Black hole mimickers from string theory

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[Based on 1705.10172, 1712.00511, and 2110.10542]

Black hole mimickers: the what and the why

Black hole information puzzle: Pure state \rightarrow Mixed state

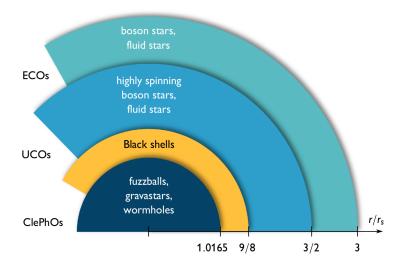
to solve, need:

- way to store information in Hawking radiation
 [claimed to be solved by string theory with branes and extra dimensions]
- why can information fall inside the horizon without violating no-cloning [claimed to be solved by BH complementarity]

No general agreement that this works.

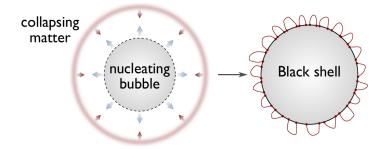
- \hookrightarrow What if there is no horizon?

Black hole mimickers: the what and the why



What is a black shell?

[Danielsson, Dibitetto, Giri '17]



Entropy of matter stuck to the shell as a finite temperature gas with a large number of degrees of freedom

Phase space enhancement of nucleation probability

How to construct a black shell in general relativity?

For the shell to be static, it must solve Israel's junction conditions:

$$\rho = \frac{1}{4\pi r} \left(\sqrt{1 + k^2 r^2} - \sqrt{1 - 2M/r} \right),$$

$$p = \frac{1}{8\pi r} \left(\frac{1 - M/r}{\sqrt{1 - 2M/r}} - \frac{1 + 2k^2 r^2}{\sqrt{1 + k^2 r^2}} \right).$$

◆ naive solution is an unstable critical point. non-trivial to stabilize
Rewrite as energy conservation:

$$4\pi r^2 - r\left(\sqrt{1 + k^2 r^2} - 1\right) = E = r\left(1 - \sqrt{1 - 2M/r}\right) \; .$$

Associate ρ_b , p_b with E: at r = 9M/4 (:= Buchdahl's radius), $\rho_b = p_b/2$. If the nucleated brane "catches" infalling matter at this radius, it can be converted to radiation directly on top of the brane.

How to construct a black shell in string theory?

Dp-brane action:

$$\begin{split} S_{BI} &= -T_p \int d^{p+1} \sqrt{-\det\left(P[G+B]_{ab} + 2\pi\alpha' F_{ab}\right)}\,, \\ S_{WZ} &= \pm T_p \int P\left[\sum C^{(n)} e^B\right] \wedge e^{2\pi\alpha' F}\,. \end{split}$$

N coincident Dp-branes: $U(1)^N \rightarrow U(N)$: Non-abelian world-vol theory

Myers' effect: dielectric effect for Dp-branes (brane polarization)

Dp-brane in non-trivial $F^{(n)}$ background \Rightarrow new minima in the scalar potential. \hookrightarrow example:

N D0 branes in a background of
$$F^{(4)}$$

D0 branes puff up into a non-commutative 2-sphere

bound state of a D2 and N D0 branes

Connecting with general relativity

D2 world volume action, gives potential:

$$V_{BI} = 4\pi T_2 \sqrt{r^4 + N^2} \xrightarrow{r \to \infty} 4\pi r^2 T_2 \left(\frac{1}{2r^4} + \frac{N^2}{2r^4} \right)$$

- \star tension: $p = -\rho$
- stiff matter: $p = \rho$ —
- ◆ radiation: $p = \frac{\rho}{2}$ ← Massless fluctuations of gauge fields: N^2 open strings

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Junction conditions:
$$\tau + \rho_g + \rho_s = \frac{k}{4\pi} + \frac{1}{8\pi k r^2} - \frac{1}{4\pi r} + \rho_b,$$
$$-\tau + p_g + p_s = -\frac{k}{4\pi} + \frac{1}{8\pi r} + p_b$$

Solution:
$$\rho_g = \rho_b - \frac{1}{12\pi r}, \quad \tau = \frac{k}{4\pi} - \frac{1}{6\pi r} + \frac{1}{16\pi k r^2}, \quad \rho_s = \frac{1}{16\pi k r^2}.$$

Junction conditions solved at Buchdahl's radius: r = 9M/4

Dp-brane wrapped on d internal dimensions

Tension:
$$\tau \sim \frac{1}{g_s \ell_s^3} \left(\frac{L}{\ell_s}\right)^d$$
.

◆ Junction condition at leading order:

$$\tau \sim \frac{k}{\ell_p^2} \sim \frac{k}{g_s^2} \frac{L^6}{\ell_s^8} \Rightarrow \frac{1}{k} \sim \frac{\ell_s}{g_s} \left(\frac{L}{\ell_s}\right)^{6-d}$$

◆ Using this, junction condition at sub-leading order gives

$$\frac{\tau N^2 \ell_s^4}{r^4} \sim \frac{1}{\ell_p^2 r^2} \frac{1}{k} \Rightarrow N \sim \frac{r}{\ell_p} \left(\frac{L}{\ell_s}\right)^{3-d}$$

For D5 brane wrapped along d=3 directions, we get the right number of degrees of freedom: $N^2 \sim r^2/\ell_b^2$.

This guarantees that energy $4\pi r^2 \rho_g = M$, with $\rho_g \sim N^2 T^3$ if $T \sim 1/M$.

Generically unstable, need energy flux to stabilize



- ⇒ heat up to local Unruh temperature
 - ◆ Energy must come from the system
 - \hookrightarrow Need energy to go into increasing N, by lowering tension

Generically unstable, need energy flux to stabilize



◆ Shell responds by relaxing tension
 ⇒ Restoring force

Continuity equation:

$$\dot{\rho} + \frac{2\dot{r}}{r} (\rho + p) = 0 \Rightarrow \dot{\rho}_{\tau} + \left(\dot{\rho}_{g} + 3\rho_{g}\frac{\dot{r}}{r}\right) + \left(\dot{\rho}_{s} + 4\rho_{s}\frac{\dot{r}}{r}\right) = 0$$

Assume the form: $j = 3\rho_g \left(\alpha \frac{\dot{T}}{T} + \beta \frac{\dot{r}}{r} \right)$.

[Danielsson, Pretorius, Lehner '21]

 \hookrightarrow Stability requires: $\alpha < 4/9, 6\alpha - \beta > 2$,

Relaxation to Buchdahl requires: $\alpha - \beta = 2/3$.

[Was solved numerically in the context of relativistic hydrodynamics]

Rotating black shells: more realistic, more complicated

[Danielsson, Giri '18]

★ Kerr metric is unique if there is a horizon.

No horizon \Rightarrow no reason to be Kerr.

- Rotating black shell with Kerr outside:
 not possible to solve similar junction conditions.
- ★ Maybe a good thing, because any deviation from Kerr, such as different multipole moments can be useful as an observational signature.

We constructed a generalization of the Kerr metric perturbatively in angular momentum, and solved the junction conditions.

- Continuity of metric: gives a 2-parameter family of solutions. Pick a physically reasonable choice: AdS inside is non-rotating
- ♦ Junction conditions: Assume that all matter components are perfect fluids: $S^{ij} = (\rho + p)u^iu^j + \rho g^{ij}$. This gives a quadrupole moment:

$$\mathcal{M}_2 = -a^2 M - \frac{2}{15} a^2 M^3 (16q - 15p) = -0.93a^2 M - 0.00575 \frac{a^2}{k} + \cdots$$

~ 7% less compared to Kerr!

How can the matter components support themselves on the shell? [Danielsson, Giri '21]

◆ Centrifugal force pushes the gas towards the equator.

Turns out that the total stress tensor on the shell is covariantly conserved

$$\nabla_{i}(\mathsf{S}_{\mathsf{brane}})_{\theta}^{i} + \nabla_{i}(\mathsf{S}_{\mathsf{gas}})_{\theta}^{i} + \nabla_{i}(\mathsf{S}_{\mathsf{stiff}})_{\theta}^{i} = \mathbf{0}$$

- ⇒ So no external force required. The components exchange energy between each other to support themselves on the shell.
- ★ This is exactly the flux that we needed for the non-rotating shell!
- \Rightarrow Instead of time component of j, we now have space components.

What condition should α and β satisfy for the shell to remain static?

♦ Turns out that $\alpha = 4/15$, $\beta = -2/5 \Rightarrow$ boundary of stability

$$j = \frac{\rho_g}{5} \left(4 \frac{\nabla_i a_R}{a_R} + 3 \frac{\nabla_i R^{(3)}}{R^{(3)}} \right)$$

What now?

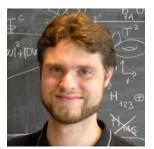
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- Top down construction in string theory necessary
 Proof of concept from IIA on T⁶/Z₂ × Z₂ in [Danielsson, Dibitetto, Giri '17]
- ◆ Fast rotating black shell to connect with observations
- Compute observables: black hole shadow [to appear soon...]

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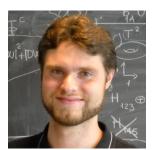
Giuseppe Dibitetto

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Thanks for listening!

