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KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Quantum Field Theory Concepts

Hitoshi Murayama (Berkeley, Kavli IPMU)

DESY Theory Workshop, September 30, 2022

THE UNIVERSITY OF CALIFORNIA BERKELEY 1868 LET THERE BE LIGHT
BERKELEY CENTER FOR THEORETICAL PHYSICS
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HIGGS, FLAVOR AND BELOW

DESY Theory Workshop
27 – 30 September 2022 at DESY Hamburg, Germany

Plenary Talks

P. Agrawal (Oxford U.)
A. El-Khadra (UIUC)
A. Greljo (Bern U.)
S. Hansmann-Menzemer (U. Heidelberg)
G. Heinrich (KIT)
S. Höche (Fermilab)
M. Kado (La Sapienza)
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G. Landsberg (Brown U.)
J. Lindert (U. of Sussex)
M. Mangano (CERN)
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H. Murayama (IPMU & Berkeley)
F. Riva (Geneva U.)
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D. Shih (Rutgers U.)
A. Vladimirov (UC Madrid)
K. Vos (U. of Maastricht)
G. Weiglein (DESY)
M. Worek (RWTH Aachen)

DESY Heinrich-Hertz-Lecture 28 September 2022

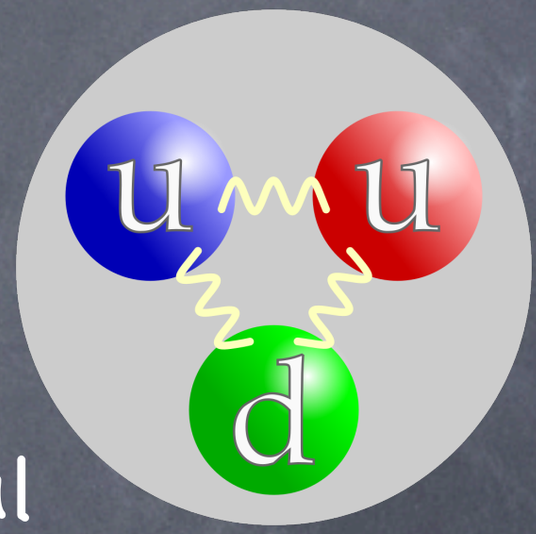
Hitoshi Murayama (IPMU & Berkeley)

references

vector-like
chiral
condensed matter

- HM, 2104.01179, PRL
- Csáki, HM, Telem, 2104.10171, PRD
- Csáki, HM, Telem, 2105.03444, PRD
- Csáki, Gomes, HM, Telem, 2106.10288, PRL
- Csáki, Gomes, HM, Telem, 2107.02813, PRD
- HM, Noether, Varier, 2111.09690
- Leedom, HM, Suter, in preparation
- Csáki, Gomes, HM, Noether, Telem, Varier, in preparation
- Kondo, HM, Sylber, 2209.09287
- HM and Wang, in preparation

I felt cheated by QCD



- When we first learn about quarks, we get told
  - Here, look at this proton.
  - There are actually colorful, beautiful quarks inside.
- But you can never see them, you can never take them out.
- Nonetheless, believe, they are in there!
- Who would buy that? Internet Scam?

Dear friend,
I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds ($11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
http://news.bbc.co.uk/1/hi/world/europe/859479.stm
I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

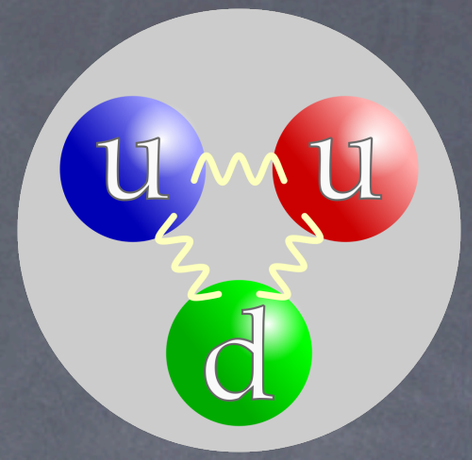
This fund ($11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.
Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

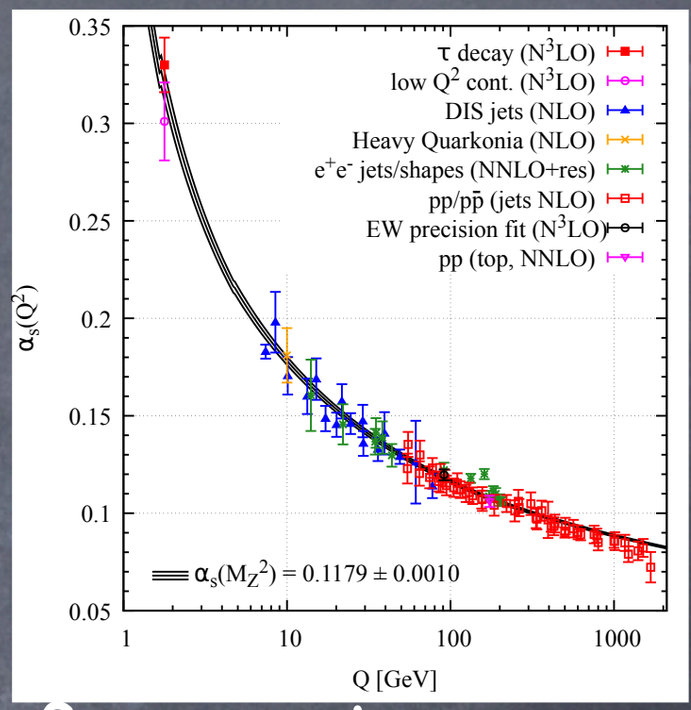
Kindly update me with the
following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name.....
2. Your age/sex .....
3. your occupation .....
4. Your residential address .....
5. Your nationality .....
6. Your private phone number .....
7. Your fax number .....

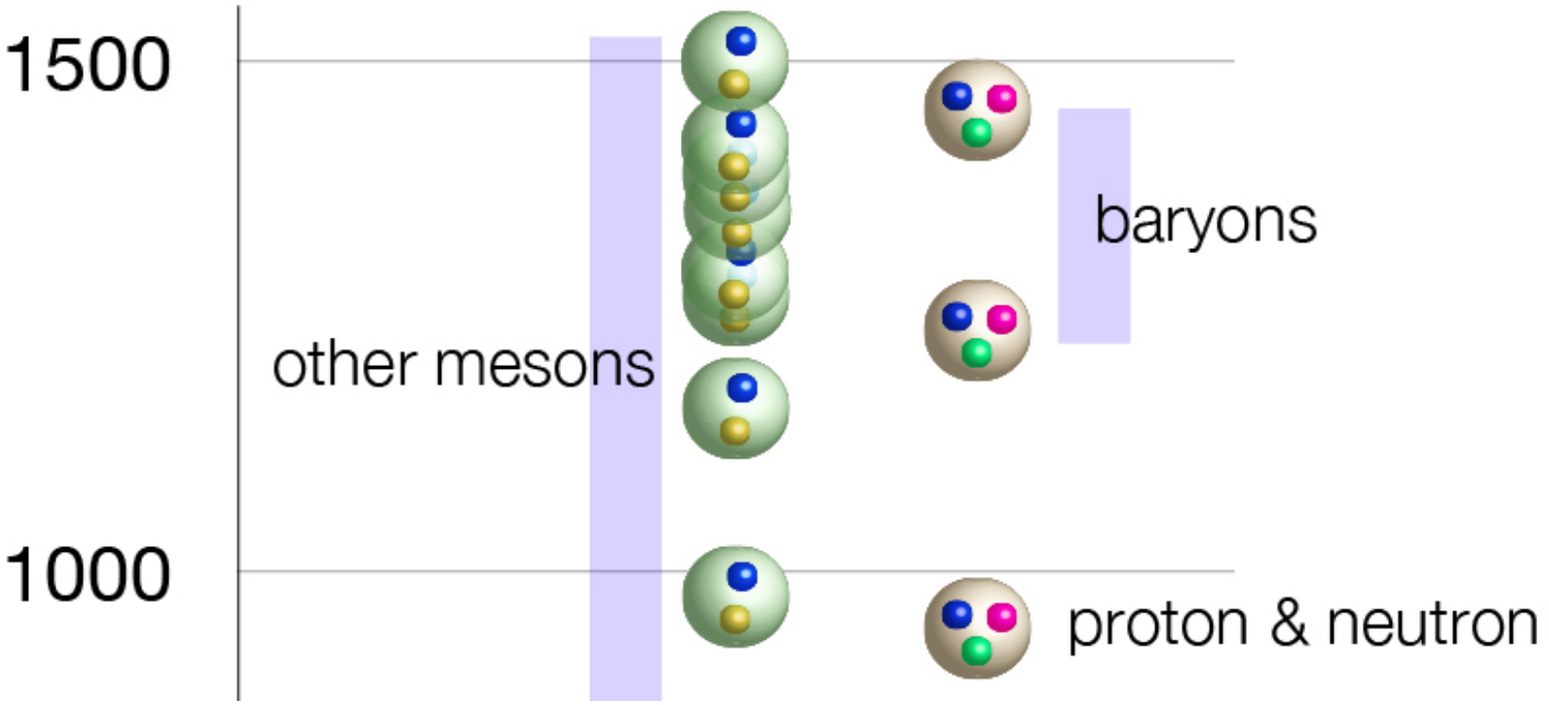
I will be waiting for your response.
Thanks for your understanding.
Have a great day.
Yours.
Andre Ouedraogo

Can we solve QCD?

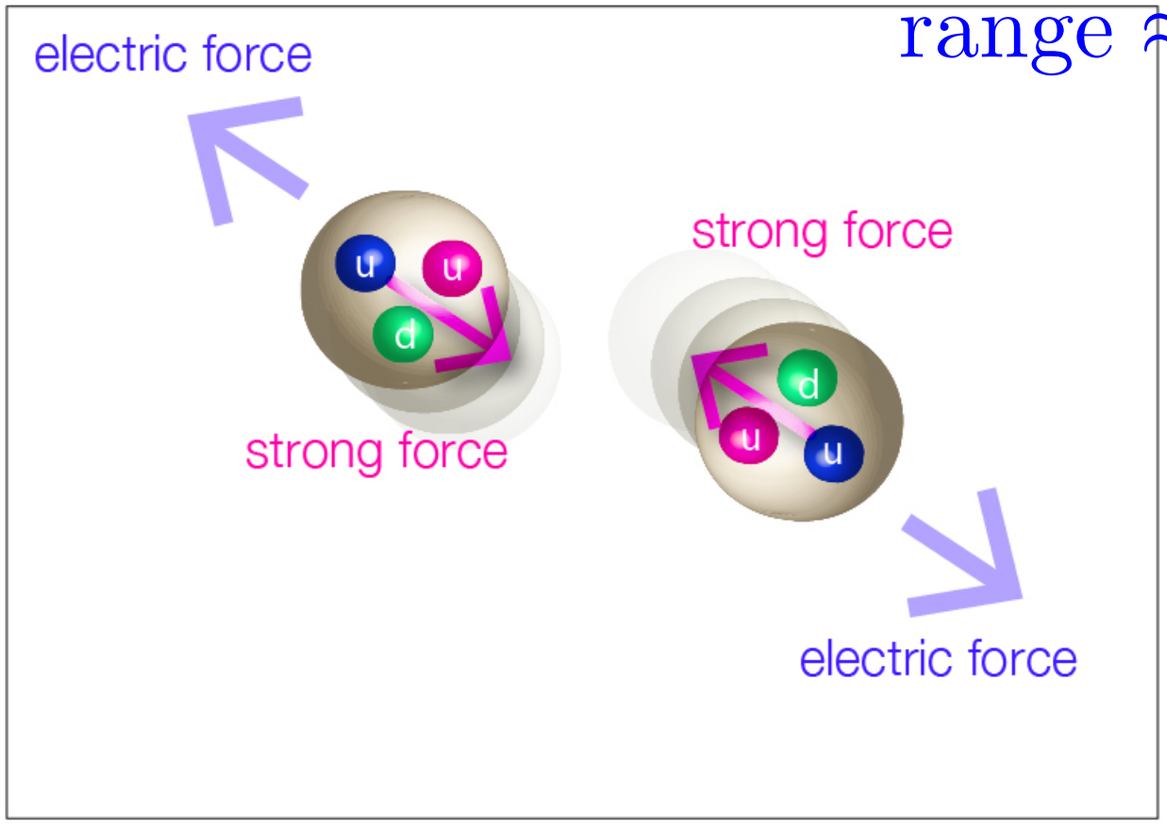




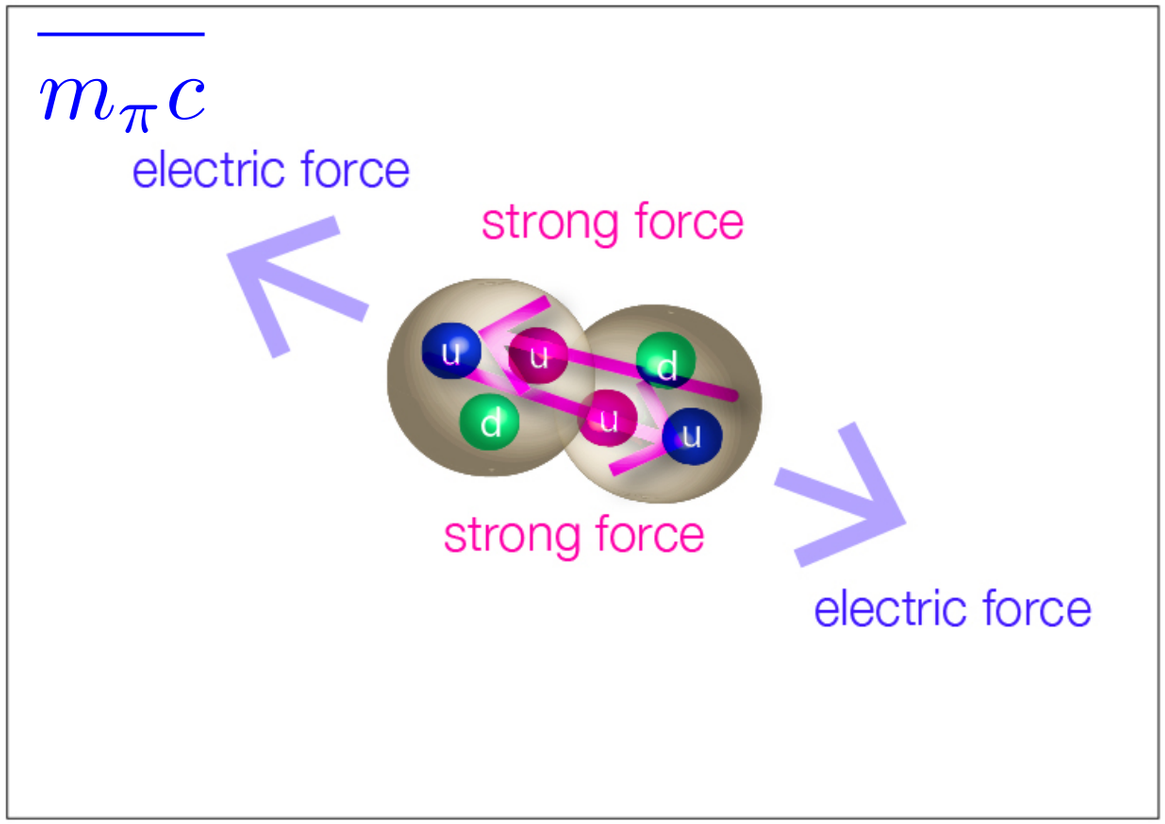
- When we first learn about quarks, we get told we can never see them
  - Internet Scam?
  - Confinement!
  - $\beta(g^2)<0$ and asymptotic freedom
  - only suggestive, doesn't prove confinement
- Another puzzle: proton and pion are made of the same quarks
  - why pion ≈ massless ≪ proton?
- very mysterious!
- Very little known about chiral gauge theories



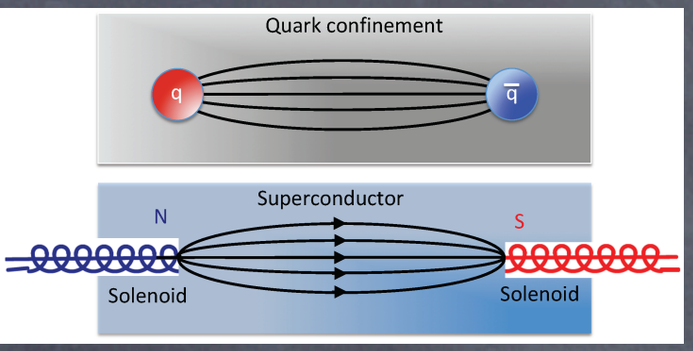
$$\text{range} \approx \frac{\hbar}{m_\pi c}$$



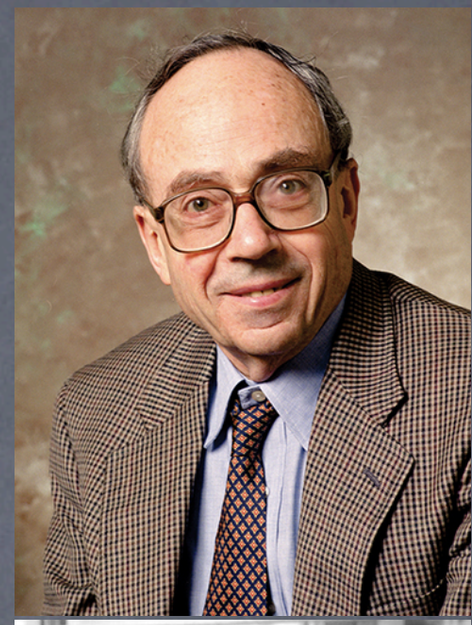
If pions are heavy



With real light-weight pions



Feeling better



- Qualitative picture makes us feel better
- Confinement
  - dual Meißner effect (Mandelstam)
  - assume monopole condensation
  - quarks confined by electric flux tube
- Chiral symmetry breaking (Nambu)
  - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
  - assume broken to $SU(N_f)_V \times U(1)_B$
  - pion = Nambu-Goldstone boson = massless
- Are there really monopoles?
- How do we know $\langle \bar{q} q \rangle \neq 0$?

Feeling even better but not there yet

- Progress in understanding QCD
- Confinement (Seiberg-Witten)
  - N=2 SYM has Coulomb branch $u=\mathrm{Tr}\Phi^2$
  - singularities = massless monopole/dyon
  - N=1 perturbation $W = \mu u - (u-\Lambda^2)M^+M^-$
  - $M^+=M^-=\sqrt{\mu}\neq 0$: monopole condensation!
  - can further perturb to N=0 with $m_\lambda \neq 0$
- Chiral symmetry breaking
  - N=2 doesn't have χS $W = \sqrt{2}\tilde{Q}_i \Phi Q^i$
  - N=1 (Seiberg) has too unusual phases

N_f

IR free

$3N_c$

IR fixed point

$\frac{3}{2}N_c$

free magnetic

$N_c + 2$

no χSB moduli

$N_c + 1$

N_c

$N_c - 1$

no ground state

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m=0

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χSB

Λ

N_f

IR free

$\frac{11}{2}N_c$ ← Banks-Saks fixed point

$\beta \sim O(1)g^4 - O(N_c)g^6$

$3N_c$

χSB

$\frac{3}{2}N_c$

???

χSB

χSB

0

m=∞

Main message

- Supersymmetric QCD is "solved" exactly by Seiberg in the 90s
  - but far removed from the real world
- Adding small SUSY breaking via anomaly mediation still allows for exact solution
- derive non-perturbative behavior analytically
  - chiral symmetry breaking
  - monopole condensation and confinement
- can solve chiral gauge theories exactly, too!
- Sometimes phase transitions, but local minima are still useful

Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
  - decouple gauginos and squarks!
  - SUSY breaking m_λ and $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- SUSY breaking effects on composites may be non-trivial
- Anomaly mediation!

Randall, Sundrum; Giudice, Luty, HM, Rattazzi (1998)

Sequestering

supergravity

$m_{3/2} = e^{K/2}|W|$

SUSY breaking

$V = e^K \left(|F|^2 - 3\,|W|^2 \right)$

no interaction

gauge theory

$m = \dfrac{W}{M_{Pl}^2}$

AMSB Summary

- Tree-level piece on dimensionful parameters

$$V_{\rm AMSB} = -m\left(\phi\frac{\partial W}{\partial\phi} - 3W\right)$$

  - loop-level piece from running

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i+\gamma_j+\gamma_k)m_{3/2}$$

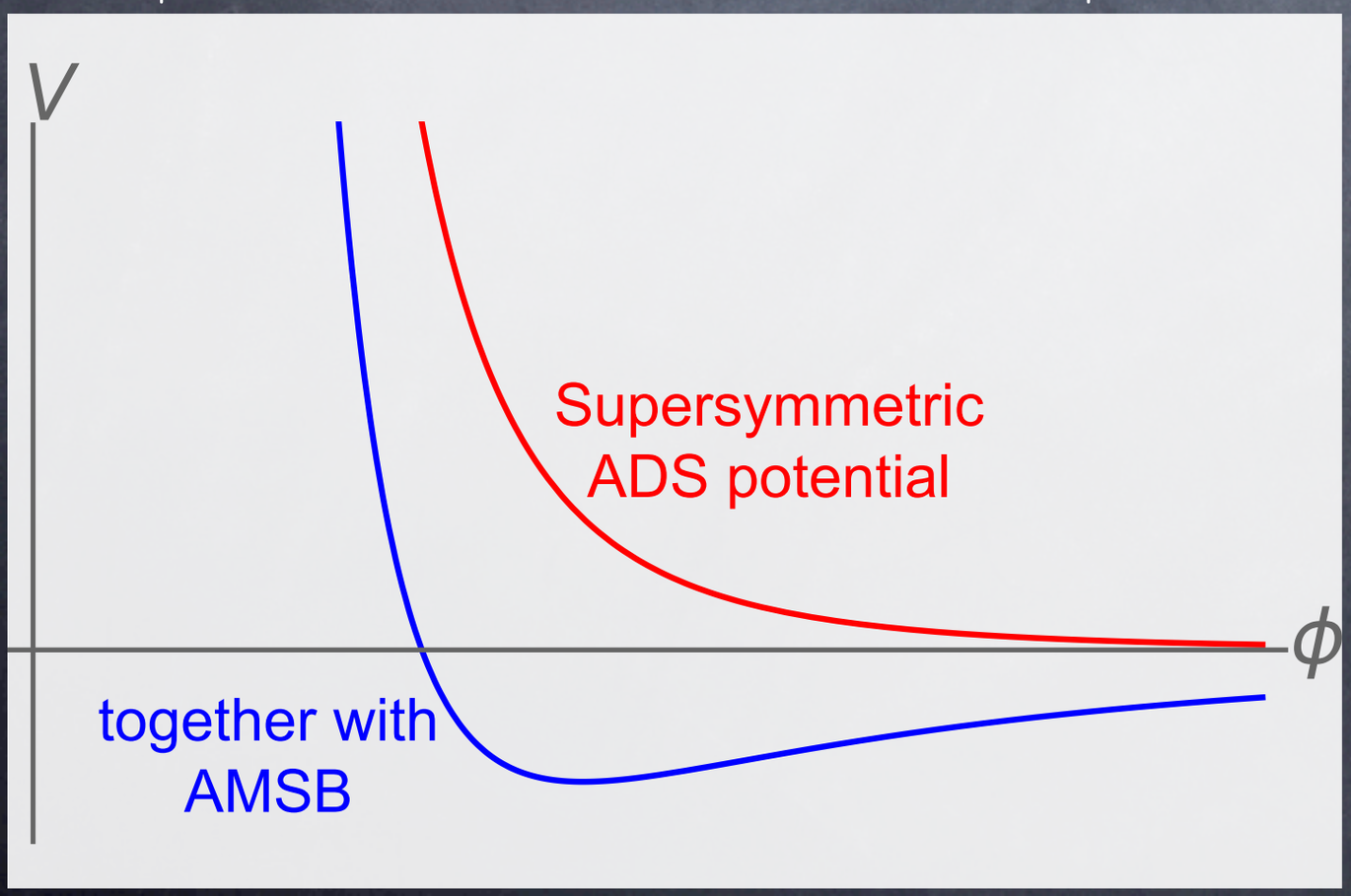
  - determined only by physics at the energy scale of interest
  - UV insensitivity!

$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$

$$V = \left|2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$



$$M_{ij} = \Lambda^2 \left(\frac{4N_f(N_c+N_f)}{3N_c - N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c} \delta_{ij}$$

$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

χSB! Proving Nambu

mesino loop→WZW term

N_f=1 special

no NGB, gapped

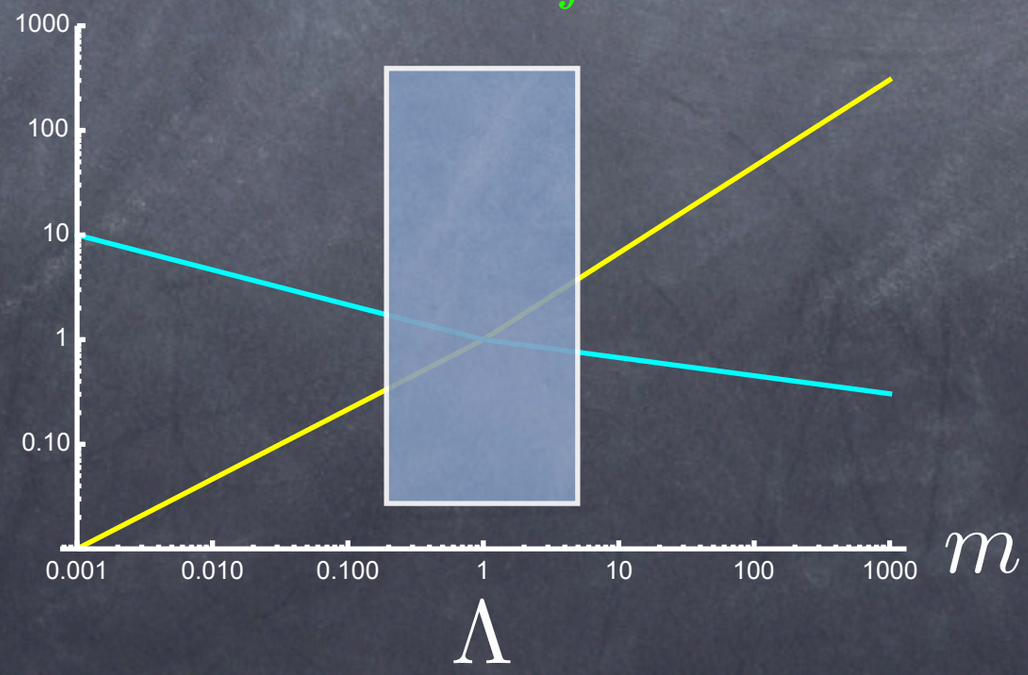
fermion bilinear

$$M^{ij} = \tilde{q}_L^{i*} \tilde{q}_R^j + \theta^2 \bar{q}_L^i q_R^j$$

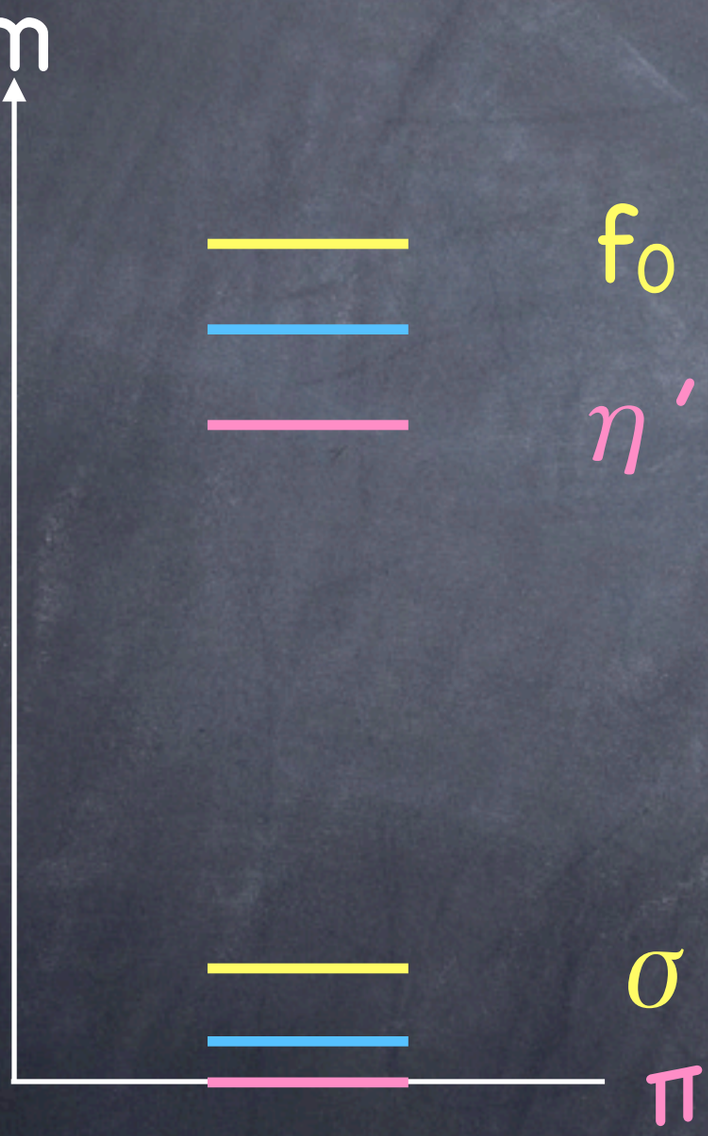
$$\tilde{q}_R^{i*} \tilde{q}_L^j \sim \left(m^{N_f - N_c} \Lambda^{3N_c - N_f} \right)^{1/N_c}$$

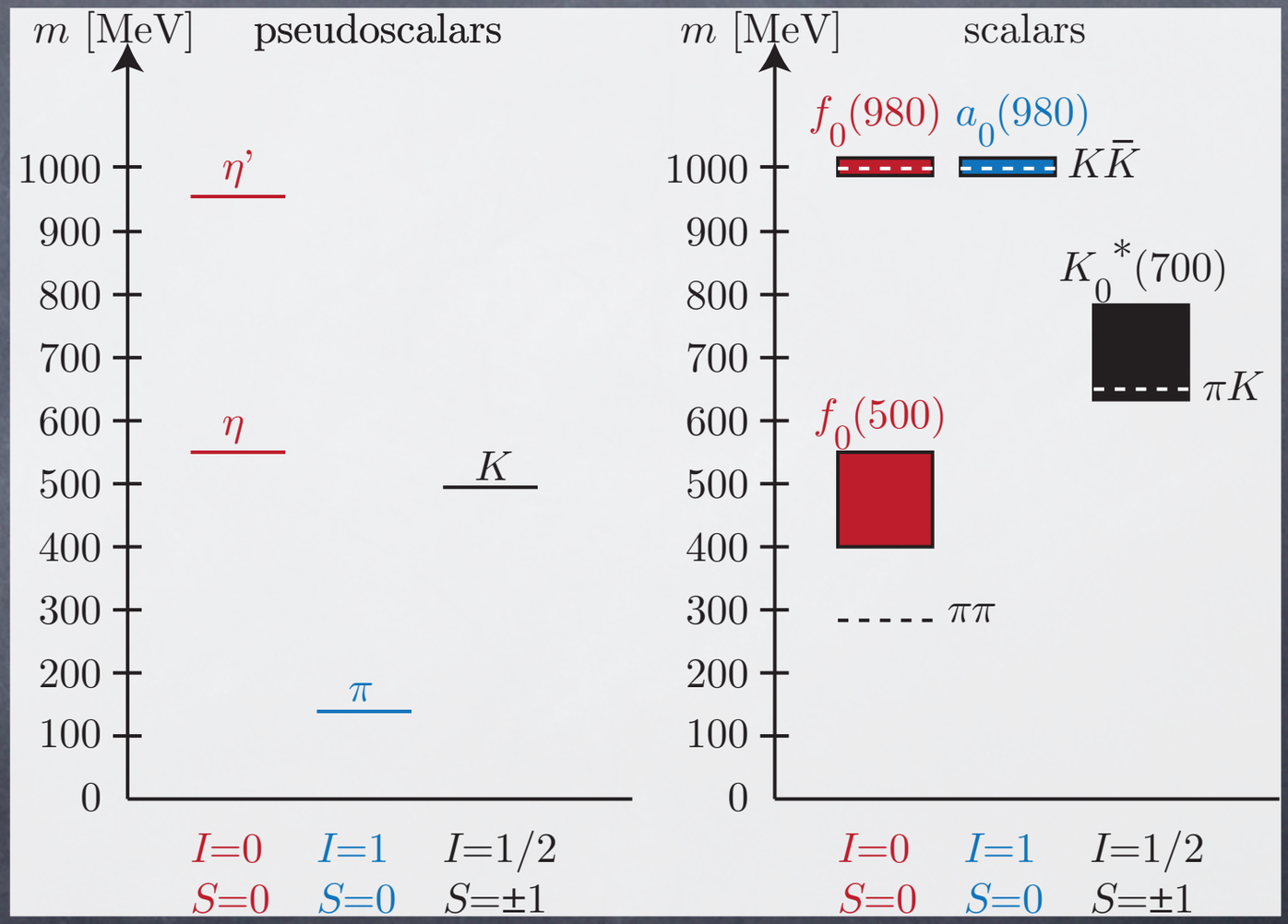
$$\bar{q}_R^i q_L^j \sim m \left(m^{N_f - N_c} \Lambda^{3N_c - N_f} \right)^{1/N_c}$$

$$N_c = 3, N_f = 2$$



Light spectrum





— fermions

confinement vs screening

- We've derived χSB in SU(N_c) QCD
  - it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law
- SO(N_c) QCD with quarks in vector rep
  - cannot screen Z_2 center (e.g. spinor rep)
  - rigorous definition of confinement
  - can we see an interplay with χSB?

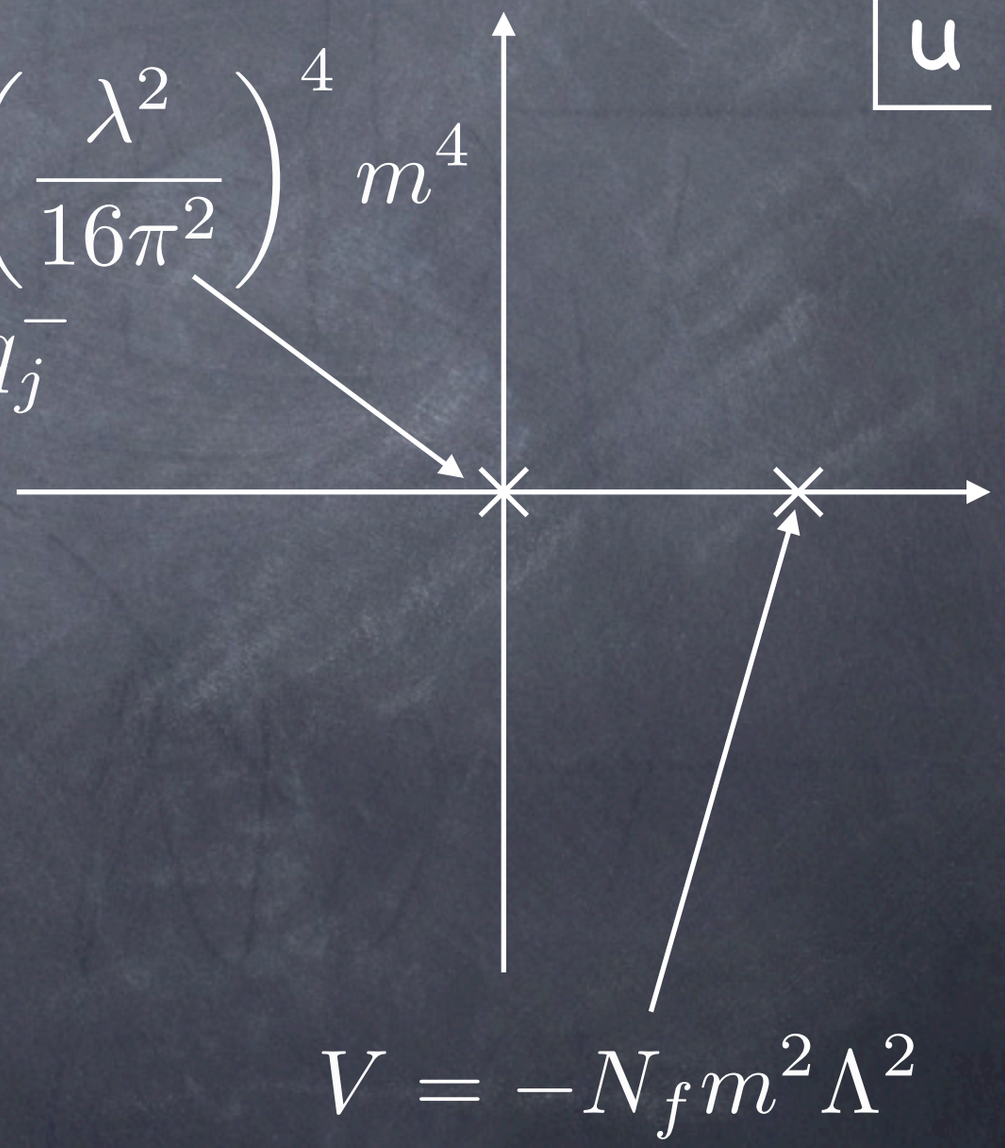
$N_f = N_c - 2$

$$Q = \begin{pmatrix} v & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & v \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{pmatrix}$$

- for $M^{ij}=Q^iQ^j\neq 0$ with rank $M=N_f$, $SO(N_c)$ is broken to $SO(2)$
- 't Hooft Polyakov monopoles!
- Coulomb branch $u = \det M$
- two singularities
- $u = \det M = 0$
  - dyons: $q_i^{\pm}$
- $u = \det M = \Lambda^{2N_f}$
  - monopoles:
    $W = (u - \Lambda^{2N_f}) E^+ E^-$
    $|E^{\pm}| = (m\Lambda)^{1/2}$
  - both monopoles and meson condense!

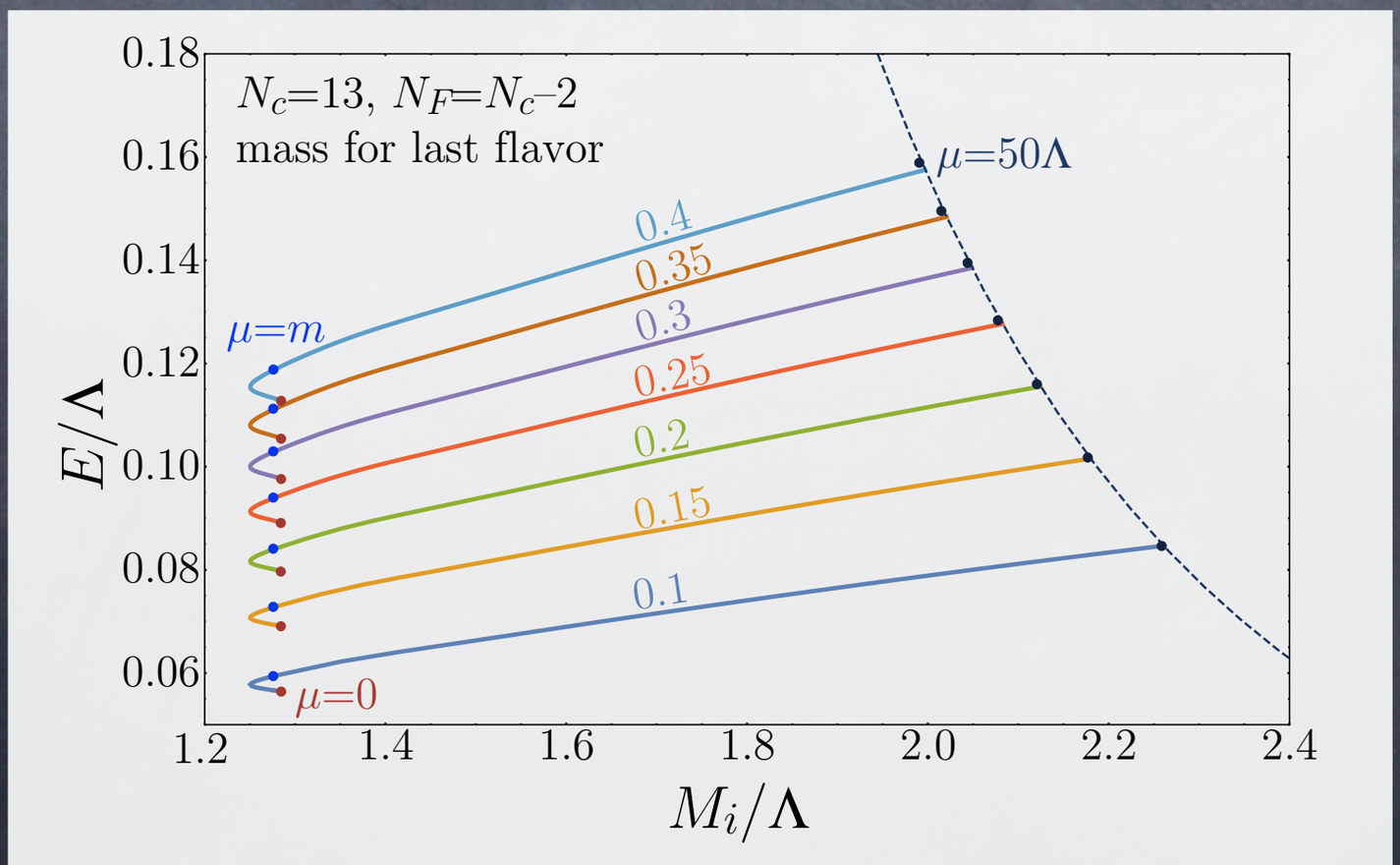
$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$

$$W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$$



$N_f < N_c-2$

- add mass m_q to some of the quarks
- can show monopole VEVs persist $m_q \to \infty$
- demonstration of confinement and chiral symmetry breaking for all $N_f \leq N_c-2$



$N_f = N_c+1$

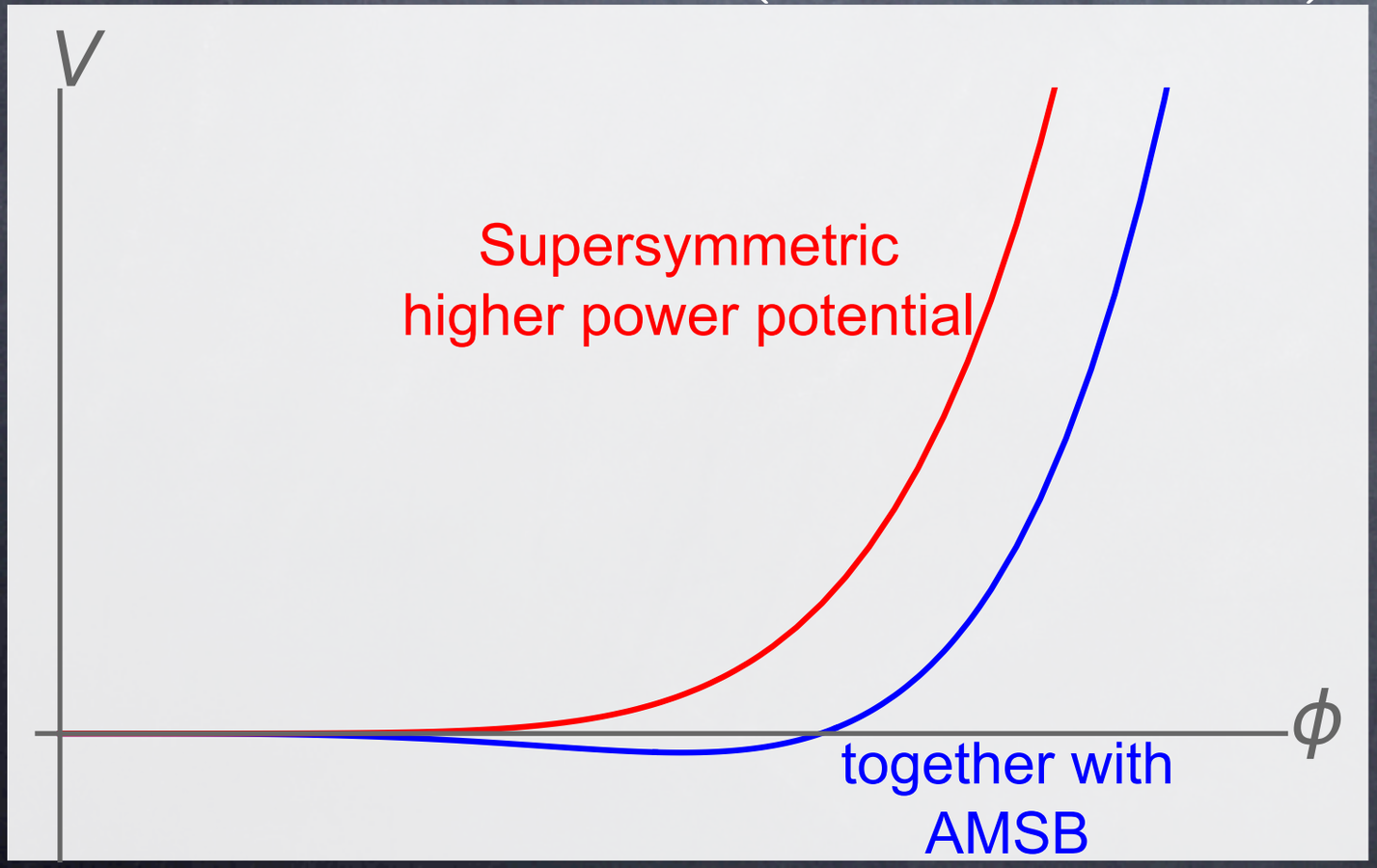
- "Confinement without χSB"

$$W = \frac{\det M - \tilde{B} M B}{\Lambda^{2N_c-1}}$$

$$W = \lambda \frac{\det M}{\Lambda^{N_f-3}} - \kappa \tilde{B} M B \qquad B_i = \epsilon_{i j_1 \cdots j_{N_c}} Q^{j_1} \cdots Q^{j_{N_c}}$$

$$V = N_f \lambda^2 \frac{|\phi|^{2N_f-2}}{\Lambda^{2N_f-6}} - \lambda (N_f - 3) m \phi^{N_f} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f-N_c)/(2N_c-N_f)} \ll \Lambda$$



$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

- χSB!
- massless pions
- Deeper minimum with $U(1)_B$ breaking

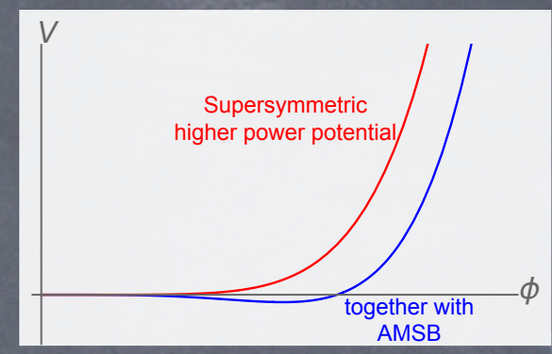
Andrea Luzio, Ling-Xiao Xu
2202.01239

Csaki, Gomes, HM, Noether, Telem, Varier, in preparation

$N_c+2 \leq N_f < 3N_c/2$

- "magnetic" IR-free SU(N_f-N_c) theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \to \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$



- go along the meson direction with rank M=N_f
- integrate out dual quarks with $M^{ij} = \phi \delta^{ij}$
- pure SU(N_f-N_c) YM forms gaugino condensate

$$W = (N_f - N_c) \left(\frac{\kappa^{N_f} \det M}{\Lambda^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)}$$

$$V = N_f \Lambda^4 \left| \frac{\kappa\phi}{\Lambda} \right|^{2N_c/(N_f-N_c)} - (2N_f - 3N_c) m \Lambda^3 \left(\frac{\kappa\phi}{\Lambda} \right)^{N_f/(N_f-N_c)} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f-N_c)/(2N_c-N_f)} \ll \Lambda$$

$$V \approx -\Lambda^4 \left(\frac{m}{\Lambda} \right)^{2N_c/(2N_c-N_f)}$$

- SU(N_f)$_L$xSU(N_f)$_R$→SU(N_f)$_V$
- deeper minimum with U(1)$_B$ breaking

Csaki, Gomes, HM, Noether, Telem, Varier, in preparation

$3N_c/2 < N_f < 3N_c$

- integrate q out like in free magnetic phase

$$W = (N_f - N_c)\left(\frac{\kappa^{N_f}\det M}{\Lambda^{3N_c-2N_f}}\right)^{1/(N_f-N_c)}$$

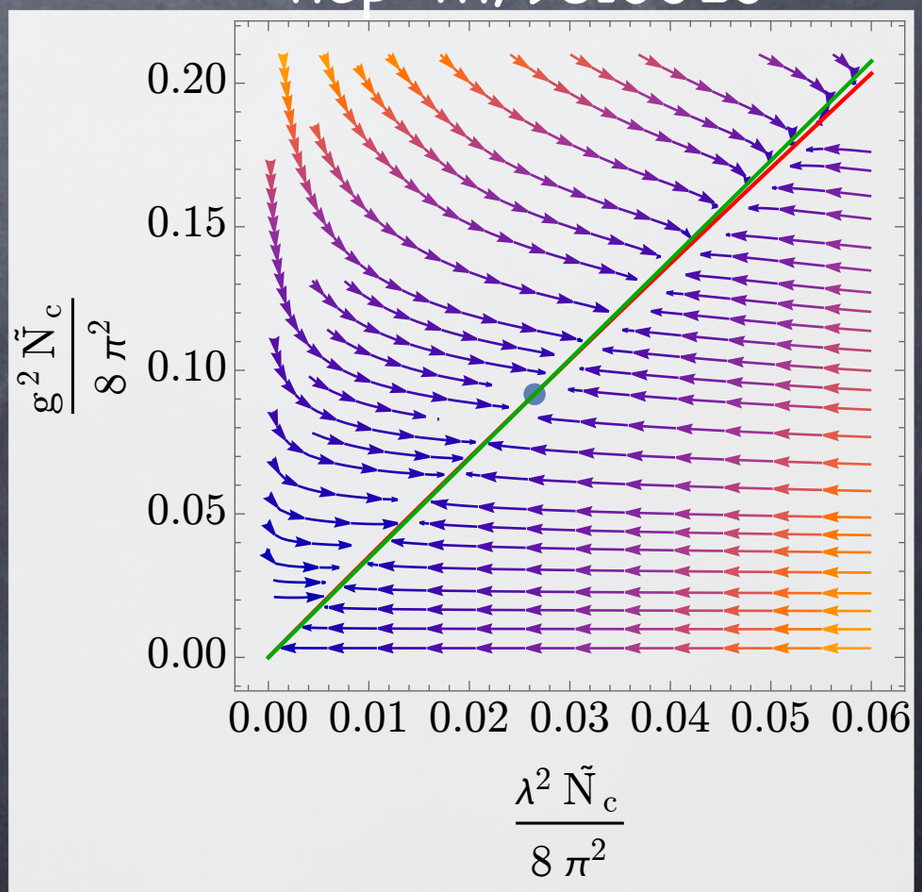
- No stable minimum for $N_f>2N_c$?
- Actually remember $Z_M(\mu) = \left(\frac{\mu}{\Lambda}\right)^{(6N_c-4N_f)/N_f}$
- Effective potential

$$V = Z_M(\phi)^{-1}\left|\frac{\partial W}{\partial M}\right|^2 + (N_f-3)mW$$

- Well-defined local minimum
- χSB up to $N_f<3N_c$

HM, Bea Noether, Digvijay Roy Varier
arxiv:2111.09690

Friedland, de Gouvêa, HM
hep-th/9810020



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χSB

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???

χSB

χSB

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m=∞

Chiral gauge theories

- It has been very difficult to formulate them on lattice because of fermion doubling
- Recent progress: domain wall fermions and "overlap lattice Dirac operator"
- Still challenging numerical problems
- Never simulated on lattice
- Only hand-waving ideas called "tumbling"
  - Theory breaks itself due to fermion bilinear condensates in the MAC

SO(10) with N_f 16's

- Yoshio Kikukawa: most likely chiral gauge theory to be simulated on lattice
- SO(10): smallest anomaly-free group with complex representations
- 16: smallest complex representation
- N_f=1: dynamical SUSY breaking, gapped
- N_f=2: gapped, SU(2) unbroken, not tumbling
- N_f=3: SU(3) broken dynamically to SO(3)
- N_f=4: SU(4)=SO(6) broken dynamically to SO(3)×SO(3) or SO(5)
- No massless composite fermions

N_f

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???

χSB

0

m=∞

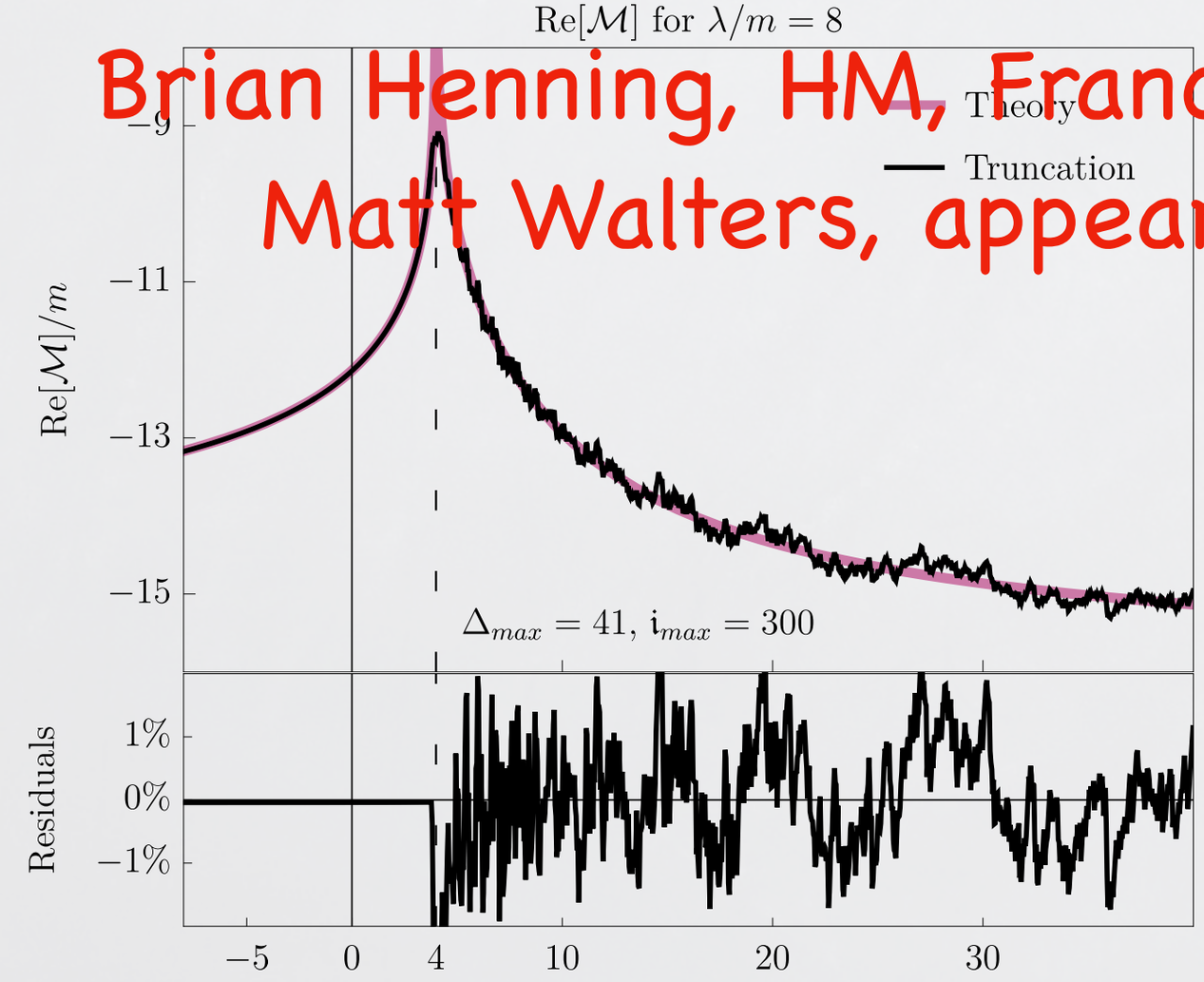
Conclusion

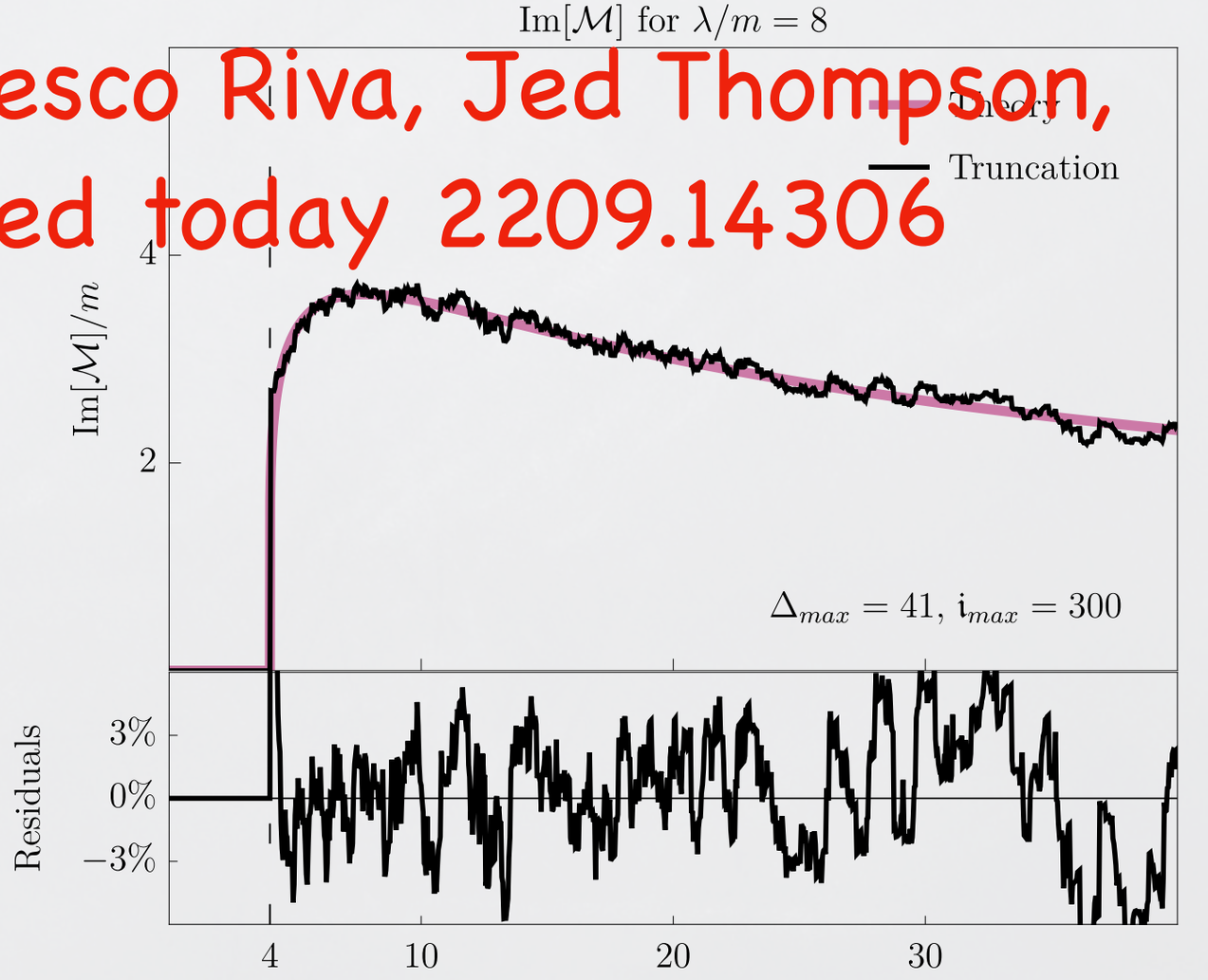
- N=1 SUSY + AMSB: a great tool to study non-SUSY gauge theories
- vector-like theories
  - Can show $\langle \bar{q} q \rangle \neq 0$
  - Monopole condensation for $SO(N_c)$
  - For $N_f > N_c$, local minima seem useful
- Chiral gauge theories can also be solved
  - Concrete predictions for SO(10) with 16's
  - Also for $SU(N_c)$ with A+Fbar, S+Fbar
  - Need to understand general symmetry breaking patterns with more examples

Hamiltonian truncation

- One can approximate QFT with a finite-dimensional Hilbert space w/o Wick rotation
- Can study scattering problems directly
- O(N) sigma model in 2+1 dim with non-perturbative Wilson-Fischer fixed point

Brian Henning, HM, Francesco Riva, Jed Thompson, Matt Walters, appeared today 2209.14306





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DESY Heinrich-Hertz-Lecture 28 September 2022

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