Gravitational Focusing of Wave Dark Matter.

DESY THEORY WORKSHOP 2022

Higgs, Flavor and Beyond

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Alessandro Lenoci

Hyungjin Kim, AL

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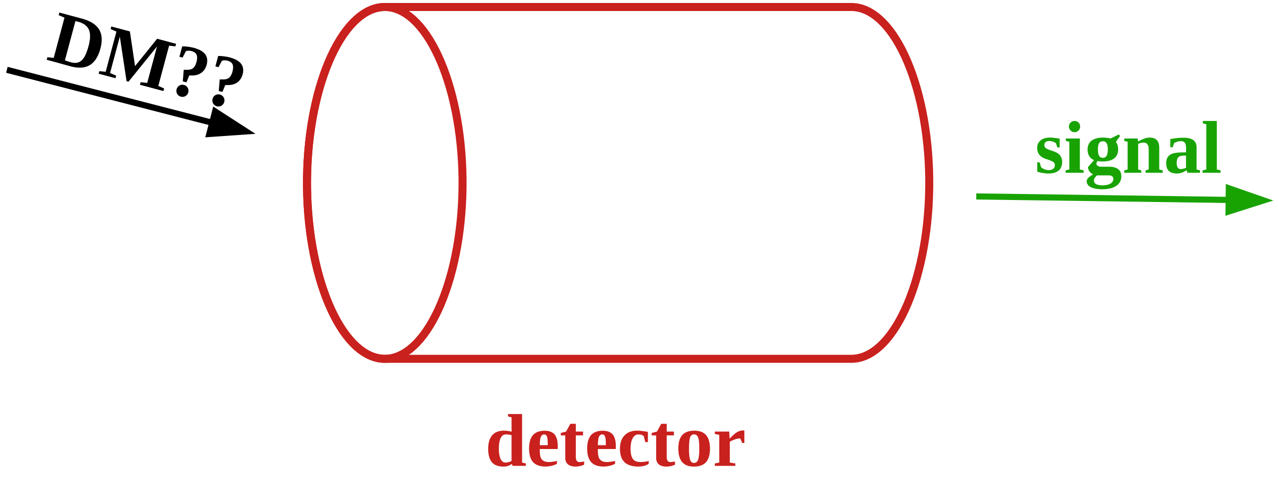
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DM Direct Detection



1) **How much DM** is there around us? signal **total power** or **rate**

2) Which are its **kinematic properties**? signal **spectral shape**

DM Direct Detection

The Standard Halo Model*

$$\rho_0 f(\mathbf{v}) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_{\rm dm})^2}{2\sigma^2}\right]$$

$$\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot = -(11, 241, 7)\ \mathrm{km/sec}$$

$$\sigma = v_C(R_\odot)/\sqrt{2} = 162\ \mathrm{km/sec}$$

$$\rho_0 = 0.3 \div 0.4\ \mathrm{GeV/cm}^3$$

Pros:

✓ Simple

✓ Reasonably accurate

*** In the Sun rest frame & galactic coordinates**

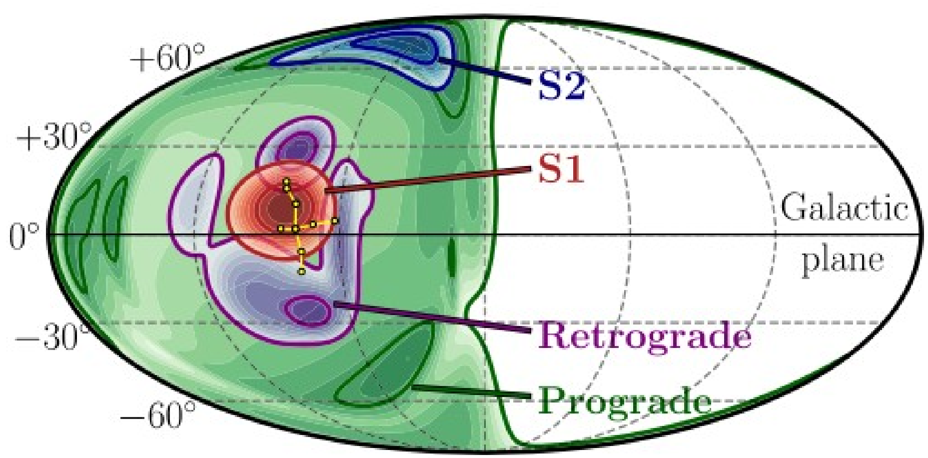
The Standard Halo Model

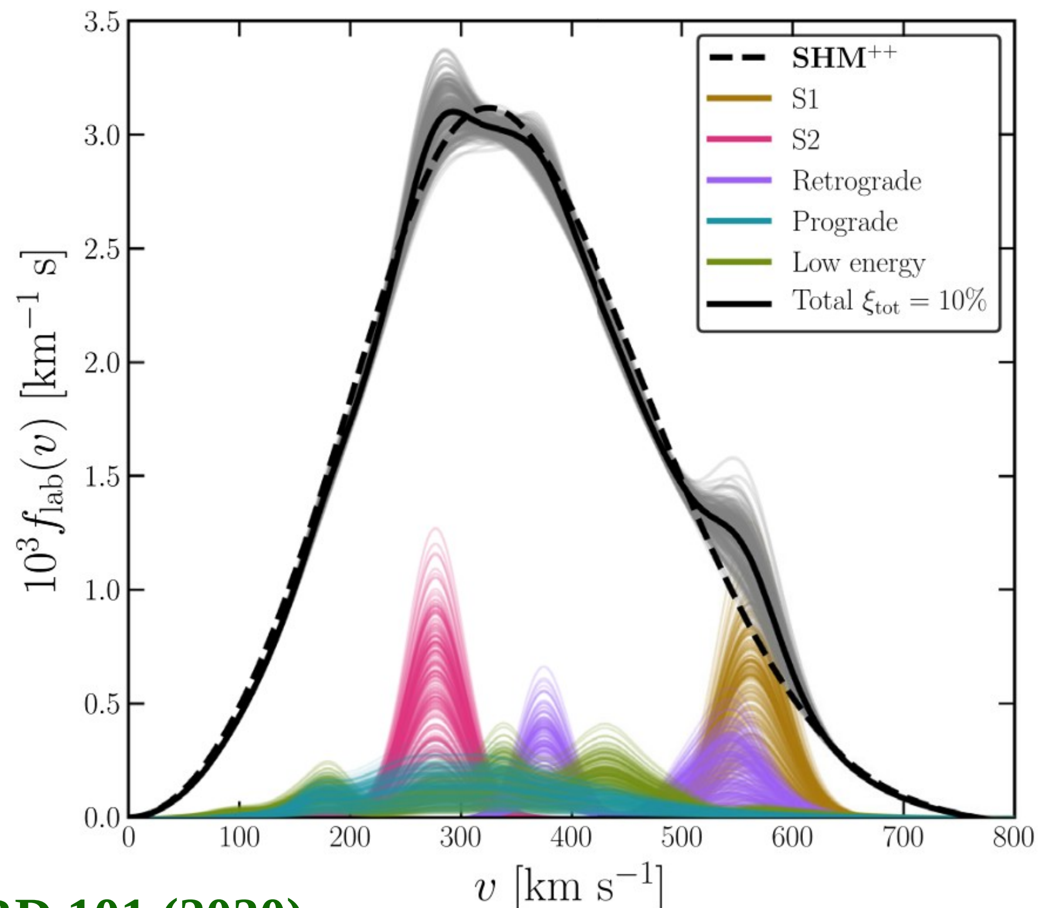
Cons:

NO local **DM substructures**, but hints from **stellar clusters**

DM usually **shares** the **stellar kinematic properties**

- ✘ GAIA stellar streams
- ✘ Thick MW stellar disk
- ✘ GAIA-Enceladus/Sausage





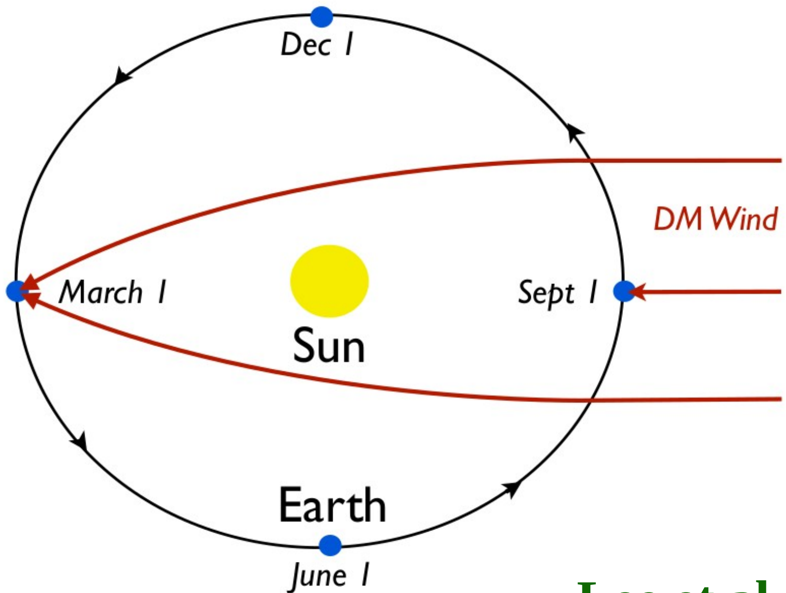
O’Hare et al. PRD 101 (2020)

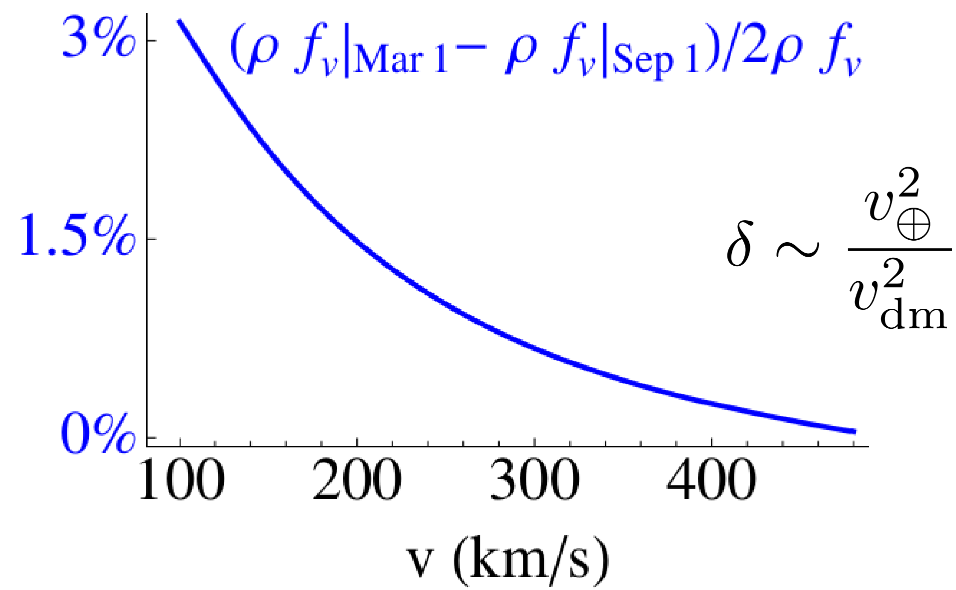
The Standard Halo Model

Cons:

Doesn't account for **gravity distortions** of local DM distribution

- ✘ Rate **modulation** effects
- ✘ Spectral shape **deformations**

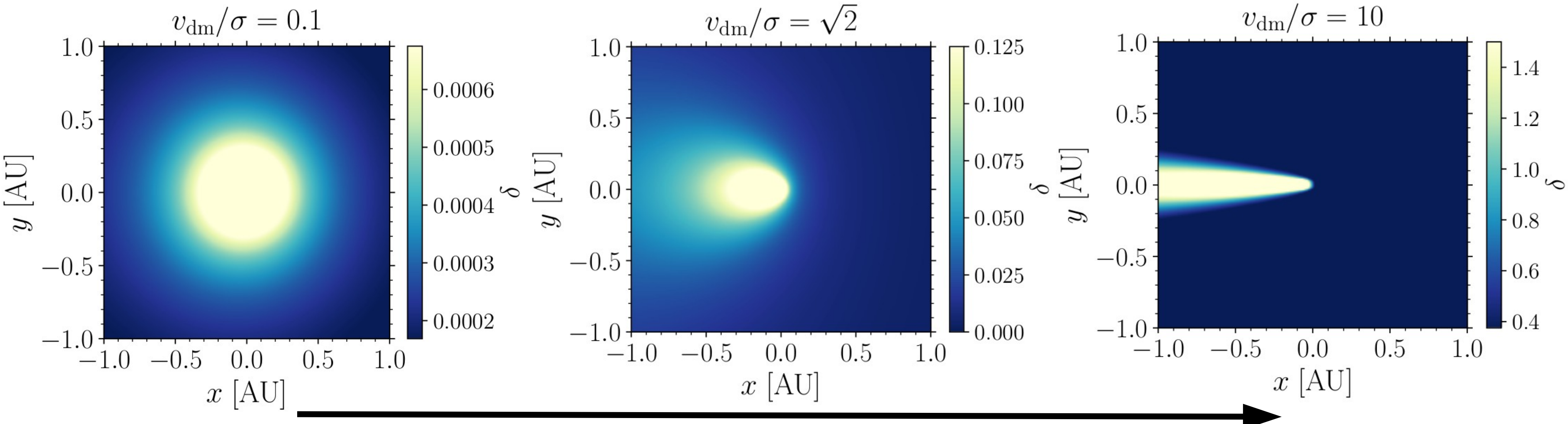




Lee et al. PRL 112 (2014)

Studied for **WIMP-like (particle) DM** but **not** for **(ultra)-light wave DM**

Particle Gravitational Focusing



colder DM

$$f(\mathbf{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_{\rm dm})^2}{2\sigma^2}\right]$$

$$\Delta\phi \propto \frac{\sigma^2}{v_{\rm dm}^2}$$

$$\delta_{\rm ds} \sim \frac{v_c^2}{\sigma^2}$$

1) **Colder DM**
- Decreases the tail's **width**
- Increases **downstream overdensity**

2) Focusing is **independent** on the **DM mass** (F/m effect)

3) Density contrast **divergent** at the origin

$$v_c = \sqrt{\frac{GM}{r}}$$

Particle vs Wave DM

de Broglie wavelength $$\lambda_{\rm dB} = \frac{2\pi}{mv}$$

Particles in a de Broglie volume $$N_{\rm dB} = \frac{(2\pi)^3 \rho}{m^4 v^3}$$

$N_{\rm dB} \gg 1$ — $N_{\rm dB} \ll 1$ — m

wave DM — $10\ \mathrm{eV}$ — **particle DM**

$L_{\rm sys} \lesssim \lambda_{\rm dB}$ **wave <u>unique</u> effects**

$L_{\rm sys} > \lambda_{\rm dB}$ **wave description = particle description**

$$m \sim 10^{-22}\ \mathrm{eV} \longrightarrow \lambda_{\rm dB} \sim 1\ \mathrm{kpc}$$ **Fuzzy DM**

$$m \sim 10^{-14}\ \mathrm{eV} \longrightarrow \lambda_{\rm dB} \sim 1\ \mathrm{AU}$$

Wave DM & gravity

We consider a **light scalar boson** in a static **Newtonian potential**

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right]$$

$$ds^2 = \left(1 + \frac{2GM}{r}\right)dt^2 - \left(1 - \frac{2GM}{r}\right)d\mathbf{x}^2$$

Wave mode **non-relativistic** expansion

$$\hat{\phi}(t,\mathbf{x}) = \frac{1}{\sqrt{2mV}}\sum_i\left[\hat{a}_i\ \Psi_i(\mathbf{x},t)e^{-imt} + \text{h.c.}\right]$$

Wave function: response of the field to gravity

Annihilation operator: statistical properties of the field

Density operator

For a simple harmonic oscillator, $\hat{H} = \omega \hat{a}^\dagger \hat{a}$

We maximize **entropy** for fixed mean occupation number $\langle n \rangle$

$$S = -\mathrm{Tr}[\hat{\rho} \log \hat{\rho}]$$

In coherent state representation $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle, \quad \alpha \in \mathbb{C}$

$$\hat{\rho} = \int d^2\alpha \, \frac{1}{\pi \langle n \rangle} \exp\left[-\frac{|\alpha|^2}{\langle n \rangle}\right] |\alpha\rangle\langle\alpha|$$

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Multi-mode DM field $1 \ll \langle n_i \rangle \propto f(\mathbf{v})$

$$\hat{\rho} = \prod_i \int d^2\alpha_i \, P(\alpha_i) |\{\alpha_i\}\rangle\langle\{\alpha_i\}|$$

Ensemble averages

$$\langle \hat{\mathcal{A}}(\hat{a}_i, \hat{a}_j^\dagger) \rangle = \mathrm{Tr}[\hat{\rho}\hat{\mathcal{A}}] \propto \prod_k \int d^2\alpha_k P(\alpha_k) \mathcal{A}(\alpha_k, \alpha_k^\star)\delta_{ij} + \mathcal{O}\left(\frac{1}{\langle n_k \rangle}\right)$$

$$\langle a_j^\dagger a_i \rangle = \delta_{ij}(\langle n_i \rangle + 1) \approx \langle a_i^\dagger a_j \rangle$$

Schrodinger equation

Klein-Gordon equation

$$(\Box + m^2)\phi = 0$$

Schrodinger equation

$$i\partial_t \Psi_i(t, \mathbf{x}) = \left[-\frac{1}{2m}\nabla^2 - \frac{\alpha_G}{r} \right] \Psi_i(t, \mathbf{x})$$

Fine structure constant $\alpha_G = GMm$

Wave function $\Psi_i = e^{-it\frac{\mathbf{k}_i^2}{2m}} \psi_{\mathbf{k}_i}(\mathbf{x})$ $\mathbf{k} = m\mathbf{v}$

$$\left[\nabla^2 + (mv)^2 + \frac{2\alpha_G m}{r} \right] \psi_{\mathbf{v}} = 0$$

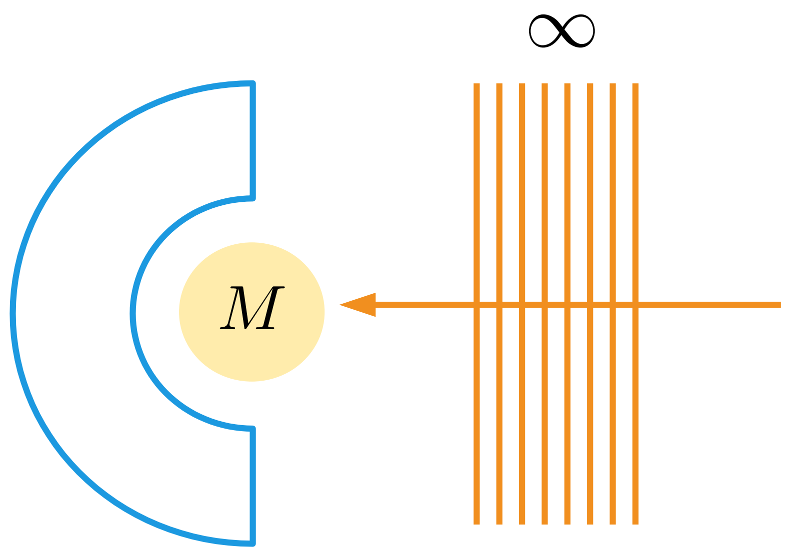
$$\psi_{\mathbf{v}} = e^{im\mathbf{v}\cdot\mathbf{x}} \Gamma(1 - i\alpha_G/v) e^{\frac{\pi}{2}\alpha_G/v} {}_1F_1[i\alpha_G/v, 1, imvr(1 - \hat{v}\cdot\hat{x})]$$

Wave Gravitational Focusing

Scattered wave DM

$$\phi(t, \mathbf{x})$$

$$f(\mathbf{v})$$



DM wave wind

$$\phi_0 = \sqrt{\frac{\rho_0}{m^2}}$$

$$f(\mathbf{v})$$

$$\psi_{\mathbf{v}}^{\infty} \sim e^{im\mathbf{v}\cdot\mathbf{x}}$$

$$\psi_{\mathbf{v}} = e^{im\mathbf{v}\cdot\mathbf{x}}\Gamma(1 - i\alpha_G/v)e^{\frac{\pi}{2}\alpha_G/v}\,{}_1F_1[i\alpha_G/v, 1, imvr(1 - \hat{v}\cdot\hat{x})]$$

Density contrast

$$\delta = \frac{\langle\phi^2\rangle}{\phi_0^2} - 1 = \int d^3v\, f(\mathbf{v})(|\psi_{\mathbf{v}}(\mathbf{x})|^2 - 1) \propto$$ **power oscillations**

Focused speed distribution

$$\Delta f(v) = v^2 \int d\Omega_v\, f(\mathbf{v})(|\psi_{\mathbf{v}}(\mathbf{x})|^2 - 1) \propto$$ **spectral distortions**

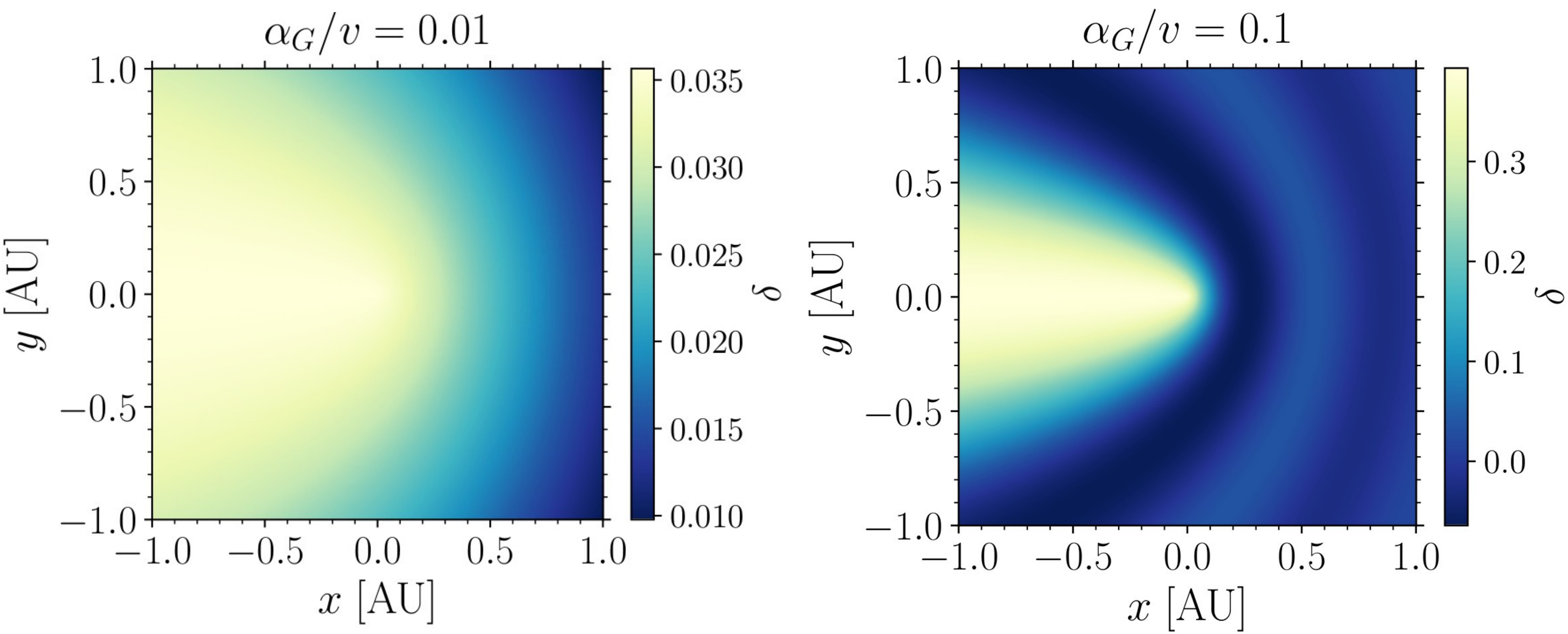
Wave Gravitational Focusing

Assume $f(\mathbf{v}) = \delta^{(3)}(\mathbf{v} - \mathbf{v}')$, **monochromatic DM**

$$1 + \delta = |\psi_{\mathbf{v}}(\mathbf{x})|^2 = |\psi_{\mathbf{v}}(0)|^2 \times |{}_1F_1[i\alpha_G/v, 1, imvr(1 - \hat{v} \cdot \hat{x})]|^2$$

$$|\psi_{\mathbf{v}}(0)|^2 = \frac{2\pi\alpha_G/v}{1 - e^{-2\pi\alpha_G/v}}$$

Sommerfeld factor



Application: Halo DM

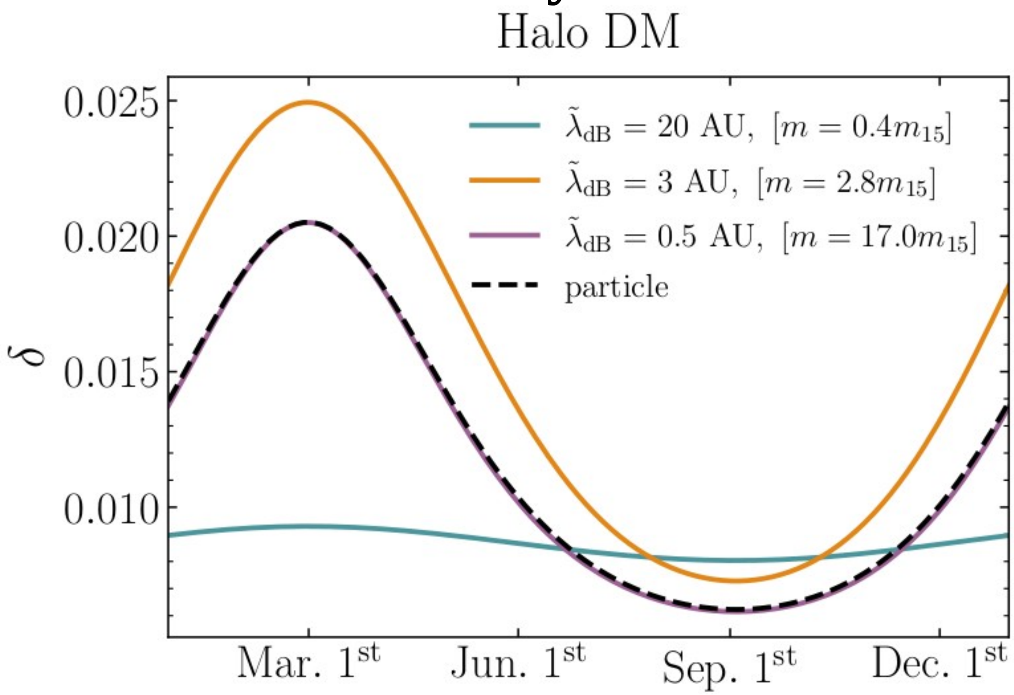
The Standard Halo Model

$$\rho_0 f(\mathbf{v}) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_{\rm dm})^2}{2\sigma^2}\right]$$

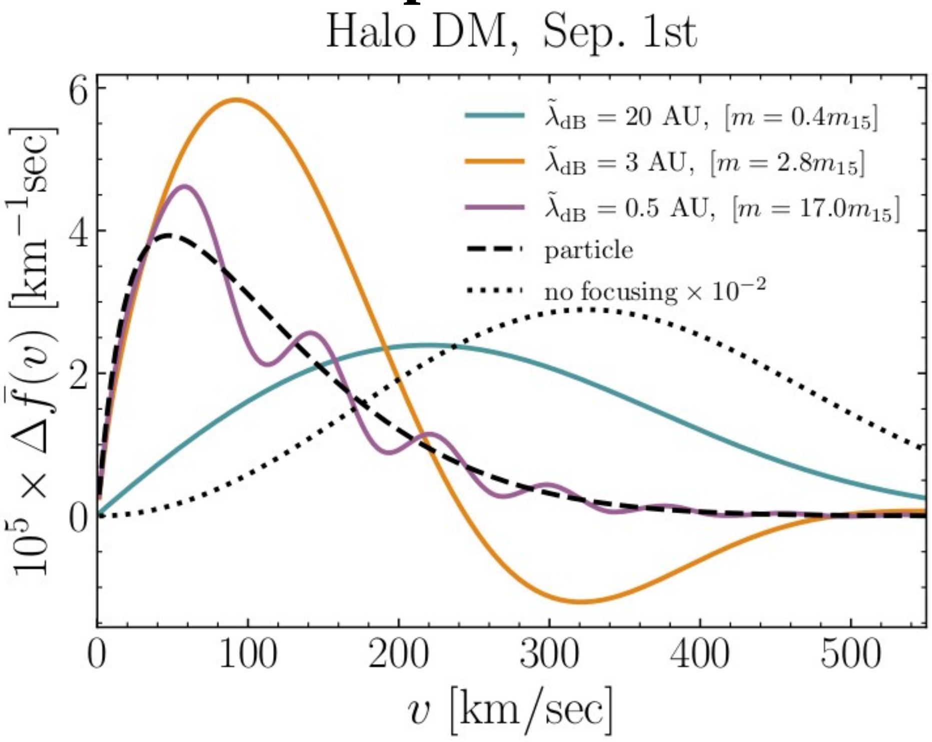
$$\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot = -(11, 241, 7)\ \mathrm{km/sec}$$

$$\sigma = v_C(R_\odot)/\sqrt{2} = 162\ \mathrm{km/sec}$$

Density contrast



Focused speed distribution

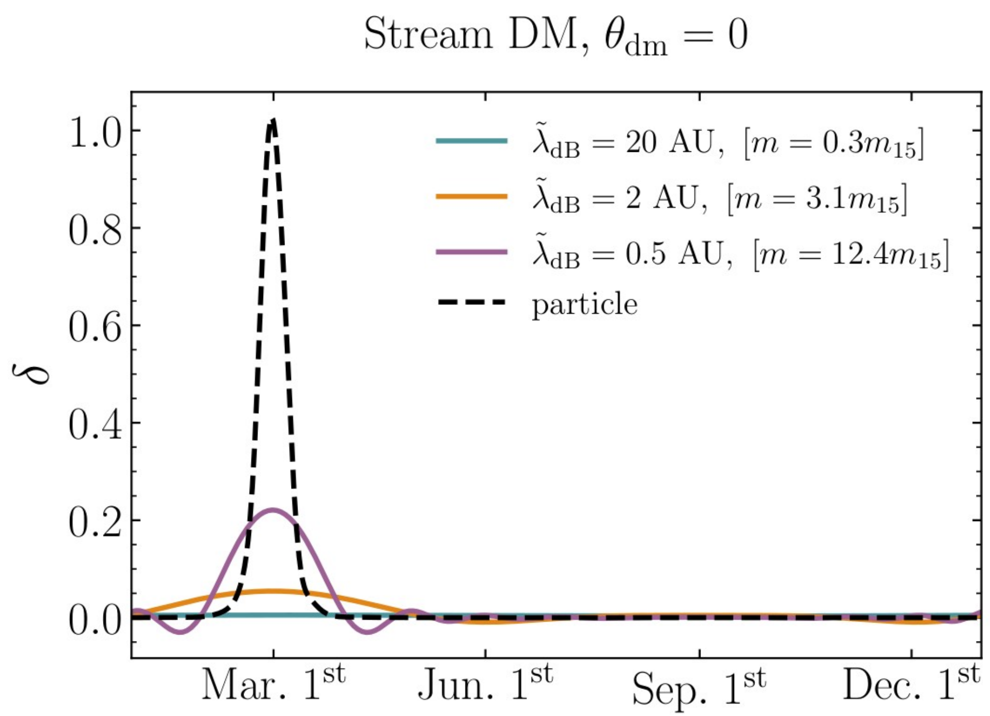


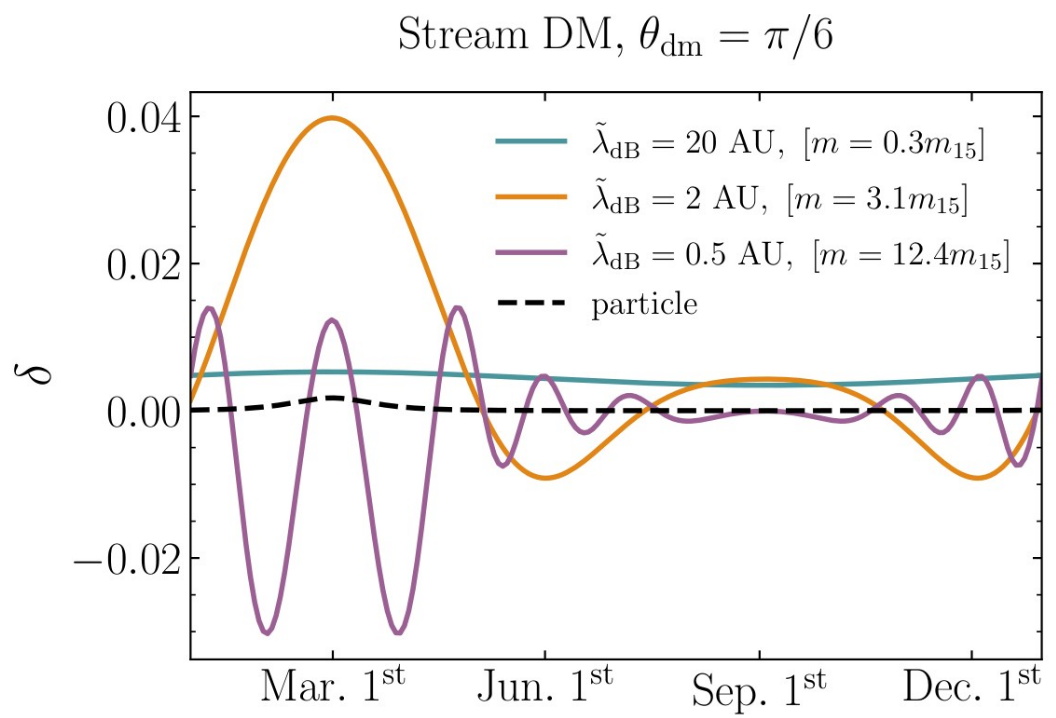
Application: Stream DM

- Dark component of stellar streams originated from dwarf galaxies
- High space & momentum **coherence**
- **Cold objects** with **large streaming velocity**

$v_{\rm dm} = 400$ km/sec

$\sigma = 30$ km/sec





$$\Delta\phi \propto \Delta t \propto \max\left[\frac{\sigma^2}{v_{\rm dm}^2},\ \frac{\tilde{\lambda}_{\rm dB}}{2\pi r}\right]$$

Why Gravitational Focusing?

- **While we still search for DM**:
  i. **Model independent** (it is a gravity effect)
  ii. Correct modeling of **direct detection** signals at % level
  iii. Other systems sensitive to the effect (binaries?)

- **Once we have detected DM**:
  i. Halo **parameter reconstruction**
  ii. Mapping of **local DM substructures**

Focusing for Wave Dark Matter

Thanks!

alessandro.lenoci@desy.de

Backup slides

Two Questions

(1) Which are the **wave effects** ?

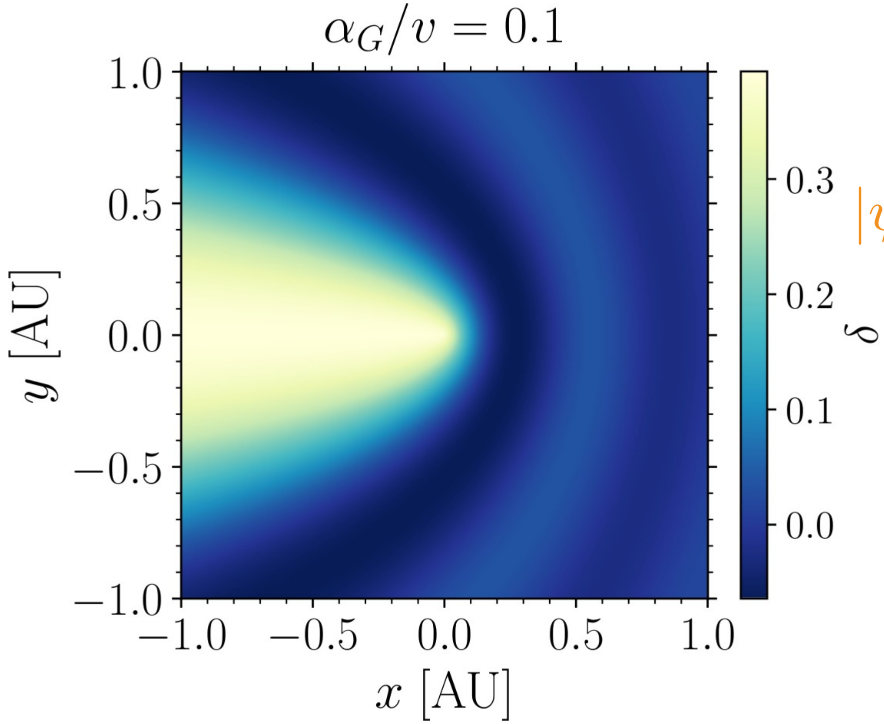
(2) Do we **retrieve particle focusing** at $r \gg \lambda_{\rm dB}$?

Assume $f(\mathbf{v}) = \delta^{(3)}(\mathbf{v} - \mathbf{v}')$, **monochromatic DM**

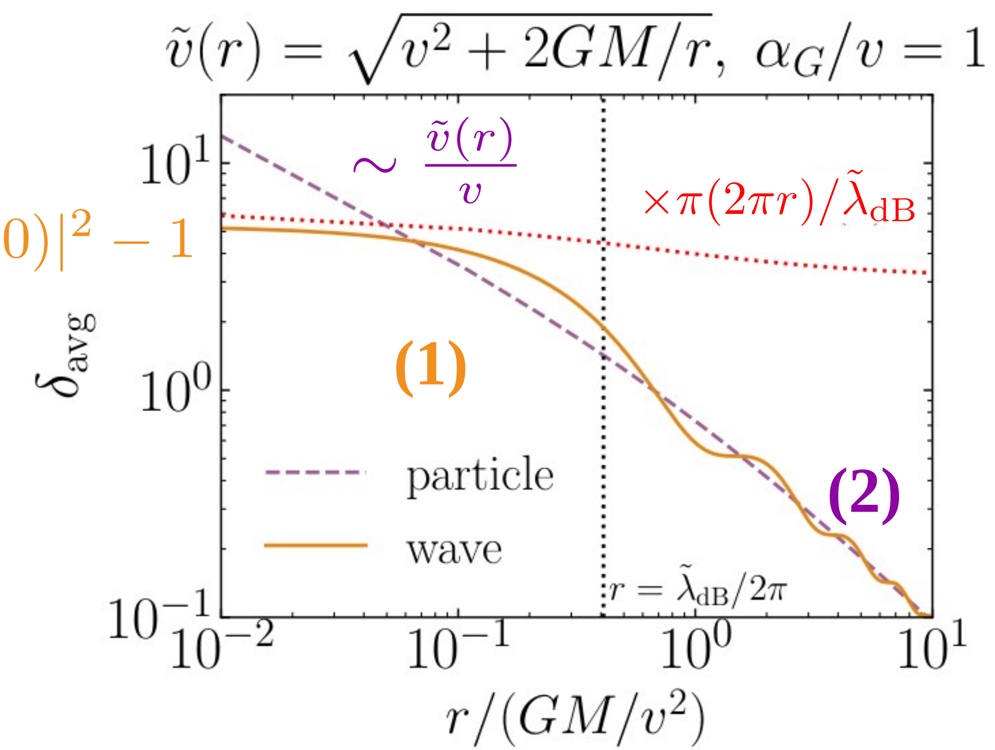
$$1 + \delta = |\psi_v(\mathbf{x})|^2 = |\psi_v(0)|^2 \times M(\dots)$$

$$\boxed{|\psi_v(0)|^2 = \frac{2\pi\alpha_G/v}{1 - e^{-2\pi\alpha_G/v}}}$$

Sommerfeld factor



$|\psi_v(0)|^2 - 1$

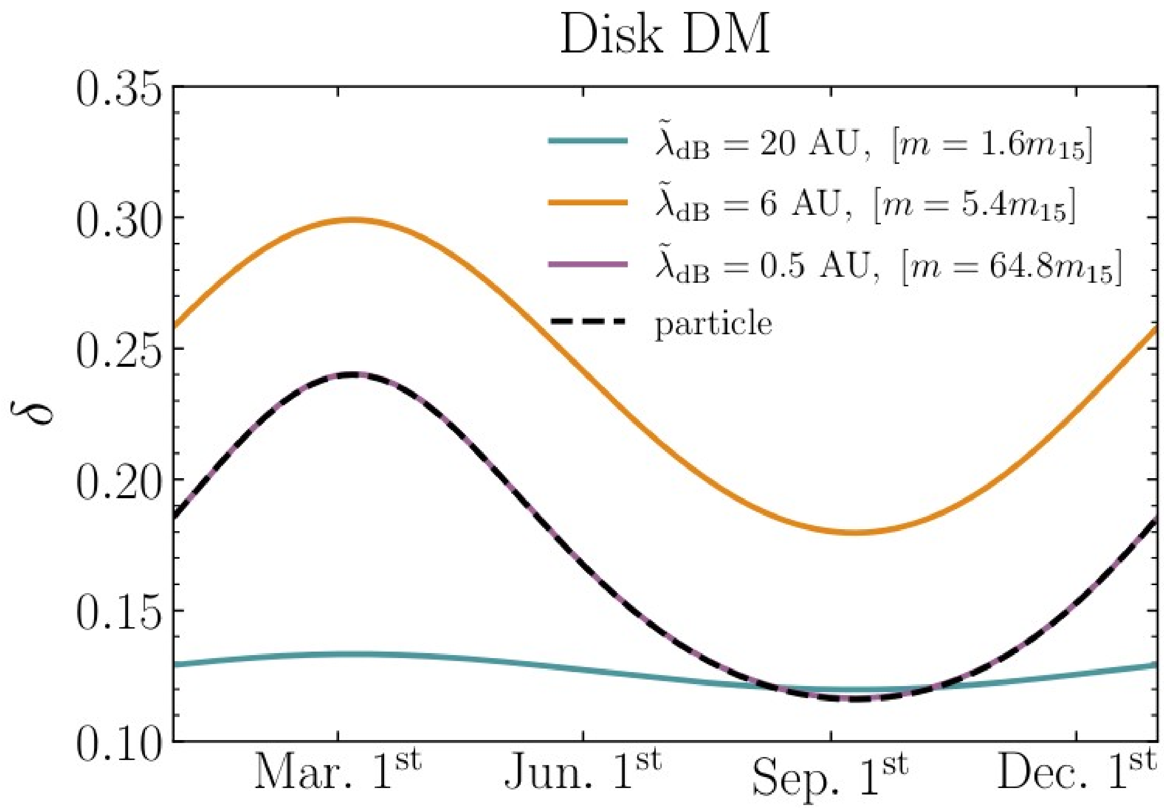


Application: (3) Thick Dark Disk

- The **thick MW stellar disk** is made of stars accreted or heated through a merger
- Dark component **co-rotating** with Galactic disk, accreted by **dynamical friction**
- More generally: any **cold substructure** with **small mean velocity**

$$f(\mathbf{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}-\mathbf{v}_{\rm dm})^2}{2\sigma^2}\right]$$

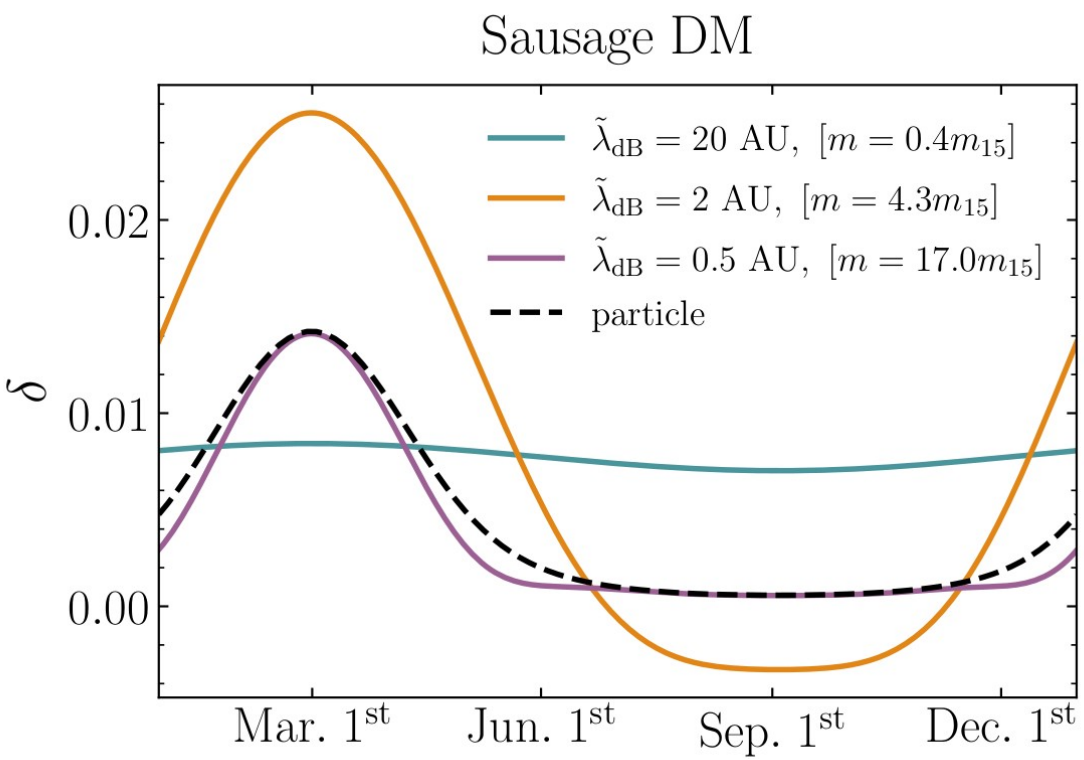
$$\mathbf{v}_{\rm dm} = (0, -50, 0)\ \mathrm{km/sec}$$
$$\sigma = 50\ \mathrm{km/sec}$$



Application: (4) GAIA-Enceladus DM

- GAIA observed a stellar population accreted through a recent merger.
- **Anisotropic** velocity structure, called **sausage**
- O(10%) ?? of local DM with similar kinematic properties

$$f(\mathbf{v}) = \frac{1}{(2\pi)^{3/2}\sqrt{\det\Sigma}} \exp\left[-\frac{1}{2}(\mathbf{v}-\mathbf{v}_{\rm dm})\Sigma^{-1}(\mathbf{v}-\mathbf{v}_{\rm dm})\right]$$

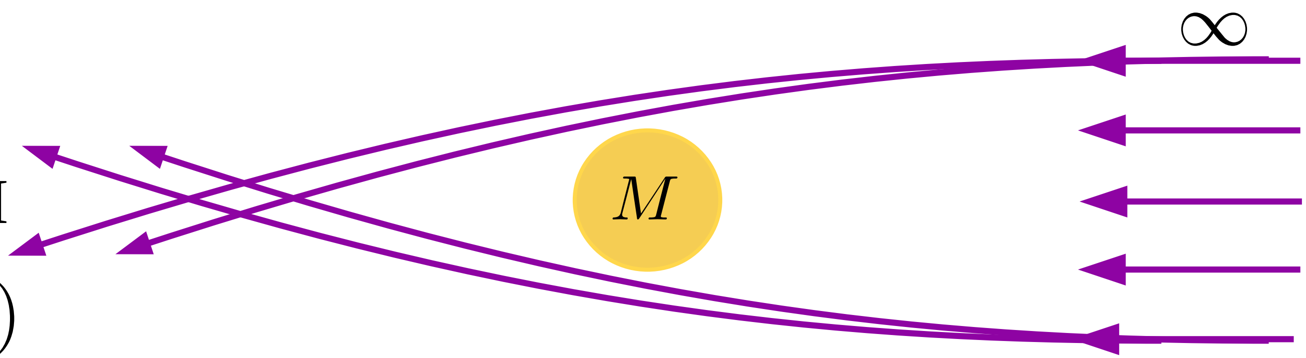


$$\Sigma = \mathrm{diag}(\sigma_r^2, \sigma_\theta^2, \sigma_\phi^2)$$

$$\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot = (11, 241, 7)\ \mathrm{km/sec}$$

$$\sigma_r = 256\ \mathrm{km/sec}, \quad \sigma_\theta = \sigma_\phi = 81\ \mathrm{km/sec}$$

Particle Gravitational Focusing



DM wind

$\rho_0,\ f(\mathbf{v}_0)$

Focused DM

$\rho(\mathbf{x}),\ f(\mathbf{v})$

$$v_e^2(r) = \frac{2GM}{r}$$

1) Energy conservation $\boxed{v(r) = \sqrt{v_0^2 + v_e^2(r)}}$

2) Liouville theorem $\boxed{\mathrm{d}^3x_0\,\mathrm{d}^3v_0 = \mathrm{d}^3x\,\mathrm{d}^3v}$

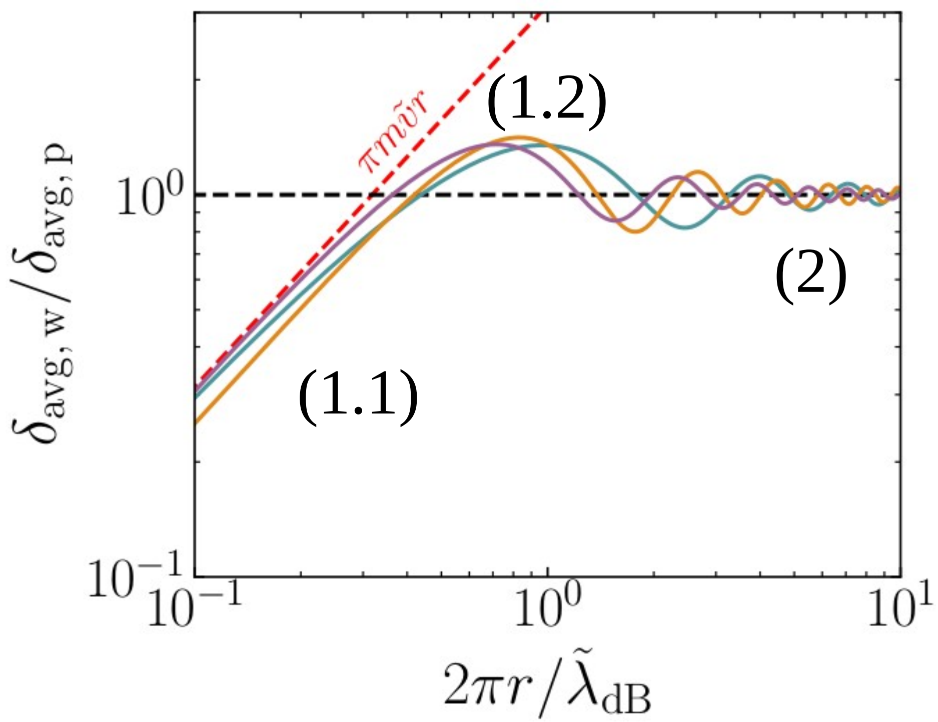
$$\delta \sim \frac{\rho}{\rho_0} - 1 \sim \boxed{\frac{\mathrm{d}^3x_0}{\mathrm{d}^3x}} - 1 \sim \boxed{\frac{v_e^2(r)}{2v_0^2}} \sim 2\times 10^{-2}$$

for halo DM @ 1 AU

Two Questions, rewind

(1) Which are the **wave effects**?
(2) Do we **retrieve particle focusing** at $r \gg \lambda_{\rm dB}$?

$$\delta_{\rm avg} = \frac{1}{4\pi}\int d\Omega_{\hat{x}}\delta$$



$$\tilde{\lambda}_{\rm dB} = \frac{2\pi}{m\tilde{v}} \qquad \tilde{v} = \sqrt{v^2+v_e^2}$$

(1.1) Suppression within $m\tilde{v}r \ll 1$
(1.2) Small enhancement $m\tilde{v}r \approx 1$

(2) Agreement with particle $m\tilde{v}r \gg 1$

For non-monochromatic waves one can replace $\tilde{v} = \sqrt{v^2+v_e^2+\sigma^2}$

Semiclassical limit

For $r \gg \lambda_{\rm dB}$ wave description should approach the particle one.

$$f_W(\mathbf{x},\mathbf{p}) = \int \mathrm{d}^3 y\, e^{i\mathbf{p}\cdot\mathbf{y}/\hbar} \int \mathrm{d}^3 v\, f(\mathbf{v}) \psi^\star_{\mathbf{v}}(\mathbf{x}+\mathbf{y}/2)\psi_{\mathbf{v}}(\mathbf{x}-\mathbf{y}/2)$$

Related to density contrast as

$$\langle \phi^2 \rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f_W(\mathbf{x},\mathbf{p})$$

From the Schrodinger equation, we get

$$\left\{ \partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}} + \frac{i}{\hbar}\left[V(\mathbf{x} + \frac{i\hbar}{2}\nabla_{\mathbf{p}}) - V(\mathbf{x} - \frac{i\hbar}{2}\nabla_{\mathbf{p}}) \right] \right\} f_W = 0$$

This is actually the Boltzmann equation for $f(\mathbf{v})$ if

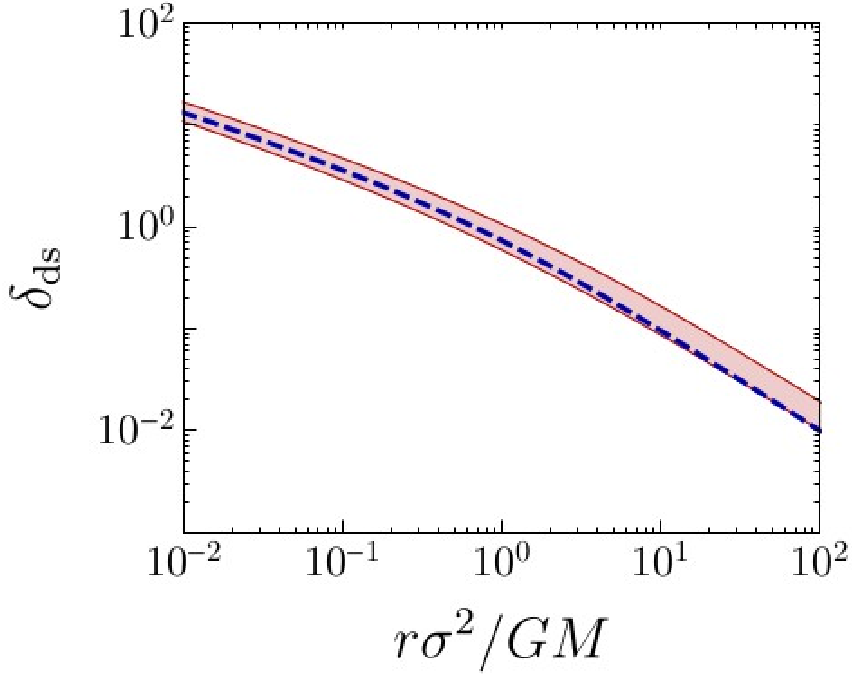
$$\hbar \to 0 \text{ or } |\mathbf{x}| \gg \lambda_{\rm dB}$$

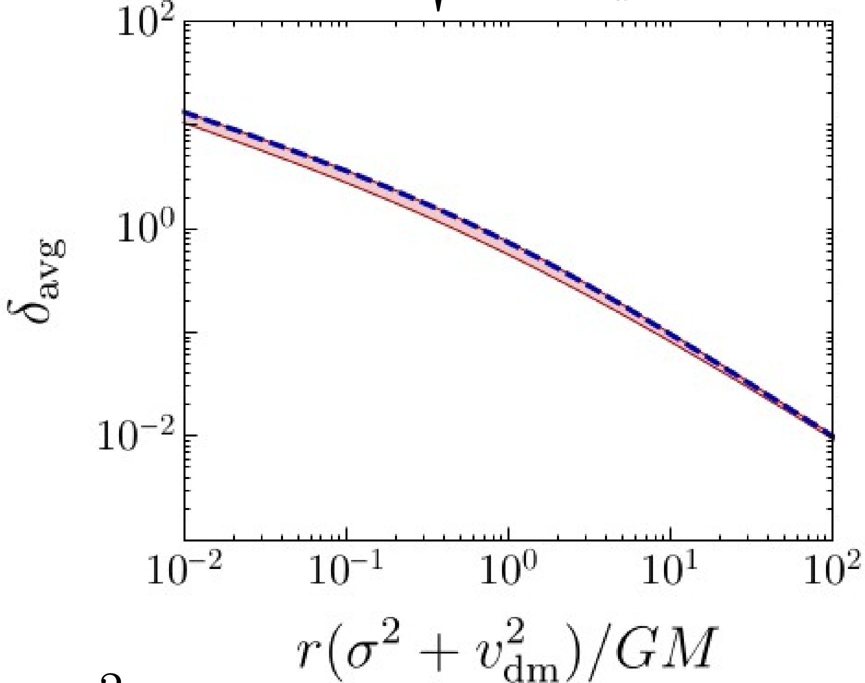
One can show that these conditions are equivalent

Particle focusing density contrast

$$1+\delta_{\rm ds} \approx \sqrt{1+\frac{v_e^2(r)}{\sigma^2}}$$

$$1+\delta_{\rm avg} \approx \sqrt{1+\frac{v_e^2(r)}{v_{\rm dm}^2+\sigma^2}}$$





$$\mu = \hat{v}_{\rm dm}\cdot\hat{x}$$

$$\Delta\mu = \frac{\delta_{\rm avg}}{\delta_{\rm ds}} \approx \frac{\sigma^2}{v_{\rm dm}^2+\sigma^2} + \mathcal{O}(v_e^2)$$

$$1+\delta = \frac{\rho(\mathbf{x})}{\rho_0} = \int \mathrm{d}^3 v f(\mathbf{v}_0(\mathbf{v},\mathbf{x})) = \int \mathrm{d}^3 v_0 d(\mathbf{v}_0,\mathbf{x}) f(\mathbf{v}_0)$$

M. S. Alenazi & P. Gondolo PRD 74 (2006)

P. Sikivie & S. Wick PRD 62 (2002)