Domain Walls in extended Higgs Sector

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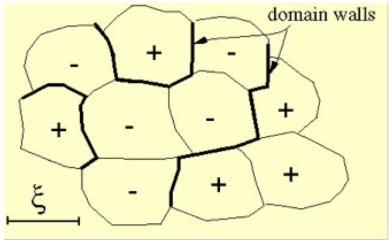
Introduction to Domain Walls

- Domain walls are a type of topological defects that arise after a spontaneous symmetry breaking of a theory with a discrete symmetry.
- After spontenous symmetry breaking, different regions of the universe can get different vacua which are degenerate with each other.



A theory with the symmetry group G **broken** to a subgroup H

$$G \longrightarrow H$$

For h element of unbroken group H and a vacuum point Φ_0: $h\Phi_0 = \Phi_0$

H keeps vacuum points invariant

For g element of symmetry group G : $g\Phi_0 = \Phi'$

Acting on Φ_0 with G generates other degenerate vacuum points

The space of all cosets G/H give the vacuum manifold of all degenerate vacuas $M = G/H$

- The vacuum manifold M of the **standard model** is a **3-Sphere**

- M is not **disconnected** and does **not contain holes** :
  No **domain walls or cosmic strings or monopoles** in the standard model.

- Beyond standard model physics can have different types of topological defects

- Presence of domain walls in a theory is problematic because they **dominate the energy of the universe** at some point in time as well as should cause **anisotropies in the CMB**
  Leads to **strong bounds** on the energy of domain walls allowed **< 0.93 MeV** (B. Zel'Dovich, I. Y. Kobzarev, L. B. Okun', Soviet Journal of Experimental and Theoretical Physics 40 (1975) 1)

- Domain walls from **approximate discrete symmetries** get **annihilated** and therefore these models are allowed if the annihilation occurs **before** dominating the energy density of the universe.

Two-Higgs-Doublet Model (2HDM) potential

$$V(\Phi_1,\Phi_2) = m_{11}^2\,|\Phi_1|^2 + m_{22}^2\,|\Phi_2|^2 - m_{12}(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$
$$+\frac{\lambda_1}{2}\,|\Phi_1|^4 + \frac{\lambda_2}{2}\,|\Phi_2|^4 + \lambda_3\,|\Phi_1|^2\,|\Phi_2|^2 + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_5\left[(\Phi_2^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_1)^2\right]$$

In the following, focus on Z_2 symmetry (softly broken by m^2_{12} term)

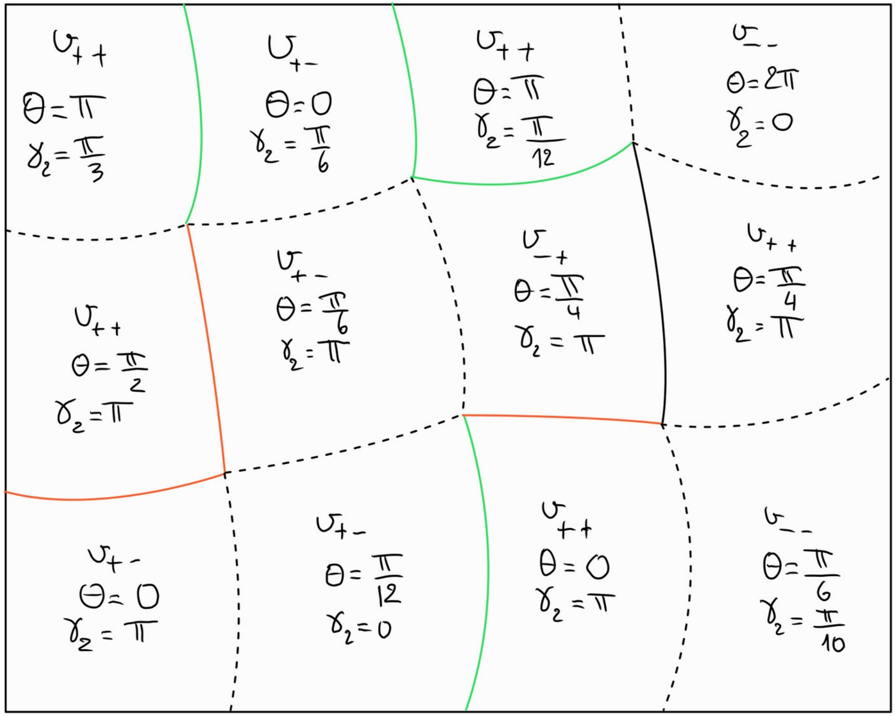
Z_2 $\quad \Phi_1 \longrightarrow \Phi_1 \qquad \Phi_2 \longrightarrow -\Phi_2$

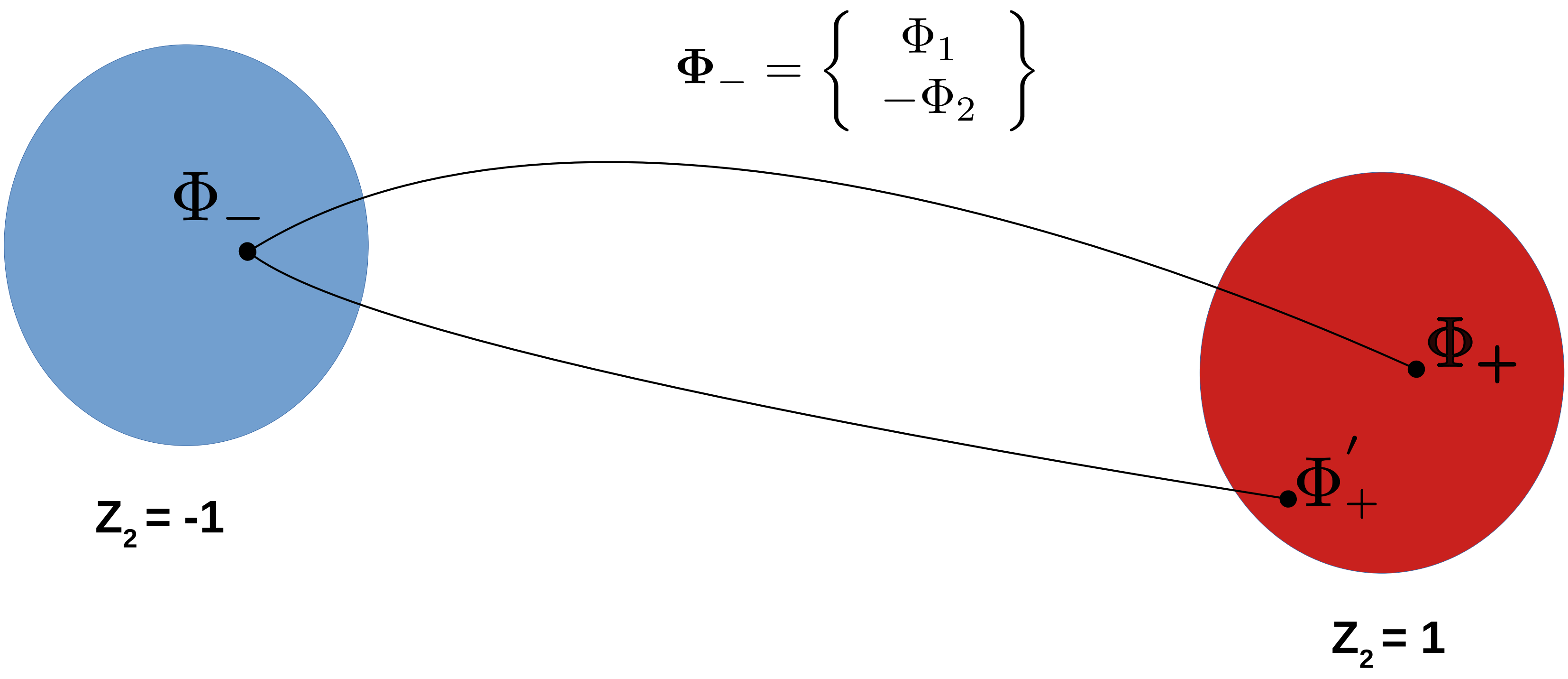
Full symmetry of the model:

$$SU_L(2)\otimes U_Y(1)\otimes Z_2 \longrightarrow U_{em}(1)$$

$M = SU(2)\otimes Z_2$ **<u>Two disconnected 3-Spheres</u>**

In the early universe after SSB, regions with a typical correlation length acquire a random VEV.





- **To obtain domain wall solutions, both minima at -∞ and +∞ should lie on different sectors of the vacuum manifold.**
- $\Phi_+ = \begin{Bmatrix} \Phi_1 \\ \Phi_2 \end{Bmatrix}$ **is a solution, then** $\Phi'_+ = U\Phi_+$ **is also a valid solution.**

Possible Parametrizations of the Higgs Vacua:

Linear Parametrization

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}$$

Can be rotated using a matrix

$U(x) \in SU_L(2) \otimes U_Y(1)$

$$U = e^{i\theta(x)} \; exp(\frac{i\gamma_i(x)\sigma_i}{2})$$

Vacuum Parametrization

$$\Phi_i(x) = U(x)\hat{\Phi}_i(x)$$

$$\hat{\Phi}_1(x) = \begin{pmatrix} 0 \\ v_1(x) \end{pmatrix}, \hat{\Phi}_2(x) = \begin{pmatrix} v_+(x) \\ v_2(x)e^{i\xi(x)} \end{pmatrix}$$

$$U = e^{i\theta} \begin{pmatrix} \cos(\gamma_1)\exp(i\gamma_2) & \sin(\gamma_1)\exp(i\gamma_3) \\ -\sin(\gamma_1)\exp(-i\gamma_3) & \cos(\gamma_1)\exp(-i\gamma_2) \end{pmatrix}$$

Possible Vacua in the 2HDM:

- Neutral Vacua $v_+ = 0, \; \xi = 0$
- CP breaking Vacua $\xi \neq 0, \; v_+ = 0$
- Charge breaking Vacua $v_+ \neq 0$

Here we only consider **neutral vacua at the boundaries** and take the general Vacuum Parametrization at each point in x (***possibility of getting CP and/or Charge violation inside the domain wall)***

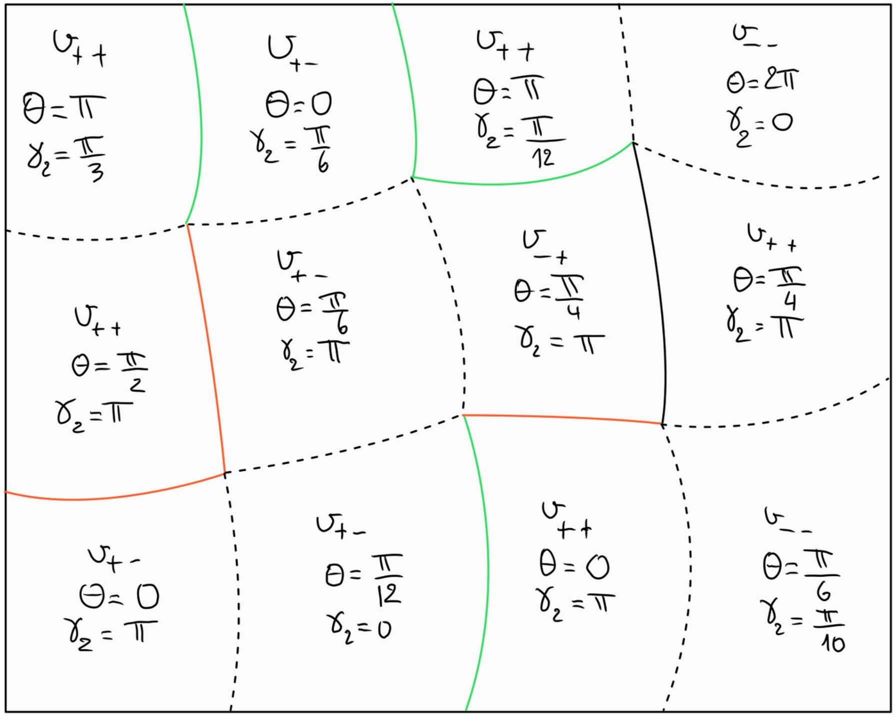
$$\Phi_1(\pm\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1^0 \end{pmatrix}, \qquad \Phi_2(\pm\infty) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \pm v_2^0 \end{pmatrix}$$

$$U = e^{i\theta}\ exp(\frac{i\gamma_i \sigma_i}{2})$$

- After SSB, **different patches of the universe with length scale ξ** (correlation length) can acquire VEVs in the **two different sectors of the vacuum manifold**.

- Because we break the **electroweak symmetry at the same time as the discrete symmetry**, we can get **Abelian and Non-Abelian domain wall solutions**. In contrast to the **standard domain walls models where only the discrete symmetry gets broken**.
  (**arXiv:hep-th/0105128** for the case of **SU(5)xZ_2 Domain Walls**)

- We end up with **several classes of domain walls** which can **break CP, Charge** or both <u>**inside**</u> the domain wall.



How to find static domain wall solutions ?

Minimize the energy functional of the vacuum configuration $\Phi(x)$

$$E = \int dx\ \mathcal{E}(x), \qquad \mathcal{E}(x) = \frac{d\Phi_i}{dx}\frac{d\Phi_i^\dagger}{dx} + V(\Phi_1, \Phi_2)$$

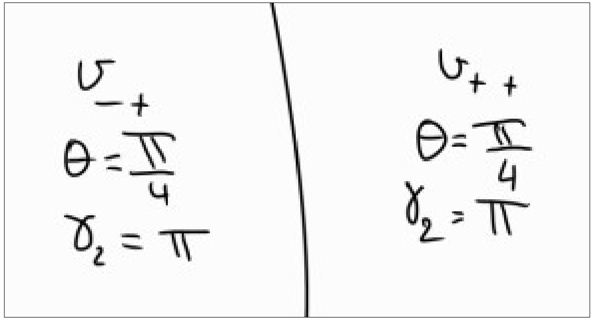
$$\text{Using } \Phi_i(x) = U(x)\hat{\Phi}(x) \quad \text{and} \quad \frac{dU}{dx} = i(\frac{d\theta}{dx} + \frac{1}{2}\frac{d\gamma_i}{dx}\sigma_i)U(x), \quad \text{we get:}$$

$$\mathcal{E}(x) = \sum_k i\frac{d\hat{\Phi}_k^\dagger}{dx}U^\dagger(x)\left(\frac{d\theta}{dx} + \frac{1}{2}\frac{\gamma_i}{dx}\sigma_i\right)U(x)\Phi_k(x) + \text{h.c} + \left\|\frac{d\hat{\Phi}_k}{dx}\right\|^2 + \left\|\hat{\Phi}_k\right\|^2\left[(\frac{d\theta}{dx})^2 + \frac{1}{4}(\frac{d\gamma_i}{dx})^2\right] + \hat{\Phi}_k\frac{d\theta}{dx}\frac{d\gamma_i}{dx}\sigma_i\hat{\Phi}_k^\dagger + V(\Phi_1, \Phi_2)$$

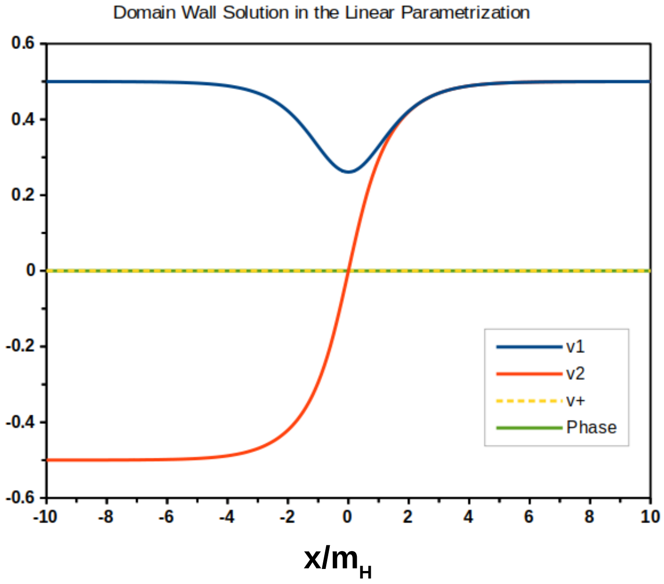
Minimization of E yields the **equation of motion for static fields** (analogous to minimizing the action)

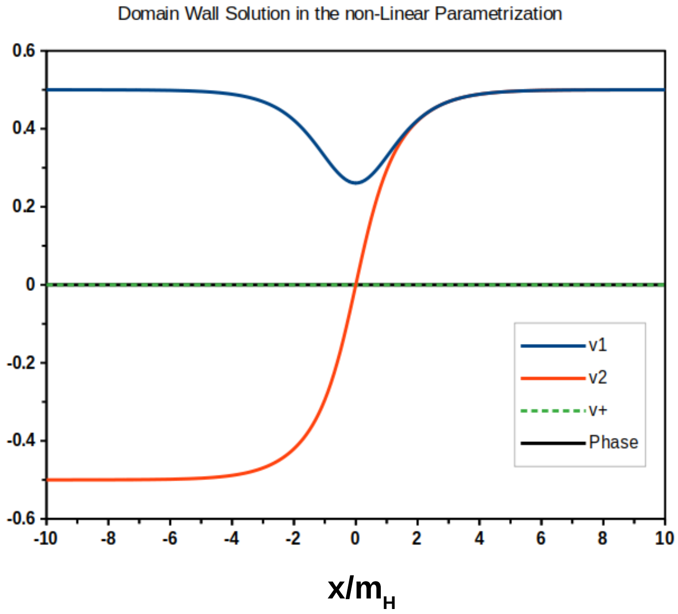
$$\frac{d}{dx}\left(\frac{\partial\mathcal{E}}{\partial\left(d\phi_n/dx\right)}\right) - \frac{\partial\mathcal{E}}{\partial\phi_n} = 0$$

Simplest case : Standard Domain Walls Solution

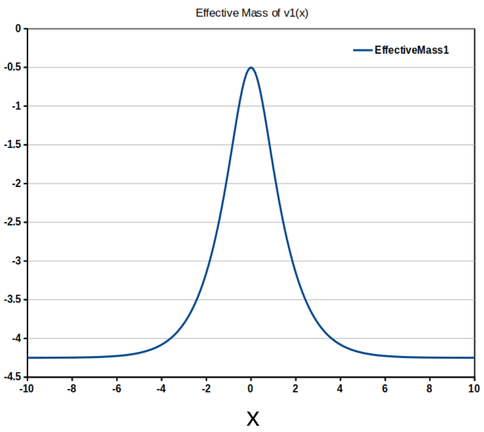


$$\mathbf{\Phi}_- = \left\{ \begin{array}{c} \Phi_1 \\ -\Phi_2 \end{array} \right\} \quad \mathbf{\Phi}_+ = \left\{ \begin{array}{c} \Phi_1 \\ \Phi_2 \end{array} \right\} \qquad U(x) = constant$$





Explaining the behavior of $v_1(x)$

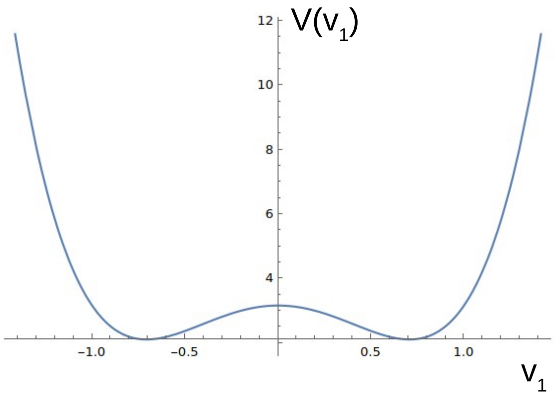


Calculate the **effective mass** term for $v_1(x)$:

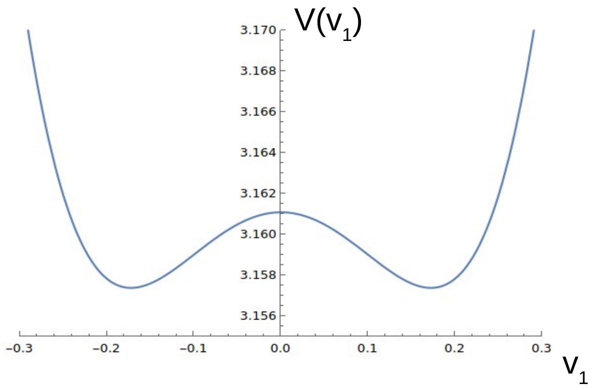
$$M_{eff1} = \frac{1}{2}m_{11}^2 + \frac{1}{4}\left[(\lambda_3 + \lambda_4)v_2^2(x) + \lambda_3 v_+^2(x) + \lambda_5 v_2^2(x)\cos(2\xi(x))\right]$$

Becomes **bigger (<u>less negative</u>) inside** the domain wall !

Potential for Φ_1 with the effective mass set to $v_2(10) = v_2$



Potential for Φ_1 with the effective mass set to $v_2(0) = 0$ (just as inside the Domain Wall)



Leads to $v_1(0)$ getting a smaller value inside the Domain Wall

Example for simplified case where only **θ(x) is non constant** :

$$U(x) = e^{i\theta(x)}$$

$$\mathcal{E}(x) = \frac{1}{2}(\frac{dv_1}{dx})^2 + \frac{1}{2}(\frac{dv_2}{dx})^2 + \frac{1}{2}(\frac{dv_+}{dx})^2 + v_2^2(\frac{d\xi}{dx})^2 + \frac{1}{2}(\frac{d\theta}{dx})^2(v_1^2 + v_2^2 + v_+^2) + v_2^2\frac{d\theta}{dx}\frac{d\xi}{dx}$$
$$+ V(v_1, v_2, v_+, \xi)$$

Leads to the equations of motion:

$$\frac{d^2v_1}{dx^2} - (\frac{d\theta}{dx})^2 v_1 - \frac{dV}{dv_1} = 0$$

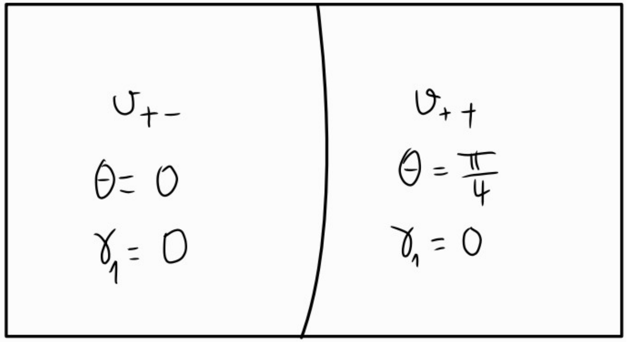
$$\frac{d^2v_2}{dx^2} - v_2(\frac{d\xi}{dx} + \frac{d\theta}{dx})^2 - \frac{dV}{dv_2} = 0$$

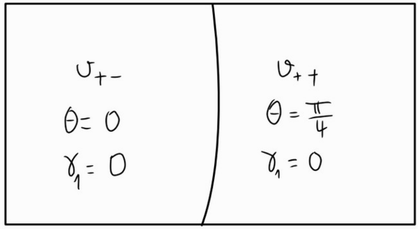
$$\frac{d^2v_+}{dx^2} - v_+(\frac{d\theta}{dx})^2 - \frac{dV}{dv_+} = 0$$

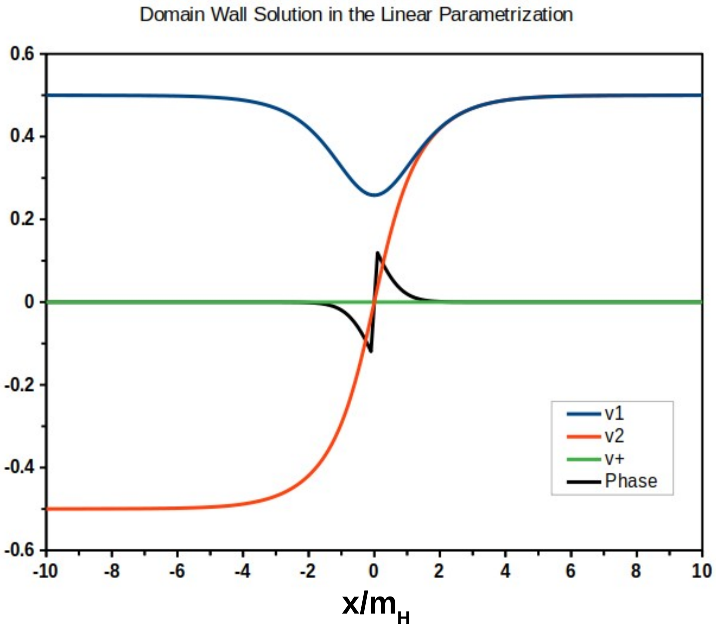
$$v_2^2\frac{d^2\xi}{dx^2} + v_2^2\frac{d^2\theta}{dx^2} + 2v_2\frac{dv_2}{dx}\frac{d\xi}{dx} + 2v_2\frac{dv_2}{dx}\frac{d\theta}{dx} - \frac{dV}{d\xi} = 0$$

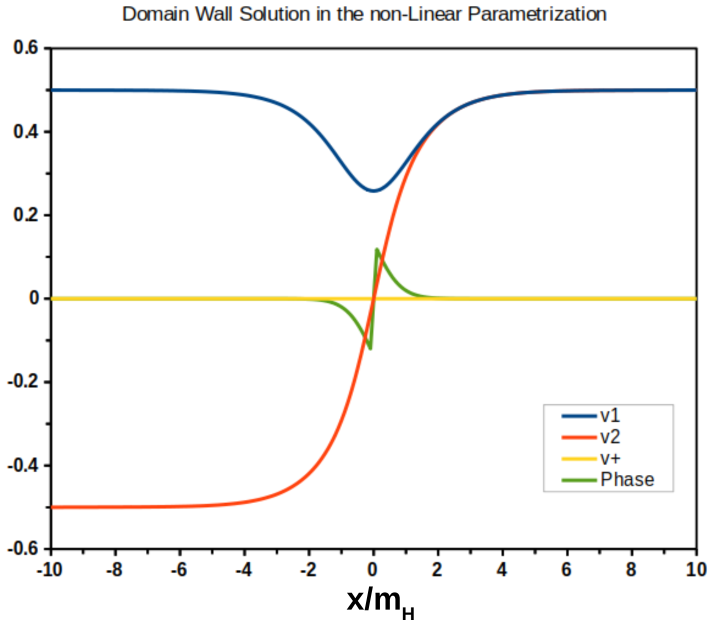
$$(v_1^2 + v_2^2 + v_+^2)\frac{d^2\theta}{dx^2} + 2\frac{d\theta}{dx}(v_1\frac{dv_1}{dx} + v_2\frac{dv_2}{dx} + v_+\frac{dv_+}{dx}) + v_2^2\frac{d^2\xi}{dx^2} + 2v_2\frac{dv_2}{dx}\frac{d\xi}{dx} = 0$$

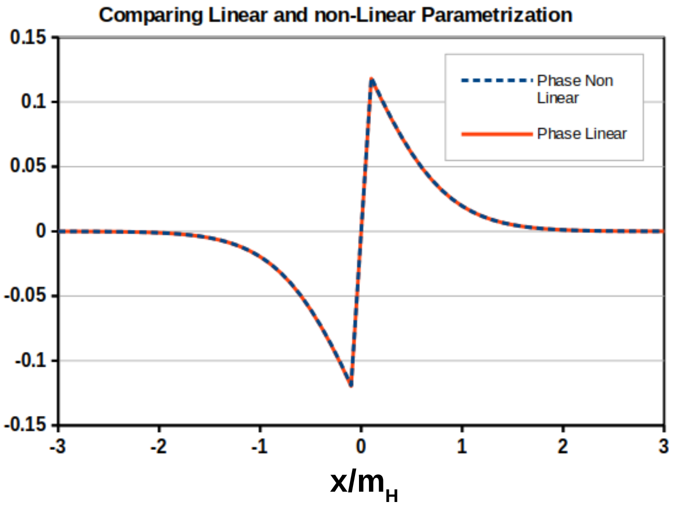
Solve numerically using
Gradient flow method
Richard A. Battye, Gary D. Brawn, Apostolos Pilaftsis (1106.3482) JHEP
Richard A. Battye, Apostolos Pilaftsis, Dominic G. Viatic (2006.13273) JHEP







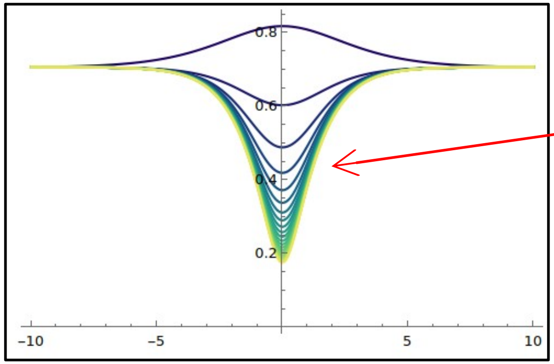




This leads to the **phase ξ(x)** between the two doublets being **non zero inside** the domain wall.

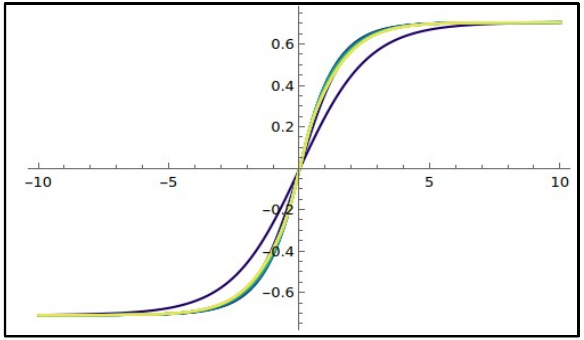
CP violation inside the domain wall

PI/4

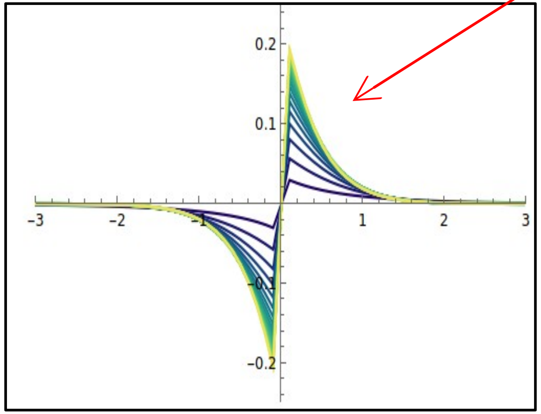


x/m_H

Increasing the mass of the **CP-Even Higgs H** makes $v_1(0)$ **smaller** and **the phase ξ** between the doublets **bigger**

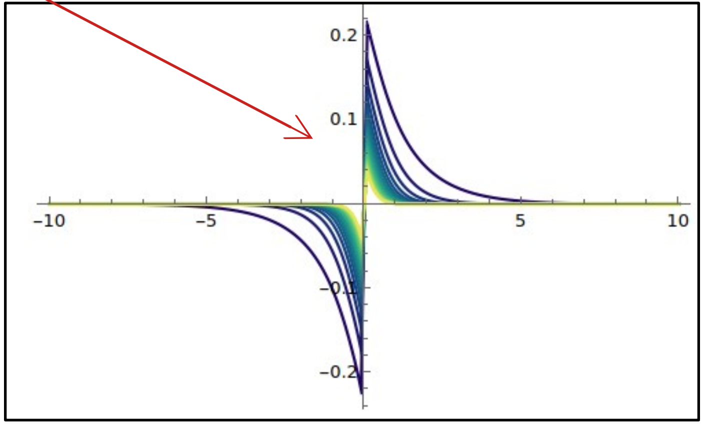


x/m_H



x/m_H

Increasing the mass of the **CP-Odd Higgs ρ** **decreases** the **phase ξ** inside the domain wall

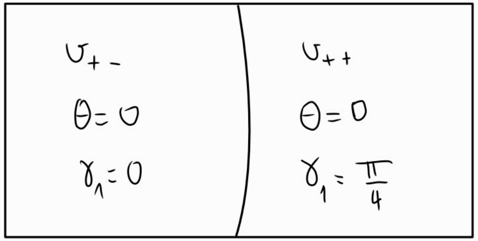


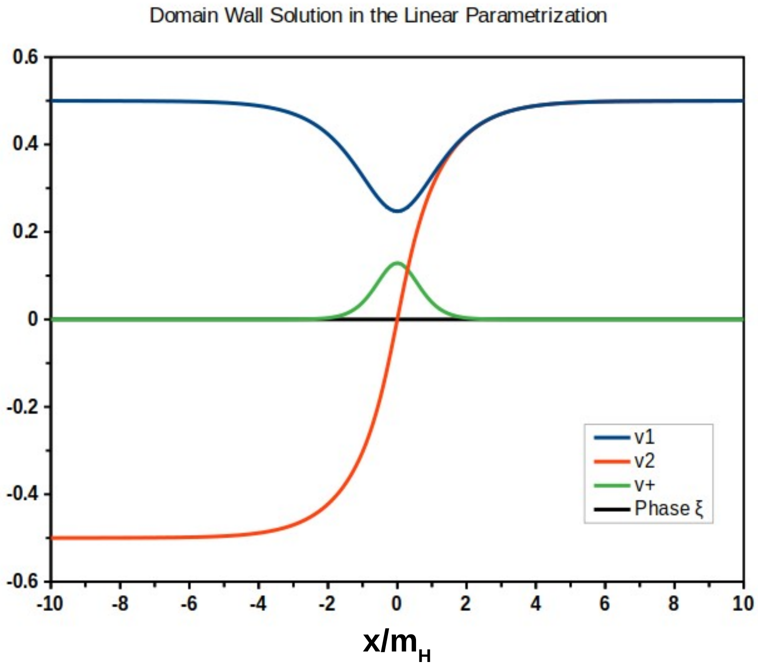
x/m_H

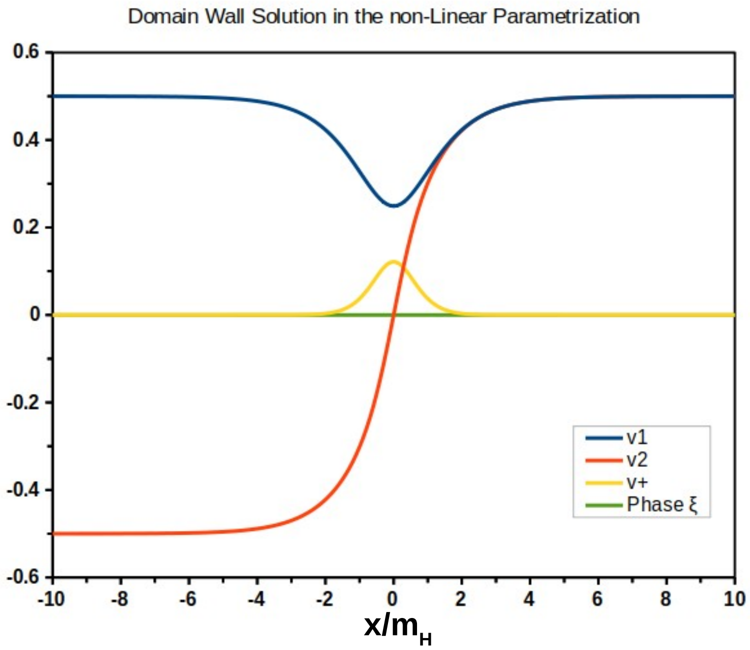
Case where γ_2 is non constant :

$$U(x) = exp(\frac{i\gamma_1(x)\sigma_1}{2})$$

As an example take $\gamma_1(x) = \left\{ \begin{array}{l} 0 \text{ at } -\infty \\ \frac{\pi}{4} \text{ at } +\infty \end{array} \right\}$



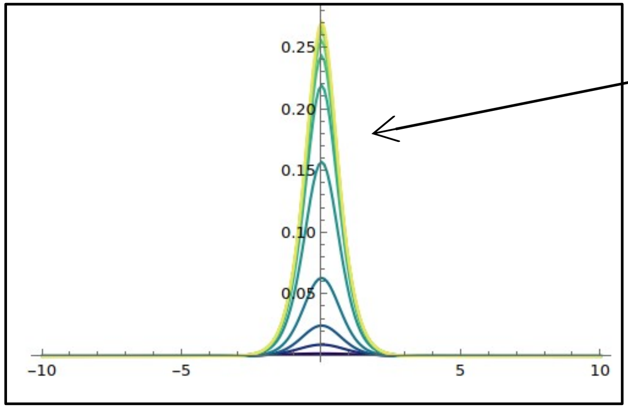




$v_+(x) \neq 0$ **inside** the domain wall.

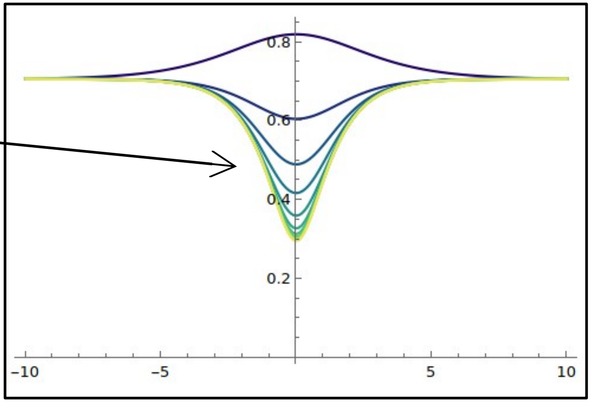
This leads to charge breaking effects and a massive photon <u>inside</u> the domain wall

Variation of mC and mH for the charge breaking DW



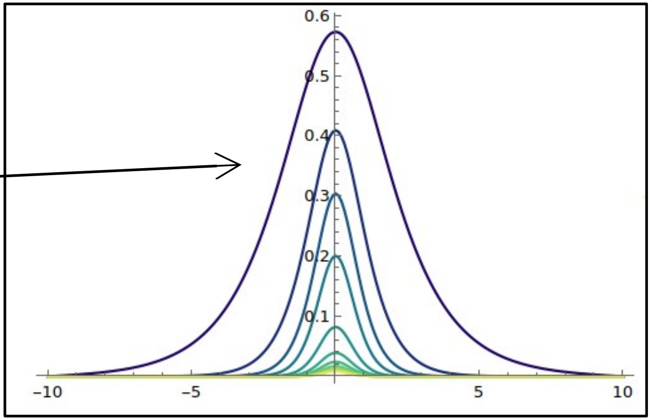
x/m_H

Increasing the mass of the **CP-Even Higgs H** makes the **maximum of the charge violating vacuum bigger**.



x/m_H

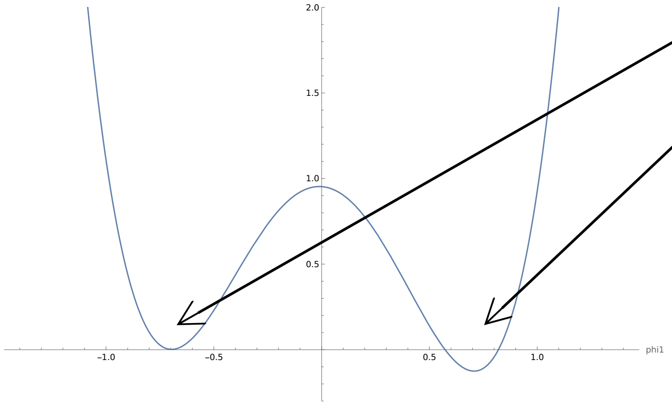
Increasing the mass of the **charged Higgs m_C** makes the maximum value of v_+ inside the domain wall **smaller**

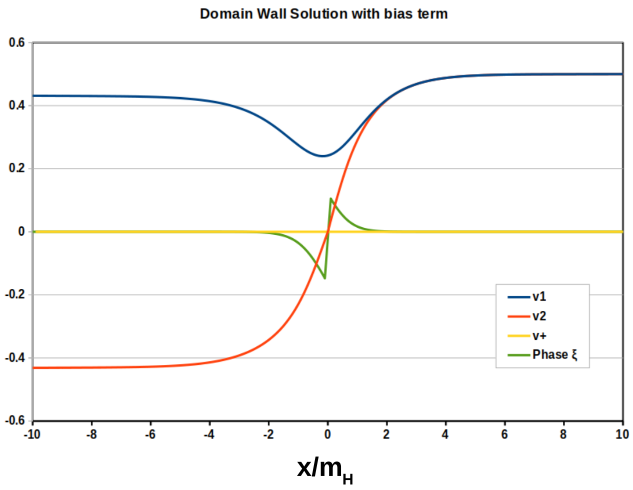


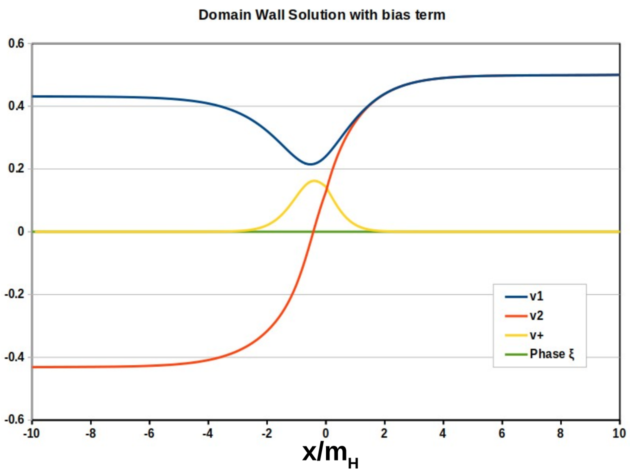
x/m_H

For $m_{12} \neq 0$, the **Z_2 symmetry is then approximate** and we get **asymmetric domain walls**:

The minima are not degenerate anymore



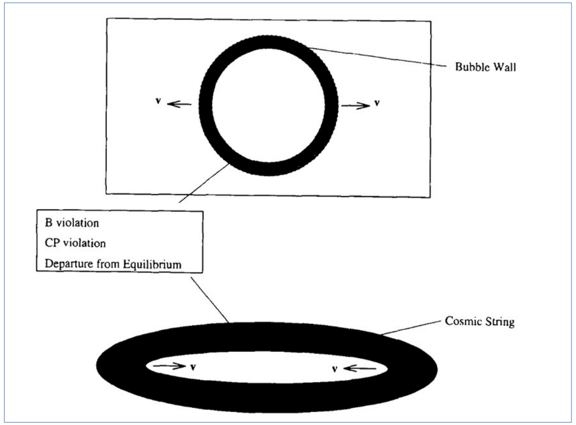




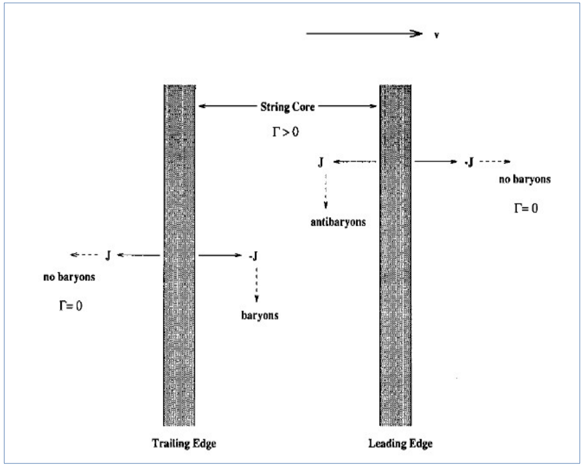
Baryogenesis with Topological Defects

Idea discussed in the 90s :

- **Local and nonlocal defect-mediated electroweak baryogenesis**
  hep-ph/9409281
- **Baryogenesis from Domain Walls in the Next-to-Minimal Supersymmetric Standard Model**
  hep-ph/9505241
- **Electroweak Baryogenesis with Cosmic Strings ?**
  Hep-ph/9901310
- **...**



From Physics Letters B 335 (1994) 123-130



Main idea:

- **The topological defect acts as the bubble wall.**
- **Sphalerons** are **less supressed** inside the topological defect
- **CP violation in the defect walls.**

Main Problems discussed in past papers:

- **Volume suppression factor** due to defect **not spanning the whole universe.**

$$\Delta n_B = \frac{1}{V}\frac{\Gamma_B}{T} V_{\rm BG}\Delta\theta$$

- **Symmetry restoration region not large enough to contain Sphalerons.**

$$R_{restoration} \sim \frac{1}{\sqrt{\lambda} v} \qquad R_{Sphalerons} \sim \frac{1}{g^2 T}$$

For N2HDM/2HDM Domain walls thickness 5-10 times smaller than Sphalerons

For cosmic strings

Very Suppressed!

String-mediated electroweak baryogenesis: A critical analysis

J. M. Cline,[1,*] J. R. Espinosa,[2,†] G. D. Moore,[1,‡] and A. Riotto[2,§]
[1]*Department of Physics, McGill University, 3600 University Street, Montréal, Québec, Canada H3A 2T8*
[2]*CERN TH-Division, CH-1211 Geneva 23, Switzerland*
(Received 6 October 1998; published 22 February 1999)

$$\left[\frac{N_B}{N_\gamma}\right]_{strings} \lesssim 10^{-10}\left[\frac{N_B}{N_\gamma}\right]_{observed}. \qquad (4)$$

That is, the mechanism just studied is uncapable of generating a sufficiently large matter-antimatter asymmetry.

What about Domain Walls ?

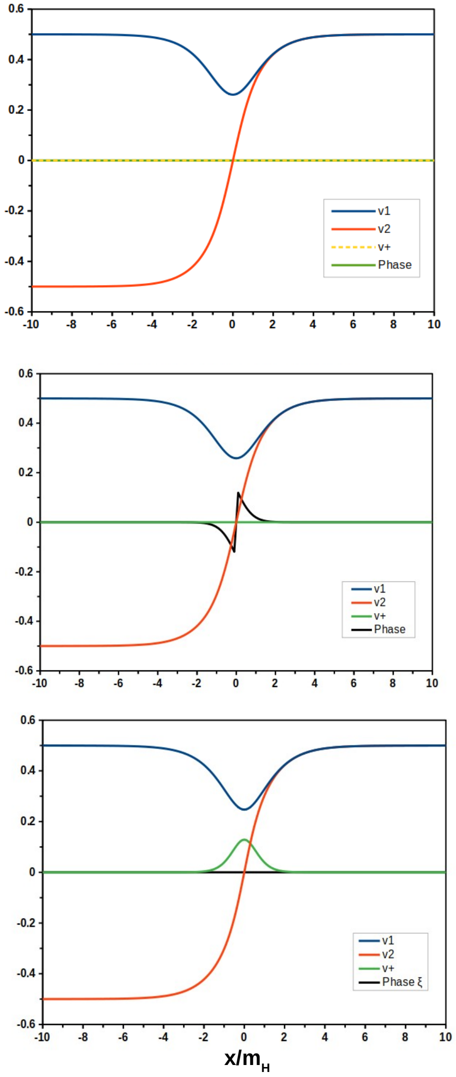
Conclusions and Outlook

- Due to **breaking SU(2)xU(1)** symmetry **along** the **Z_2 symmetry**, the domain walls in the 2HDM can have **CP and Charge violation inside the wall**.

- **Bias term for the Z_2 symmetry** leads to **asymmetry** in the breaking of CP in the different parts of the wall.

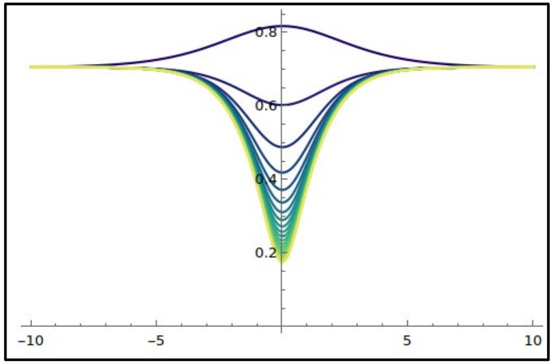
- Possible future directions include :

1) **Electroweak Baryogenesis** using the **domain walls**.
2) **Using charge violation** inside the domain wall to **constrain** the model.
3) **Probing gravitational wave** spectrum from **annihilating domain walls.**



Backup

Increasing m_H leads to effective mass becoming **more negative** outside the Domain wall and **less negative inside the domain wall**, which leads to $v_1(0)$ becoming smaller with higher m_H. However, for the **first parameter point (m_H = 80 GeV)**, the **effective mass is more negative inside the domain wall.**

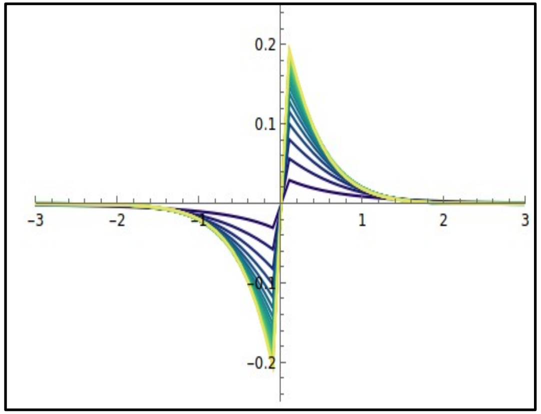


Terms influencing ξ(x) :

$$\mathcal{E}_\xi(x) = \frac{1}{2} v_2^2(x) (\frac{d\xi}{dx})^2 + v_2^2(x) \frac{d\theta}{dx} \frac{d\xi}{dx} + \frac{\lambda_5}{4} v_1^2(x) v_2^2(x) \cos(2\xi(x))$$

Always positive

Negative



The second term needs to be negative to make sure that the behaviour of ξ(x) minimizes the energy.

We find from the equation of motion for θ(x) :

$$\frac{d\theta}{dx} = -\frac{v_2^2}{v_1^2 + v_2^2 + v_+^2} \frac{d\xi}{dx}$$

Second term is more **negative for smaller $v_1(x)$** and **bigger gradient for ξ(x), gives a more negative contribution than the third term for bigger ξ(x)**