

# Techniques for model-independent Interpretations of Light Hidden Particle Searches

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Higgs, Flavour and Beyond

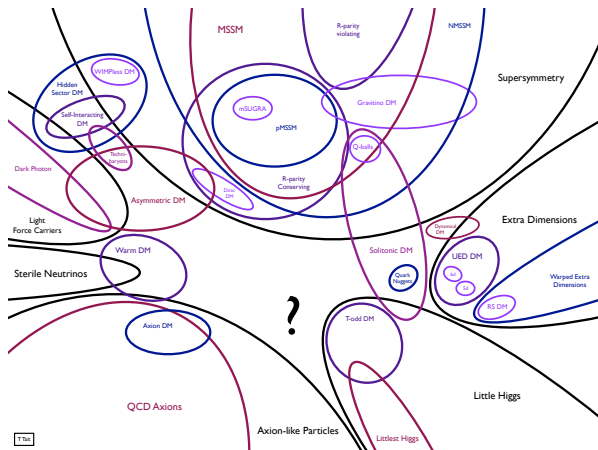
based on

arXiv:2105.06477 / arXiv:2203.02229

Published in Collaboration with Chiara Arina, Jan Hajer

# Effective Field Theories: Good But Not Perfect

- EFTs fine for heavy new physics (NP)

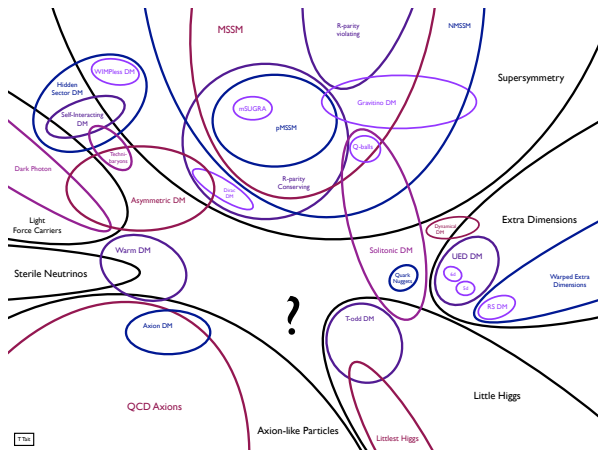


Made by T. Tait, see arXiv:1401.6085

What about more complicated, realistic hidden sectors?  
⇒ Use factorization!

# Effective Field Theories: Good But Not Perfect

- EFTs fine for heavy new physics (NP)
- EFTs, simplified models for light NP useful but simplistic



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What about more complicated, realistic hidden sectors?  
⇒ Use factorization!

# Small Portal Couplings Ensure Factorization

See [arXiv:2203.02229](#)

- Most general SM extension:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{hidden}} + \epsilon \mathbf{A}_d \mathbf{B}^d$$

$\mathbf{A}_d$  = SM operator /  $\mathbf{B}^d$  = hidden operator /  $\epsilon$  = coupling /  $d \geq 4$  allowed

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$$\Gamma(\text{SM} \rightarrow \text{SM}' + \text{hidden}) \propto \epsilon^2 \mathbf{M}_d \mathbf{M}_e^\dagger \mathbf{J}^{de} + \mathcal{O}(\epsilon^3) \quad \mathbf{J}^{de} = \sum \mathbf{J}^d \mathbf{J}^{e\dagger}$$

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Indistinguishable final states

$\mathbf{M}$  = reduced matrix elements (SM only, model-independent)

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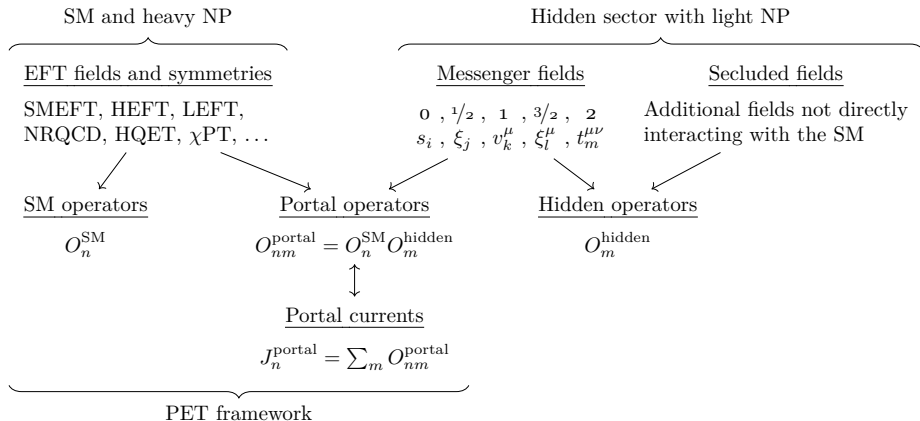
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- 2 With portal EFTs: Model-independent constraints

# PETs maximize Model-independence

See [arXiv:2105.06477](https://arxiv.org/abs/2105.06477)

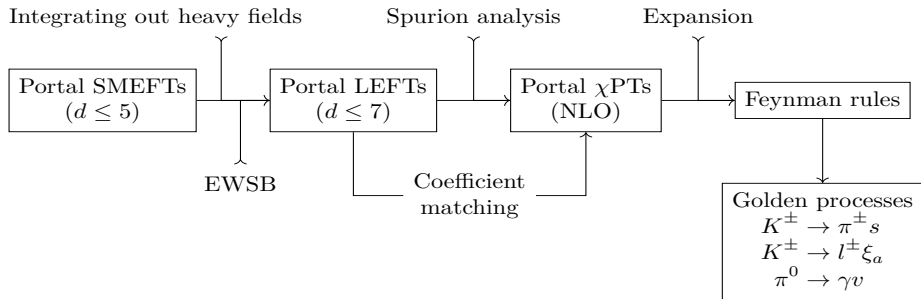
We developed framework to construct **portal effective theories (PETs)** that couple SM EFTs to generic hidden sectors





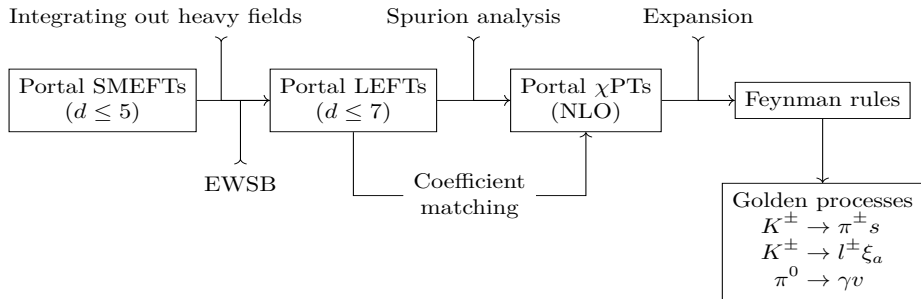
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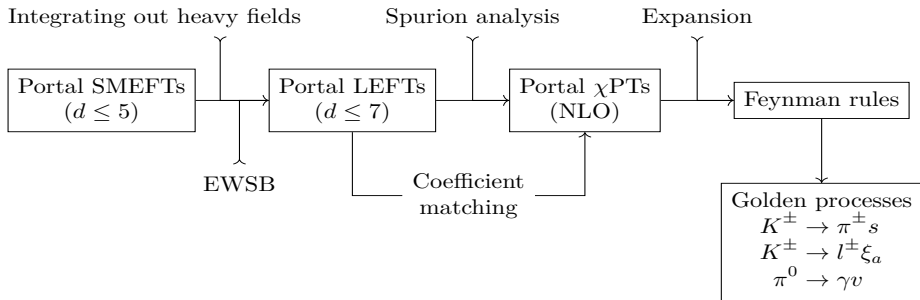
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- We constructed **portal SMEFTs** / **LEFTs** / **ChPTs**  
( $S = 0, 1/2, 1$  messengers /  $\Delta F \neq 0$  allowed / all leading portal operators)

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- We computed benchmark master decay rates  
( $\pi^0 \rightarrow \gamma \nu$  /  $K^+ \rightarrow \pi^+ s$  /  $K^+ \rightarrow \ell^+ + \text{anything}$ )

# Summary and Outlook

## 1 Hidden Particle Production Rates Factorize via

$$\Gamma(\text{SM} \rightarrow \text{SM}' + \text{hidden}) \propto \epsilon^2 \mathbf{M}_d \mathbf{M}_e^\dagger \mathbf{J}^{de} + \mathcal{O}(\epsilon^3)$$

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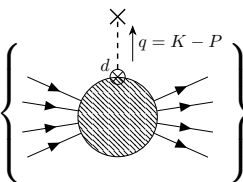
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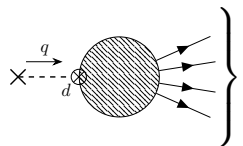
- Factorizing decay / scattering / finite temperature rates
- PETs for heavy mesons, (p)NRQED *etc.*
- Computing interaction rates, hidden currents

**Thank you for your attention!**



# $M_d, J^d$ are Feynman Diagram Sums

$$i M_d = \mathcal{K} \left\{ \begin{array}{c} \text{Diagram 1} \end{array} \right\} \mathcal{P}$$


$$i J_d = \left\{ \begin{array}{c} \text{Diagram 2} \end{array} \right\} \mathcal{Q}$$


# EW Scale PETs: Portal Operators

	$d$	Higgs	Yukawa + h.c.	Fermions	Gauge bosons
	3	$s_i  H ^2$			
	4	$s_i s_j  H ^2$			
$s_i$		$s_i s_j s_k  H ^2$	$s_i q_a \bar{u}_b \tilde{H}^\dagger$		$s_i G_{\mu\nu}^a G_a^{\mu\nu}$
		$s_i D^\mu H^\dagger D_\mu H$	$s_i q_a \bar{d}_b H^\dagger$		$s_i W_{\mu\nu}^a W_a^{\mu\nu}$
	5	$s_i  H ^4$	$s_i \ell_a \bar{e}_b H^\dagger$		$s_i B_{\mu\nu} B^{\mu\nu}$
					$s_i G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$
					$s_i W_{\mu\nu}^a \tilde{W}_a^{\mu\nu}$
					$s_i B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\xi_a$	4		$\xi_a \ell_b \tilde{H}^\dagger$		
$+$ h.c.	5	$\xi_a \xi_b  H ^2$	$\xi_a^\dagger \bar{\sigma}^\mu \ell_b D_\mu \tilde{H}^\dagger$		$\xi_a \sigma^{\mu\nu} \xi_b B_{\mu\nu}$
$v^\mu$		$v^\mu v^\mu  H ^2$		$v^\mu q_a^\dagger \bar{\sigma}_\mu q_b$	
		$\partial_\mu v^\mu  H ^2$		$v^\mu \bar{u}_a^\dagger \sigma_\mu \bar{u}_b$	
	4	$v^\mu H^\dagger \overleftrightarrow{D}_\mu H$		$v^\mu \bar{d}_a^\dagger \sigma_\mu \bar{d}_b$	
				$v^\mu \ell_a^\dagger \bar{\sigma}_\mu \ell_b$	
				$v^\mu \bar{e}_a^\dagger \sigma_\mu \bar{e}_b$	

# Strong Scale PETs: $d = 6, 7$ and $|\Delta F| = 1$ Portal Operators

$d$	Two quarks	Quark dipole	Four fermions
6	$s_i s_j s_k \bar{d} d$	$s_i F^{\mu\nu} \bar{d} \sigma_{\mu\nu} d$	
	$\partial^2 s_i \bar{d} d$	$s_i G^{\mu\nu} \bar{d} \sigma_{\mu\nu} d$	
	$s_i \partial_\mu s_j d^\dagger \bar{\sigma}^\mu d$		
7	$s_i s_j s_k s_l \bar{d} d$		$s_i d^\dagger \bar{q}^\dagger \bar{q} d$
			$s_i q^\dagger \bar{\sigma}^\mu q q^\dagger \bar{\sigma}_\mu q$
			$s_i d^\dagger \bar{\sigma}^\mu d \bar{q} \sigma_\mu \bar{q}^\dagger$
			$s_i e^\dagger \bar{\sigma}_\mu \nu u^\dagger \bar{\sigma}^\mu d$
			$s_i \nu^\dagger \bar{\sigma}_\mu \nu d^\dagger \bar{\sigma}^\mu d$
$\xi_a$ h.c.	$\xi_a^\dagger \bar{\sigma}_\mu e d^\dagger \bar{\sigma}^\mu u$		
	$\xi_a^\dagger \bar{\sigma}_\mu \nu d^\dagger \bar{\sigma}^\mu d$		

# Strong Scale QCD & $\chi$ PT Portal Interactions

SM: **4 currents** capture quark masses, photons,  $\theta$  angle:

$$\delta\mathcal{L}_{\text{QCD}} = -q^\dagger \bar{\sigma}_\mu \textcolor{blue}{I}^\mu q - \bar{q} \sigma_\mu \textcolor{blue}{r}^\mu q^\dagger - \theta G_{\mu\nu} \tilde{G}^{\mu\nu} - [\bar{q} \textcolor{blue}{m} q + \text{h.c.}]$$

QCD Flavour symmetry determines meson to current coupling  
 $\Rightarrow$  spurion analysis

Our work: **10 portal currents** parametrize new physics:

$$\delta\mathcal{L}_{\text{QCD}} \rightarrow -q^\dagger \bar{\sigma}_\mu \textcolor{red}{L}^\mu q - \bar{q} \sigma_\mu \textcolor{red}{R}^\mu q^\dagger - \Omega G_{\mu\nu} G^{\mu\nu} - \Theta G_{\mu\nu} \tilde{G}^{\mu\nu} \\ - [\bar{q} \textcolor{red}{M} q + \bar{q} \bar{\sigma}^{\mu\nu} \textcolor{red}{T}_{\mu\nu} q - \bar{q} \textcolor{red}{F} \bar{\sigma}^{\mu\nu} G_{\mu\nu} q + \text{h.c.}] + \mathcal{L}_{\bar{q}q\bar{q}q}[\textcolor{red}{H}_l, \textcolor{red}{H}_r, \textcolor{red}{H}_s]$$

## Portal $\chi$ PT: Couple mesons to **all portal currents**

- We extend spurion approach to include new currents
- We estimate resulting new  $\chi$ PT coefficients  
(Using scale anomaly, QCD condensates, large  $n_c$  limit)

# 1) Model-independent $\pi^0 \rightarrow \gamma \nu$ ; Width

Interaction:

$$\mathcal{L}_{\text{portal}} \supset \frac{2}{(4\pi)^2 f} (2\partial^\mu \mathbf{V}_{\nu u}^\nu + \partial^\mu \mathbf{V}_{\nu d}^\nu) \frac{\pi^0}{\sqrt{2}} e \tilde{F}_{\mu\nu}, \quad \mathbf{V}_\nu^\mu = \epsilon_{UV} \left( \mathbf{c}_\nu^L + \mathbf{c}_\nu^R \right) \nu^\mu$$

Width:

$$\Gamma(\pi^0 \rightarrow \gamma \nu_i) = 2\epsilon_{\text{eff}}^2 \Gamma_{\pi^0 \rightarrow \gamma\gamma} \left( 1 - \frac{m_\nu^2}{m_\pi^2} \right)^3, \quad \epsilon_{\text{eff}} = \epsilon_{UV} \frac{2(\mathbf{c}_\nu^R + \mathbf{c}_\nu^L)_u + (\mathbf{c}_\nu^R + \mathbf{c}_\nu^L)_d}{2e(2q_u + q_d)}$$

## 2) $K^+ \rightarrow \pi^+ s_i$ Portal Interactions

Currents:

$$\begin{aligned}\Omega &\supset \frac{\epsilon_{UV}}{v} c_i^{S_\omega} s_i, & \mathbf{M} &\supset \epsilon_{UV} \left( \mathbf{c}_i^{S_m} + \mathbf{c}_{\partial^2 i}^{S_m} \frac{1}{v^2} \partial^2 \right) s_i, & H_x &\supset h_{xi} \frac{\epsilon_{UV}}{v} s_i, \\ \Theta &\supset \frac{\epsilon_{UV}}{v} c_i^{S_\theta} s_i, & \mathbf{\Gamma} &\supset \epsilon_{UV} \left( \lambda_d^s c_{i\bar{s}d}^\gamma + \lambda_s^d c_{ids}^\gamma \right) s_i\end{aligned}$$

Scalar current coupling to Kaons:

$$\mathcal{L}_{\text{portal}} \supset -\frac{b}{2} \epsilon_{UV} K^+ \pi^- \left( c_{K\pi s_i} + \text{Re } \mathbf{c}_{\partial^2 i s}^{S_m d} \frac{\partial^2}{v^2} \right) s_i,$$

where

$$\begin{aligned}c_{K\pi s_i} = & \text{Re } \mathbf{c}_i^{S_m d} + \frac{\epsilon_{EW}}{2} \left( (m_K^2 - m_\pi^2) \text{Re } \mathbf{c}_i^{S_m u} + m_K^2 \text{Re } \mathbf{c}_i^{S_m d} - m_\pi^2 \text{Re } \mathbf{c}_i^{S_m s} \right) \theta_{K^\pm \pi^\mp} \\ & - \frac{\epsilon_{EW}}{2} \left( 2v h_b \left( \frac{m_{ud}}{m_s} \left( \mathbf{c}_i^{S_m d} - \mathbf{c}_i^{S_m s^\dagger} \right) + \mathbf{c}_i^{S_m d^\dagger} \right) + \frac{m_{ud} + m_s}{v} h_{bi} - \kappa_\gamma \left( c_{ids}^\gamma + c_{i\bar{s}d}^{\gamma\dagger} \right) \right)\end{aligned}$$

$\theta_{K^\pm \pi^\mp}$  is kaon to pion mixing angle,  $m_{ud}$  is light quark mass

## 2) Model-independent $K^+ \rightarrow \pi^+ s_i$ Amplitude

$$\mathcal{A}(K^+ \rightarrow \pi^+ s_i) = \mathcal{A}_{\text{Re } S_m}(K^+ \rightarrow \pi^+ s_i) + \mathcal{A}_{\text{Im } S_m}(K^+ \rightarrow \pi^+ s_i) \\ + \mathcal{A}_\omega(K^+ \rightarrow \pi^+ s_i) + \mathcal{A}_\theta(K^+ \rightarrow \pi^+ s_i) + \mathcal{A}_{8+27}(K^+ \rightarrow \pi^+ s_i)$$

where

$$\mathcal{A}_{\text{Re } S_m}(K^+ \rightarrow \pi^+ s_i) = -\frac{\epsilon_{\text{UV}} b}{2} \left( c_{K\pi s_i} - \text{Re } \mathbf{c}_{\partial^2 i s}^{S_m d} \frac{m_s^2}{v^2} \right)$$

$$\mathcal{A}_\omega(K^+ \rightarrow \pi^+ s_i) = \frac{\epsilon_{\text{UV}} \epsilon_{\text{EW}} c_i^{S_\omega}}{\beta_0 v} \left( h'_b m_K^2 - \frac{1}{2} (h_8 + (3-1)h_{27}) (m_K^2 + m_\pi^2 - m_s^2) \right)$$

$$\mathcal{A}_{8+27}(K^+ \rightarrow \pi^+ s_i) = -\frac{\epsilon_{\text{UV}} \epsilon_{\text{EW}}}{4v} (h_{8i} + (3-1)h_{27i}) (m_K^2 + m_\pi^2 - m_s^2)$$

$$\mathcal{A}_{\text{Im } S_m}(K^+ \rightarrow \pi^+ s_i) = -i \epsilon_{\text{UV}} \epsilon_{\text{EW}} b \left( c_{s_i \pi} \frac{V_{K\pi\pi}}{m_s^2 - m_\pi^2} + c_{s_i \eta} \frac{V_{K\pi\eta}}{m_s^2 - m_\eta^2} + c_{s_i \eta'} \frac{V_{K\pi\eta'}}{m_s^2 - m_{\eta'}^2} \right)$$

$$\mathcal{A}_\theta(K^+ \rightarrow \pi^+ s_i) = i \frac{\epsilon_{\text{UV}} \epsilon_{\text{EW}} c_i^{S_\theta} m_0^2}{v} \left( c_\eta \frac{V_{K\pi\eta'}}{m_s^2 - m_{\eta'}^2} - s_\eta \frac{V_{K\pi\eta}}{m_s^2 - m_\eta^2} \right) .$$

- $c_\eta, s_\eta$  are (co-)sine of  $\eta - \eta'$  mixing angle,  $V_{\phi_a \phi_b \phi_c}$  are SM 3 meson vertices
- $c_{K\pi s_i}, c_{s_i \phi_a}$  parametrize coupling to hidden sectors

### 3) Factorization example: $K^+ \rightarrow \ell^+ + \text{anything}$

- Relevant portal interactions (see arXiv:2105.06477):

$$\mathcal{L}_{\text{portal}} \supset \underbrace{\nu \nu \mathbf{B}_\nu + \frac{V_{\text{us}}}{v^2} (s^\dagger \bar{\sigma}_\mu u) (\mathbf{B}_\ell^\dagger \bar{\sigma}^\mu \ell)}_{n=2 \text{ portal operators}} \Rightarrow \begin{array}{l} \text{Two portal vertices} \\ \text{(Missing mass } q^2) \end{array}$$



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- Compute  $M_\ell / M_\nu \Rightarrow$  Master decay rate:

$$\frac{d}{dx_q} \frac{\Gamma(K^+ \rightarrow \ell^+ + \text{NP})}{\Gamma(K^+ \rightarrow \ell^+ + \nu)} = \frac{\rho(x_q)}{\rho(0)} \underbrace{\frac{1}{2\pi x_q} \text{tr}_D \left\{ \not{q} \mathbf{J}^{\ell\ell} - 2 \text{Re } \nu \mathbf{J}^{\ell\nu} + \frac{\not{q}}{q^2} \nu^2 \mathbf{J}^{\nu\nu} \right\}}_{= \frac{F(x_q)}{2\pi}}$$

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Use constraints on  $F(x_q)$  to compare and contrast different models!

### 3) Re-interpreting a prior HNL search

(See also [arXiv:2005.09575](https://arxiv.org/abs/2005.09575))

Master decay rate:

$$\frac{d}{dx_q} \frac{\Gamma(K^+ \rightarrow \ell^+ + \text{NP})}{\Gamma(K^+ \rightarrow \ell^+ + \nu)} = \frac{\rho(x_q)}{\rho(0)} \underbrace{\frac{1}{2\pi x_q} \text{tr}_D \left\{ \not{q} \boldsymbol{J}^{\ell\ell} - 2 \text{Re } \nu \boldsymbol{J}^{\ell\nu} + \frac{\not{q}}{q^2} \nu^2 \boldsymbol{J}^{\nu\nu} \right\}}_{= \frac{F(x_q)}{2\pi}}$$

General form factor structure:

$$\frac{F(x)}{2\pi} = \sum_i A_i \delta(x - x_i) + B \qquad x_i = \frac{m_i^2}{m_K^2}$$

Resulting bounds:

$$\rho(x_e, x_i) A_i \lesssim 7 \cdot 10^{-11} \qquad \rho(x_e, x_q) B(x_q) \lesssim 2 \cdot 10^{-4}$$

### 3) HNL Hidden Currents

$$\mathbf{B}_d = \sum_i c_{di} \xi_i \quad d = \nu, \ell$$

$$J_{\dot{\beta}\alpha}^{\nu\nu} = \sum_i \frac{c_{\nu i}^\dagger c_{\nu i}}{2\omega_i} (q_i^\mu \bar{\sigma}_\mu)_{\dot{\beta}\alpha} \bar{\delta}(q_0 - \omega_i) \quad J_{\beta\alpha}^{\ell\nu} = \sum_i \frac{c_{\ell i}^\dagger c_{\nu i}}{2\omega_i} m_i \epsilon_{\beta\alpha} \bar{\delta}(q_0 - \omega_i)$$

$$J_{\beta\dot{\alpha}}^{\ell\ell} = \sum_i \frac{c_{\ell i}^\dagger c_{\ell i}}{2\omega_i} (q_i^\mu \sigma_\mu)_{\beta\dot{\alpha}} \bar{\delta}(q_0 - \omega_i)$$

$$\frac{F_\ell(x_q)}{2\pi} = \sum_i U_i^2 \Theta(q_0) \delta(x_q^2 - x_i^2) \quad x_i = \frac{m_i^2}{m_K^2} \quad U_i^2 = \left| c_{\ell i} - \frac{v c_{\nu i}}{m_i} \right|^2$$