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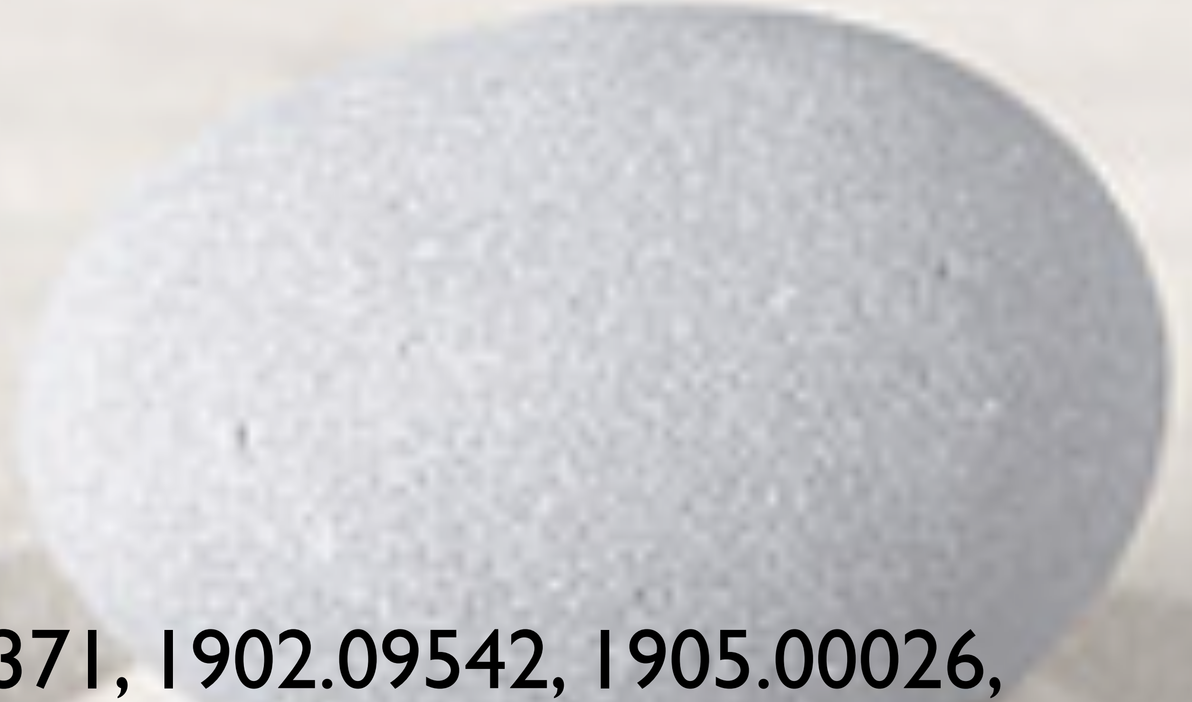
FOR FUNDAMENTAL PHYSICS

EPS - HEP
QFT and String Theory
The Large Charge Expansion

Susanne Reffert
University of Bern



based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371, 1902.09542, 1905.00026, 1909.08642,1909.02571, 2008.03308, 2010.07942, 2102.12488, 2110.07617, 2110.07616, 2203.12624 + work in progress

L. Alvarez-Gaume (SCGP), D. Banerjee (Kolkata),
Sh. Chandrasekharan (Duke), N. Dondi (Bern), S. Hellerman (IPMU),
I. Kalogerakis (Bern), R. Moser (Bern), Daniil Krichevskiy, O. Loukas,
D. Orlando (INFN Torino), V. Pellizzani (Bern), F. Sannino (Odense/Napoli),
T. Schmidt (Bern), Ian Swanson, M. Watanabe (Kyoto)

Introduction

Strongly coupled physics is notoriously difficult to access, especially analytically.
We do not have small parameters in which to do a perturbative expansion. Our most basic notions of field theory are of a perturbative nature.

Make use of symmetries, look at special limits/subsectors where things simplify.

Examples:

- large-N limit, ’t Hooft limit
- ε expansion
- supersymmetric sectors
- large spin
- …

Introduction

Here: study theories with a global symmetry group.
Hilbert space of the theory can be decomposed into sectors of fixed charge Q.
Study subsectors with large charge Q.

Large charge Q becomes controlling parameter in a perturbative expansion!

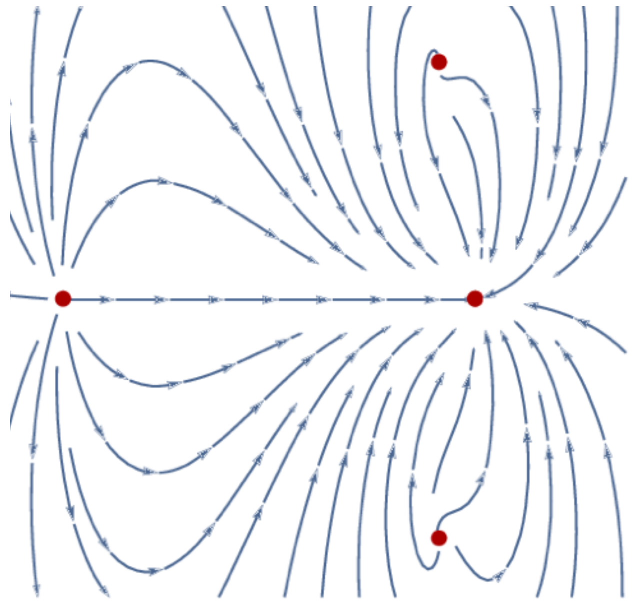
Effective theory at large Q:

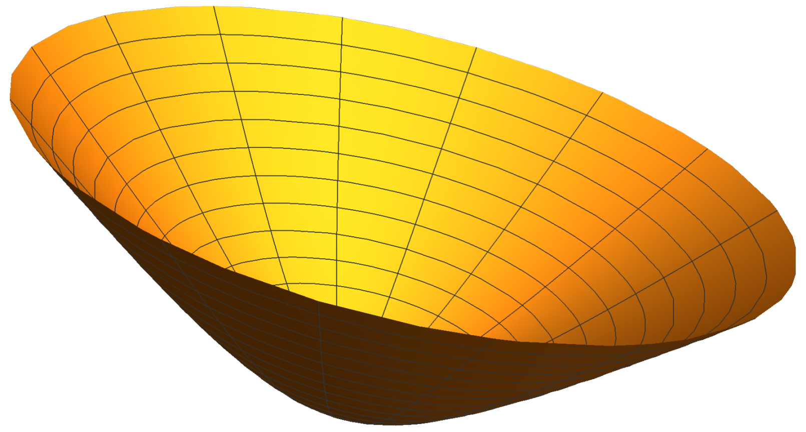
vacuum + Goldstone + 1/Q-suppressed corrections

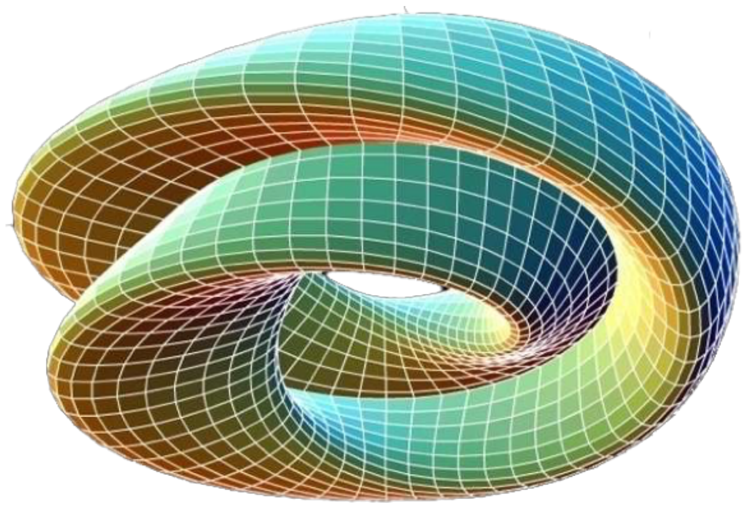
Introduction

Conformal field theories (CFTs) play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity (via AdS/CFT)
- string theory (WS theory)







But: CFTs do not have any intrinsic scales, most have by naturalness couplings of O(1).

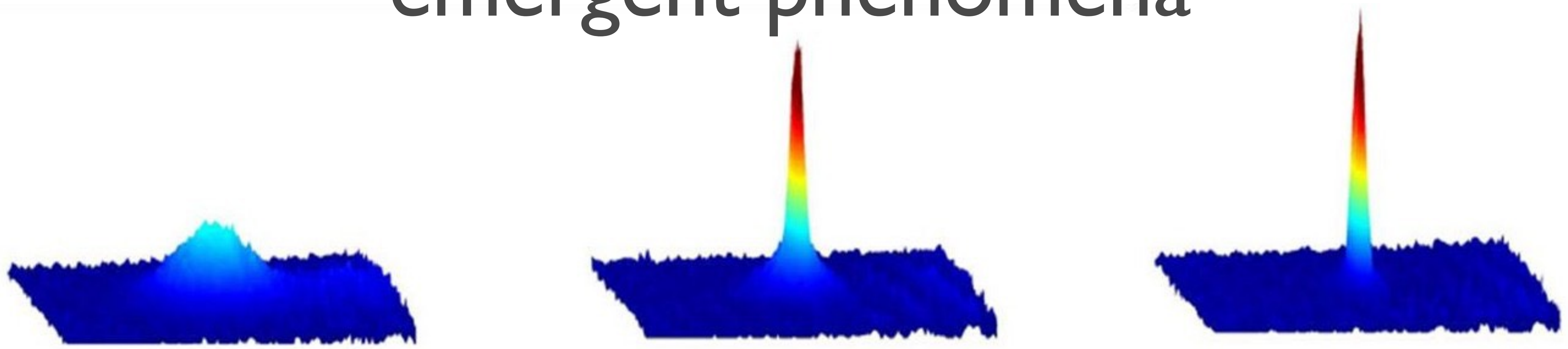
Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…

Prime candidate for the large-charge approach.

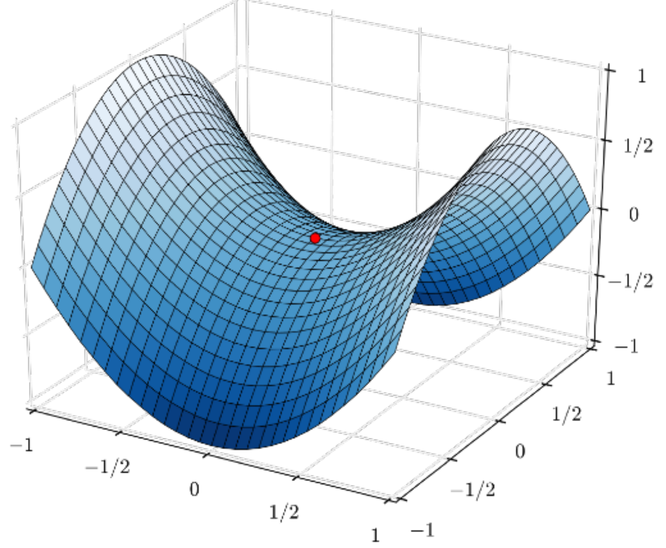
Introduction

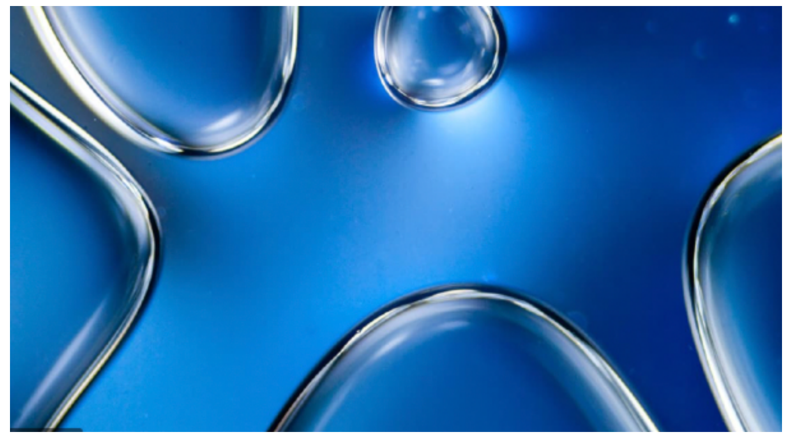
Consider systems with large quantum number
many degrees of freedom

emergent phenomena



semiclassical description





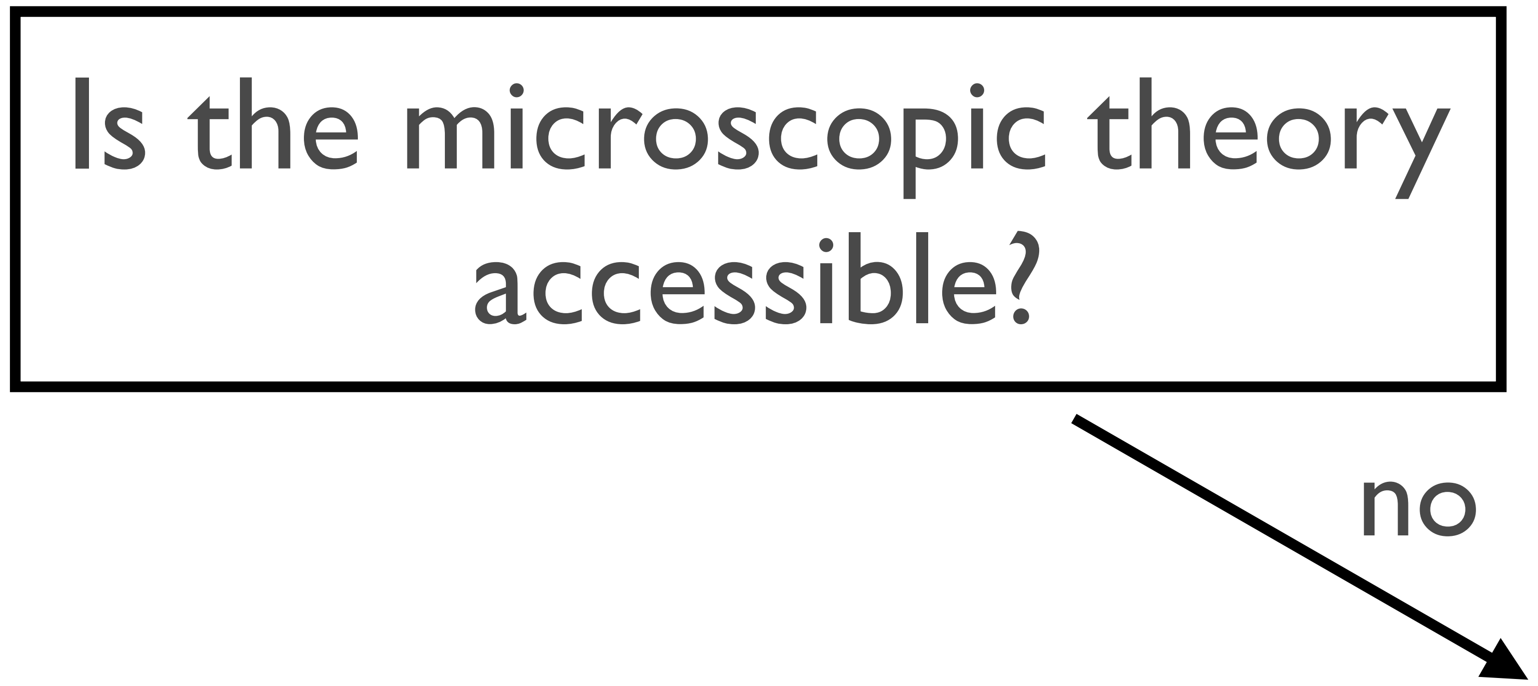
e.g. superfluid

works especially well for strongly coupled systems!

Introduction

Is the microscopic theory accessible?

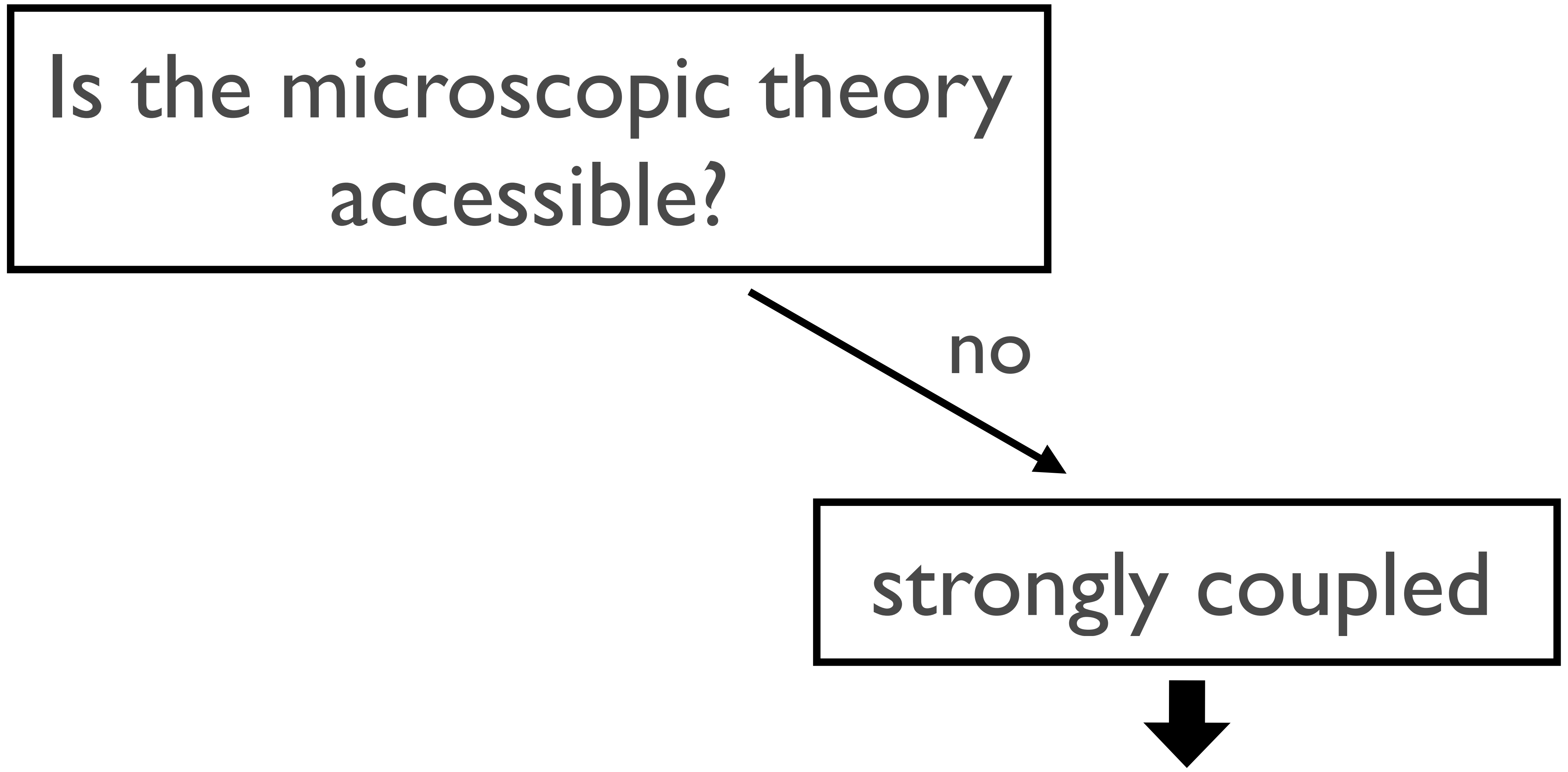
Introduction



Is the microscopic theory accessible?

no

Introduction



Is the microscopic theory accessible?

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strongly coupled

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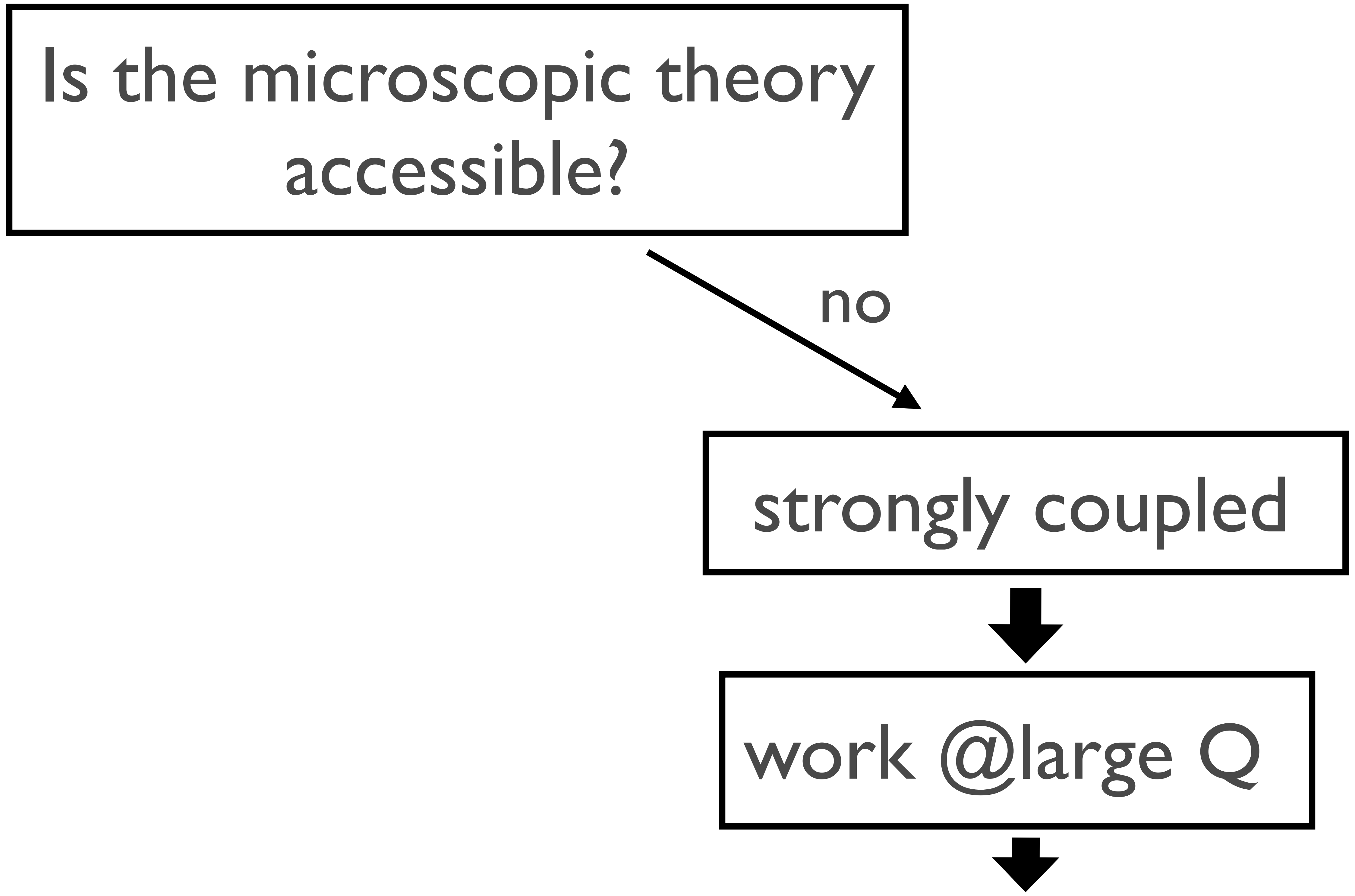


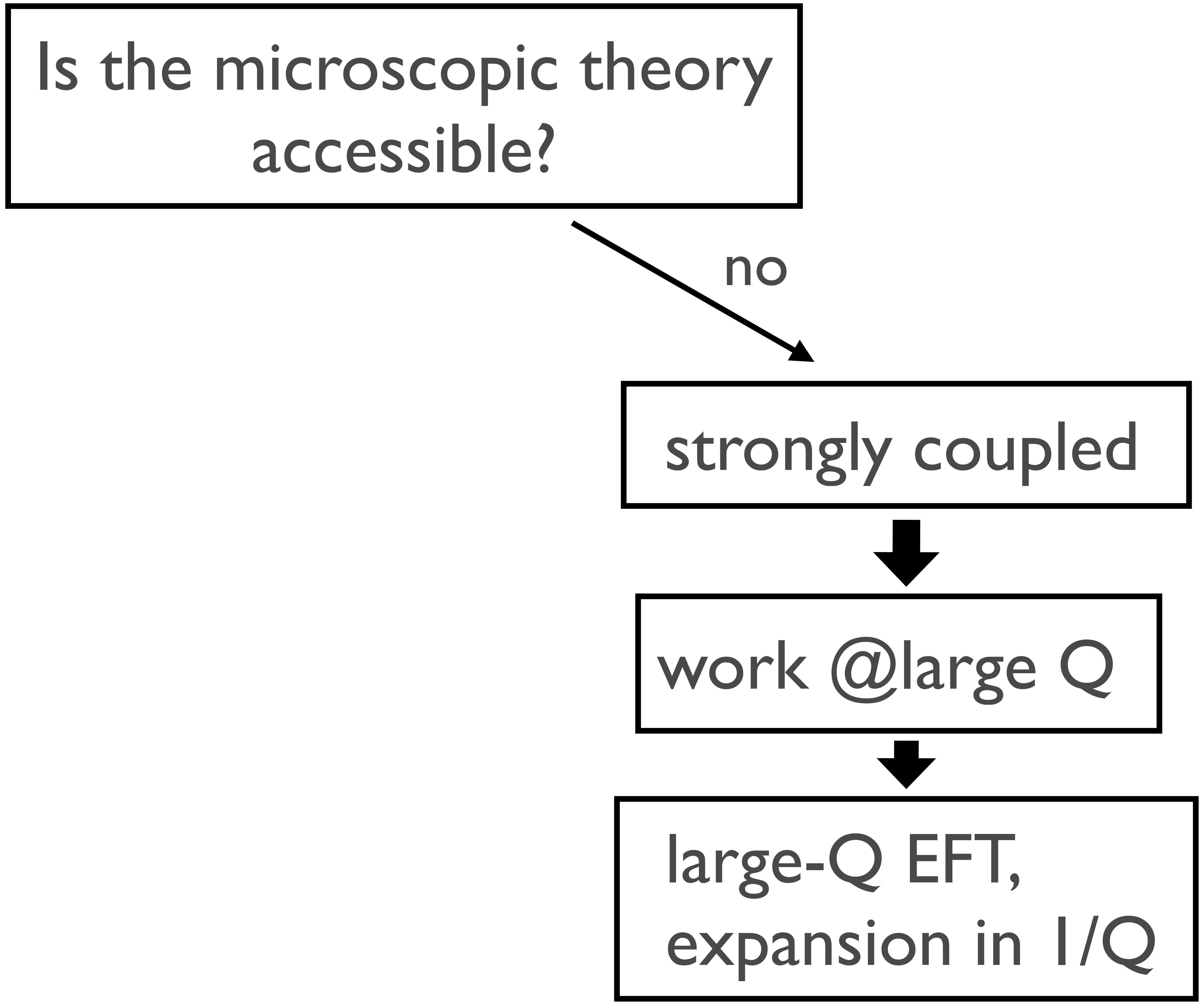
Is the microscopic theory accessible?

no

strongly coupled

work @large Q

Introduction



Is the microscopic theory accessible?

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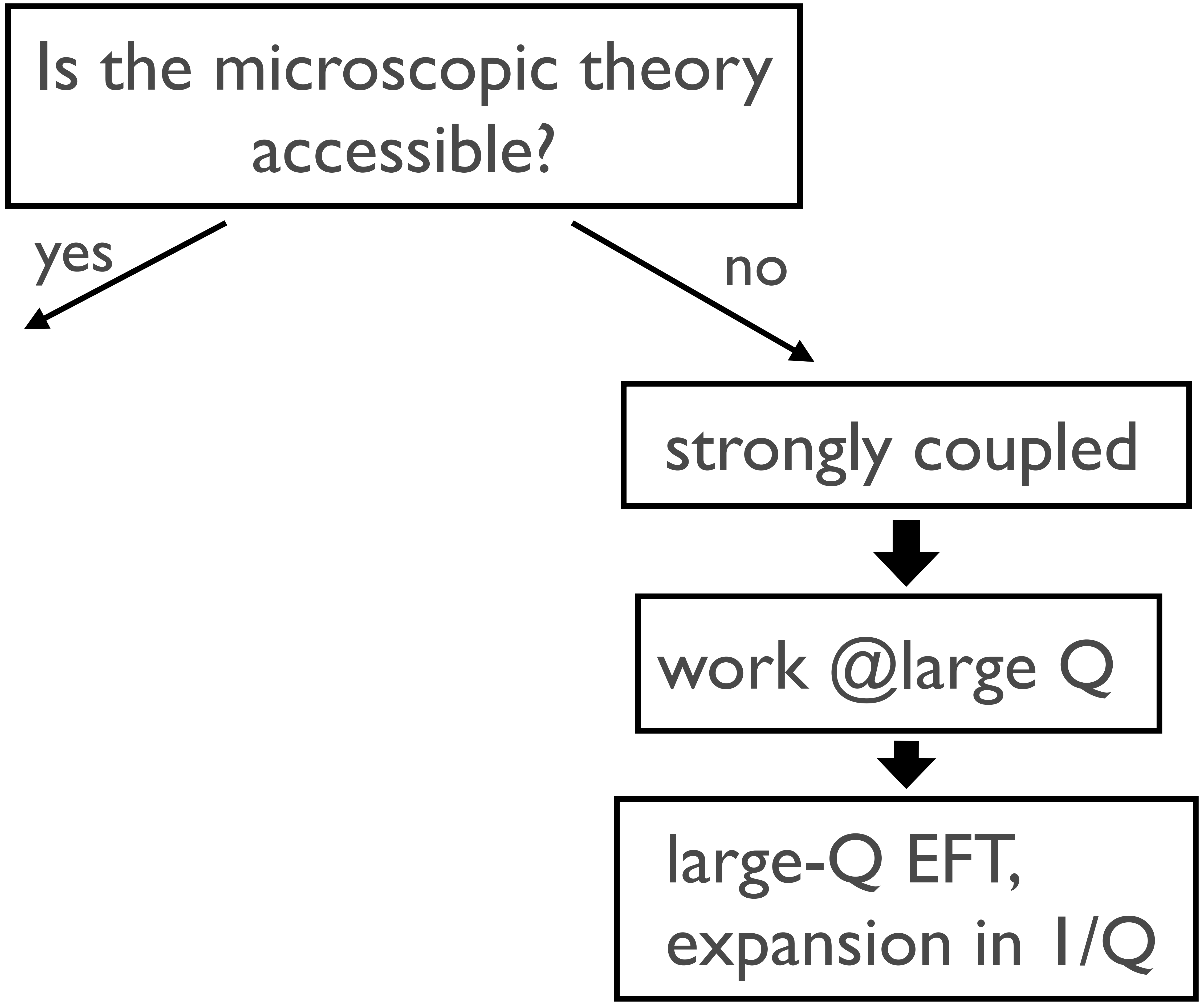
large-Q EFT,
expansion in 1/Q

Introduction



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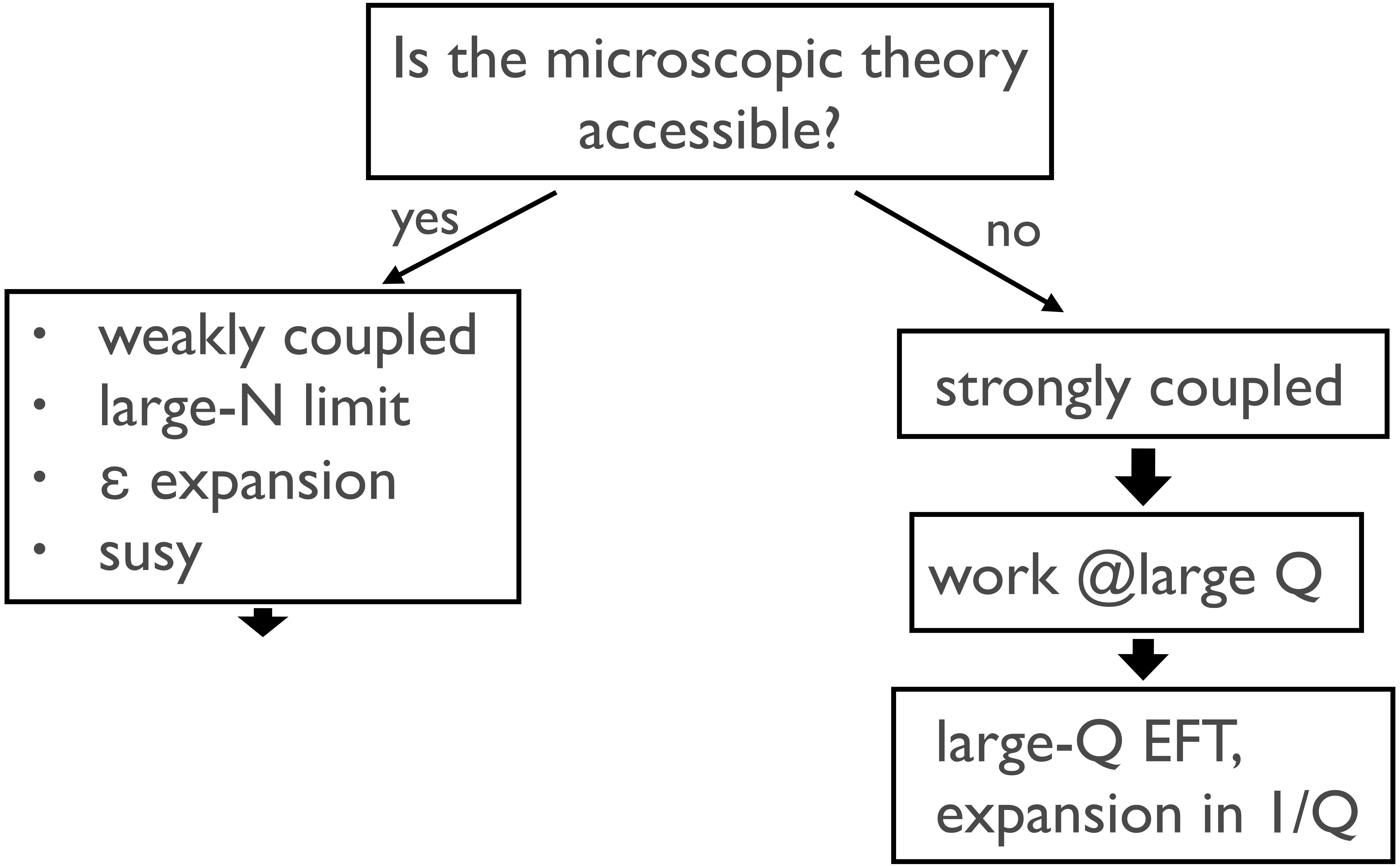


Is the microscopic theory accessible?

yes

- weakly coupled
- large-N limit
- ε expansion
- susy

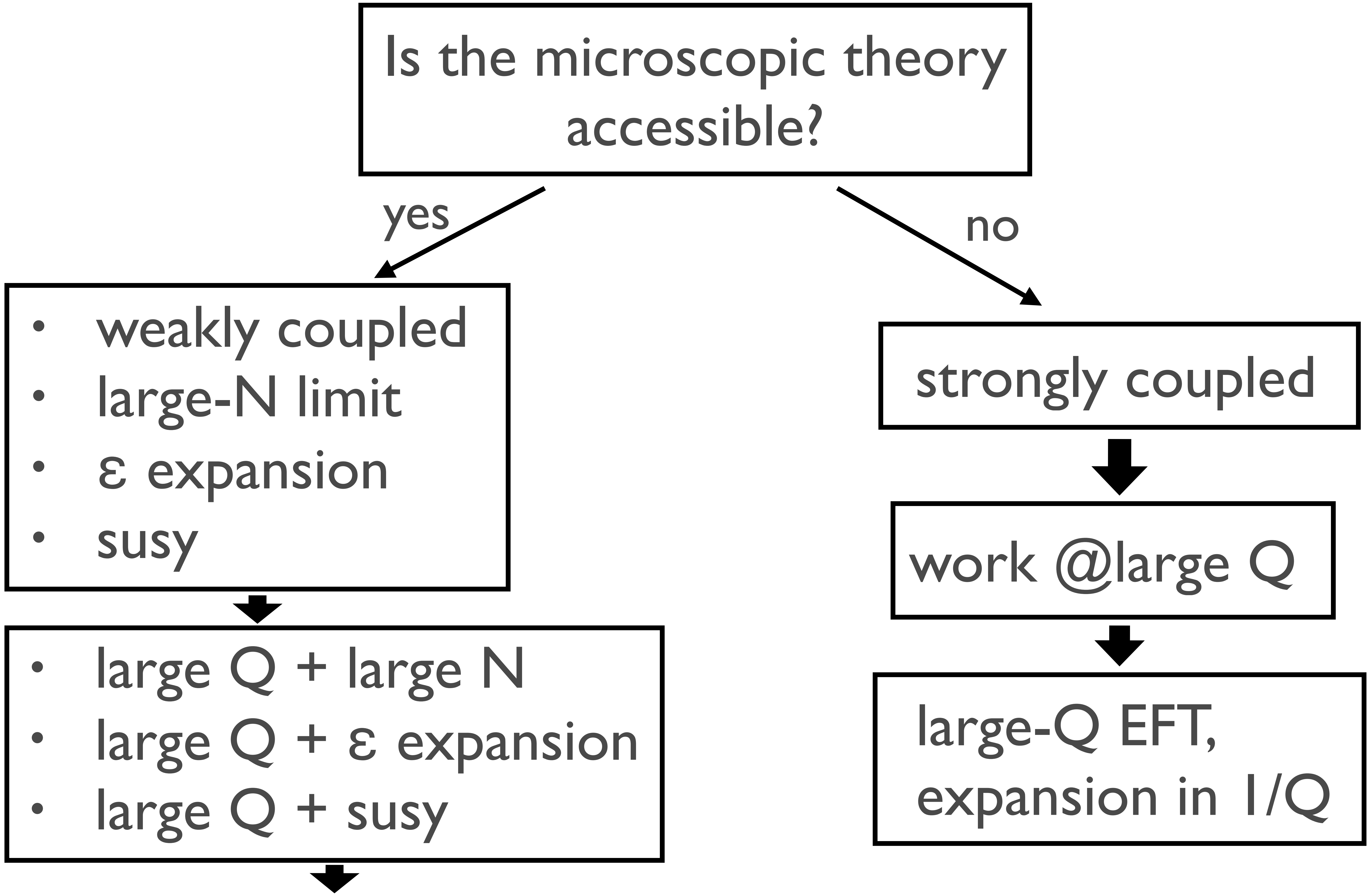
no

strongly coupled

work @large Q

large-Q EFT,
expansion in 1/Q

Introduction



Is the microscopic theory accessible?

yes

- weakly coupled
- large-N limit
- ε expansion
- susy

- large Q + large N
- large Q + ε expansion
- large Q + susy

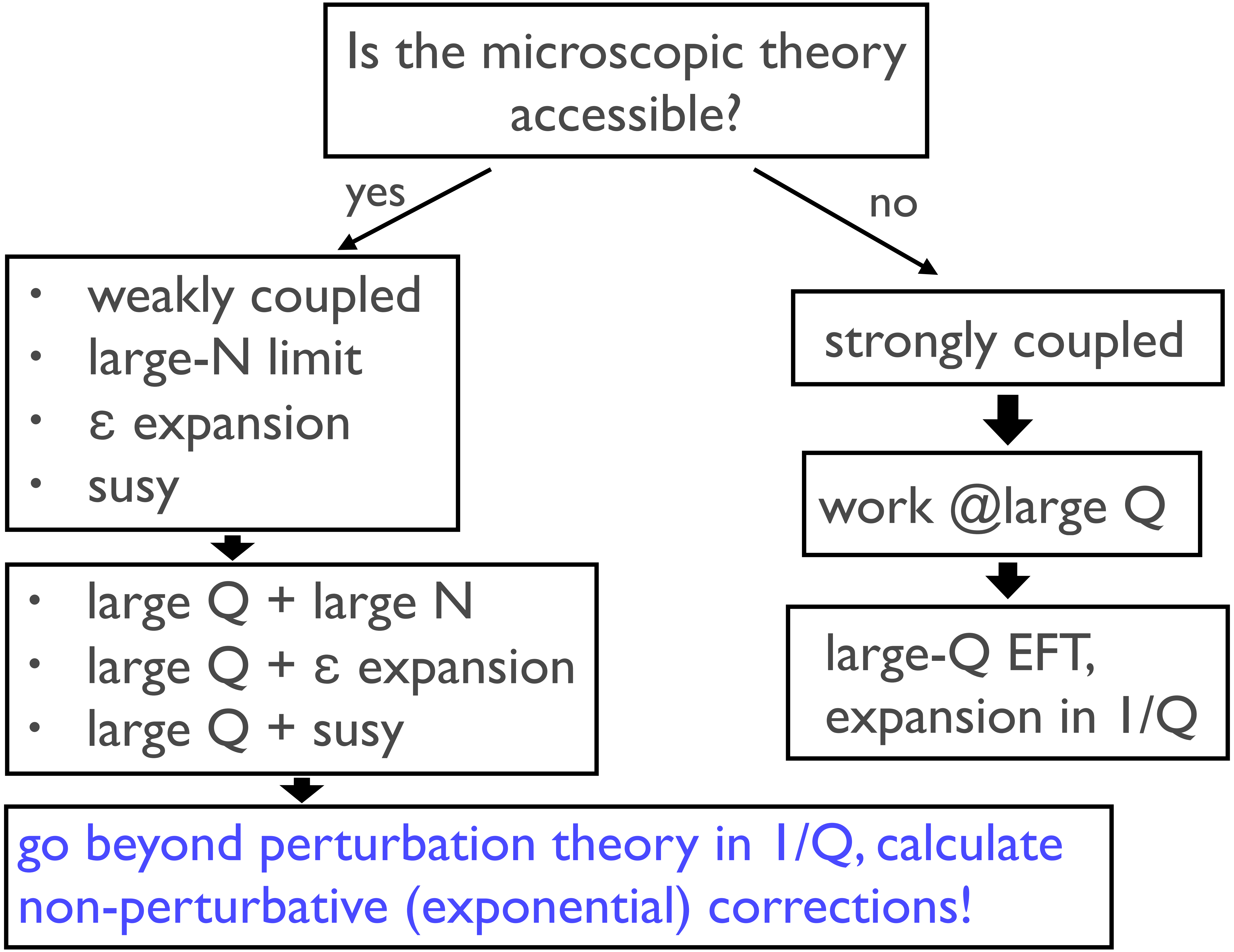
no

strongly coupled

work @large Q

large-Q EFT,
expansion in 1/Q

Introduction



Is the microscopic theory accessible?

yes

- weakly coupled
- large-N limit
- ε expansion
- susy

- large Q + large N
- large Q + ε expansion
- large Q + susy

no

strongly coupled

work @large Q

large-Q EFT, expansion in 1/Q

go beyond perturbation theory in 1/Q, calculate non-perturbative (exponential) corrections!

Introduction

The seem to be 2 main categories of behavior for systems at large quantum number:

Superfluid

isolated vacuum

- Wilson-Fisher CFT
- NRCFT (unitary Fermi gas)
- N=2 SCFT in 3d
- asymptotically safe model in 4d
- NJL model

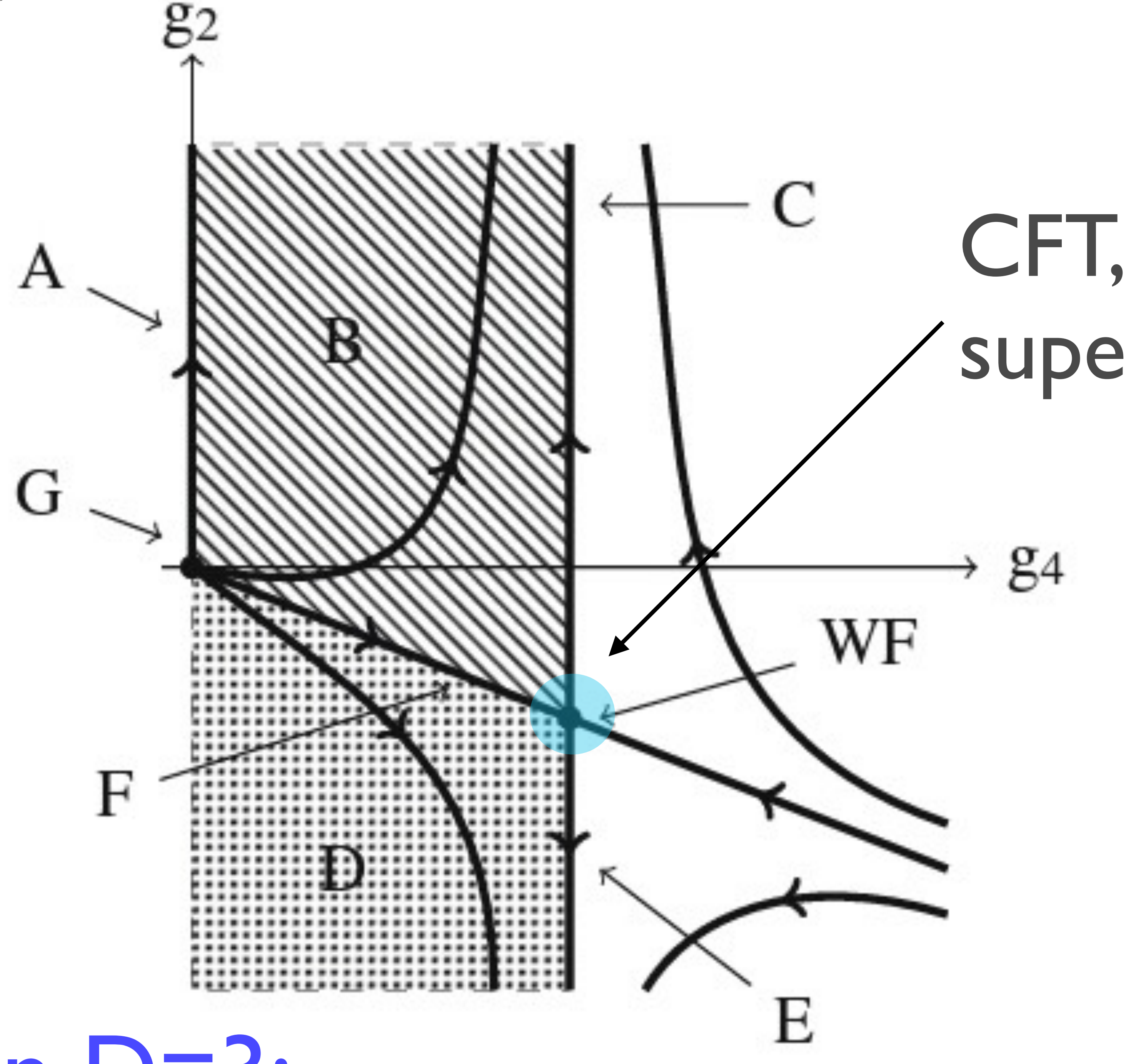
EFT of the moduli space

moduli space of vacua

- free boson
- N=2 theories in 4d

Introduction

Example: Scalar field theories in 2<D<4 have a strongly-coupled interacting fixed point, the Wilson-Fisher FP.



CFT, strongly coupled superfluid at large Q

O(2N) vector model in D=3:

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (\partial_\mu^i \phi_i)^\dagger (\partial_\nu^i \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N} (\phi_i^\dagger \phi_i)^2 \right]$$

For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

The O(2) model

Simplest example: O(2) model in (2+1) dimensions

$$\mathcal{L}_{\mathrm{UV}} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

$$\varphi_{\mathrm{IR}} = a \, e^{i\chi}$$ Global U(1) symmetry: $$\chi \to \chi + \mathrm{const.}$$

Look at scales: put system in box (2-sphere) of scale R

Second scale given by U(1) charge Q:

$$\rho^{1/2} \sim Q^{1/2}/R$$

Study the CFT at the fixed point in a sector with

UV scale

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

cut-off of effective theory

The O(2) model

Fixing the charge breaks symmetries:

$$SO(3,2) \times O(2) \to SO(3) \times D \times O(2) \rightsquigarrow SO(3) \times D'$$

$D' = D - \mu O(2)$

Broken U(1) - superfluid!

Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi \, \partial^\mu \chi)^{3/2}$$

can get this purely by dimensional analysis

Lowest-energy solution: homogeneous ground state

$$\chi = \mu t,$$

non-const. vev

Beyond LO: use dimensional analysis, parity and scale invariance to determine (tree-level) operators in effective action (Lorentz scalars of scaling dimension 3, including couplings to geometric invariants)

The O(2) model

Use ρ-scaling to determine which terms are not suppressed:

$$\partial\chi \sim \rho^{1/2}, \quad \partial\ldots\partial\chi \sim \rho^{-1/4}$$

Result for NLSM action in D=3:

LO Lagrangian

curvature coupling

$$\mathcal{L} = k_{3/2}(\partial_\mu\chi\partial^\mu\chi)^{3/2} + k_{1/2}R(\partial_\mu\chi\partial^\mu\chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by inverse powers of Q

Energy of classical ground state at fixed charge:

2 dimensionless parameters

cannot be calculated within EFT!

$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}}Q^{3/2} + \frac{c_{1/2}}{2}R\sqrt{V}Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

dependence on manifold

The O(2) model

Expand action around GS to second order in fields: $\chi = \mu t + \hat{\chi}$

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2}R\mu + (\partial_t\hat{\chi})^2 - \frac{1}{2}(\nabla_{S^2}\hat{\chi})^2 + \dots$$

Compute zeros of inverse propagator for fluctuations and get dispersion relation:

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

← dictated by conf. invariance $1/\sqrt{d}$

⇒ χ is indeed a "conformal" Goldstone

Are also the quantum effects controlled?

Yes! All effects except Casimir energy of χ are suppressed (negative ρ-scaling).

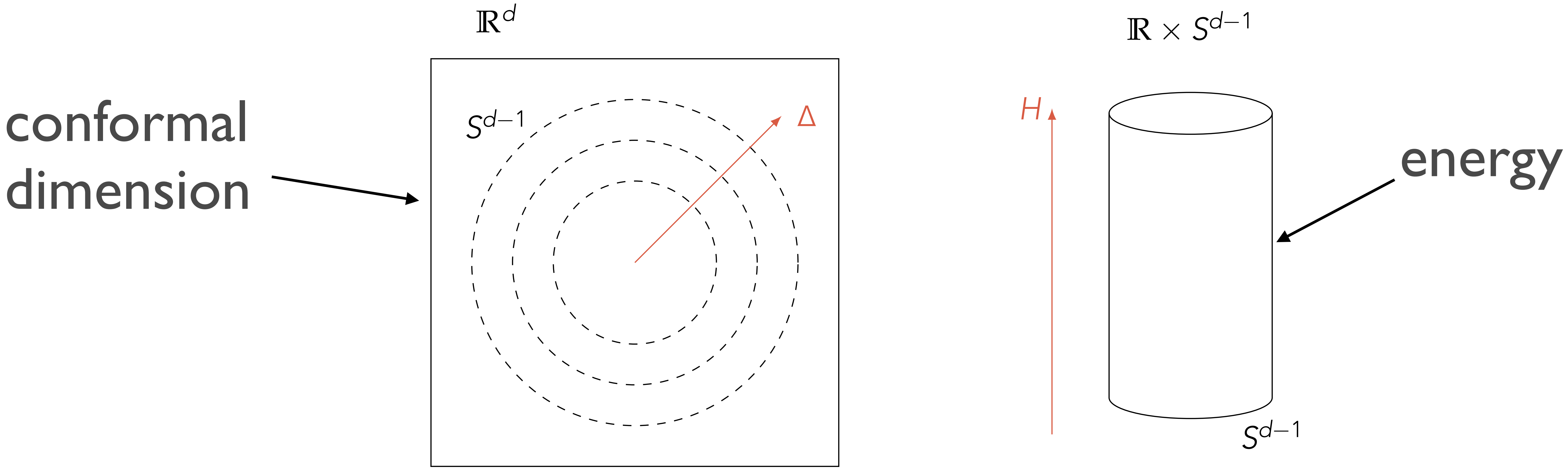
Effective theory at large Q:

vacuum + Goldstone + 1/Q-suppressed corrections

The O(2) model

We're ready to calculate observables:
CFT: conformal data (scaling dim. + 3pt coefficients)!

Use state-operator correspondence of CFT:



Scaling dimension of lowest operator of charge Q:

energy of class. ground state

$$D(Q) = R_0(E_0 + E_{Cas}) = c_{3/2}Q^{3/2} + c_{1/2}Q^{1/2} - 0.0937 \cdots + \mathcal{O}(Q^{-1/2})$$

quantum correction from Casimir energy of Goldstone

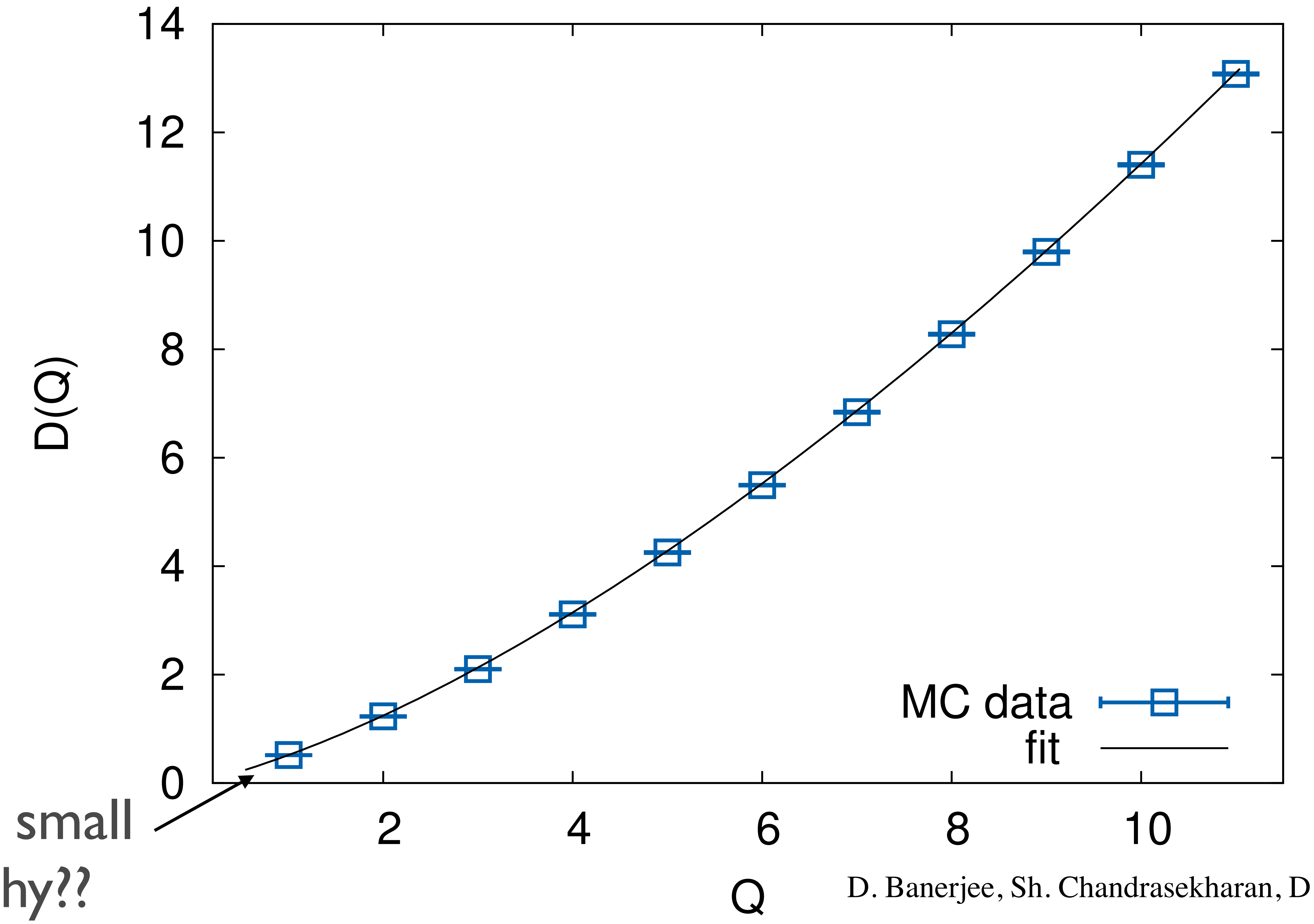
S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

The O(2) model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent calculation on the lattice:



works for small charge. Why??

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711]

Excellent agreement!!

$$c_{3/2} = 1.195(10)$$
$$c_{1/2} = 0.075(10)$$

Large-charge expansion works extremely well for O(2).

Beyond O(2)

Where else can we apply the large-charge expansion?

Obvious generalization in 3d: O(2N) vector model
non-Abelian global symmetry group: new effects

Different symmetry breaking patterns possible, inhomogeneous ground states possible.

Homogeneous case: same form of ground state,

$$SO(3,2) \times O(2N) \to SO(3) \times D \times U(N) \to SO(3) \times D' \times U(N-1)$$

We expect dim[U(N)/U(N-1)] = 2N-1 Goldstone d.o.f.

On top of the conformal Goldstone of O(2), a new sector with N-1 non-relativistic type II Goldstones and N-1 massive modes with m=2μ appears.

The O(2N) vector model

Dispersion relation:

$$\omega = \frac{p^2}{2\mu} + \mathcal{O}\mu^{-3}$$

The non-relativistic Goldstones count double.

Nielsen and Chadha; Murayama and Watanabe

Counting type I and type II modes, indeed,

$$1 + 2(N-1) = 2N - 1 = \dim(U(N)/U(N-1))$$

Non-relativistic Goldstones contribute to the conformal dimensions only at higher order.
The ground-state energy is again determined by a single relativistic Goldstone!

Same formula for scaling dimensions as for O(2):

N-dependent

universal for O(2N)

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

verified at large N for CP(N-1) model de la Fuente

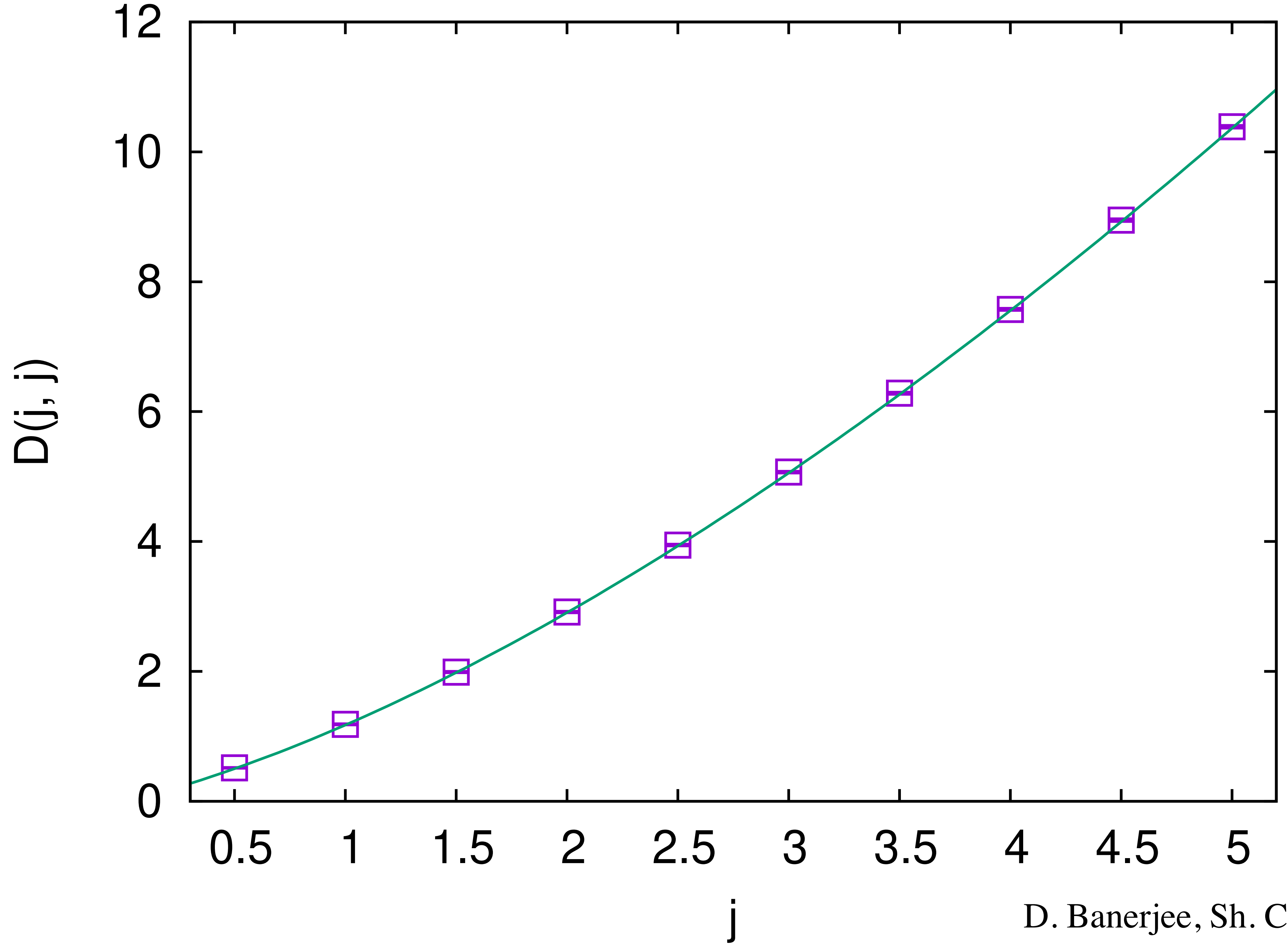
L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

The O(2N) vector model

Testing our prediction:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

New lattice data for O(4) model:



$$c_{3/2} = 1.068(4)$$
$$c_{1/2} = 0.083(3)$$

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. 1902.09542

Again excellent agreement with large-Q prediction!

The large-N limit

Standard large-N methods, expand path integral around saddle point (no EFT!)

Extra control parameter at large N: can go further!

Start in the UV with

$$S[\phi_i] = \sum_{i=1}^{N} \int \mathrm{d}t\mathrm{d}\Sigma \left[g^{\mu\nu} (\partial_\mu^i \phi_i)^\dagger (\partial_\nu^i \phi_i) + r(\phi_i^\dagger \phi_i) + \frac{u}{2N} (\phi_i^\dagger \phi_i)^2 \right]$$

For r=R/8, this flows to the WF fixed pt in the IR, $u \to \infty$

Scaling dimension for Q/N>>1:

$$\frac{\Delta(Q)}{2N} = \frac{2}{3} \left(\frac{Q}{2N} \right)^{3/2} + \frac{1}{6} \left(\frac{Q}{2N} \right)^{1/2} - \frac{7}{720} \left(\frac{Q}{2N} \right)^{-1/2} - \frac{71}{181440} \left(\frac{Q}{2N} \right)^{-3/2} + \dots$$

same Q-scaling as in EFT

L. Alvarez-Gaume, D. Orlando, S.R. 1909.02571

Small charge limit, Q/N<<1:

engineering dimension of ϕ

$$\frac{\Delta(Q)}{Q} = \frac{1}{2} + \frac{4}{\pi^2} \frac{Q}{N} + \mathcal{O} \left(\frac{Q}{2N} \right)^2$$

In this limit, the operator of charge Q is φ^Q.

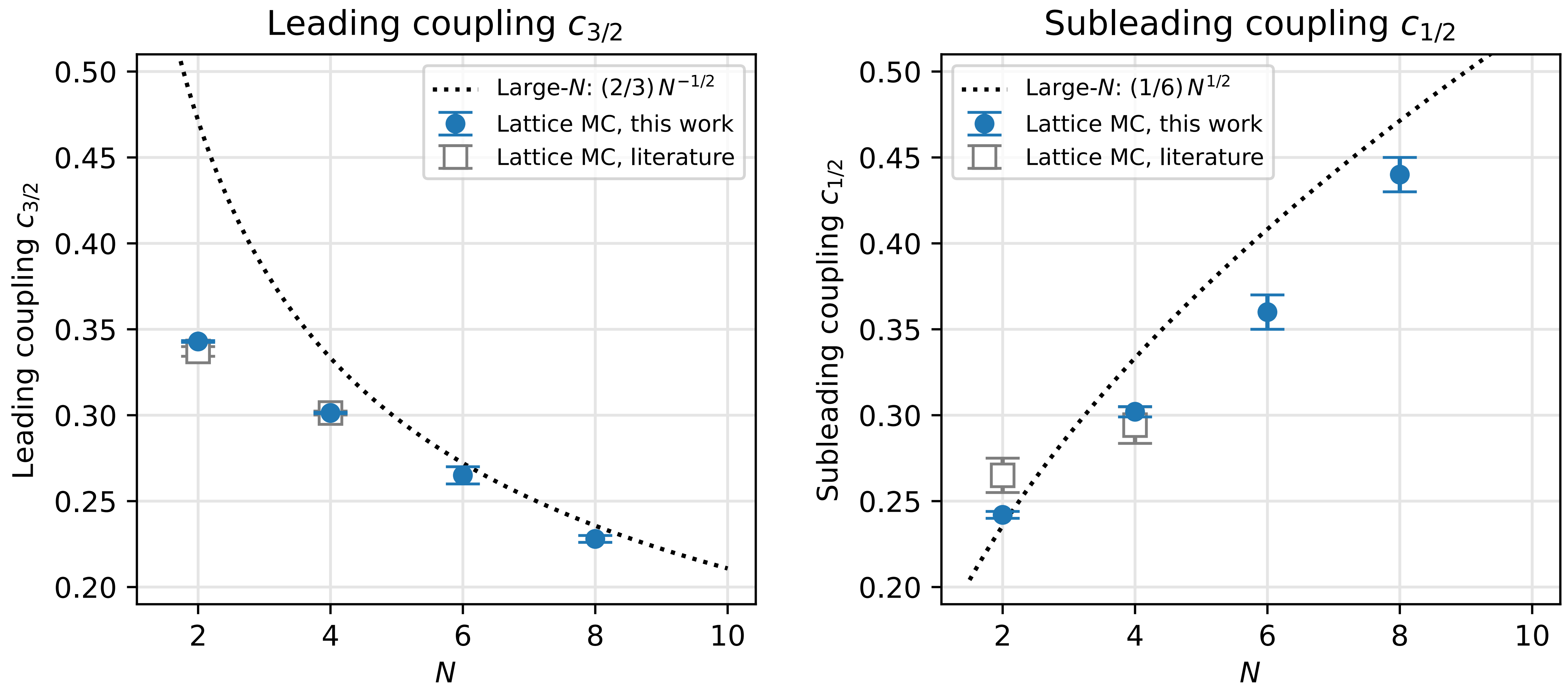
The large-N limit

NLO in N: reproduce dispersion relations of Goldstones.

Find coefficients of the expansion (leading order in N):

$$c_{3/2} = \frac{1}{3}\sqrt{\frac{2}{N}} \qquad c_{1/2} = \frac{1}{3}\sqrt{\frac{N}{2}}$$

Comparison to lattice data:



Singh, arXiv:2203.00059 [hep-lat]

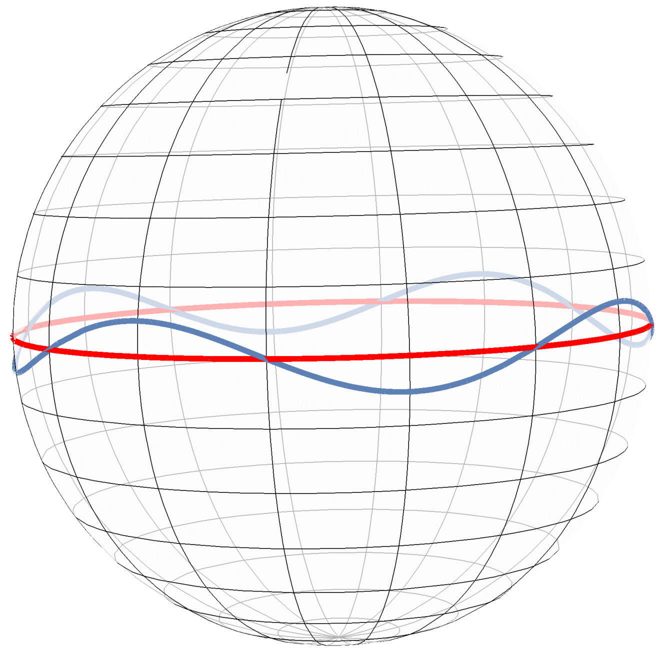
Resurgence analysis

Since we can compute all the coefficients of the large-Q expansion, we can do a resurgence analysis to relate the large and small-charge regimes. Asymptotic series which diverges as (2L)!

We can write the transseries. Find non-perturbative corrections:

$$e^{-2\pi k\sqrt{Q/(2N)}}$$

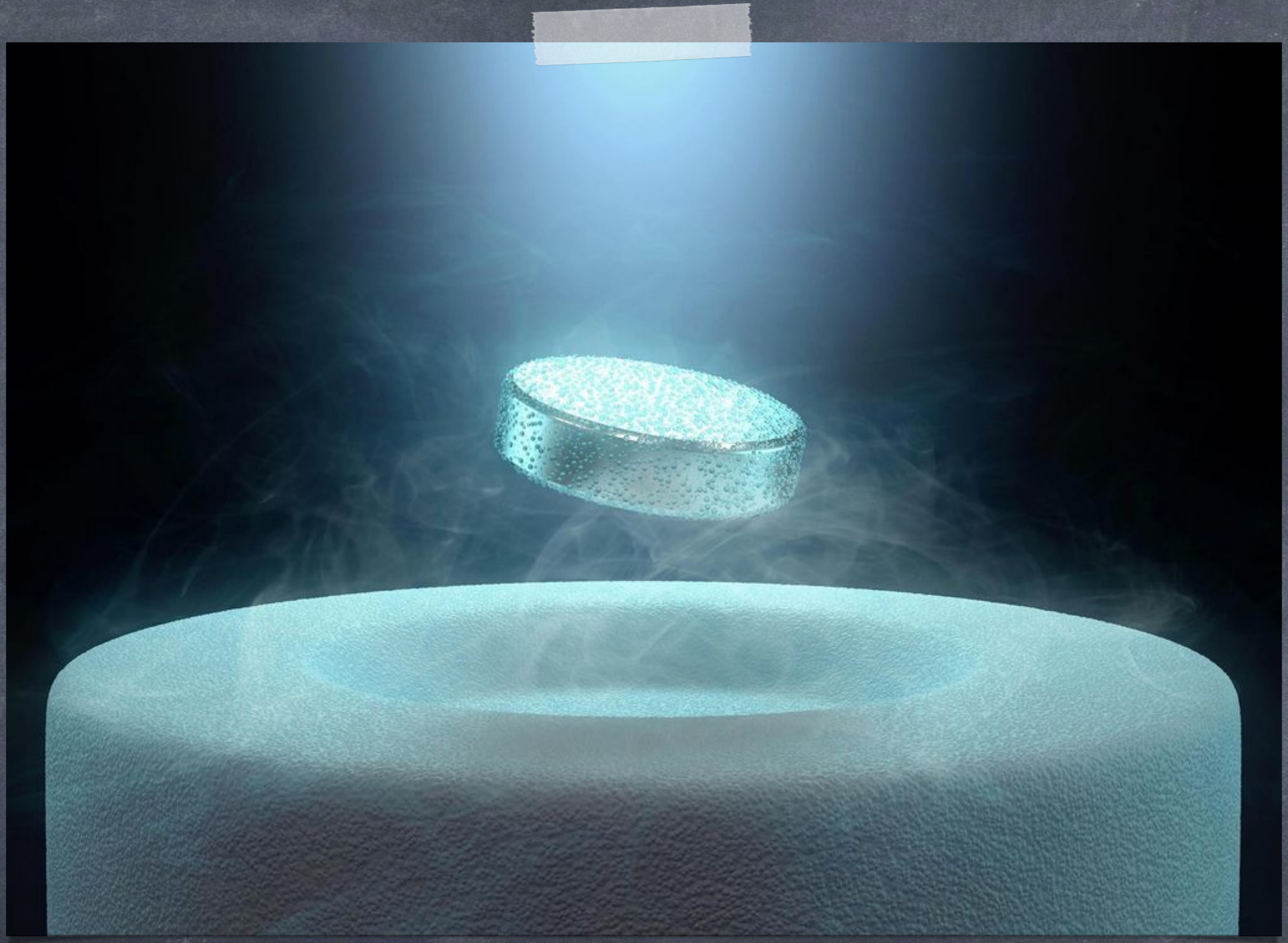
Geometric interpretation: particles of mass μ propagating on the equator of the 2-sphere.



CFT + resurgence: This picture must work for any N!

The optimal truncation is $\mathcal{O}(\sqrt{Q})$ terms. This explains why the comparison to the lattice calculation works so well.

A. Dondi, I. Kalogerakis, D.Orlando, S.R, arXiv: 2102.12488 [hep-th]



Fermions@large Q

Fermions@large Q

Will large Q work for fermionic models?

Antipin, Bersini, Panopoulos;

Let’s start with the multicomponent Nambu-Jona-Lasinio (NJL) model, also known as the chiral Gross-Neveu (GN) model in 3D:

$$S_{\text{cGN}} = -\int \mathrm{d}^3x \left[\bar{\psi}_a i\partial\!\!\!/ \psi_a + \frac{g}{2N}\left(\left(\bar{\psi}_a\psi_a\right)^2 + \left(\bar{\psi}_a i\gamma_5\psi_a\right)^2\right)\right]$$

There are two conserved currents:

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \qquad\qquad j^{5\mu} = \bar{\psi}\gamma^\mu\gamma^5\psi$$

We can study this model at large N with standard methods.

We find that only the axial charge gives rise to a condensate at criticality.

Scaling dimension:

large Q/N

$$\frac{\Delta}{N} = \frac{\sqrt{2}}{3}\left(\frac{Q}{\kappa N}\right)^{3/2} + \frac{1}{3\sqrt{2}}\left(\frac{Q}{\kappa N}\right)^{1/2} + \dots$$

small Q/N

$$= \frac{1}{2}\frac{Q}{N} + \frac{1}{\pi^2}\left(\frac{Q}{N}\right)^2 + \dots$$

Fermions@large Q

Like for the scalar case, we get a condensate at fixed charge, but not WF universality class.

Can go to a different frame using the Pauli-Gürsey transformation: condensate is due to Cooper pairs!

The end result is similar in the sense that we have an EFT in terms of Goldstones fluctuating around a condensate.

Fermions@large Q

What happens if there is not axial charge to fix?

Study standard Gross-Neveu model:

$$S_{GN} = -\int \mathrm{d}^3x \left[\bar{\psi}_a i\partial\!\!\!/ \psi_a + \frac{g}{2N}(\bar{\psi}_a\psi_a)^2\right]$$

Only one current, can fix its associated charge.

Result@leading order in N: the fixed-charge ground state is not a condensate, but a Fermi surface.

Interaction is exponentially suppressed in N, behaves like a free fermion.
SSB is a non-perturbative effect.

Summary

Summary

Concrete examples where a strongly-coupled CFT simplifies significantly at large charge.

O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

- Excellent agreement with lattice results for O(2), O(4)
- large Q and large N: path integral at saddle pt., more control than in EFT, can calculate coefficients
- can follow the flow away from conformal point, find the full effective potential

NJL model: similar results, condensate due to Cooper pairs.

GN model at large N: condensate suppressed at large N.

Further directions

- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces)

  Hellerman, Maeda, Orlando, Reffert, Watanabe;
  Argyres et al.

- Connection to holography (gravity duals)

  Loukas, Orlando, Reffert, Sarkar;
  De la Fuente, Zosso;
  Giombi, Komatsu, Offertaler;
  Perlmutter et al.

- Operators with spin; connection to large-spin results

  Cuomo, de la Fuente, Monin, Pirtskhalava, Rattazzi; Cuomo

- Use/check large-charge results in conformal bootstrap

  Jafferis and Zhiboedov

- Further lattice simulations: inhomogeneous sector, general O(N)

  Chandrasekharan et al.;
  Singh

- CFTs in other dimensions (2, 5, 6)

  Komargodski, Mezei, Pal, Raviv-Moshe;
  Araujo, Celikbas, Reffert, Orlando;
  Moser, Orlando, Reffert

Further directions

- Chern-Simons matter theories @large charge

Watanabe

- 4-ε expansion @large charge

Arias-Tamargo, Rodriguez-Gomez, Russo;
Badel, Cuomo, Monin, Rattazzi; Watanabe;
Antipin et al.

- going away from the conformal point

Orlando, Reffert, Sannino;
Orlando, Pellizzani, Reffert

- non-relativistic CFTs

Favrod, Orlando, Reffert; Kravec, Pal;
Orlando, Pellizzani, Reffert;
Hellerman, Swanson; Pellizzani

- Boundary CFTs at large Q

Cuomo, Mezei, Raviv-Moshe

- Swampland, weak gravity conjecture

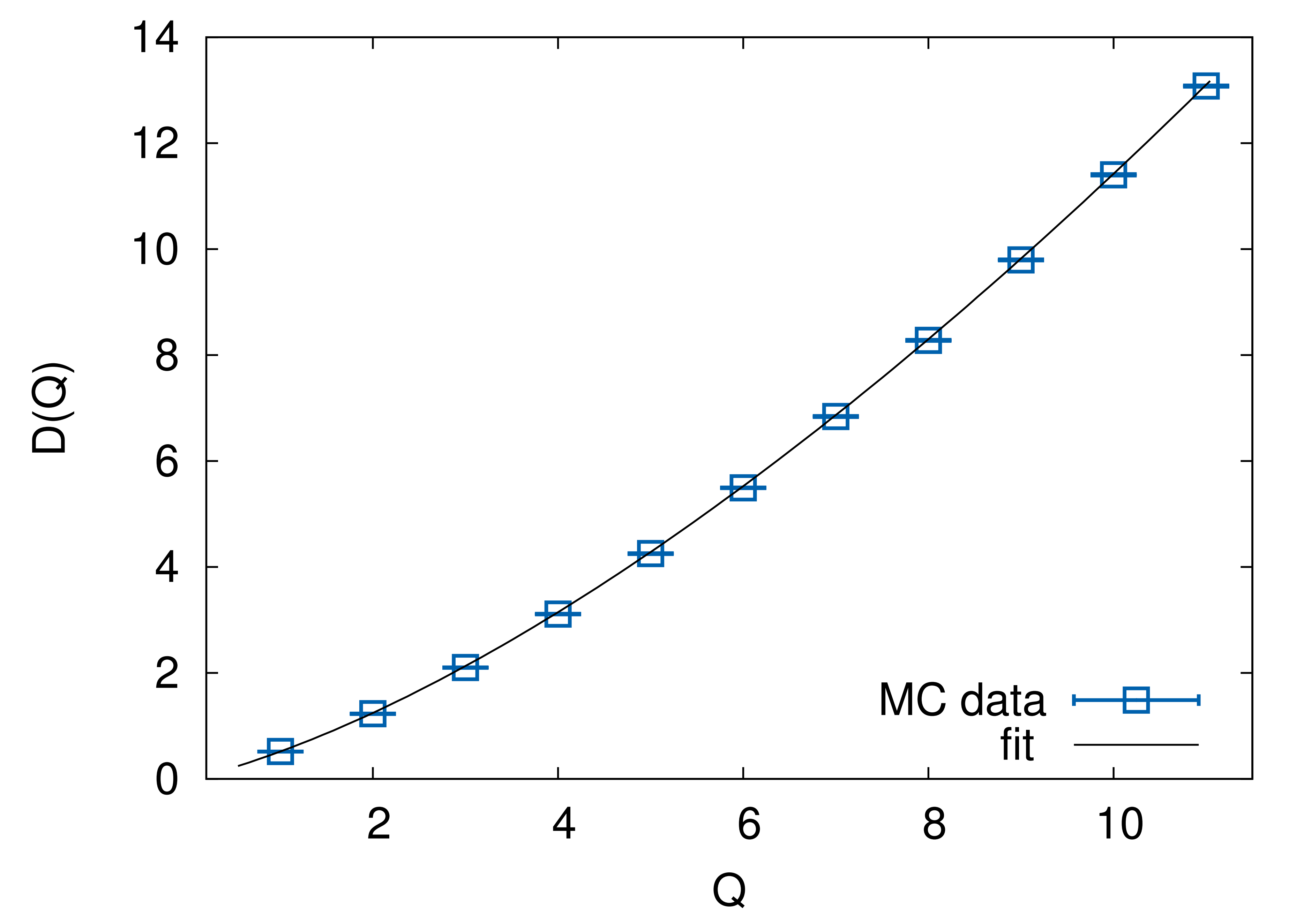
Aharony, Palti; Antipin et al.
Orlando, Palti

- Study fermionic theories. Can large-charge approach be used for QCD (e.g. large baryon number)?

Komargodski, Mezei, Pal, Raviv-Moshe;
Antipin, Bersini, Panopoulos;
Dondi, Hellerman, Kalogerakis, Moser, Orlando, Reffert;

- Gauge theories @large charge

Antipin et al.



Thank you for your attention!