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Track reconstruction at the LUXE experiment using quantum algorithms

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ABSTRACT

LUXE (Laser Und XFEL Experiment) is a proposed experiment at DESY which will study Quantum Electrodynamics (QED) in the strong-field regime, where QED becomes non-perturbative. The measurement of the rate of electron-positron pair creation, an essential ingredient to study this regime, is enabled by the use of a silicon tracking detector. Precision tracking of positrons traversing the four layers of the tracking detector becomes very challenging at high laser intensities due to the high rates, which can be computationally expensive for classical computers. In this paper, a preliminary study of the potential of quantum computers to reconstruct positron tracks is presented. The reconstruction problem is formulated in terms of a Quadratic Unconstrained Binary Optimisation (QUBO), and solved using simulated quantum computers and hybrid quantum-classical algorithms such as Variational Quantum Eigensolver (VQE). Different ansatz circuits and optimisers are studied. The results are discussed and compared with classical track reconstruction algorithms using Graph Neural Network and Combinatorial Kalman Filter.

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1 Introduction

LUXE [1] is a planned experiment at DESY in Hamburg to study the transition far into the strong-field regime of QED, where QED becomes non-perturbative. In this experiment, the high-energy electron beam from the European XFEL is used together with a high-power laser. Both the interaction of the laser beam with the electron beam as well as with a beam of bremsstrahlung photons are studied. Processes of interest are Compton scattering and Breit-Wheeler pair creation. In the Compton process in a plane wave background of a laser field,

$$e^- + n\gamma_L \rightarrow e^- + \gamma, \tag{1}$$

an electron emits a high-energy photon, where n is the number of laser photons γ_L participating in the process. Breit-Wheeler pair creation,

$$\gamma + n\gamma_L \rightarrow e^+ + e^-, \tag{2}$$

in the presence of a strong electromagnetic field is the decay of a high energy photon into electron-positron pairs. The classical non-linearity parameter,

$$\xi = \frac{m_e}{\omega_L}\frac{\mathcal{E}_L}{\mathcal{E}_{cr}}, \tag{3}$$

with m_e as the electron mass, ω_L as the laser frequency, $\mathcal{E}_L$ as the instantaneous laser field strength and $\mathcal{E}_{cr} = m_e^2c^3/e\hbar$ as the critical field strength, known as Schwinger-Limit, is used to demarcate the regime of strong-field QED in particle-laser and photon-laser interactions ($\xi \gg 1$).

2 Experimental setup

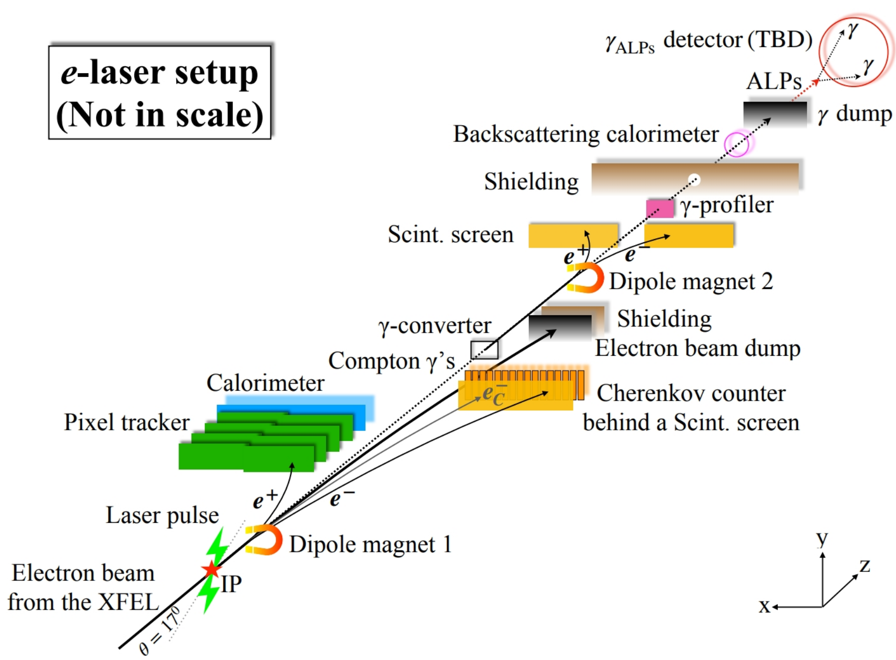

Figure 1: LUXE setup in e-laser mode. Recreated from [1].

Figure 1 shows the experimental setup of LUXE for the e-laser mode. The electron beam from the European XFEL is guided to the interaction point (IP) and crossed with the high-power laser. In the initial phase-0 of LUXE, a laser with 40 TW is used. For phase-1, an upgrade of the laser up to 350 TW is planned. Utilising a dipole magnet, electrons and positrons are deflected to their respective detector systems for energy and position measurements. The track reconstruction study which is presented in the following focuses on positrons detected by a silicon pixel tracking detector. The tracking detector system consists of eight staves partially overlapping and forming four layers with respect to the beam-axis. Each layer covers a length of approximately. 50 cm in x-direction. The staves are populated by 9 sensor chips, each containing 1024×512 pixels with a size of $29 \times 27\ \mu\text{m}^2$.

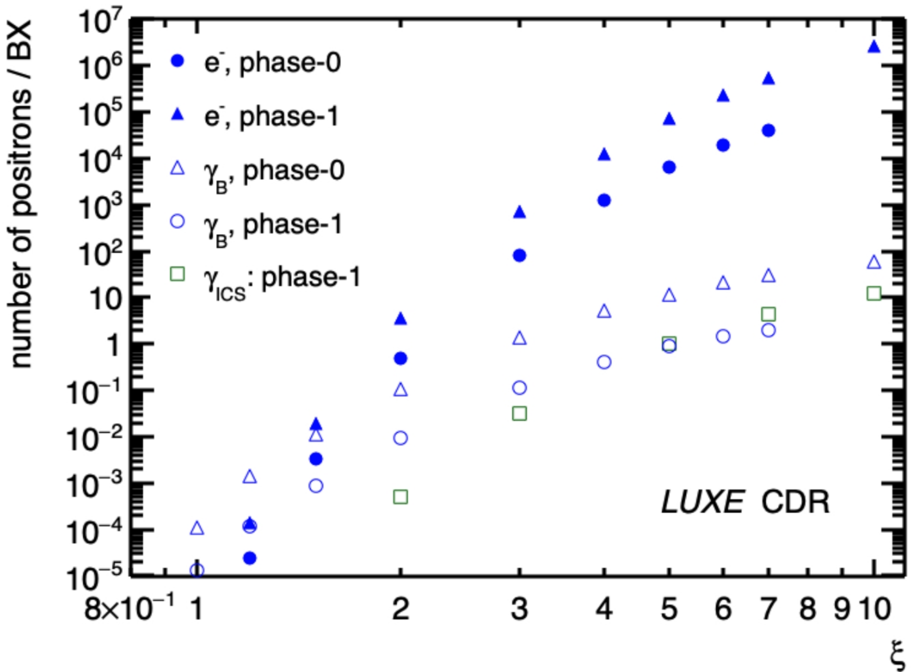

Figure 2: Number of expected positrons per interaction (BX) of the laser with both the electron beam and the Bremsstrahlung photon source as a function of the classical nonlinearity parameter ξ. Reproduced from [1].

To study the transition far into the non-perturbative regime of QED, a key concept is to measure the number of positrons generated from the Breit-Wheeler process with respect to the parameter ξ. The number of expected positrons ranges from less than 10^{-4} up to 10^{6} in the e-laser setup, displayed in Figure 2. Both a low background rate ($< 10^{-3}$) at low ξ and good linearity up to a high multiplicity are essential for track reconstruction. For this challenging task, we explore the potential of using quantum computing algorithms for track reconstruction. Previous results of our work can be found in Ref. [2]. An overview of possible quantum algorithms suitable for charged particle tracking is given in Ref. [3].

3 Data sets and selection

In this study simulated data are used. Signal interactions at the IP are generated with PTARMIGAN [4], a custom Monte Carlo event generator. Positrons stemming from PTARMIGAN are propagated through the dipole magnet and the positron tracking system using a simplified simulation. In the simplified simulation, parameters such as position and resolution of detector layers, as well as scattering processes can be tuned to explore the impact on the tracking approach. Within the frame of this study the detector geometry of the simplified simulation is reduced to a set of four non-overlapping layers.

The used simulated data predicts future measurements of phase-0 of LUXE for the e-laser setup for $\xi \in \{4,\ 5,\ 7\}$, which corresponds to 800 to 60,000 expected positrons. Only the 500 particles per laser beam interaction that are closest to the beamline are considered for the track reconstruction task in order to equalize the size of all used data sets. Both the track density and the complexity of the track reconstruction task increase with ξ.

Our starting point for track reconstruction is either doublet or triplets. Doublets are a set of two hits from exclusively consecutive layers, while triplets are a set of two doublets with exactly one shared hit. With respect to the beam line, an angle-based pre-selection procedure is applied to the doublets based on the experiment geometry. Triplets are formed if the angles between two doublets with one shared hit are not exceeding the expected maximum multiple scattering in the detector. In this procedure, the combinatorial candidates are reduced without lowering the efficiency.

4 Methodology

4.1 Classical tracking

As a benchmark a classical tracking approach with a combinatorial Kalman Filter (CKF) technique is used. For this, the A Common Tracking Software (ACTS) toolkit [5] is employed. Triplets are used as seeds to find an initial estimate of the track parameters. Scanning for matching hit candidates, the initial estimate is updated and the measurement search is performed at the same time. Eventually, after track finding and fitting is completed, an ambiguity-solving step is applied to remove tracks with shared hits.

4.2 Graph neural network

Another method which is explored in this work is the use of a graph neural network (GNN) [6, 7]. Hits are represented as nodes. Edges are connections between nodes, forming doublet-like structures, called segments, and are only kept if they satisfy the pre-selection criteria. The GNN consists of alternating EdgeNetwork and NodeNetwork and is trained to optimize the edge connections, thus learning which segments should be chosen to be a part of track candidates. Furthermore, there is a hybrid quantum-classical version of the GNN-based tracking [8], but this is not examined in this work.

4.3 Quantum algorithm

When using the quantum algorithm, the tracking task is approached by first encoding triplets as binary variables and then deciding which triplets to keep or discard in the subsequent track reconstruction process. An objective function is defined, called quadratic unconstrained binary optimization (QUBO), similar to Ref. [9]. The goal is to minimise the objective

$$O = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i, \qquad T_i, T_j \in \{0, 1\}, \tag{4}$$

with T_i and T_j representing triplets on position i and j of a possible solution vector, and a_i and b_{ij} as coefficients.

The quadratic term describes the relation between triplets. This relation is quantified by the parameter b_{ij}, which has a negative value if triplets form a track candidate, a positive value if they are in conflict and zero if they do not share a hit. The parameter a_i rates a triplet based on the angle between the two doublets that make up the triplet. In contrast to our previous results, we are focusing entirely on the relation term in this work, discarding the linear term completely.

Solving the QUBO directly on a quantum device is not possible, therefore the objective has to be mapped to an Ising hamiltonian. Finding the ground state of the Ising hamiltonian is equivalent to minimizing the QUBO and thus finding an optimal solution to the track reconstruction task. The Ising Hamiltonian,

$$\mathcal{H} = -\sum_{n=1}^{N} \sum_{m<n} \overline{b}_{nm} \sigma_n^x \sigma_m^x - \sum_{n=1}^{N} \overline{a}_n \sigma_n^x, \tag{5}$$

is solved using the Variation Quantum Eigensolver (VQE), a hybrid quantum-classical algorithm. For this task, the Qiskit [10] toolkit is employed. As a benchmark for VQE, an analytical solution can be obtained by using Numpy Eigensolver. In this study noise is excluded for VQE. As optimiser the Nakanishi-Fujii-Todo (NFT) [11] algorithm is selected.

We have improved VQE's hyperparameters to boost performance compared to our previous results. NFT is chosen instead of Constrained Optimization by Linear Approximation (COBYLA). The quantum circuit following the *TwoLocal* ansatz scheme is altered to a linear entanglement from a circular entanglement scheme. The circuit depth is increased to three (see previous results in [2]).

The quantum circuit is shown in Fig. 3.

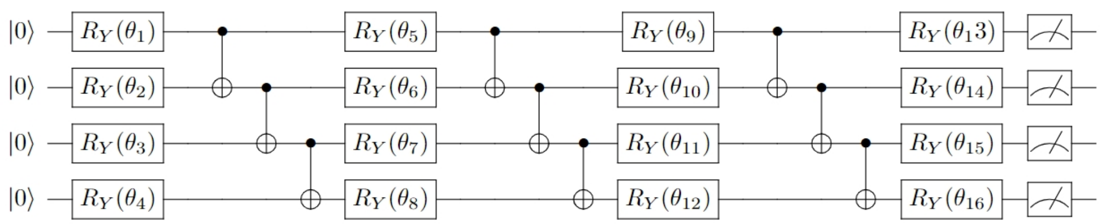

Figure 3: Variational quantum circuit layout. The *TwoLocal* ansatz is used with three repetitions of R_Y and CNOT gates for entanglement and an additional final rotation layer. For simplicity, only a four qubit system is shown.

To solve the QUBO an initial guess of the solution in the form of a string representation of the set of triplets, assuming values {0, 1} is made. For solving the QUBO in one step, the number of available qubits for the computation has to be the same as the number of triplets participating in the QUBO. Since sizes of quantum devices of this magnitude are not available and simulating huge devices is computationally infeasible, the problem has to be broken down into smaller parts, called sub-QUBOs, which are solved sequentially in each iteration. A sub-QUBO size of 7 is chosen. The order of triplets used in the sub-QUBO process is determined by their impact on the Hamiltonian energy if the binary representation of the triplet in the QUBO is flipped. A sketch of the QUBO solving process with a focus on the sub-QUBO routine is shown in Fig. 4.

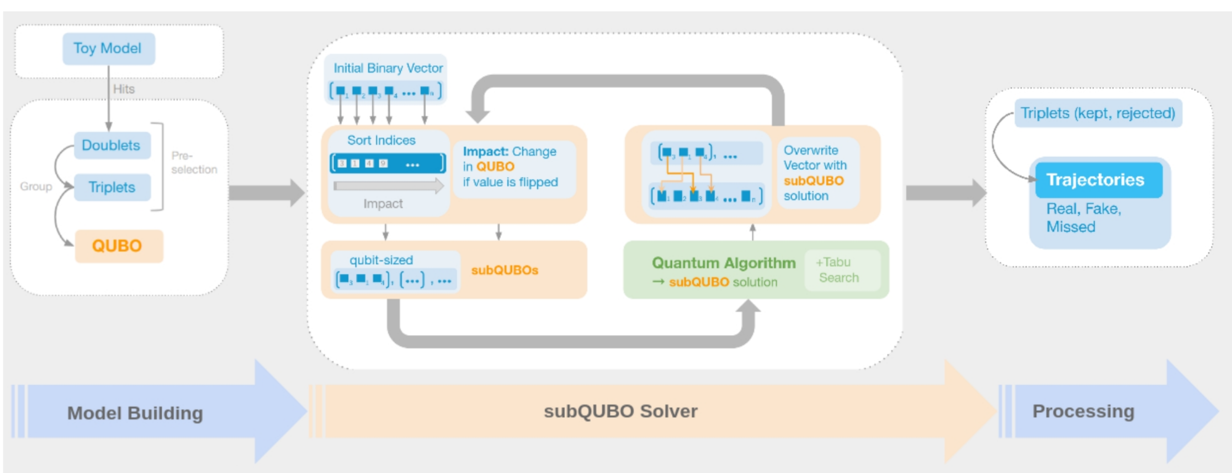

Figure 4: Sketch of the QUBO solving approach with focus on the sub-QUBO routine [2].

5 Results

Comparing the performance of the track reconstruction approaches is done on track level. Efficiency and fake rate are used as metrics. A track is defined as a set of four hits of consecutive layers which is either combining doublets and triplets into quadruplets or is found directly with the classical CKF-based tracking method. A matched track stems from exactly one particle.

Efficiency and fake rate are defined as

$$\text{Efficiency} = \frac{N_{\text{tracks}}^{\text{matched}}}{N_{\text{tracks}}^{\text{generated}}} \qquad \text{and} \qquad \text{Fake rate} = \frac{N_{\text{tracks}}^{\text{fake}}}{N_{\text{tracks}}^{\text{reconstructed}}} . \tag{6}$$

In Fig. 5 efficiency and fake rate for 500 tracks is displayed as a function of the classical non-linearity parameter ξ. Conventional CKF-based tracking is used as a benchmark to show what can actually be achieved in terms of efficiency and fake rate, and compared to GNN-based tracking and VQE. Eigensolver results are added as a benchmark for VQE approach for a sub-QUBO size of seven.

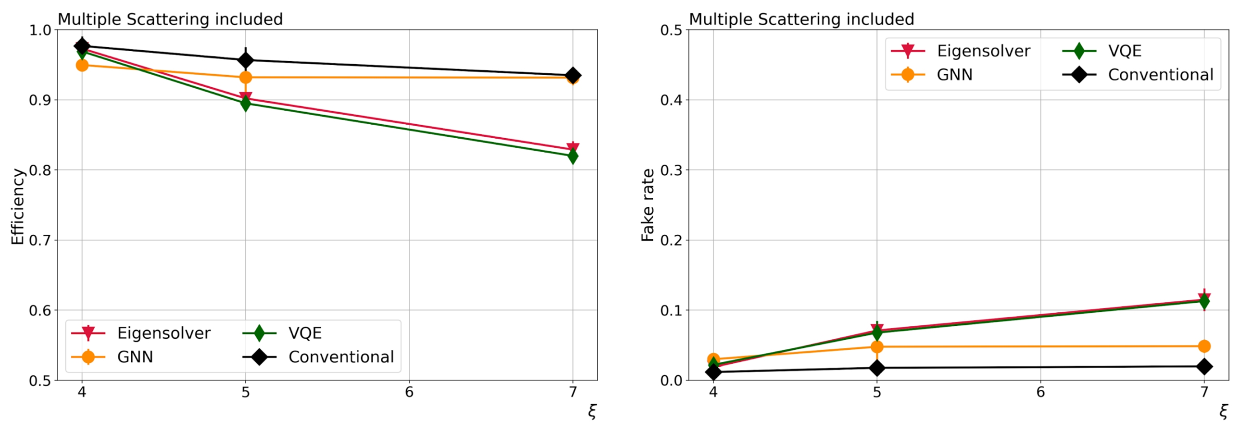

Figure 5: Track reconstruction efficiency and fake rate as a function of ξ.

CFK-based tracking efficiency decreases with ξ but is still performant at the highest shown track density at $\xi = 7$. GNN-based tracking shows nearly ξ-independent efficiency. VQE and Eigensolver deteriorate strongly at high ξ values while being comparable with CKF-based tracking and GNN at $\xi = 4$. While GNN is believed to profit from more training examples, hence possibly increasing its performance further, VQE is likely limited by the set of parameters and the size of the sub-QUBOs, therefore both approaches can be further optimized. To investigate the impact of the sub-QUBO size on the performance, only the Eigensolver is used, because simulating VQE for 16 qubits is computational very costly. In Fig. 6 the impact of the size of the sub-QUBO on the efficiency and fake rate is shown for 1000 tracks, that are closest to the beamline. Increasing the size of the sub-QUBOs from 7 to 16 results in an imrpovement of the efficiency up to 10% at high ξ. This indicates, that the available sub-QUBO size is a limiting factor of the optimization algorithm.

Using advanced entanglement structures is a way to improve the results of VQE on the sub-QUBO level. Four different entanglement structures are compared in Fig. 7. Linear entanglement is shown in Fig. 3. Circular entanglement has an additional CNOT entanglement gate from the last to the first qubit. Full entanglement refers to each qubit being entangled with every other qubit. An hamiltonian driven approach is used, if qubits are only entangled if they are representing triplets, which actually have a shared hit, thus an immediate connection. Linear and Hamiltonian driven entanglement show a performance similar to the Eigensolver, which is an upper limit for the performance of the sub-QUBO approach. Full entanglement performs slightly worse, whereas circular entanglement performs very poor. This significant difference in performance is unexpected and will be a subject of further investigations.

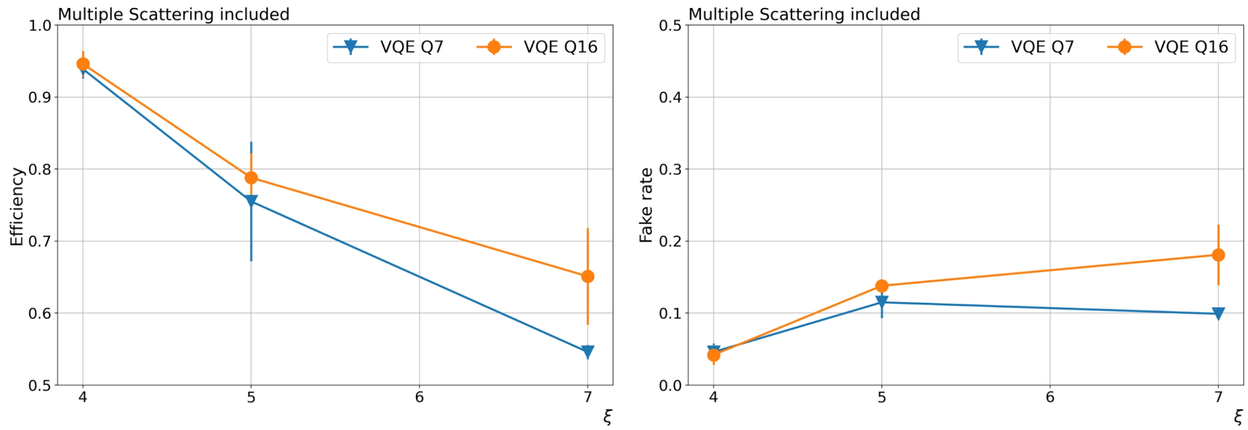

Figure 6: Track reconstruction efficiency and fake rate for 1000 tracks as a function of ξ for sub-QUBO sizes 7 (Q7) and 16 (Q16).

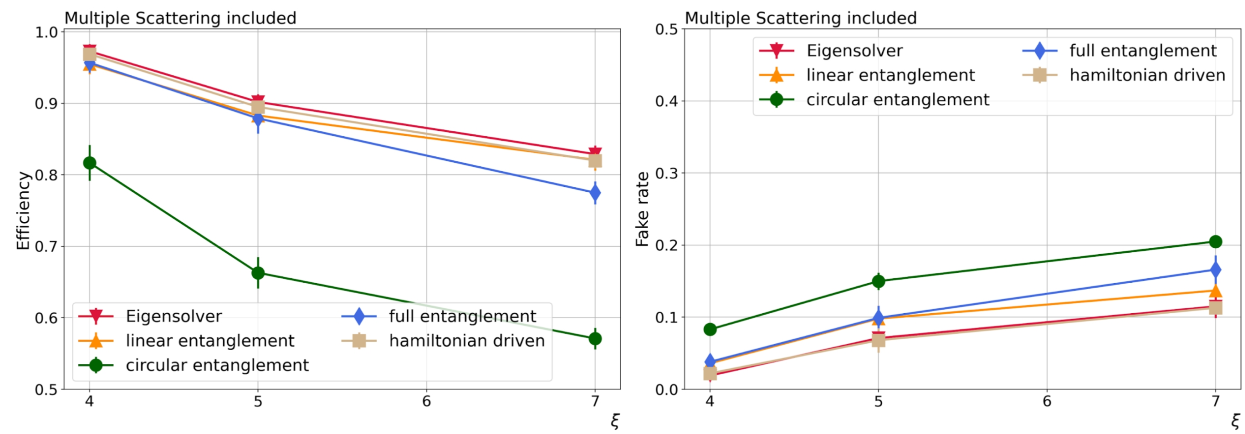

Figure 7: Efficiency and fake rate as a function of ξ. The Eigensolver result is the upper limit of what can be achieved by employing VQE and using the sub-QUBO subroutine approach.

6 Conclusions

Using a hybrid quantum-classical algorithm for track reconstruction in the LUXE experiment is studied, as well as a GNN-based tracking approach. As a benchmark conventional CKF-based tracking is used. Currently the performance of the quantum approach is poorer than GNN-based and conventional tracking but clues for optimization on the quantum part as well as on the classical part are identified and will be investigated in the future. Especially the optimization of the sub-QUBO routine and the possible decoupling of its dependency on the sub-QUBO size are strong candidates for major improvements.

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