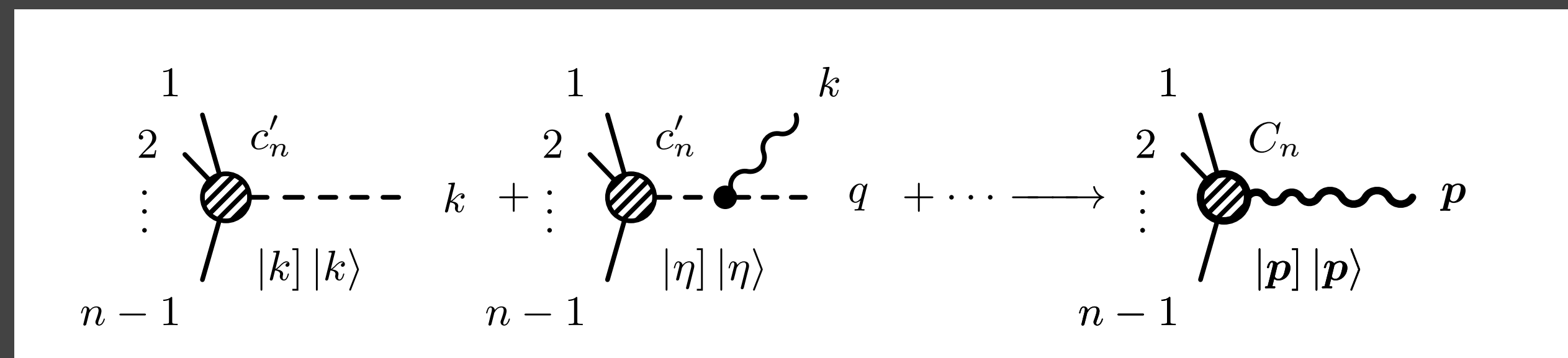
$\mathbf{p}] \mathbf{p}] :$



soft Higgs leg supplies
second lightlike
momentum to form
massive momentum

$$\mathbf{p} = k + q$$

$\mathbf{p}] \mathbf{p}\rangle :$



SMEFT amplitudes from on-shell Higgsing

Balkin Durieux Kitahara YS Weiss '21

$$M_n^{ct}(1, \dots, n) \leftarrow A_n(1, \dots, n) + v \lim_{q \sim v \rightarrow 0} A_{n+1}(1, \dots, n; H(q)) + \dots$$

SMEFT amplitudes from on-shell Higgsing

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in practice: to get massive contact-term: bold spinor structures = covariantize wrt massive LG

eg:

massless vector $i]i] \rightarrow \mathbf{i}]\mathbf{i}]$

massless scalar with momentum insertion $p_i = i]\langle i$

—> 1. massive *scalar* with momentum insertion \mathbf{p}_i

—> 2. massive *vector* $p_i = i]\langle i \rightarrow \mathbf{i}]\langle \mathbf{i}$ (longitudinal vector from Goldstone boson)

SMEFT amplitudes from on-shell Higgsing

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Liu Ma YS Waterbury, in progress

—> 4-point contact terms corresponding to the *SMEFT* at dim-6, 8

can compare with *most general* amplitudes

circumvents derivation of Feynman rules in broken phase: with everything given in terms of physical quantities

contact-terms are first (easy) step towards seeing Higgsing on-shell: ?extend to full amplitude

also derived all 3-points in a Higgsed U(1) toy model (from massless amplitudes
with up to two additional Higgs legs)

to conclude:

lots of developments via amplitudes

some: relearning QFT we know:

(different perspectives useful for anything of importance;
hopefully also get to physics we don't know..)

eg: unbroken gauge symmetry from interactions of spin-1 particles (Lorentz, LG)

similarly for broken gauge symmetry: Higgsing as bolding = covariantizing wrt massive LG

(IR unification of massless amplitudes Arkani-Hamed et al '17)

one theory we don't know: underlying theory of EWSB: EFT parametrization

bootstrapping amplitudes: can capture most general EFTs/SMEFT (no field redefinitions etc, purely physical)

model-independent parametrization of LHC observables relating purely physical quantities:

help identify useful observables

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