Updated predictions for $R(D^{(*)})$ using the residual chiral expansion

BLPRXP Form Factors

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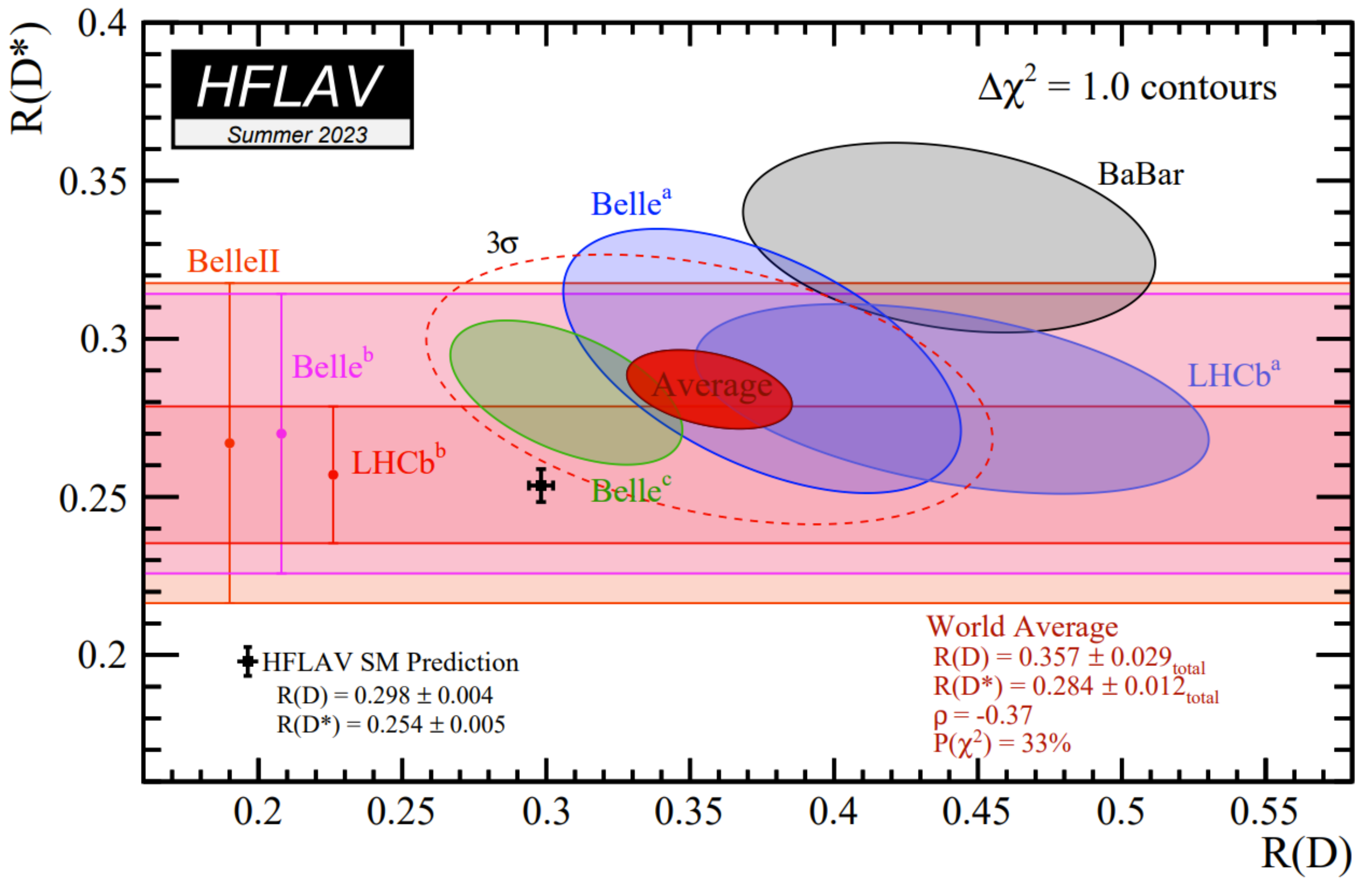
Experimental Status of $R(D^{(*)})$



The difference with the SM prediction corresponds to 3.3σ (HFLAV '23)

$R(D)$ exceeds SM by 2.0σ

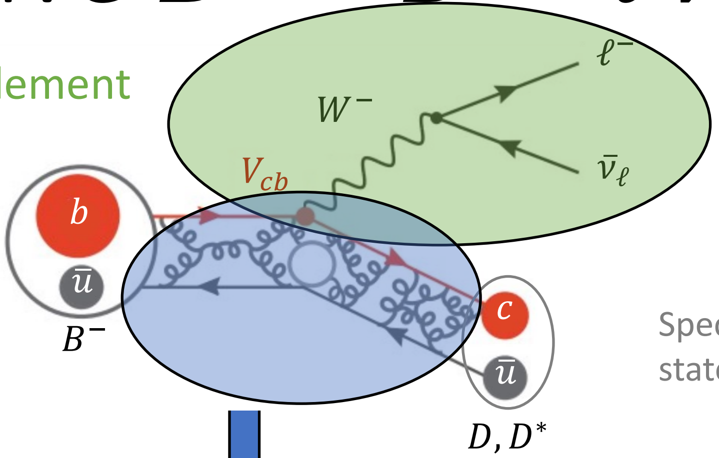
$R(D^*)$ exceeds SM by 2.2σ

Update of the work in Phys.Rev.D 106 (2022) 9, 096015

- New experimental data
- New nonzero-recoil lattice QCD data

Exclusive $B \to D^{(*)} \ell \bar{\nu}_\ell$

Leptonic Matrix Element



Specific final state meson

$$\Gamma(B \to D\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{G}(1) \qquad \mathcal{G}(1) = h_+(1)$$
$$\Gamma(B \to D^*\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{F}(1) \qquad \mathcal{F}(1) = h_{A_1}(1)$$

Hadronic Matrix Elements can not be calculated from first principles
→ Can be parameterized with form factors $h_X = h_X(w)$ and extracted from data
→ Lattice QCD must provide (at least) inputs on their normalization

$$\frac{\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_D}} = h_+(v+v')^\mu + h_-(v-v')^\mu$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V \epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu\gamma^5 b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu$$

Heavy Quark Symmetry Basis

BLPRXP Form Factors for $B \to D^{(*)} \ell \bar{\nu}_\ell$

Form factors in the framework of HQET:
$B \to D \ell \bar{\nu}_\ell$ and $B \to D^* \ell \bar{\nu}_\ell$ are linked

Expansion to order $\mathcal{O}\left(1/m_{b,c}^{(2)}\right)$, $\mathcal{O}(1/(m_b m_c))$

$$\frac{\langle H_c | \bar{c} \Gamma b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} \propto 1 + \underbrace{\frac{1}{2m_c} + \frac{1}{2m_b}}_{+3} + \overbrace{\underbrace{\frac{1}{4m_c^2}}_{+20 \to +1} + \frac{1}{2m_b^2} + \frac{1}{4m_c m_b}}^{+32 \to +3}$$

Proliferation of non-perturbative parameters

Supplemental power counting in the transverse residual momentum $\gamma_\mu D_\perp^\mu$
→ Drastic reduction of the non-perturbative parameters
Leftover Isgur-Wise functions to be fitted determined by Nested Hypothesis Test on data
→ Determination of $|V_{cb}|$ and the $B \to D^{(*)} \ell \bar{\nu}_\ell$ form factors
→ Prediction of $R(D^{(*)})$

Nested Hypothesis Test

- The free parameters in our model are
  - entering at zero-recoil: $|V_{cb}|, m_b^{1S}, \delta m_{bc}, \rho_1, \lambda_2, \rho_*^2, c_*, \hat{\eta}(1)$
  - and beyond: $\hat{\eta}'(1), \hat{\chi}_2(1), \hat{\chi}_2'(1), \hat{\chi}_3'(1), \hat{\varphi}_1'(1), \hat{\beta}_2(1), \hat{\beta}_3'(1)$
- Nested Hypothesis Test to determine optimal set of fit parameters
  - Starting point are the parameters contribution at zero-recoil
  - Subsequently add parameters to the model in all combinations
  - Test alternative fit hypothesis with cut-off $\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 < 1$
  - Reject combinations with highly correlated parameters

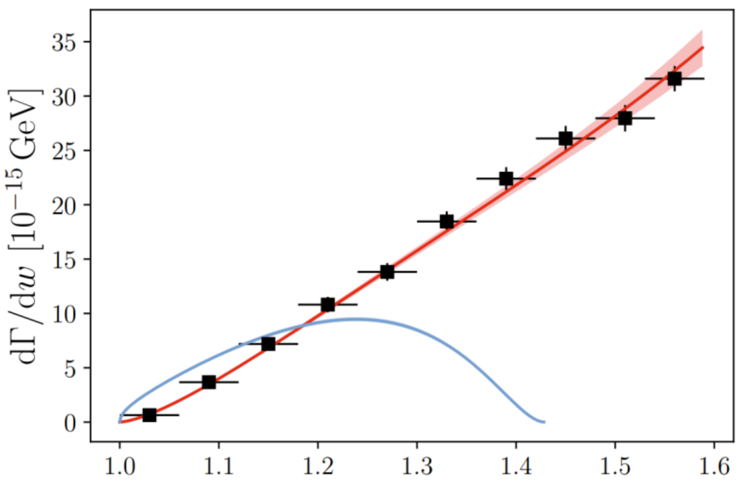
Experimental Inputs

- A) Belle $B \to D\ell\bar{\nu}_\ell$ tagged ’15 → Only use shape and BR world average
- B) Belle $B \to D^*\ell\bar{\nu}_\ell$ untagged ’19
- C) Belle $B \to D^*\ell\bar{\nu}_\ell$ tagged ’23 → Updated measurement wrt ‘17
- Today: Only use measured hadronic recoil spectra

BLPRXP ’23 Fit $B \to D^{(*)}\ell\bar{\nu}_\ell$

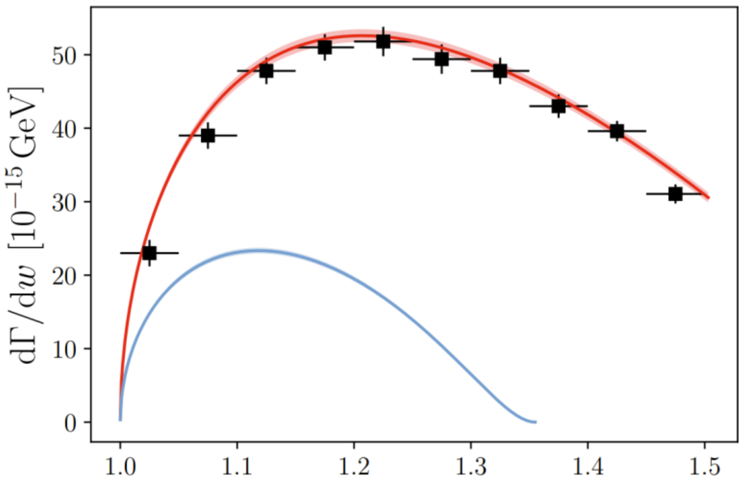
BLPRXP ’23 $B \to D^{(*)}\tau\bar{\nu}_\tau$

A) $B \to D\ell\bar{\nu}_\ell$



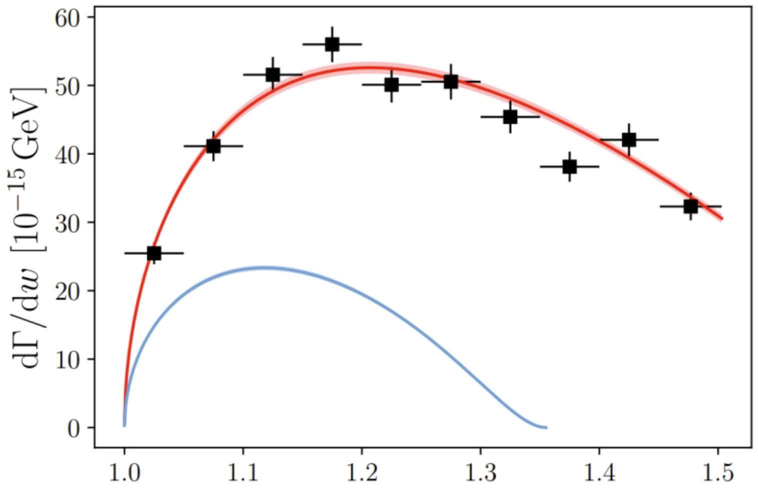
$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

B) $B \to D^*\ell\bar{\nu}_\ell$



$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

C) $B \to D^*\ell\bar{\nu}_\ell$

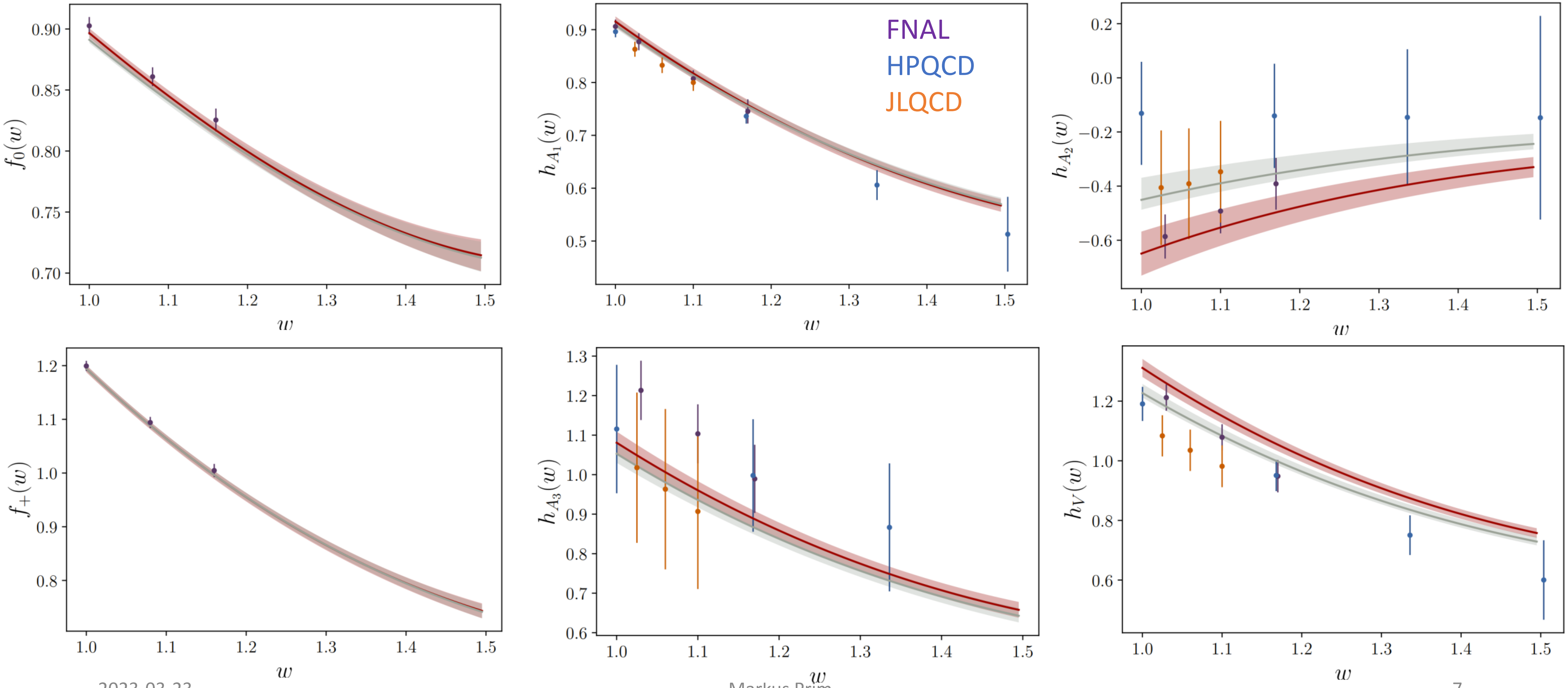


$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

Lattice Inputs

BLPRXP ’23 $f_{+/0}, h_{A_1}(1), B \to D^{(*)}\ell\bar{\nu}_\ell$

BLPRXP ’23 $f_{+/0}\ h_X(w), B \to D^{(*)}\ell\bar{\nu}_\ell$



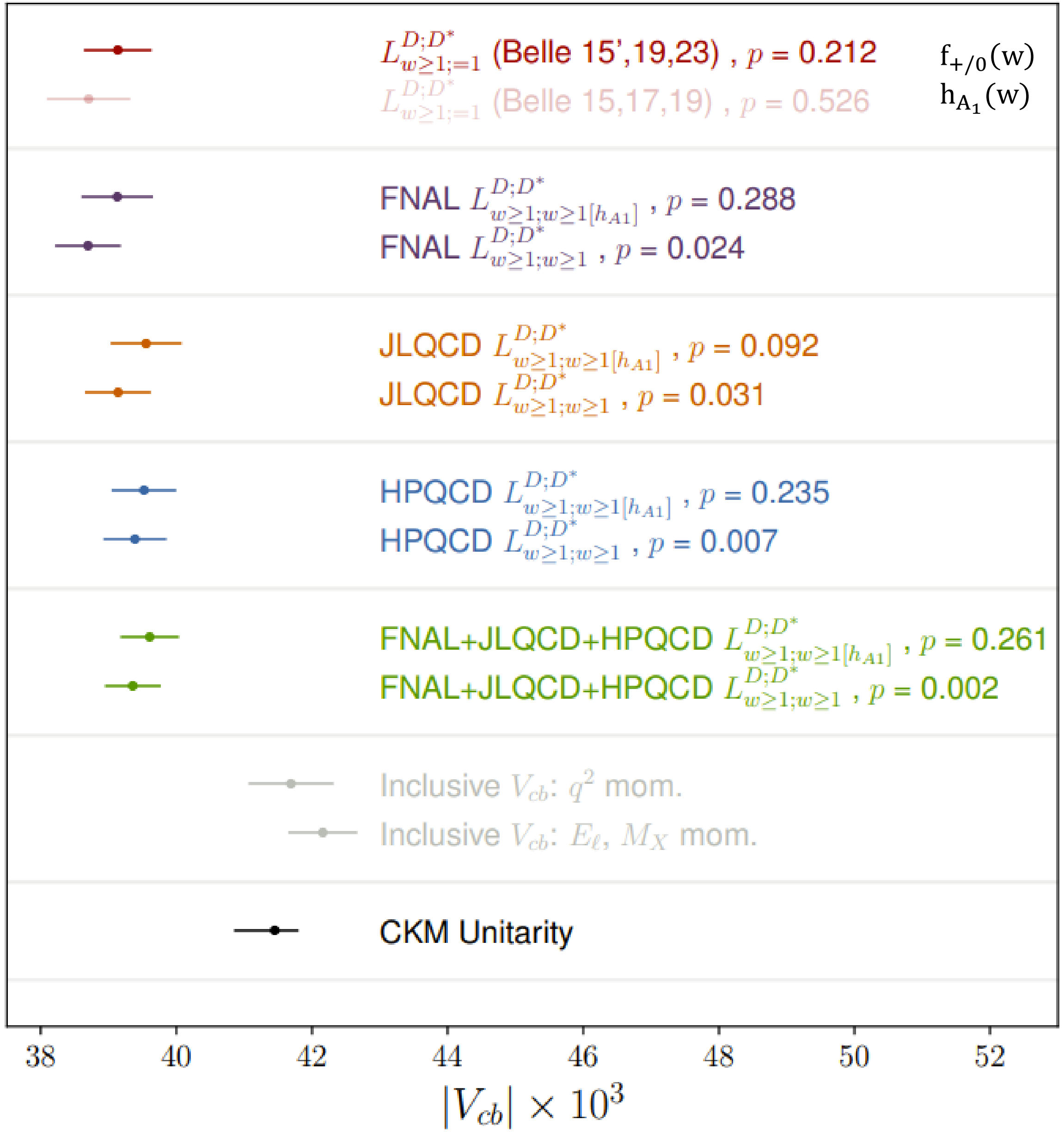
$|V_{cb}|$ from BLPRXP

non-zero recoil lattice inputs:

- $h_{A_1}(w)$ only has good p-values
- full set $h_X(w)$ results in worse p-values

$|V_{cb}| = (39.1 \pm 0.5) \times 10^{-3}$
using $\mathrm{f}_{+/0}(\mathrm{w}), \mathrm{h}_{\mathrm{A}_1}(\mathrm{w})$
$|V_{cb}| = (38.7 \pm 0.6) \times 10^{-3}$

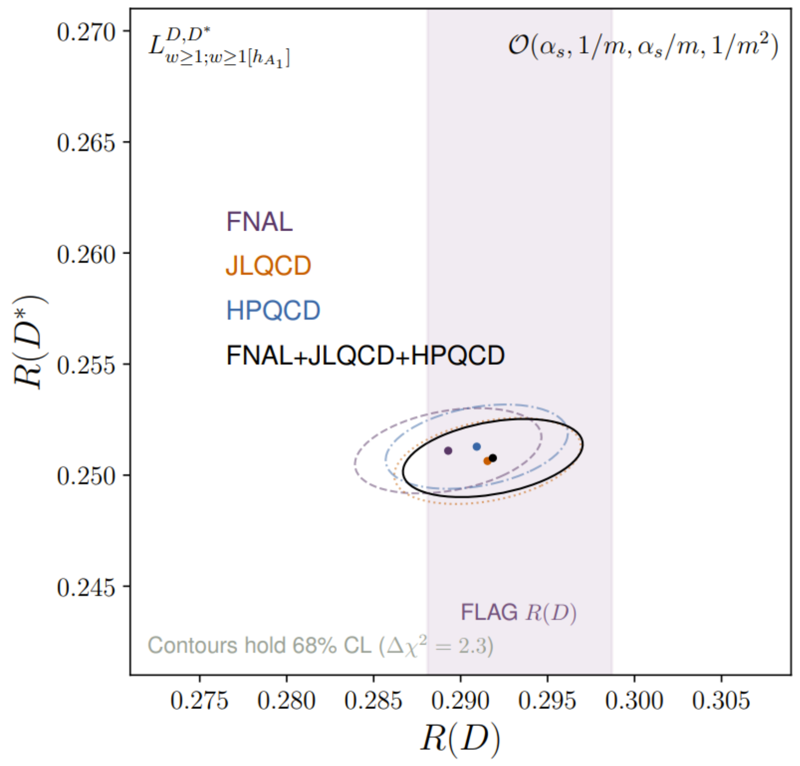
$|V_{cb}| = (39.6 \pm 0.4) \times 10^{-3}$ $[h_{A_1}(w)]$
$|V_{cb}| = (39.4 \pm 0.4) \times 10^{-3}$ $[h_X(w)]$
with the same NHT hypothesis



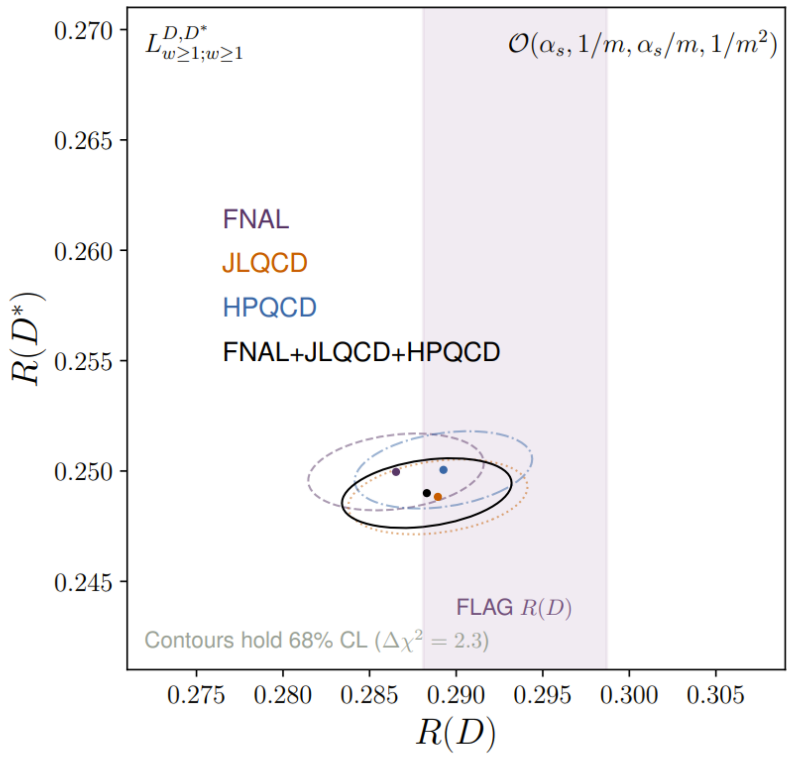
$R(D^{(*)})$ Predictions – Lattice Inputs

Prediction depends on the lattice input, but is compatible within uncertainties

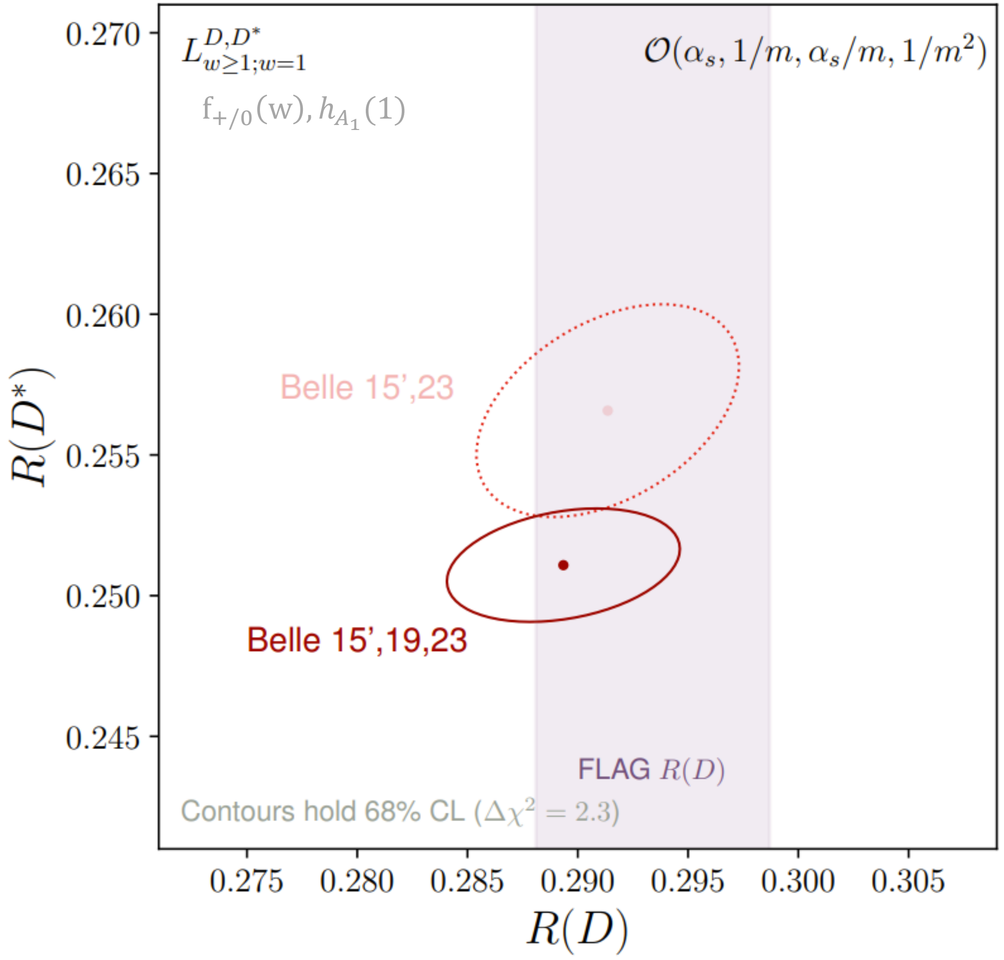
$f_{+/0}(w), h_{A_1}(w)$



$f_{+/0}(w), h_X(w)$

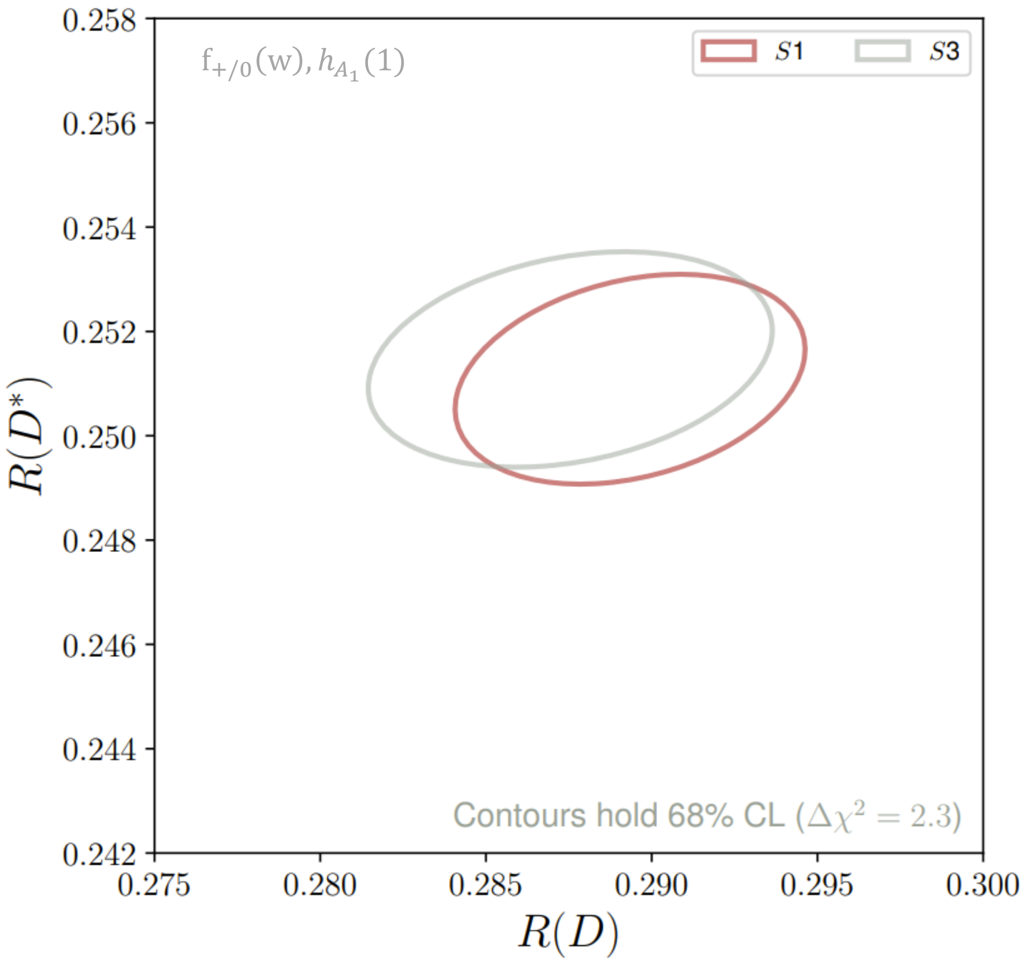


$R(D^{(*)})$ Predictions – Experimental Inputs



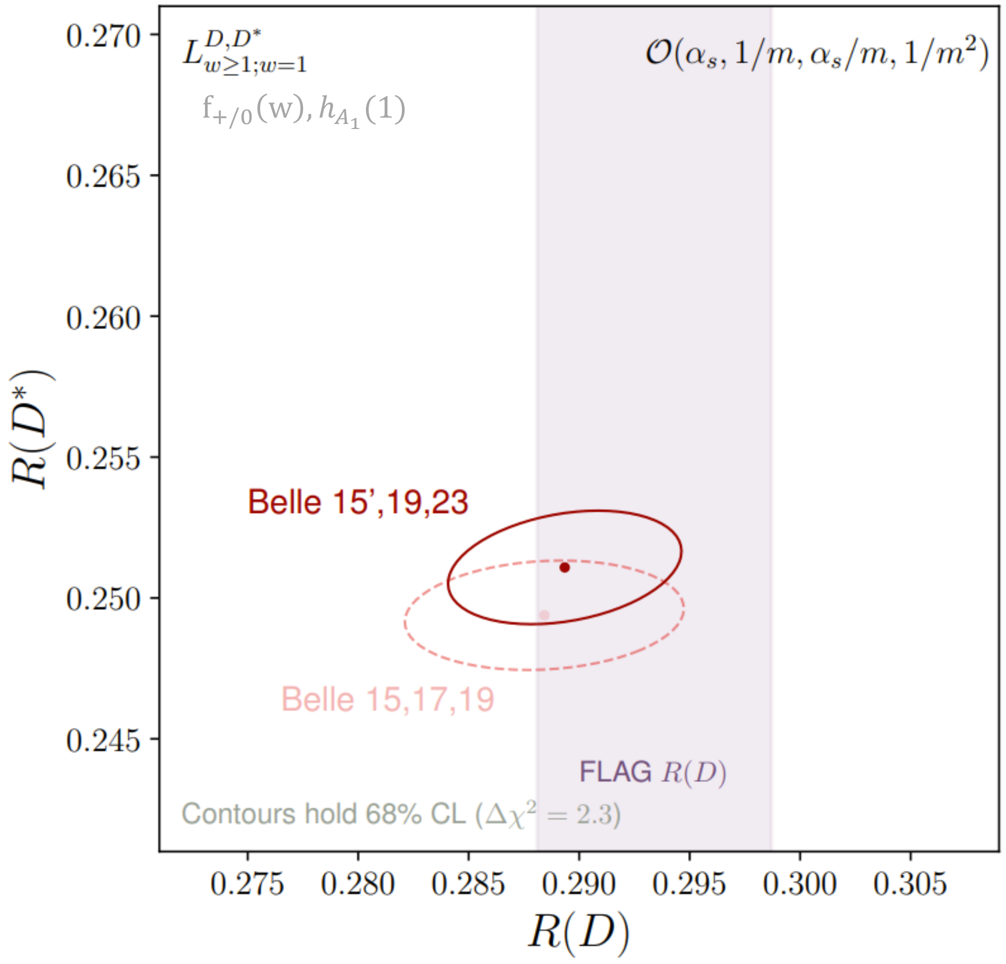
Small tension in the $R(D^*)$ prediction using different inputs.
→ Safety factor of 2 applied in our final prediction

$R(D^{(*)})$ Predictions – Model Dependence



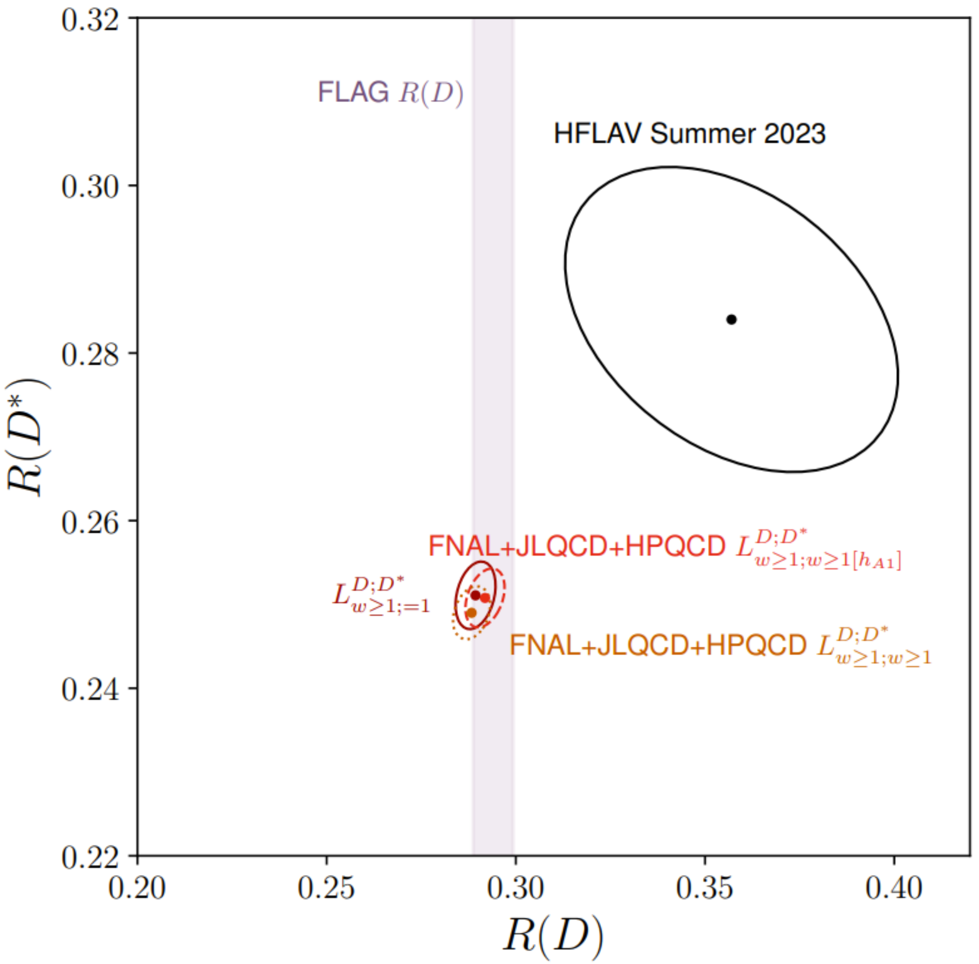
$R(D^{(*)})$ dependence on selected hypothesis from the NHT is small and compatible within the uncertainty.

$R(D^{(*)})$ Predictions – Impact of Updates



1σ shift in $R(D^*)$

$R(D^{(*)})$ Predictions – vs. Experiment



The picture with respect to the experimental measurements did not change, still a strong tension!

$R(D) = 0.289 \pm 0.003$
$R(D^*) = 0.251 \pm 0.003$
$\rho = 0.286$ using $f_{+/0}(w), h_{A_1}(1)$

Summary & Outlook

- $R(D^{(*)})$ tension with the experiment remains
- Include new experimental inputs next
  - Belle (See T08 23.08.2023, 09:12)
  - Belle II (See T08 22.08.2023, 17:55)

Backup

$\hat{h}(w) \equiv h(w)/\xi(w)$

$$\hat{h}_+ = 1 + \hat{\alpha}_s \left[C_{V_1} + \frac{w+1}{2} (C_{V_2} + C_{V_3}) \right] + \sum_{Q=c,b} \varepsilon_Q \hat{L}_1^{(Q)} - \varepsilon_c \varepsilon_b \hat{M}_8 \, ,$$

$$\hat{h}_- = \hat{\alpha}_s \frac{w+1}{2} (C_{V_2} - C_{V_3}) + \varepsilon_c \hat{L}_4^{(c)} - \varepsilon_b \hat{L}_4^{(b)} \, ,$$

$$\hat{h}_V = 1 + \hat{\alpha}_s C_{V_1} + \varepsilon_c [\hat{L}_2^{(c)} - \hat{L}_5^{(c)}] + \varepsilon_b [\hat{L}_1^{(b)} - \hat{L}_4^{(b)}] + \varepsilon_c \varepsilon_b \hat{M}_9 \, ,$$

$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s C_{A_1} + \varepsilon_c \left(\hat{L}_2^{(c)} - \hat{L}_5^{(c)} \frac{w-1}{w+1} \right) + \varepsilon_b \left(\hat{L}_1^{(b)} - \hat{L}_4^{(b)} \frac{w-1}{w+1} \right) + \varepsilon_c \varepsilon_b \hat{M}_9 \, ,$$

$$\hat{h}_{A_2} = \hat{\alpha}_s C_{A_2} + \varepsilon_c [\hat{L}_3^{(c)} + \hat{L}_6^{(c)}] - \varepsilon_c \varepsilon_b \hat{M}_{10} \, ,$$

$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s (C_{A_1} + C_{A_3}) + \varepsilon_c [\hat{L}_2^{(c)} - \hat{L}_3^{(c)} + \hat{L}_6^{(c)} - \hat{L}_5^{(c)}] + \varepsilon_b [\hat{L}_1^{(b)} - \hat{L}_4^{(b)}] + \varepsilon_c \varepsilon_b [\hat{M}_9 + \hat{M}_{10}] \, ,$$

Leading

$\mathcal{O}\left(\frac{1}{m_{b,c}^{(2)}}\right)$

$\mathcal{O}\left(\frac{1}{m_b m_c}\right)$

Parametric form of the Leading Order Isgur-Wise function

Optimized conformal variable

$$z_*(w) = \frac{\sqrt{w+1} - \sqrt{2}a}{\sqrt{w+1} + \sqrt{2}a}, \quad \text{with} \quad a^2 \equiv \frac{w_0+1}{2} = \frac{1+r_D}{2\sqrt{r_D}}. \tag{1}$$

Leading order Isgur-Wise function parametrized as polynomial in z_*.

$$\frac{\xi(w)}{\xi(w_0)} = 1 - 8a^2\rho_*^2 z_* + 16(2c_* a^4 - \rho_*^2 a^2) z_*^2 + \dots \tag{2}$$

No sensitivity to cubic terms given the current experimental and lattice data.

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Parametric form of $\mathcal{G}(1)$

$$\frac{\mathcal{G}(w)}{\mathcal{G}(w_0)} = 1 - 8a^2\tilde{\rho}_*^2 z_* + 16(2\tilde{c}_* a^4 - \tilde{\rho}_*^2 a^2) z_*^2 + \ldots\,, \tag{3}$$

Major axis of doom:

$$\tilde{c}_* \simeq [(V_{21} + 16a^2)\tilde{\rho}_*^2 - V_{20}]/32a^4\,, \tag{4}$$

$$\rho_*^2 = \tilde{\rho}_*^2 + \frac{\hat{h}'_+(w_0) - \rho_D \hat{h}'_-(w_0)}{\hat{h}_+(w_0) - \rho_D \hat{h}_-(w_0)}\,, \tag{5}$$

$$c_* = \tilde{c}_* + 2\rho_*^2(\rho_*^2 - \tilde{\rho}_*^2) - \frac{\hat{h}''_+(w_0) - \rho_D \hat{h}''_-(w_0)}{\hat{h}_+(w_0) - \rho_D \hat{h}_-(w_0)}\,,$$

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