PARISI-SOURLAS SUPERSYMMETRY

in RFIM and branched polymers

Apratim Kaviraj
DESY

Based on
1912.01617 , 2009.10087, 2112.06942, 2203.12629 w/ S. Rychkov & E. Trevisani

Invitation

[1912.01617] AK, Rychkov, Trevisani, JHEP
[2009.10087] AK, Rychkov, Trevisani, JHEP
[2112.06942] AK, Rychkov, Trevisani , PRL
[2203.12629] AK, Trevisani , JHEP

A class of disordered (impure) QFTs near criticality has a mysterious hidden supersymmetry (Parisi-Sourlas SUSY).

The SUSY results in the phenomenon of "dimensional reduction" of the disordered CFT (Parisi-Sourlas conjecture).

Some findings from experiments/numerics in this model have not been explained till date.

Main results:

- Systematic set-up of RG flow, understanding of the SUSY and its emergence
- Mechanism of how SUSY "breaks" and consistent explanation of all experimental/numerical findings.

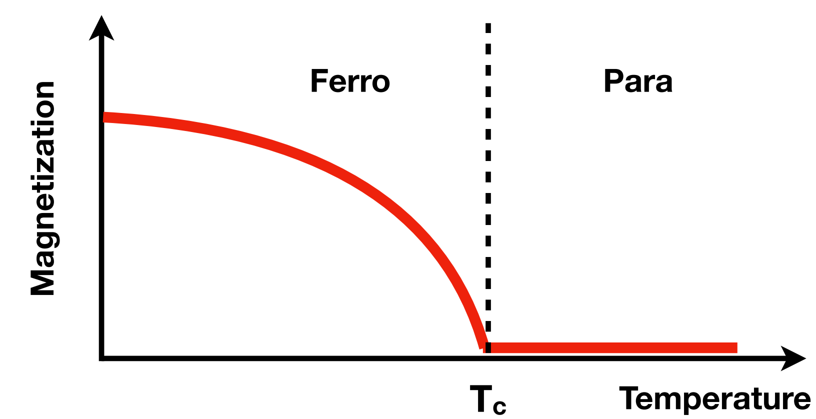
Disordered model	Dim reduced model	Dim. red. (Numerical finding)	Our explanation
RFIM	Ising	$6 \leq d < 4$	✓
Branched poly	Lee-Yang	$8 \leq d < 2$	✓

Resolution of a 46 year old problem !

RG flow and critical points

Conformal Field Theories (CFTs) are scale invariant theories, important in most aspects of theoretical physics (String Theory, AdS/CFT, Statistical Physics, etc).

CFTs describe critical behavior of systems, which is independent of microscopic details.



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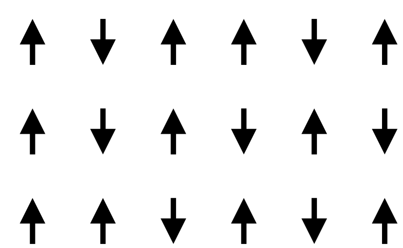
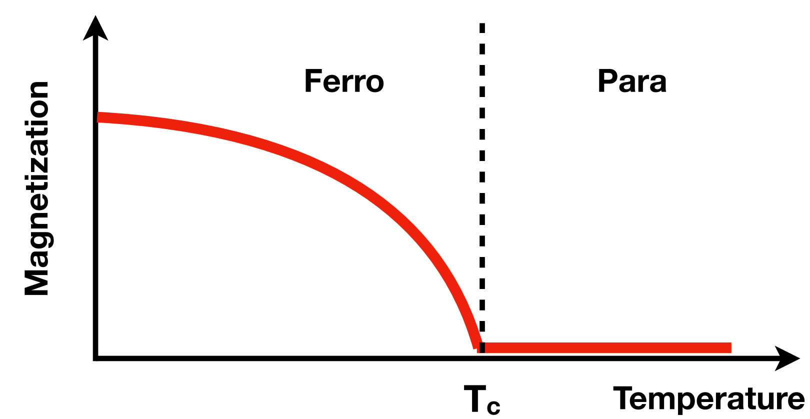
A simple example: Ising model

$$H(J) = -\sum_{<ij>} J s_i s_j$$





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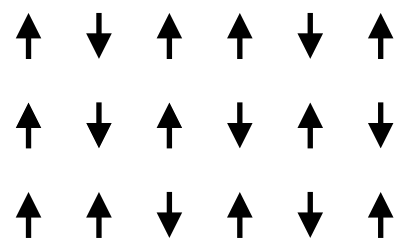

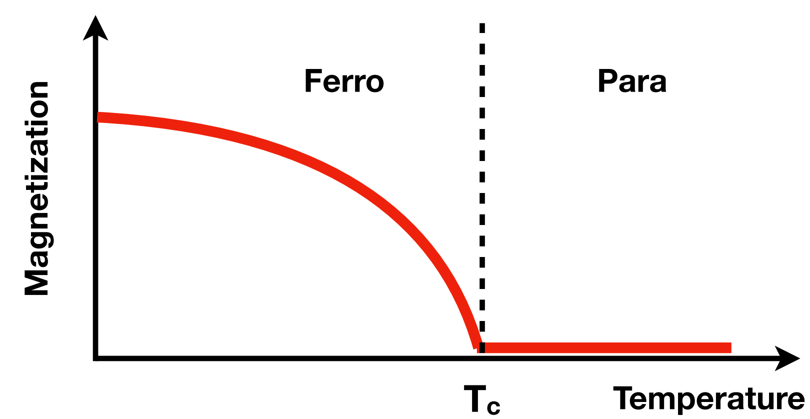

RG flow of Ising model is often described by a scalar quantum field theory ($Z_{\phi^4} = Z_{\text{Ising}}$)

$$S[\phi] = \int d^d x \left[(\partial \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]$$

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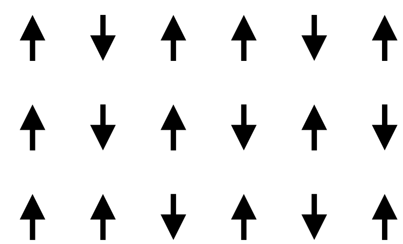

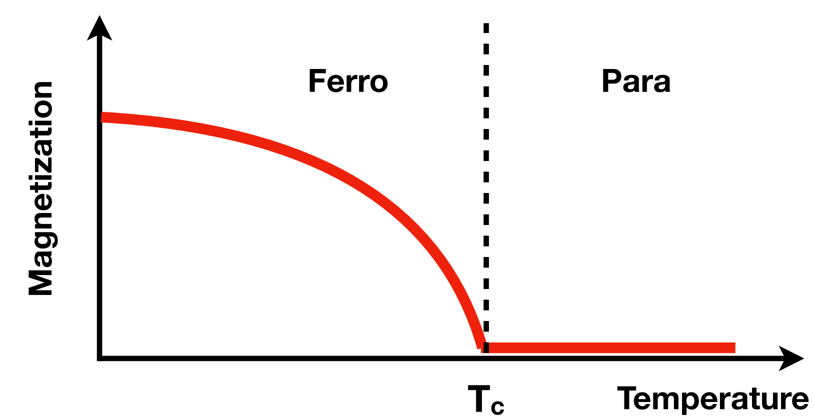

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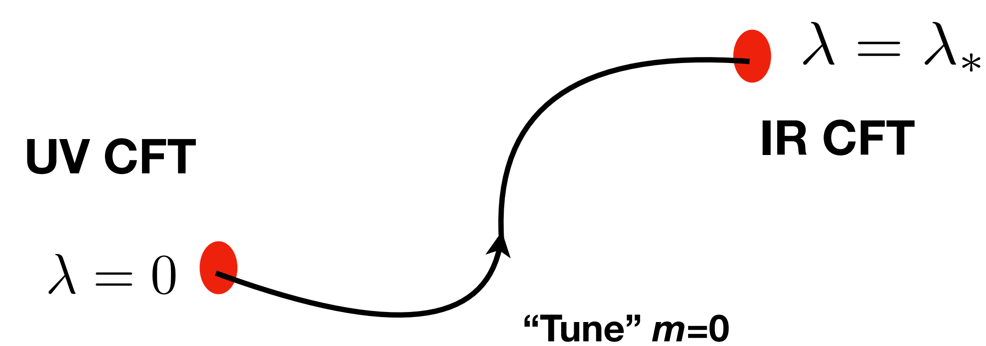
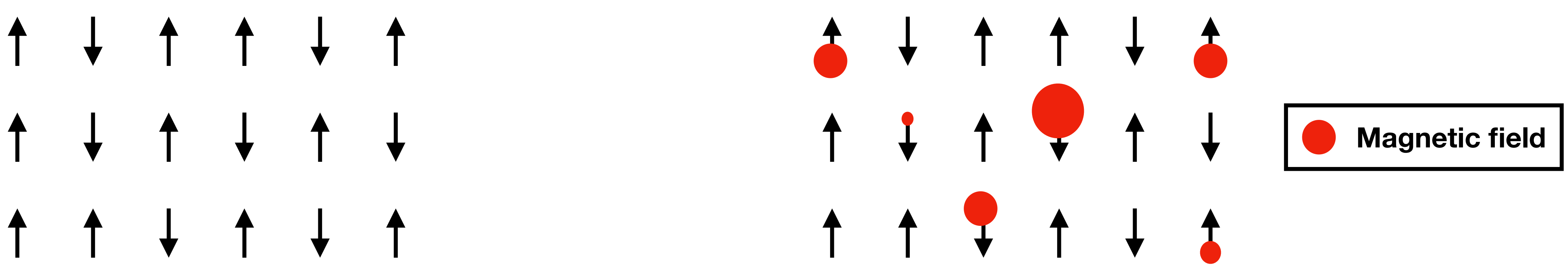
Beta function $\beta_\lambda = -\epsilon\lambda + \frac{3\lambda^2}{16\pi^2}$



Pure vs impure CFTs

At large correlation lengths (near critical point) the effect of impurities become prominent.

Quenched disorder: When impurities are frozen - not in thermal equilibrium with system.



Pure **Impure**

Random field models

Definition: $$S[h] = \int d^d x \Big[(\partial\phi)^2 + V(\phi) + h(x)\phi(x) \Big]$$

Impurity distribution:
(Gaussian)

$$P(h) = \exp\left[-\frac{1}{2H} \int d^d x h^2 \right]$$

H = strength of disorder.

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Disorder averaging (quenched)

$$\overline{\langle \phi(x_1)\phi(x_2)\phi(x_3)\cdots \rangle} = \int \mathcal{D}h(x)\, \langle \phi(x_1)\phi(x_2)\cdots \rangle_h \;\; P[h(x)]$$

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Mostly studied:

$V(\phi) \sim \phi^4$ Random field Ising Model (RFIM)

$V(\phi) \sim \phi^3$ Random field ϕ^3 Model

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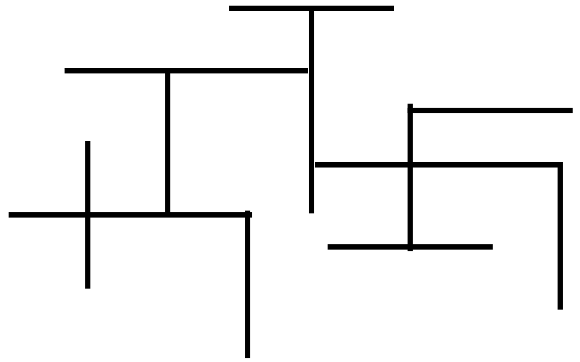
$\big\downarrow$ Lattice model



Branched polymers

Random field models

Definition: $$S[h] = \int d^d x \Big[(\partial\phi)^2 + V(\phi) + h(x)\phi(x) \Big]$$

Disorder-averaged correlators have a <u>new</u> critical point !

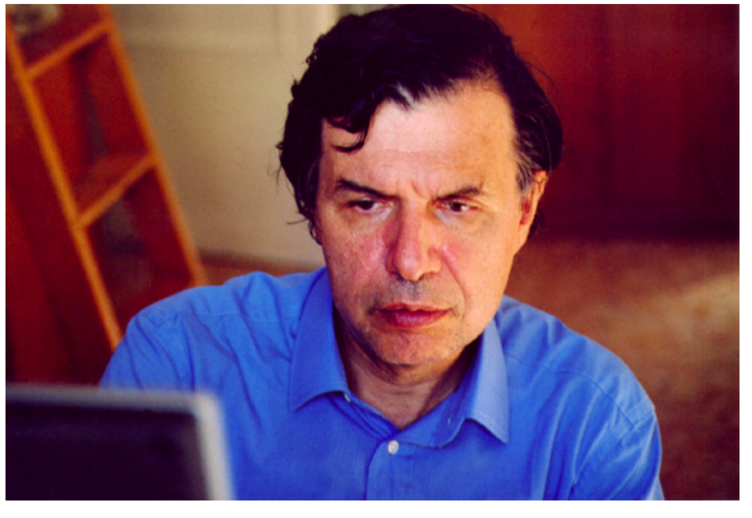
What CFT describes the critical point of random field models?

Parisi-Sourlas conjecture

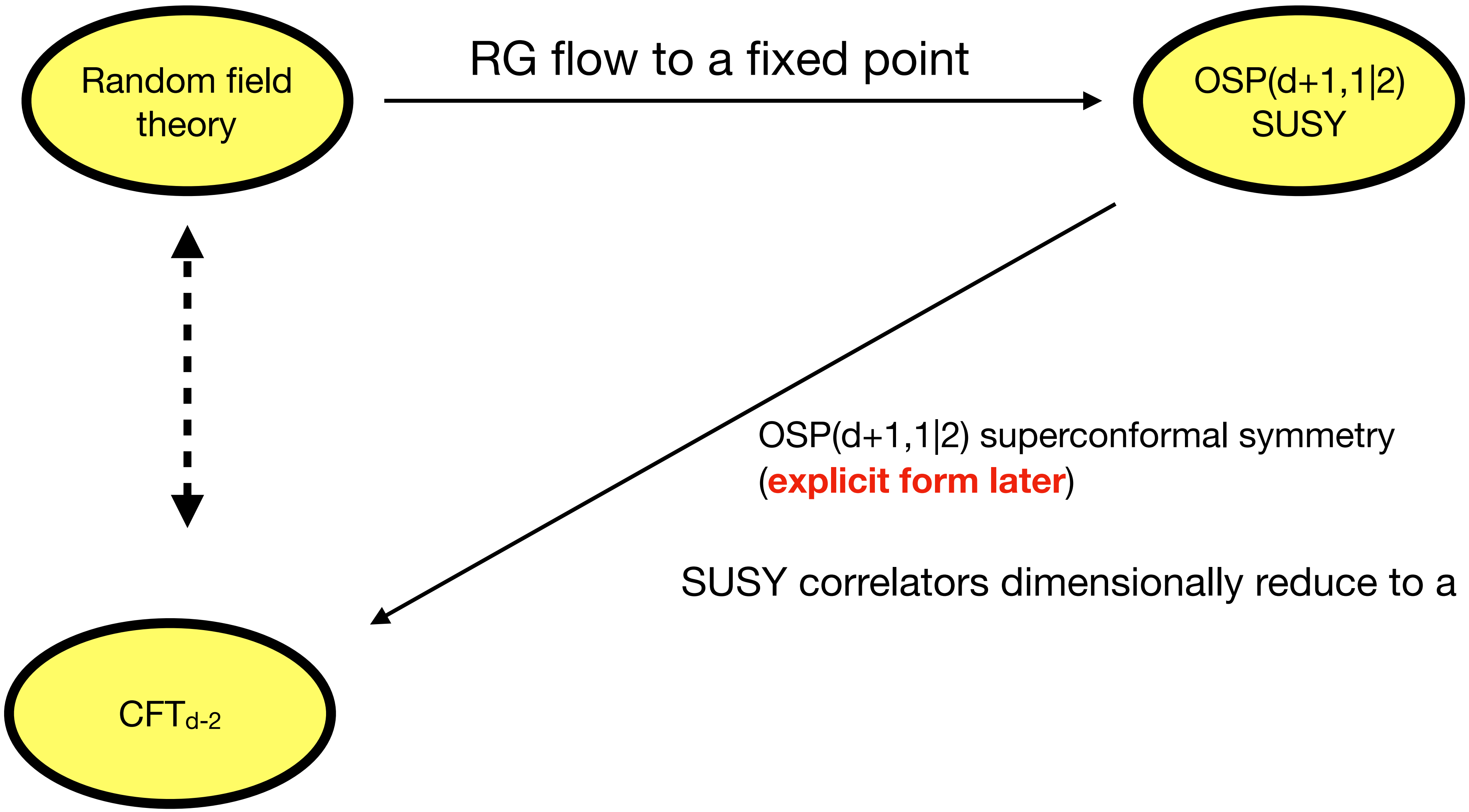
[Parisi, Sourlas, 1979]

1. **Parisi-Sourlas SUSY**: The fixed point of a random field theory is described by a supersymmetric CFT.

2. **Dimensional reduction:** Observables in the SUSY CFT are same as a (d-2)-dimensional CFT.

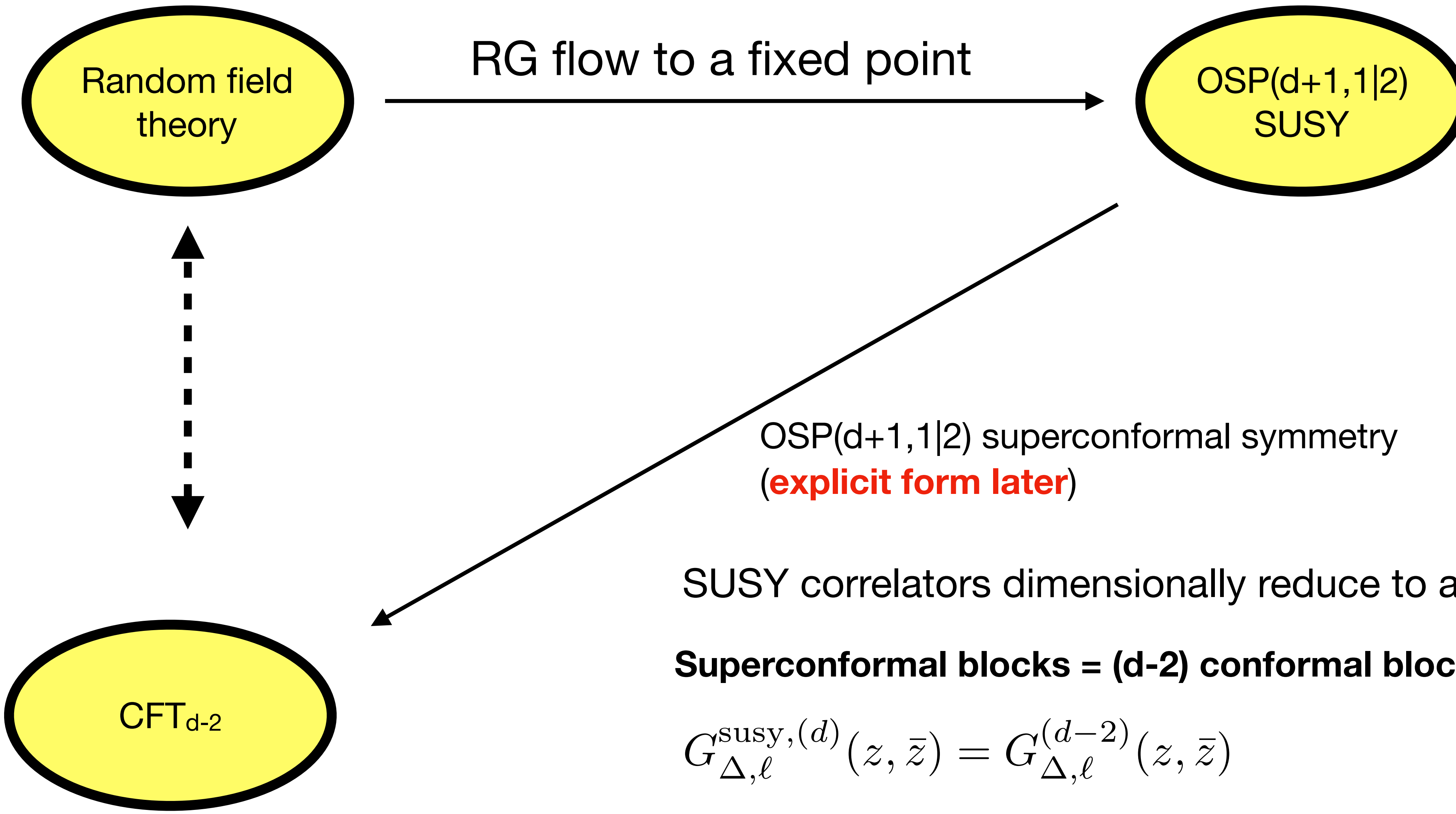


Parisi-Sourlas conjecture



SUSY correlators dimensionally reduce to a **local CFT$_{d-2}$** .

Parisi-Sourlas conjecture



SUSY correlators dimensionally reduce to a **local CFT$_{d-2}$** .

Superconformal blocks = (d-2) conformal blocks

$$G^{\text{susy},(d)}_{\Delta,\ell}(z,\bar{z}) = G^{(d-2)}_{\Delta,\ell}(z,\bar{z})$$

Susy stress tensor => conserved (d-2) stress tensor

[1912.01617] AK, Rychkov, Trevisani

It does not always work !!

Numerical check of dimensional reduction (Monte Carlo)

[Fytas, Martin-Mayor, Parisi, Picco, Sourlas 2013-2019]

$$V(\phi) = \lambda\phi^4$$

Lattice model (Disordered CFT) d dimension	Lattice model (Pure CFT) d-2 dimension	Dimensional reduction
6d free theory	4d free theory	✓
5d RFIM	3d Ising	✓
4d RFIM	2d Ising	✗
3d RFIM	No Ising CFT	✗

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$V(\phi) = g\phi^3$

Lattice model (Disordered CFT) d dimension	**Lattice model (Pure CFT) d-2 dimension**	**Dimensional reduction**
8d free theory	6d free theory	✓
7d Branched polymer	5d Lee-Yang	✓
6d Branched polymer	4d Lee-Yang	✓
5d Branched polymer	3d Lee-Yang	✓
4d Branched polymer	2d Lee-Yang	✓
3d Branched polymer	1d Lee-Yang	✓

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$V(\phi) = g\phi^3$	8d free theory	6d free theory	✓
	7d Branched polymer	5d Lee-Yang	✓
	6d Branched polymer	4d Lee-Yang	✓
	5d Branched polymer	3d Lee-Yang	✓
	4d Branched polymer	2d Lee-Yang	✓
	3d Branched polymer	1d Lee-Yang	✓

How does the Parisi-Sourlas conjecture work/fail?

Main question: how does SUSY emerge in the RG flow?

To answer: <u>We need a well-defined QFT</u>

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We are interested in disorder-averaged correlation functions

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$$\overline{\log Z(h)} = \lim_{n\to 0} \frac{\overline{Z^n} - 1}{n}$$

Enter the **Replica Method**

Replica Method

$$\overline{Z^n} = \int \prod_{i=1}^{n} \mathcal{D}\phi_i \, e^{-S_r}$$

$$S_r = \int d^dx \Big[\frac{1}{2} \sum_{i=1}^{n} (\partial \phi_i)^2 - H \Big(\sum_{i=1}^{n} \phi_i \Big)^2 + \sum_{i=1}^{n} V(\phi_i) \Big]$$

This action at $n \longrightarrow 0$

is a QFT that describes random field models

Replica action in Cardy variables

$$\sum_i \big(\partial\phi_i\big)^2 + H(\sum_i \phi_i)^2 + \sum_i (m^2\phi_i^2 + \lambda\phi_i^4)$$

$\phi_1 = \varphi + \frac{\omega}{2}$ $\phi_i = \varphi - \frac{\omega}{2} + \chi_i \quad [i = 2, \cdots, n]$ **(Gives good scaling operators)** [Cardy, 1985]

$$= \partial\varphi\partial\omega - H\omega^2 + \frac{1}{2}\sum_{i=2}^{n}(\partial\chi_i)^2 + m^2(\varphi\omega + \frac{\chi_i^2}{2}) + \lambda(4\varphi^3\omega + 6\varphi^2\chi_i^2)$$

$$+ \ O(n) \ + \ \text{irrelevant}$$

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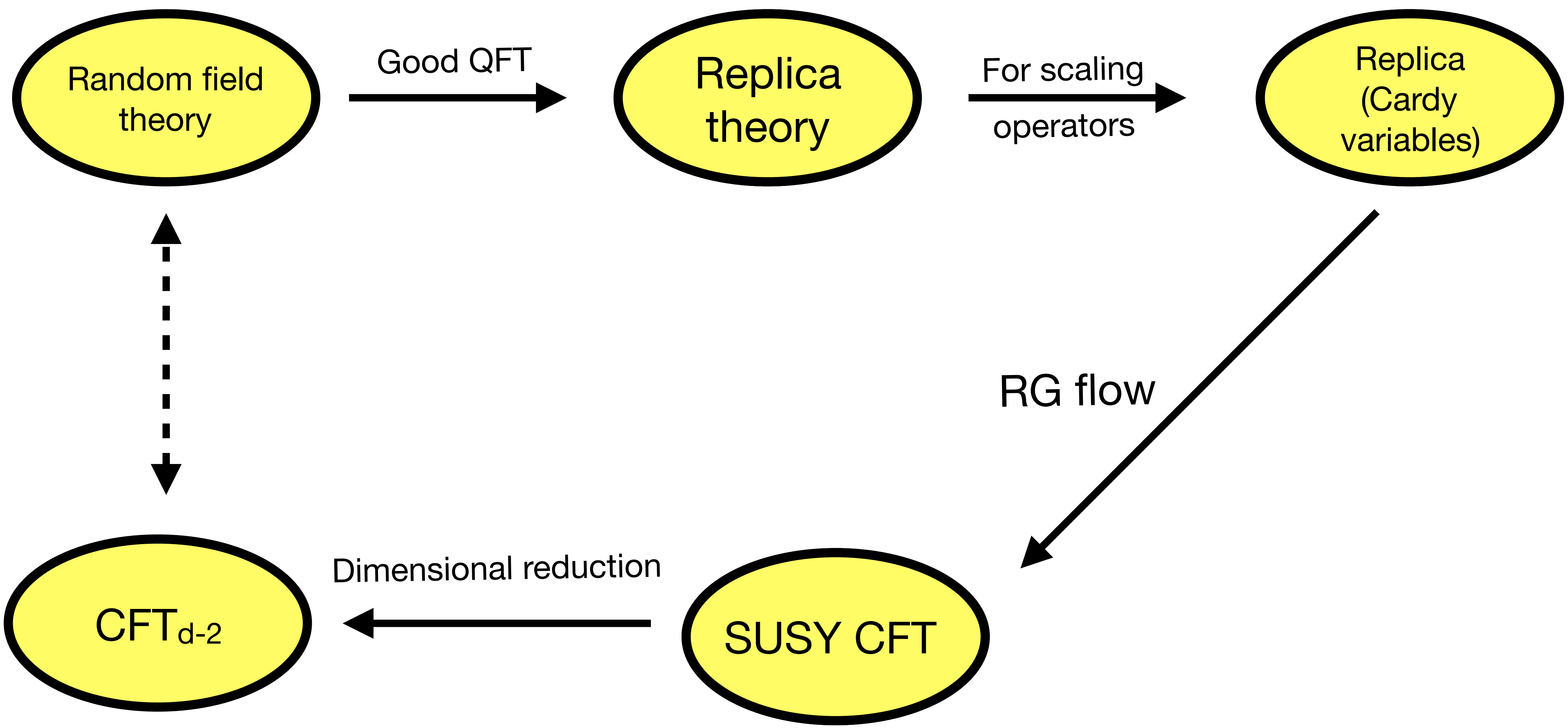
$$+ O(n) \; + \; \text{irrelevant}$$

$$\boxed{\chi_i \overset{n\to 0}{\to} \psi, \bar{\psi} \text{ (fermions)}}$$

→ **OSp supersymmetric theory**

Emergence of SUSY



$$S_{\rm susy} = \partial\varphi\partial\omega - H\omega^2 + \partial\psi\partial\bar{\psi} + \lambda(4\varphi^3\omega + 12\varphi^2\psi\bar{\psi})$$

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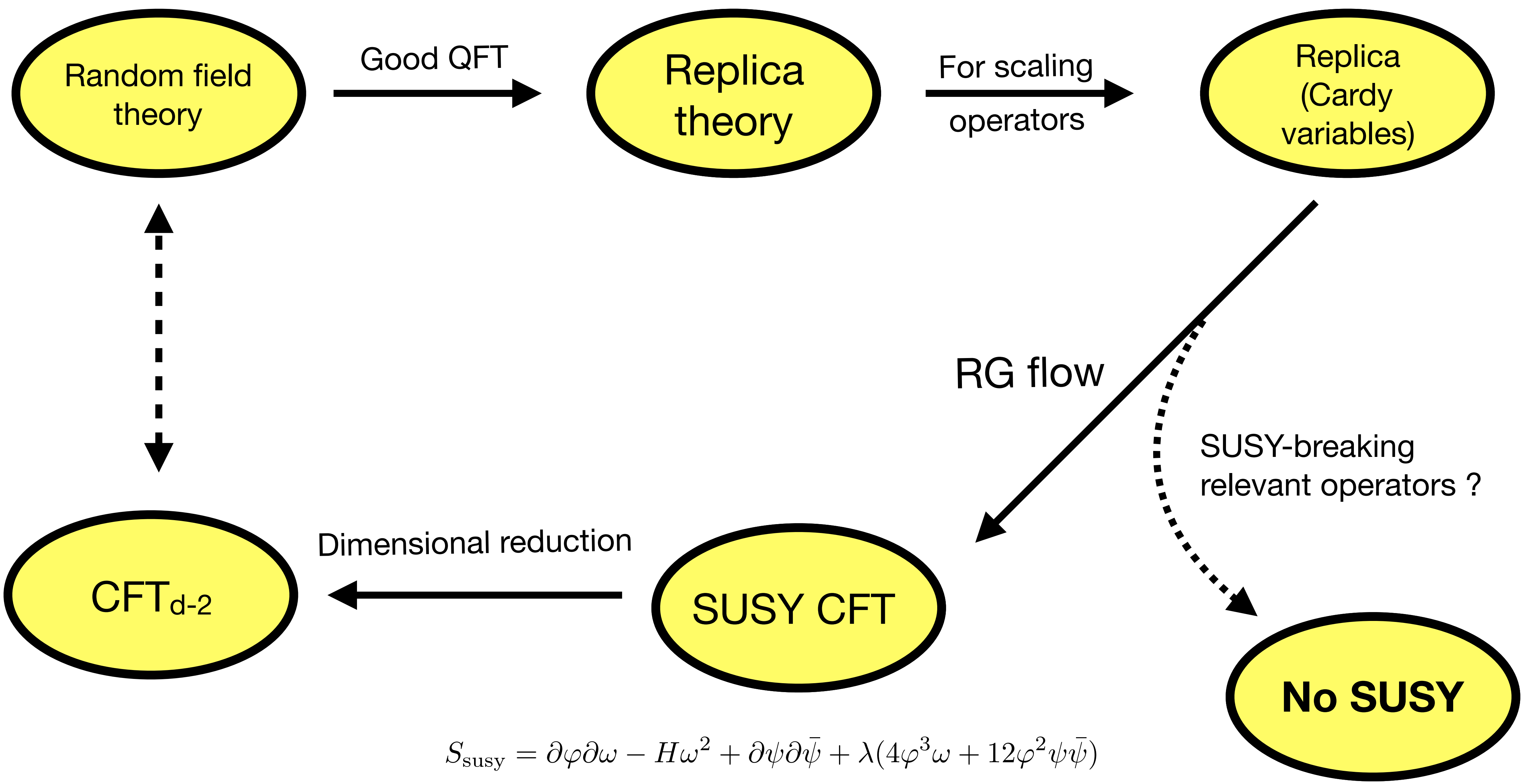
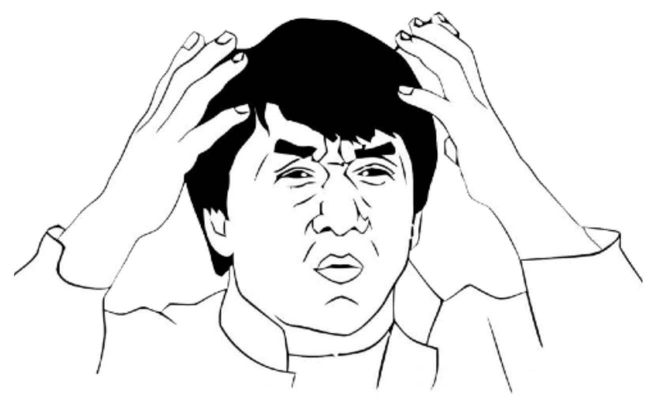


$$S_{\rm susy} = \partial\varphi\partial\omega - H\omega^2 + \partial\psi\partial\bar{\psi} + \lambda(4\varphi^3\omega + 12\varphi^2\psi\bar{\psi})$$

A paradox!!

UV
Replica theory —— RG ——▶ **IR SUSY CFT**

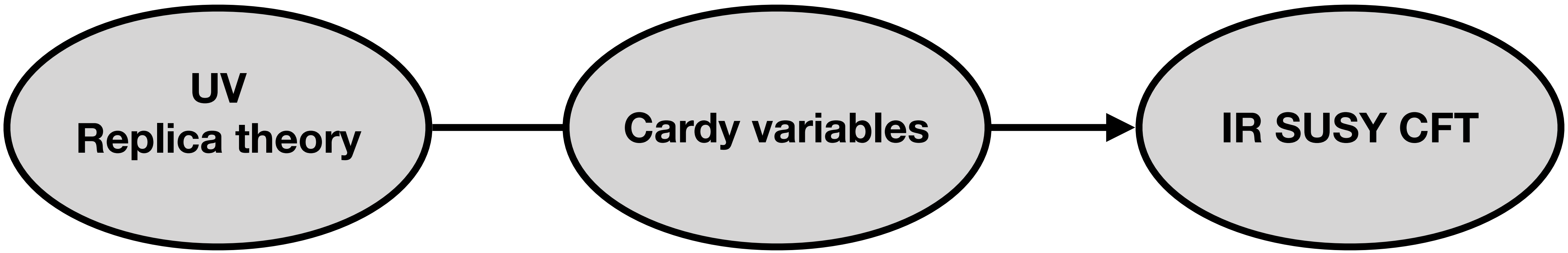
Permutation symmetry of n replicas (S_n symmetry)



No S_n symmetry

But RG should preserve all symmetries !!

Resolution

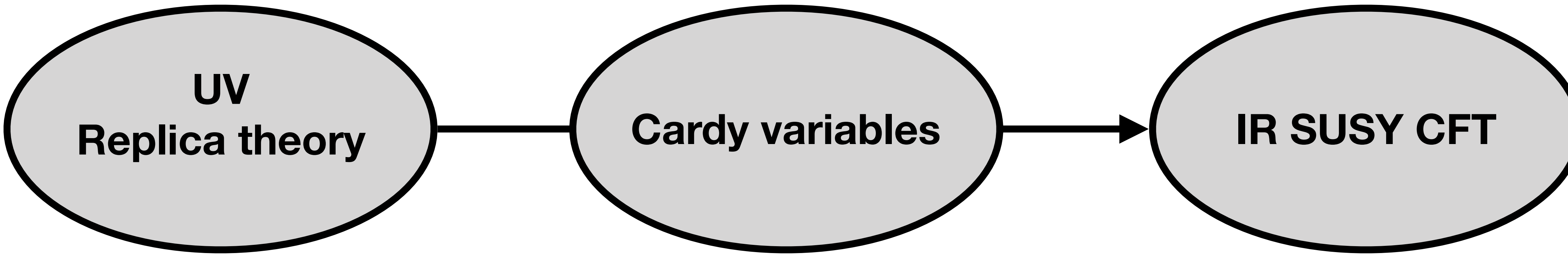


Obscures S_n symmetry
to get scaling operators.

$$[\varphi] = \frac{d}{2} - 2,\ [\chi_i] = \frac{d}{2} - 1,\ [\omega] = \frac{d}{2}$$

$$\sum_{i=1}^{n} \phi_i^4 \overset{\text{Cardy}}{=} \underbrace{4\varphi^3\omega + 6\varphi^2\chi_i^2}_{\text{Relevant}} + \underbrace{\frac{1}{H}(\chi_i^4 - 6\varphi\chi_i^2\omega)}_{\text{Irrelevant}} + \underbrace{\frac{1}{H^2}\varphi\omega^3}_{\text{More irrelevant}} + \cdots$$

Resolution

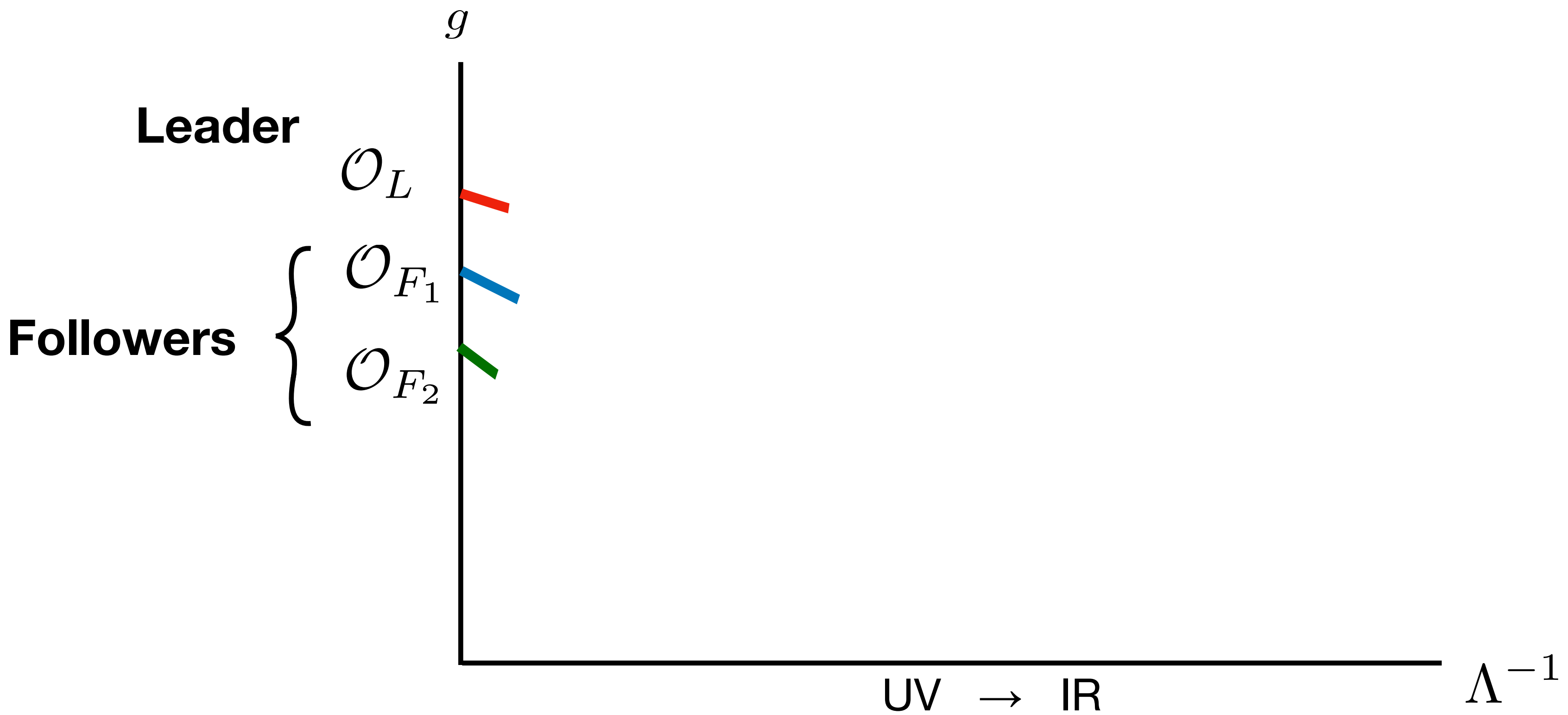


Obscures S_n symmetry
to get scaling operators.

$$\mathcal{O}_{S_n\,\mathrm{singlet}} \overset{\text{Cardy}}{=} \mathcal{O}_L + \mathcal{O}_{F_1} + \mathcal{O}_{F_2} + \cdots$$

same RG correction (anomalous dimension)
but different overall scaling

RG evolution of leader and followers



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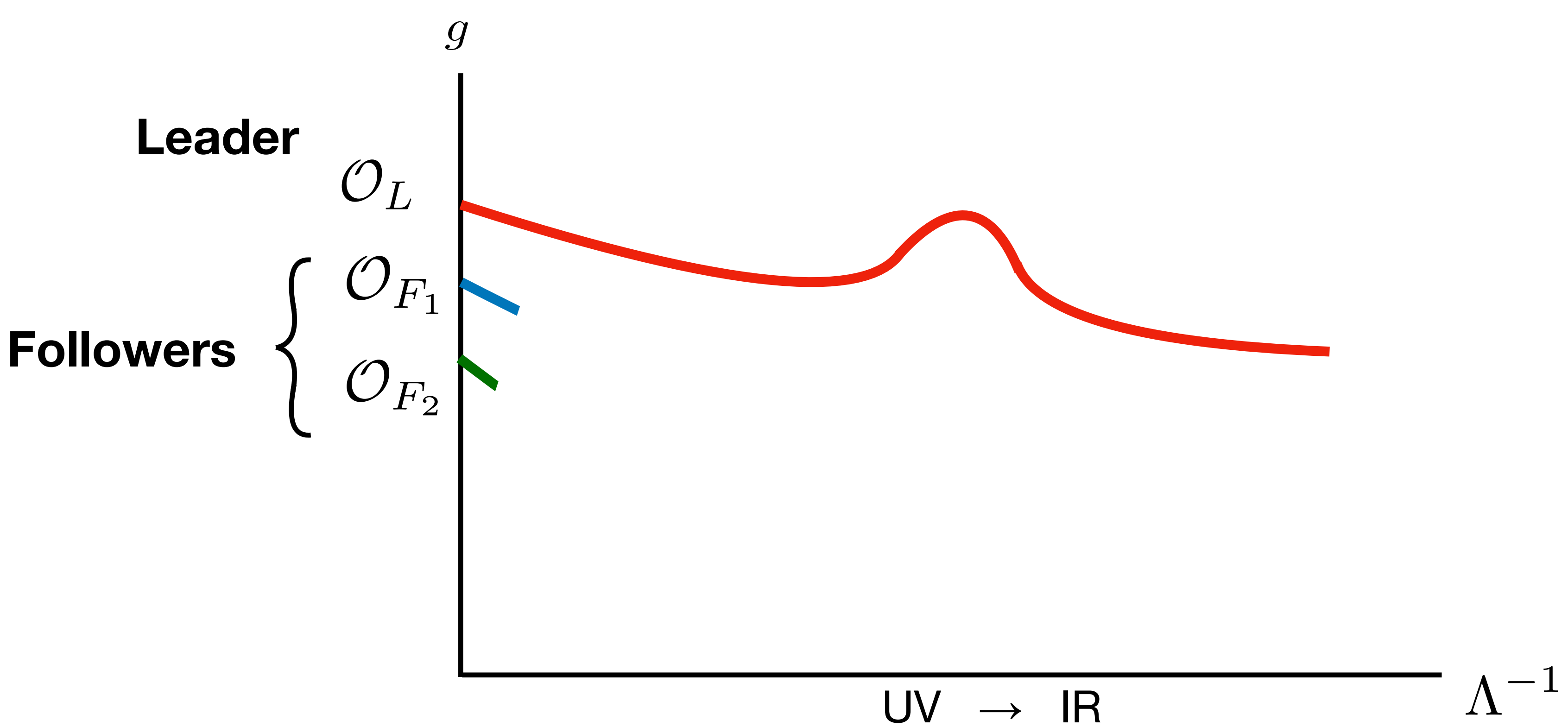

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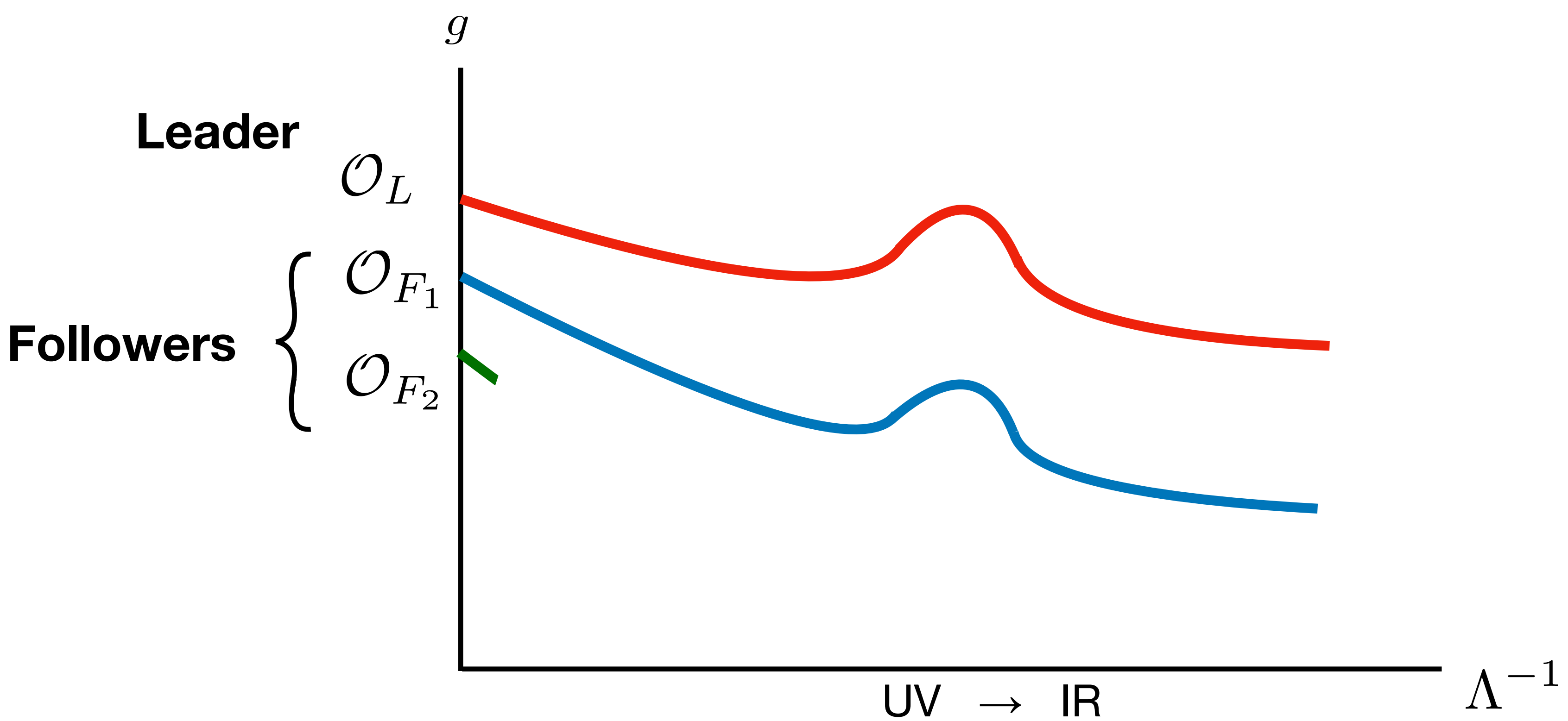

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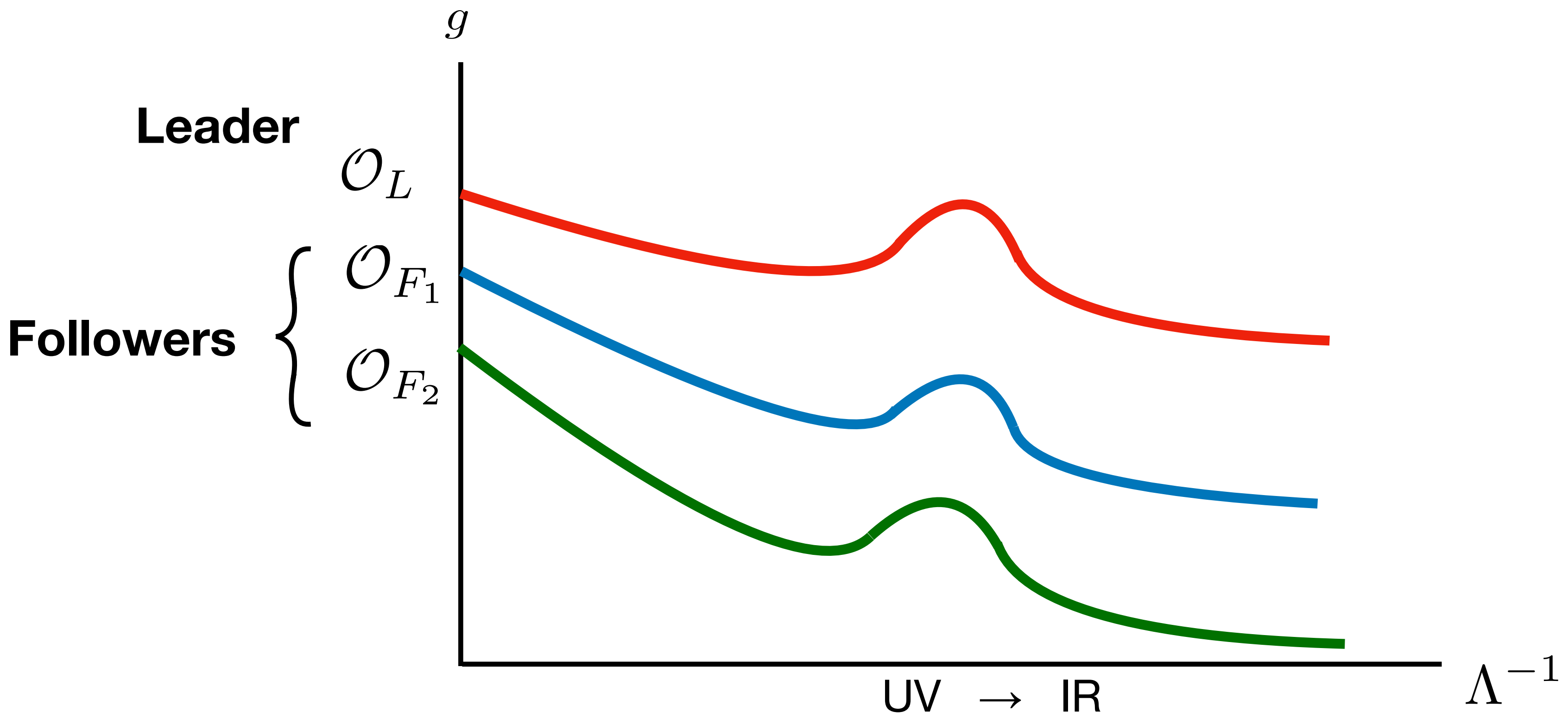

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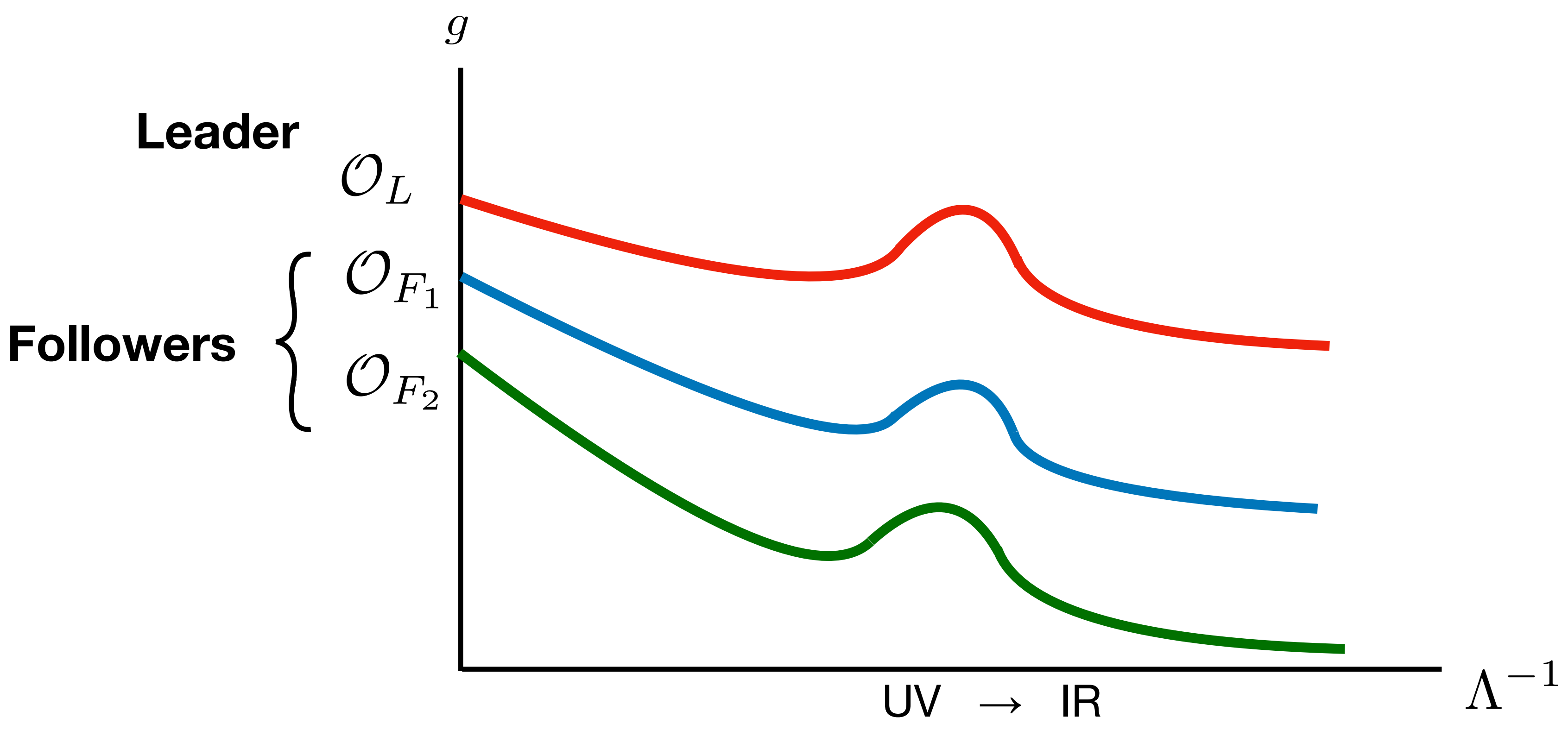


The leaders may flow to (an emergent) SUSY CFT

….or may not !!

We classified and computed anomalous dimensions of low lying leaders in perturbation theory

[2009.10087] AK, Rychkov, Trevisani

[2112.06942] AK, Rychkov, Trevisani

Findings

For $V(\phi) = \lambda\phi^4$ we found SUSY breaking leader operators become relevant in $d \leq d_c \in [4.2, 4.5]$ (unstable fixed point)

Numerical findings of SUSY or dimensional reduction

d=6	✓
d=5	✓
d=4	✗
d=3	✗

For $V(\phi) = g\phi^3$ we found no SUSY breaking leader operators becoming relevant.

d=8	✓
d=7	✓
d=6	✓
d=5	✓
d=4	✓
d=3	✓

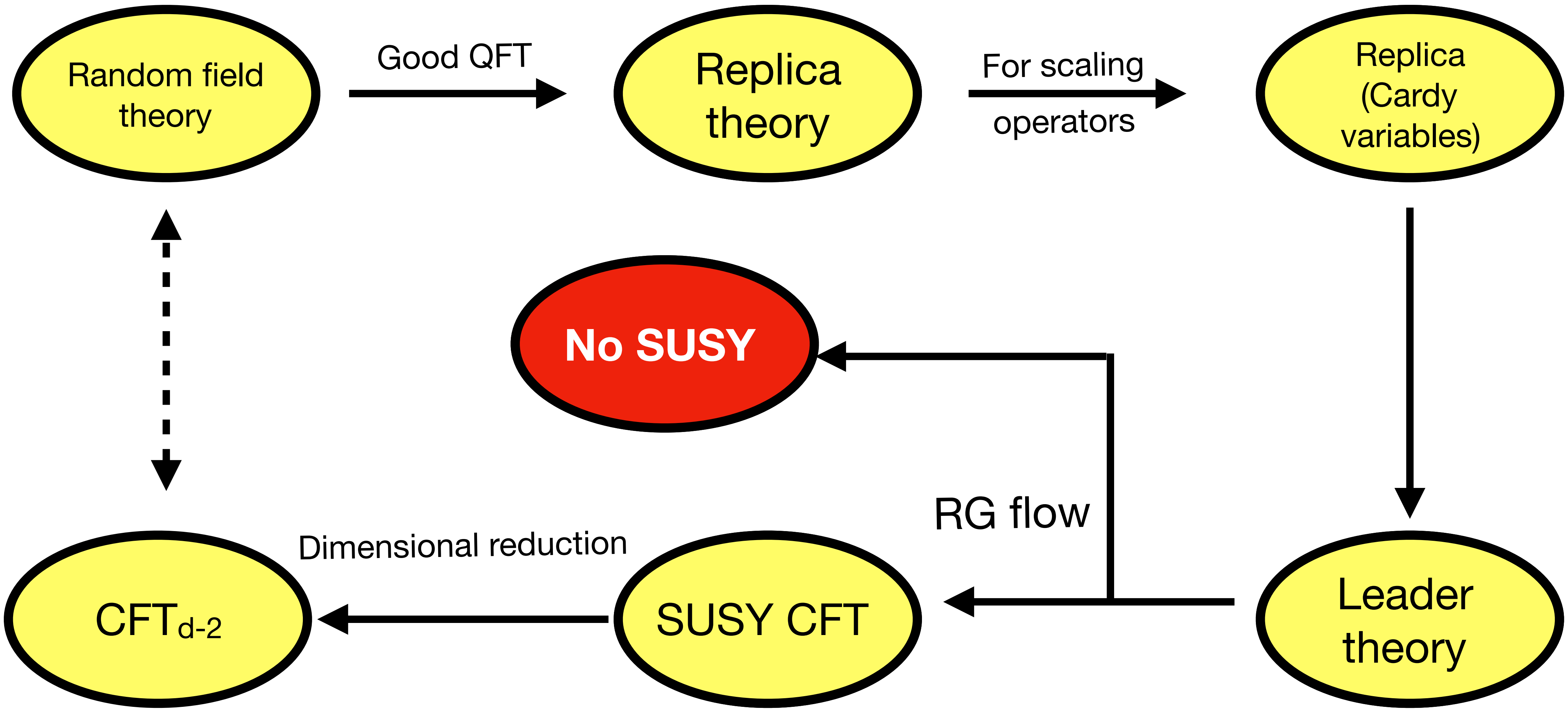
[2009.10087] AK, Rychkov, Trevisani

[2112.06942] AK, Rychkov, Trevisani

[2203.12629] AK, Trevisani

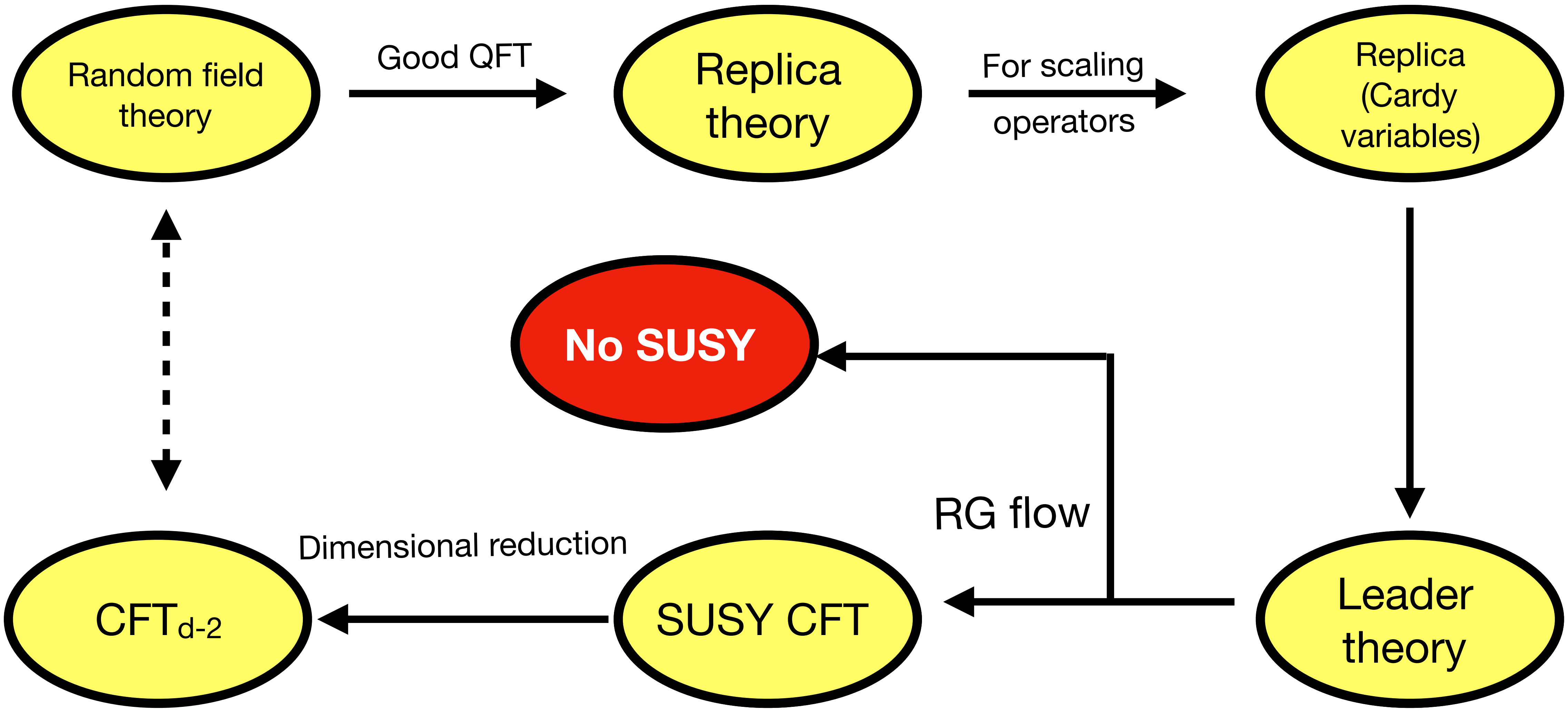
(JHEP, PRL)

Summary



Random field theory
Good QFT
Replica theory
For scaling operators
Replica (Cardy variables)
No SUSY
RG flow
Leader theory
SUSY CFT
Dimensional reduction
CFT$_{d-2}$

Summary



Future directions

- Tuning to an unstable SUSY fixed point in d=4 RFIM (confirmation of our scenario).
- Conformal bootstrapping the RFIM (Difficulty: taking limits)
- Unexplored features of logarithmic CFTs.
- Understanding the implications SUSY and dimensional reduction.

Thanks