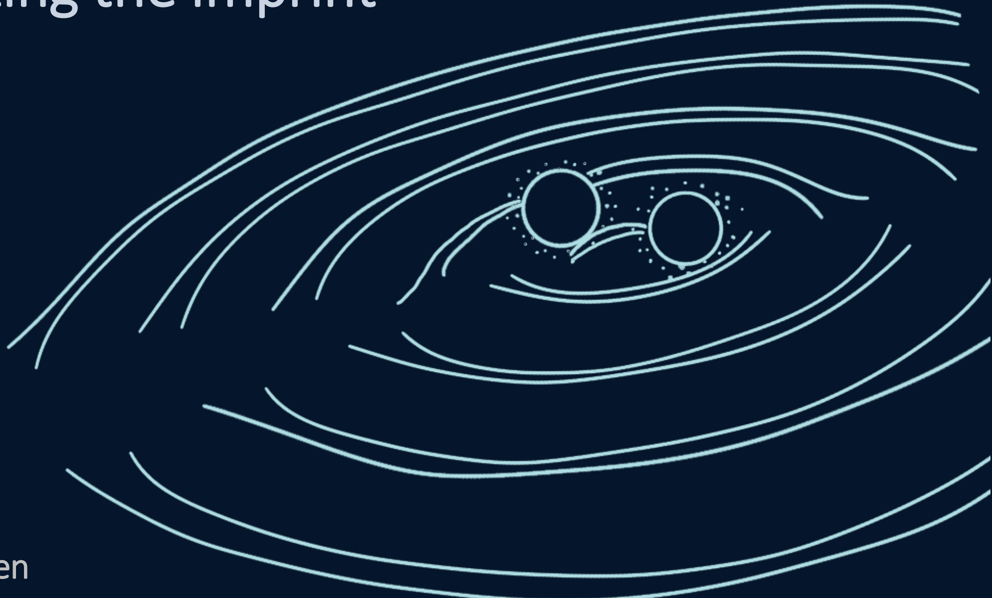
Utrecht University

Based on: arxiv 2302.08408

Black holes letting their hair down

Probing hairy black holes in sGB gravity with scalar tidal effects and analyzing the imprint on the GW signature.

EPS-HEP 2023, Hamburg



Iris van Gemeren B. Shiralilou, T. Hinderer, S. Vandoren

Testing (modified) gravity

- One of the main open questions in theoretical physics : nature of gravity
- Phenom POV: Testing modified gravity models with GWs
- Specifying to promising theory: **scalar Gauss Bonnet gravity**

$$S = \frac{c^3}{16\pi G}\int d^4x\,\sqrt{-g}\left[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \alpha f(\phi)R^2_{GB}\right] + S_m$$

$$R^2{}_{GB} = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

P. Pani and V. Cardoso (2009) arXiv:0902.1569

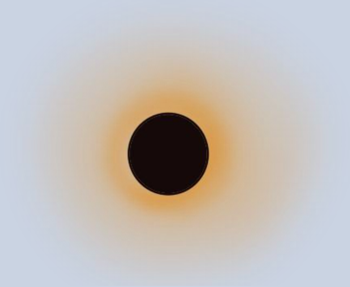
Scalarized black holes

$$\Box\phi = V'(\phi) - \alpha f'(\phi) R^2_{GB}$$

- sGB evades **no hair theorem** -> hairy BH solutions
- Spontaneous vs nonvanishing -> $f(\phi)$
- Properties of hair depend on parameters of the theory

    Massless vs massive

    Regularity at the horizon

1) 

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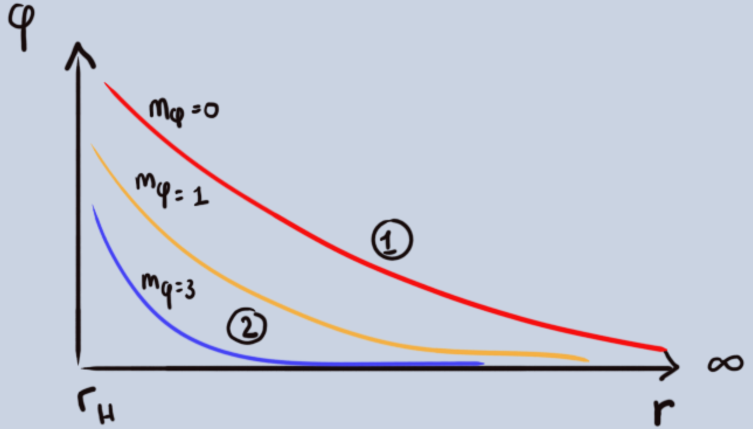
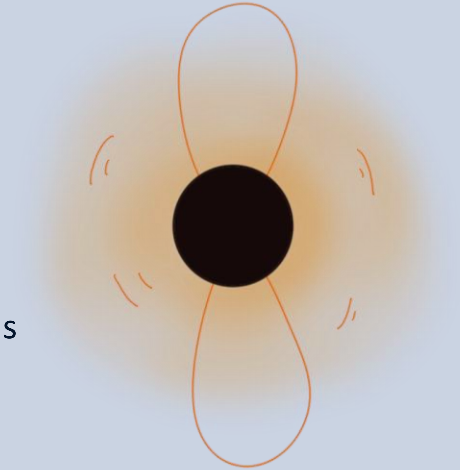


P. Kanti, e.a. (1996), arXiv:9511071
F.-L. Julie, E. Berti (2019) arXiv:1909.05258
D.D. Doneva, K.V.Staykov,S.S.Yazadjiev (2019) arXiv:1903.08119
IG, T.Hinderer, S.Vandoren (in prep)

Scalar tidal effects



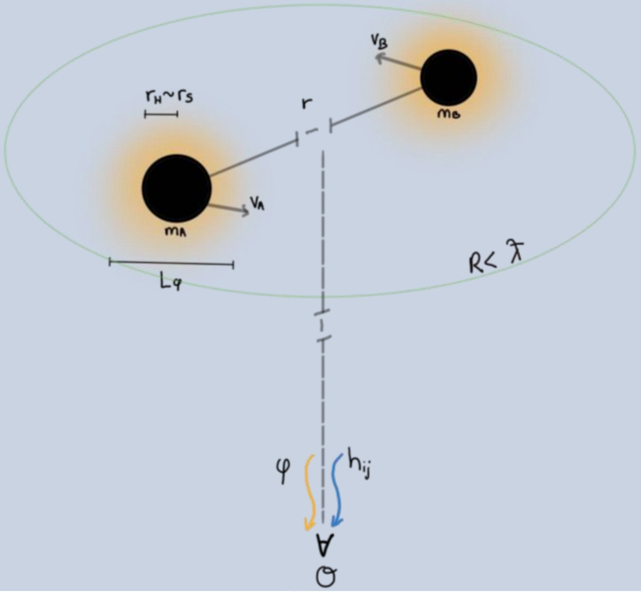
- In binary: leading order **interaction effect** of (massless) scalar clouds
- Tidal field $\boldsymbol{E_\mu}$ of the scalar field is given by **gradient**
- Tidal field deforms scalar field → **induced dipole moment** $\boldsymbol{Q_\mu}$
- Key parameter characterising tidal effects **:** $\boldsymbol{\lambda^s} = -\frac{\boldsymbol{Q_\mu}}{\boldsymbol{E_\mu}}$

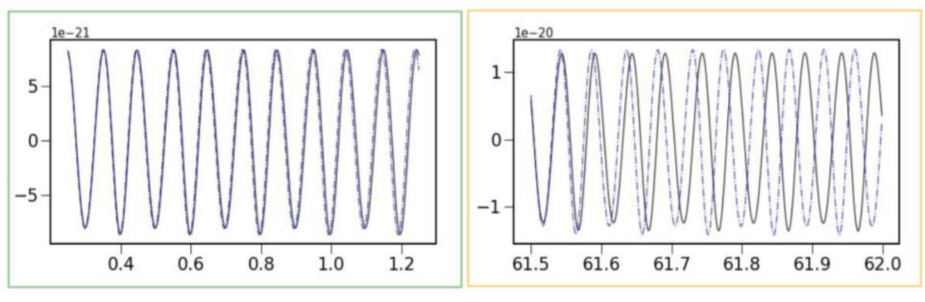
$$\lambda^s = \frac{7}{6} M\alpha f''(\phi_\infty)$$

L. Bernard (2020) arXiv:1906.10735
T. Hinderer (2008) arXiv:0711.2420
IG, B. Shiralilou, T. Hinderer (2023) arXiv:2302.08480

Dynamics & Inspiral modelling binary

- Change in energetics + additional radiative energy loss
- Exploiting **hierarcy of scales**; matching approximation schemes, **PN** near the sources (up to 1PN)**, multipolar PM** for asymptotic radiation
- Incorporating tidal terms as **finite size corrections** of **skeletonized point particle** description
- Analysis GW **phase evolution**→ main interest for data analysis





Credit: B. Shiralilou, T. Hinderer, S. Nissanke, N. Ortiz, H. Witek (2021) arXiv:2105.13972

F.-L. Julie, E. Berti (2019) arXiv:1909.05258
IG, B. Shiralilou, T. Hinderer (2023) arXiv:2302.08480
G.Creci, IG, T.Hinderer (in prep)

Contributions to the phase evolution

- Focus on phase evolution in fourier domain
- Tidal & GB contributions are supressed
- Degenerate in the frequency
- Tidal contributions from dynamics and induced dipole come with opposite sign

$$\psi \propto (\rho_{GB} + \rho_{tid}) f^{\#}$$

$$\rho_{tid} \propto \#\zeta_{dynamics} + \#\zeta_{induced\ dipole}$$

Phase analysis

- Main findings:

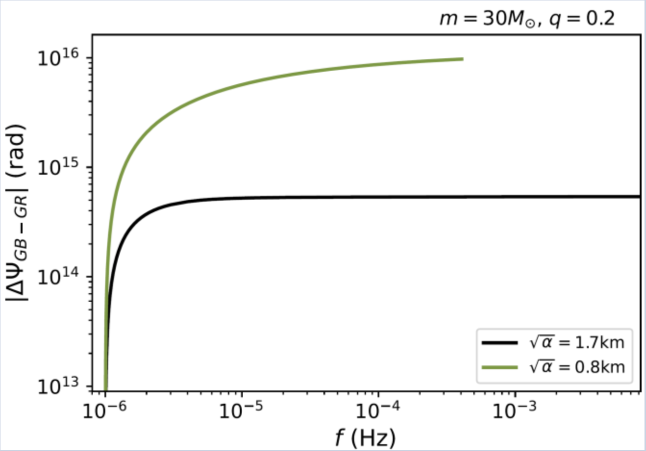
  - Early vs late inspiral difference in tidal & GB contributions, opposite sign

  - In Fourier domain early inspiral : **smaller** coupling, **larger** tidal contributions

early inspiral

$$\frac{\psi_{tid}}{\psi_{GB}} > 0$$

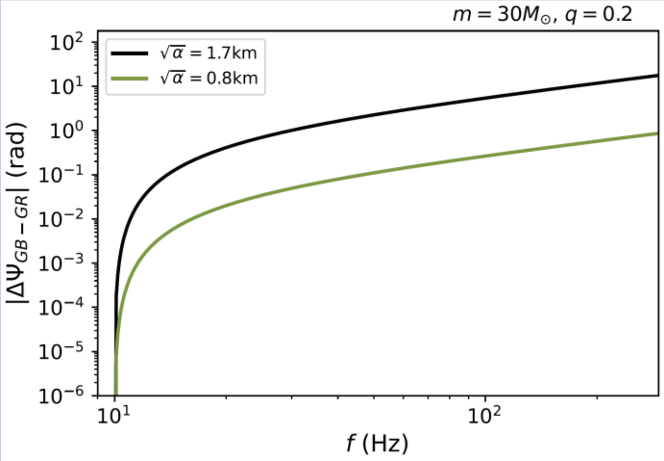
$$|\psi_{tid}| > |\psi_{GB}|$$



late inspiral

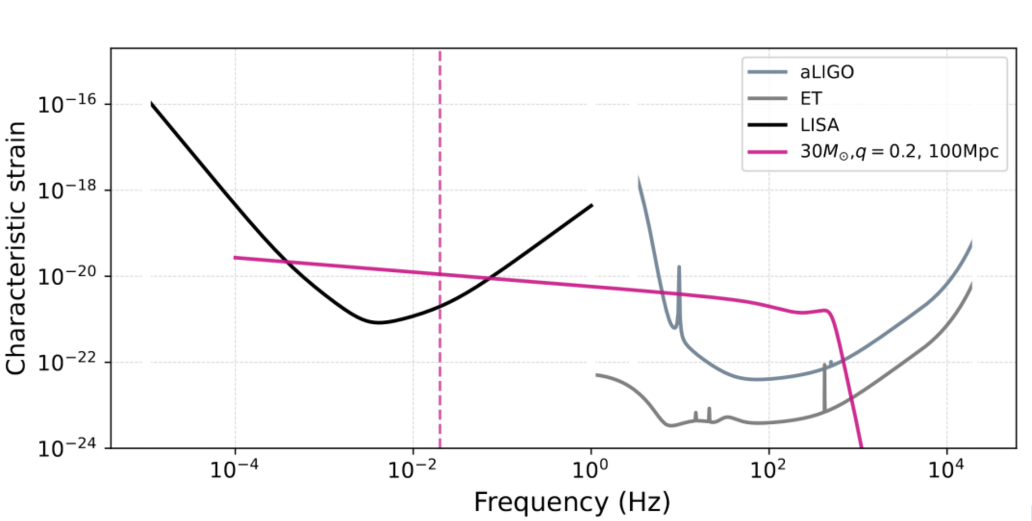
$$\frac{\psi_{tid}}{\psi_{GB}} < 0$$

$$|\psi_{tid}| < |\psi_{GB}|$$



Prospects for detection

- Opportunities for multiband detection
- LISA & ET/aLIGO
- Mitigating degeneracies

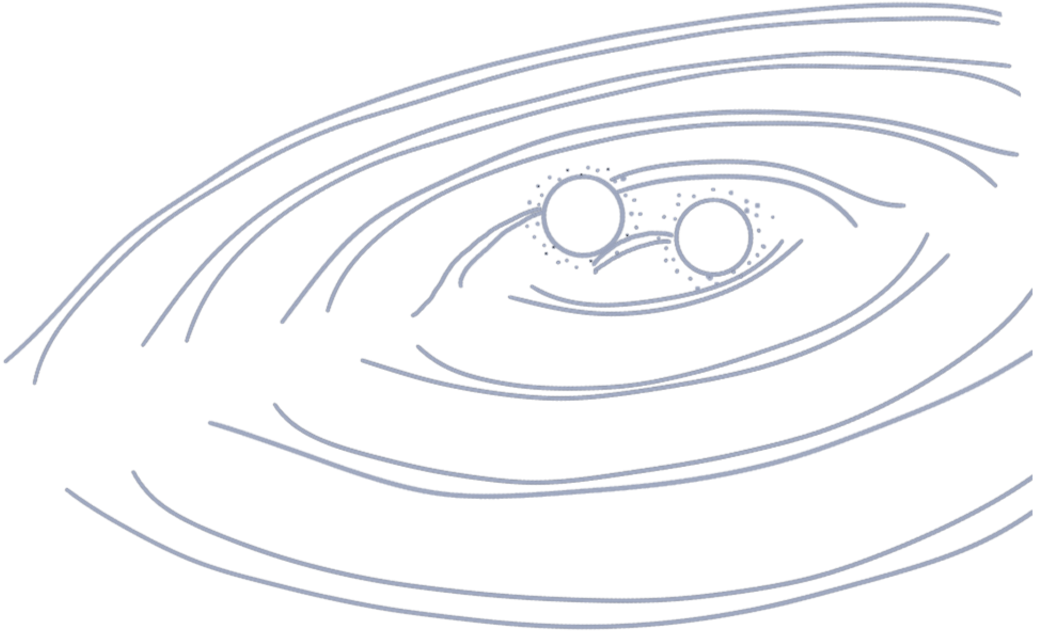


Take home message

- Interesting BH scalar configurations
- Effects of the additional higher curvature and scalar tidal corrections in GWs
- Different contributions and dependencies on the parameters early inspiral vs late inspiral
- Outlook:
  - extended analysis massive scalar field
  - Including mixed tidal effects
  - bias on parameter estimation
- Sparked your interest? Check out our paper: arxiv 2302.08408

Thank you!

Backup slides



Scalarized black holes

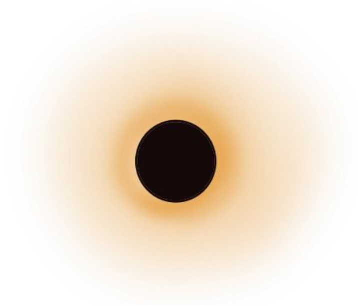
$$\Box\phi = V'(\phi) - \alpha f'(\phi) R^2_{GB}$$

1) $\phi\big|_{r\to\infty} \propto \phi_\infty + \frac{D}{r} + ..$ $D = \frac{\alpha}{2M} f''(\phi_\infty)$

$$f'(\varphi_h)^2 < \frac{r_h^4}{96\alpha^2}$$

2) $\phi\big|_{r\to\infty} \propto \alpha \frac{e^{-m\,r}}{\mathrm{r}} + \ldots$

$$|\varphi_h| < |\frac{12\hat{\alpha} f'(\varphi_0)}{\hat{m}_{dl}}| = |\frac{12\alpha f'(\varphi_0)}{\hat{m} r_H^4}|$$

1) 

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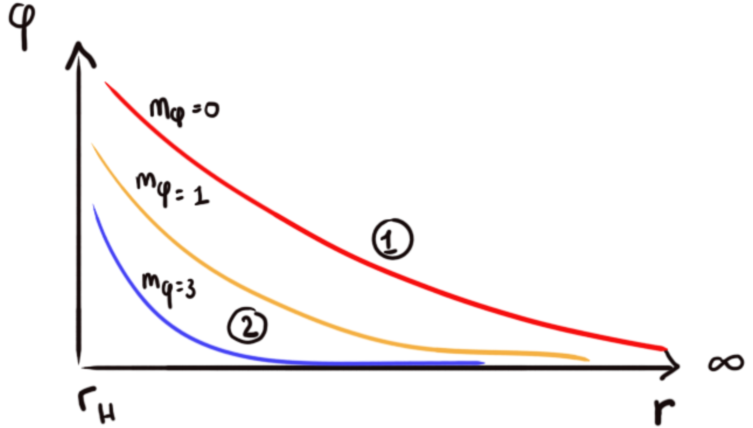


P. Kanti, e.a. (1996), arXiv:9511071
F.-L. Julie, E. Berti (2019) arXiv:1909.05258
D.D. Doneva, K.V.Staykov,S.S.Yazadjiev (2019) arXiv:1903.08119

Massive scalar field

$$S_{msGB} = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}(R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\hat{m}^2\varphi^2 + \alpha f(\varphi)R^2_{GB}),$$

with $\hat{m} = \frac{m_\varphi c}{\hbar}$.

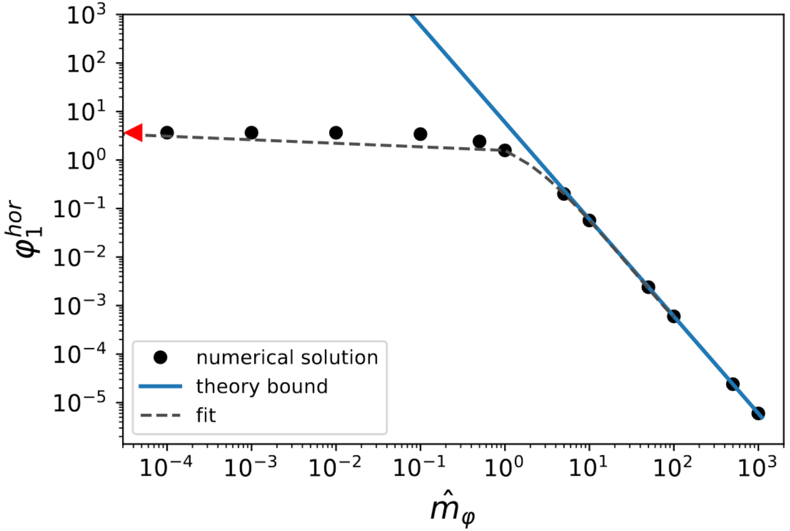
$$G_{\mu\nu} = T_{\mu\nu},$$

$$T_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi - \frac{1}{4}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi - \frac{1}{4}g_{\mu\nu}\hat{m}^2\varphi^2 - 4\alpha^* R^{*\alpha\mu\nu\beta}\nabla_{\alpha\beta}f(\varphi),$$

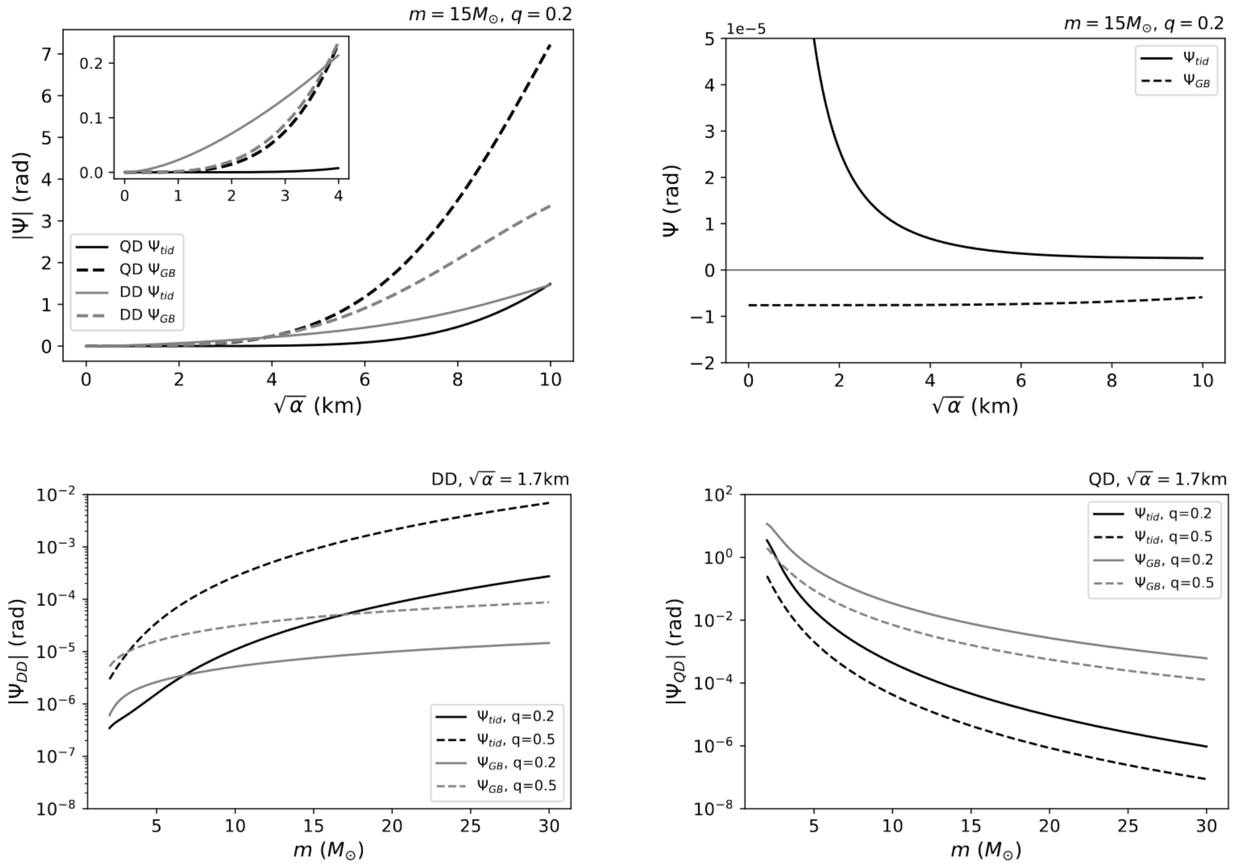
$$\Box\varphi = \hat{m}^2\varphi - \alpha f'(\varphi)R^2_{GB}.$$

$$\varphi = c_1\frac{e^{-\hat{m}_{dl}\frac{r}{rH}}}{r} + c_2\frac{e^{\hat{m}_{dl}\frac{r}{rH}}}{2\hat{m}_{dl}r}.$$

$$\varphi \sim \varphi_h + (\frac{\hat{m}^2_{dl}\varphi_h}{r_H} - \frac{12\hat{\alpha}f'(\varphi_0)}{r_H})(r - r_H) + \ldots,$$



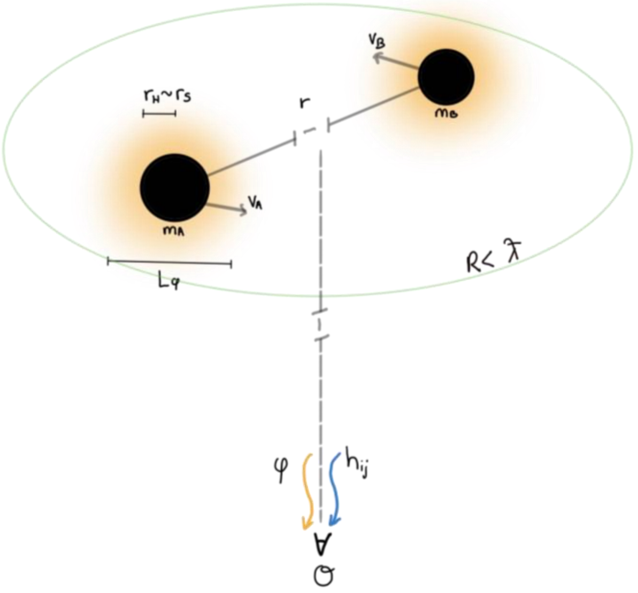
Coupling & mass dependencies tidal and GB contributions

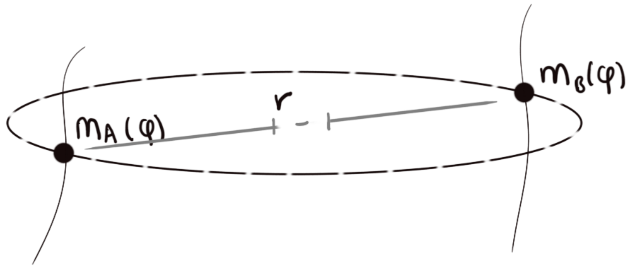


Skeletonization

$$S = S_{\text{gravity}} + S_{\text{pp}} + S_{\text{tid}}$$

$$S_{tid} = c \int ds_A \frac{\lambda_A}{2} \partial_\mu \varphi_B \partial^\mu \varphi_B$$





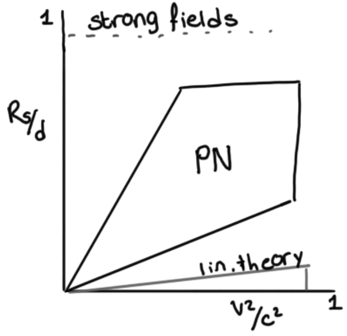
$$\bar{M}_A = m_A^0 \left\{ 1 + \frac{\alpha_A^0}{c^2} \delta\varphi^{(1)} + \frac{1}{c^4} \left(\alpha_A^0 \delta\varphi^{(2)} + \left[\left(\alpha_A^0\right)^2 + \beta_A^0 \right] \delta\varphi^{(1)} \right) \right\} + \mathcal{O}(c^{-6}) \,,$$

$$\alpha_A^0 = \left. \frac{d \, \ln[\bar{M}_A(\varphi)]}{d\varphi} \right|_{\varphi=\varphi_0} \qquad \beta_A^0 = \left. \frac{d \, \alpha_A(\varphi)}{d\varphi} \right|_{\varphi=\varphi_0} .$$

Dynamics, PN expansion & 2 body lagrangian

$$R_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{g_{\mu\nu}}{2}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi)$$
$$+\frac{8\pi G}{c^4}\left(T^{\rm pp}_{\mu\nu} - \frac{g_{\mu\nu}}{2}T^{\rm pp}\right) + \frac{8\pi G}{c^4}\left(T^{\rm tid}_{\mu\nu} - \frac{g_{\mu\nu}}{2}T^{\rm tid}\right),$$

$$\Box\varphi = -\frac{1}{4}\alpha f'(\varphi)R^2_{GB} - \frac{4\pi G}{c^4}(\bar{\delta}S_{\rm pp} + \bar{\delta}S_{\rm tid}),$$



1 PN expanded metric

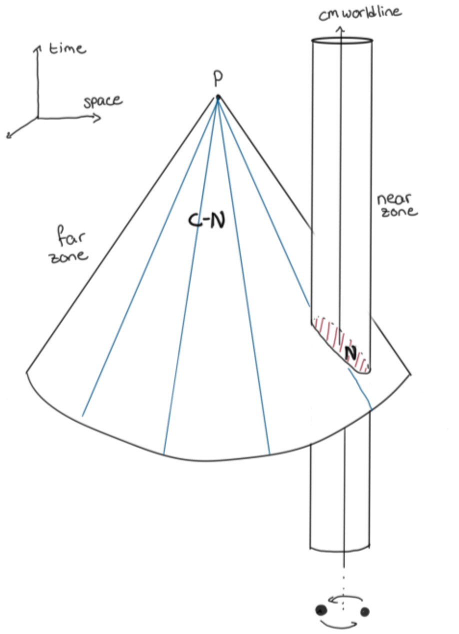
$$g_{00} = e^{-2U/c^2} + \mathcal{O}\left(c^{-6}\right),$$
$$g_{0i} = -4g_i/c^3 + \mathcal{O}\left(c^{-5}\right),$$
$$g_{ij} = \delta_{ij}e^{2U/c^2} + \mathcal{O}\left(c^{-4}\right),$$
$$\varphi = \varphi_0 + c^{-2}\delta\varphi^{(1)} + \mathcal{O}\left(c^{-4}\right),$$

$$L_{AB} = -m^0_A c^2 + \frac{1}{2}m^0_A\mathbf{v}^2_A + \frac{G\bar{\alpha}\, m^0_A m^0_B}{2r} + \frac{1}{8c^2}m^0_A\mathbf{v}^4_A$$
$$+\frac{G\bar{\alpha}\, m^0_A m^0_B}{rc^2}\Bigg[-\frac{G\bar{\alpha}m^0_A}{2r}\left(1+2\bar{\beta}_B\right) + \frac{3}{2}\left(\mathbf{v}^2_A\right)$$
$$-\frac{7}{4}\left(\mathbf{v}_A\cdot\mathbf{v}_B\right) - \frac{1}{4}\left(\mathbf{n}\cdot\mathbf{v}_A\right)\left(\mathbf{n}\cdot\mathbf{v}_B\right) + \frac{\bar{\gamma}}{2}\left(\mathbf{v}_A - \mathbf{v}_B\right)^2\Bigg]$$
$$+\frac{\alpha f'\left(\varphi_0\right)}{r^2}\frac{G^2 m^0_A m^0_B}{r^2c^2}\left[m^0_A\left(\alpha^0_B + 2\alpha^0_A\right)\right]$$
$$-\frac{1}{4}\frac{G^2\bar{\alpha}^2\mu m}{r^4c^2}\zeta + (A\leftrightarrow B),$$

$$\zeta \equiv -\lambda^{(s)}_A\frac{m^0_B\alpha^{0\,2}_B}{\bar{\alpha}^2 m^0_A} - \lambda^{(s)}_B\frac{m^0_A\alpha^{0\,2}_A}{\bar{\alpha}^2 m^0_B}$$

Blanchet and T. Damour, “Post-newtonian generation of gravitational waves,”Annales de l’I.H.P. Physique th´eorique, vol. 50, no. 4, pp. 377–408, 1989.

Calculation waveforms



Gothic metric & Harmonic gauge

$$\mathfrak{g}^{ab} = \sqrt{-g}\, g^{ab}$$

$$\partial_\nu \mathfrak{g}^{\mu\nu} = 0$$

Multipole expansion

$$\phi(x) = -\frac{G}{c^4}\sum_{l=0}^{\infty} \frac{(-1)^l}{l!}\,\partial_L\left(\frac{1}{R} I_s^L(\tau)\right)$$

$$I_s^L(\tau) = \int_{\mathcal{M}} d^3x'\, \mu_s(\tau, x')\, x'^L$$

$$\Box\, h^{\alpha\beta} = \frac{16\pi G}{c^4}\mu^{\alpha\beta}$$

$$\mu^{\alpha\beta} = (-g)T_m^{\alpha\beta} + \frac{c^4}{16\pi G}\left(\Lambda_{GB}^{\alpha\beta} + \Lambda_{GR}^{\alpha\beta}\right)$$

$$\longrightarrow \quad h_{\alpha\beta} = -\frac{4G}{c^4}\int d^4x' \frac{\mu^{\alpha\beta}(t', x')\,\delta(t' - t + |x - x'|/c)}{|x - x'|}$$

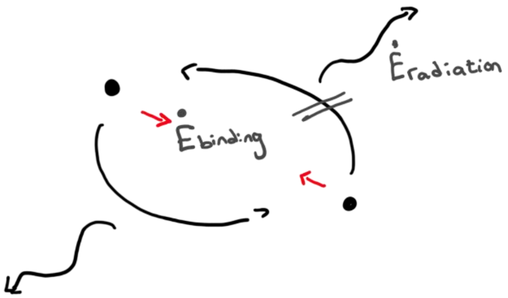
$$\Box\, \phi = \frac{4\pi G}{c^4}\mu_s$$

$$\mu_s = -\frac{\delta S_{pp}}{\delta\phi\sqrt{-g}} - \frac{c^4}{16\pi G}\alpha f'(\phi)\widehat{\mathcal{R}_{GB}^2}$$

$$\longrightarrow \quad \phi = -\frac{G}{c^4}\int \frac{\mu_s\left(t', \mathrm{x}'\right)\delta\left(t' - t + \left|\mathrm{x} - \mathrm{x}'\right|/c\right)}{\left|\mathrm{x} - \mathrm{x}'\right|} d^4x'$$

M. E. Pati and C. M. Will, “Post-Newtonian gravitational radiation and equations of motion via direct integration of the relaxed Einstein equations: Foundations,” ,vol. 62, p. 124015, Dec. 2000
K. S. Thorne, “Multipole expansions of gravitational radiation,” Rev. Mod. Phys., vol. 52, pp. 299–339, Apr 1980

From waveform to phase



$$\frac{dE(\omega)}{dt} = -F(\omega)$$

$$\dot{\phi} = \omega$$

$$\frac{d\omega}{dt} + \frac{F(\omega)}{E'(\omega)} = 0$$

$$\frac{d\phi}{dt} - \omega = 0$$

$$E(x) = -\frac{\mu c^2 x}{2}\left[1 + x\left(-\frac{3}{4} - \frac{\eta}{12} + E_{\rm s}\right) + x^3 c^4 \left(E_{\rm GB} + E_{\rm tid}\right)\right],$$

$$E_{\rm GB} = -\frac{10}{3}\frac{\alpha f'(\varphi_0)}{G^2\bar{\alpha}^{7/2}m^2}\left(3S_+ + \frac{\Delta m}{m}S_-\right),$$

$$E_{\rm tid} = \frac{5}{3}\frac{\zeta}{G^2 m^3 \bar{\alpha}^2}.$$

$$\mathcal{F}_S(x) = x^4 c^5\left[S4 + x(S5 + S5_{\rm GB}x^2c^4 + S5_{\rm tid}x^2c^4)\right],$$

$$S5_{\rm GB} = \left(\frac{4\alpha f'(\varphi_0)\eta^2 S_-}{3\bar{\alpha}^{7/2}G^3m^2}\right)\left[\frac{8S_-}{3\bar{\alpha}}(3S_+ + \frac{\Delta m}{m}S_-) - 2S_+(S_+ + \frac{\Delta m}{m}S_-)\right],$$

$$S5_{\rm tid} = \left(\frac{4\eta S_-}{3G^3\bar{\alpha}m^3}\right)\left(\frac{\bar{\zeta}}{G\bar{\alpha}^{3/2}m} - \frac{4\eta S_-\zeta}{3}\right)$$

$$\mathcal{F}_T(x) = x^5 c^5\left[T5 + x(T6 + T6_{\rm GB}x^2c^4 + T6_{\rm tid}x^2c^4)\right]$$

$$T6_{GB} = \left(\frac{128\alpha f'(\varphi_0)\eta^2}{5\bar{\alpha}^{9/2}G^3m^2}\right)\left[\frac{\Delta m}{m}S_-\left(\frac{25}{14} + \frac{4}{3\bar{\alpha}}\right) + S_+\left(\frac{25}{14} + \frac{4}{\bar{\alpha}} + \frac{3}{7}\eta\right)\right],$$

$$T6_{\rm tid} = -\frac{256\eta^2\zeta}{15G^3\bar{\alpha}^4m^3}.$$

Construction fs action

- Scalar multipole moments Q and external fields E
- Up to second order derivatives in the field
- Time derivatives supressed over spatial ones (PN approximation)
- Adiabatic limit
- Field redefinitions

$$\mathcal{L}_{\rm FS} = c_1 Q_I^{(s)} \mathcal{E}^I_{(s)} + c_2 \mathcal{E}_I^{(s)} \mathcal{E}^I_{(s)} + \mathcal{L}^{\rm int}(Q_I^{(s)}, \dot{Q}_I^{(s)}) + \dots \qquad \mathcal{L}^{\rm int} = -\frac{1}{2\lambda_s} Q^I_{(s)} Q_I^{(s)} + c_4 \dot{Q}^I_{(s)} \dot{Q}_I^{(s)},$$

$$Q_I^{(s)} = -\lambda_s \mathcal{E}_I^{(s)},$$

$$S_{\rm FS} = c \int ds \left[\frac{\lambda_s}{2} \mathcal{E}_I^{(s)} \mathcal{E}^I_{(s)} + c_2 \mathcal{E}_I^{(s)} \mathcal{E}^I_{(s)} + \dots \right].$$

On EdGB in string theory

- Low energy limit in heterotic string thoery
- Scalar field corresponds to dilaton
- Lagrangian:

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}(\partial_\mu \phi)^2 + \frac{\alpha'}{8g^2} e^{\phi} R^2_{\mathrm{GB}},$$

- Where alpha relates to the regge slope in the string tension

$$T = \frac{1}{2\,\pi\,\alpha'}$$

On R^2 making GR renormalizable

Introducing quantum fluctuations h_μν, arbitrary background:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu};$$

Counterterm one loop

$$\delta\mathcal{L} = \frac{\sqrt{|g|}}{8\pi^2(d-4)}\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right)$$

K.S. Stelle, “Renormalization of higher derivative quantum gravity” ,Phys. Rev. D 16, 953 – Published 15 August 1977
A. Y. Petrov, “Introduction to Modified Gravity,” SpringerBriefs in Physics, Springer,5 2020.