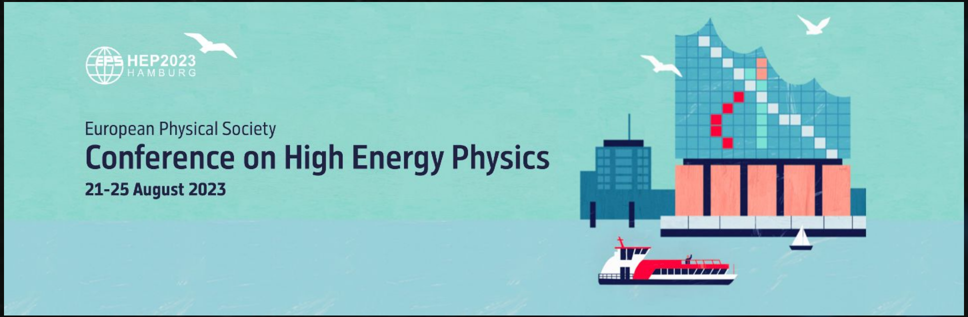


Search for gravitational wave signals from known pulsars in O3 data using the 5n-vector ensemble method

Luca D’Onofrio

INFN
Istituto Nazionale di Fisica Nucleare

SAPIENZA
UNIVERSITÀ DI ROMA

Continuous gravitational waves (CWs)

SOURCES : Isolated spinning neutron stars with non-axisymmetric mass distribution (and not only [1])

- CWs are "long-lived" signals.
- CW frequency is linked to the source rotation frequency
- CW amplitude is expected much weaker than that generated by binary BH/NS coalescences

8 parameters for CW signal :

$f_{rot} \quad \dot{f}_{rot} \quad \alpha \quad \delta$ → **EXTRINSIC**

$h_0 \quad \phi \quad \eta \quad \psi$ → **INTRINSIC**

Different strategies considering source assumptions:

- **Targeted search**;
- Narrow-band search;
- Directed search;
- All-sky search;

[1] Piccinni, Galaxies 2022, 10(3), 72

CW Signal

Source as triaxial “bumpy” neutron star rotating around a principal axis of inertia :

$$\boxed{f_{gw} = 2f_{rot}}$$

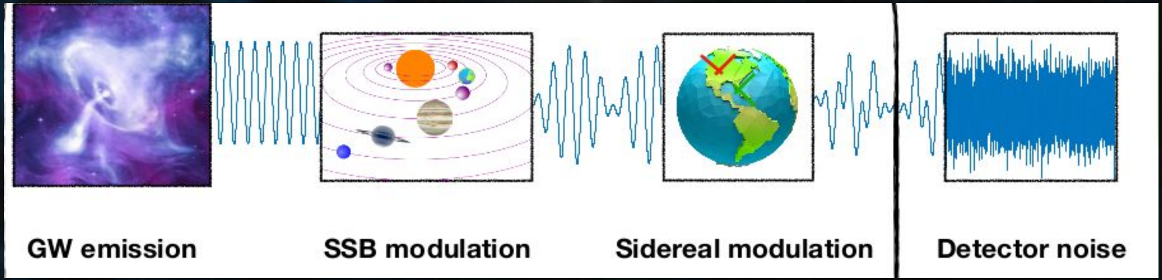
$$h_0 \simeq 10^{-27} \left[\frac{f_{gw}}{100\,\mathrm{Hz}}\right]^2 \left[\frac{10\,\mathrm{kpc}}{d}\right] \left[\frac{I}{10^{38}\,\mathrm{kg}\cdot\mathrm{m}^2}\right] \left[\frac{\epsilon}{10^{-6}}\right]$$

with

$$\epsilon = \frac{|I_x - I_y|}{I_z} \approx \frac{\Delta R}{R}$$

$$h_0^{SD} = \frac{1}{d}\left(\frac{5}{2}\frac{GI_z}{c^3}\frac{\dot{f}_{rot}}{f_{rot}}\right)^{1/2}$$

→ **Spin-down limit**: theoretical upper limit

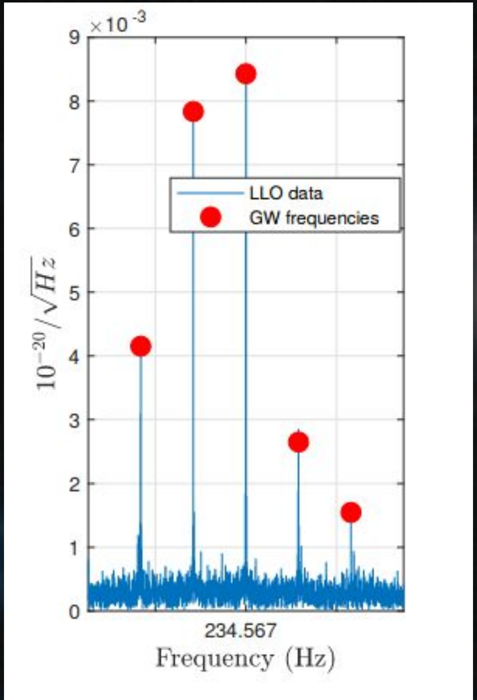


- Doppler correction
- Spin-down correction
- Glitches (Ask me!)

The 5-vector method

P Astone et al 2012 J. Phys.: Conf. Ser.363 012038

$h \sim 1.6\times10^{-25}$ SNR(1yr) ~ 68



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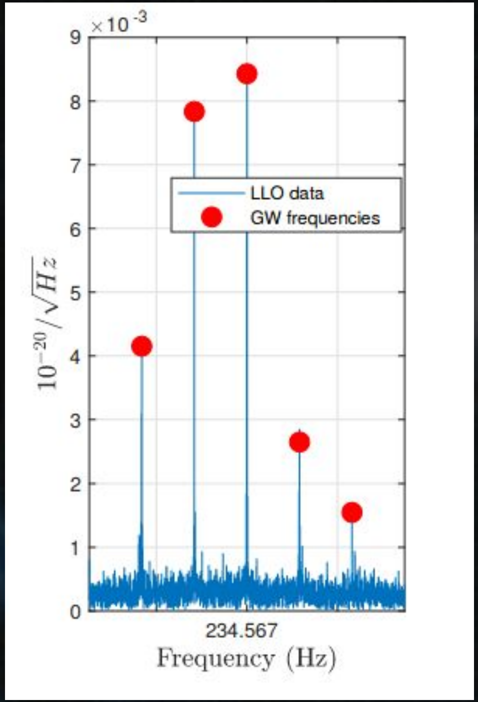


$$x(t) = h(t) + n(t)$$

$$h(t) = H_0(H_+ A^+ + H_\times A^\times)e^{\jmath\omega_0 t+\gamma_0}$$

h(t) can be rewritten in terms of the 5-vectors: **X A⁺ Aˣ**

$$\mathbf{X} = \int_T x(t) e^{-i(\omega_0 t - \mathbf{k}\Omega t)} dt \qquad \mathbf{k} = 0, \pm1, \pm2$$

$$\hat{H}_{+/x} = \frac{\mathbf{X}\cdot\mathbf{A}^{+/x}}{|\mathbf{A}^{+/x}|^2} \rightarrow H_0 e^{i\gamma} H_{+/x}$$

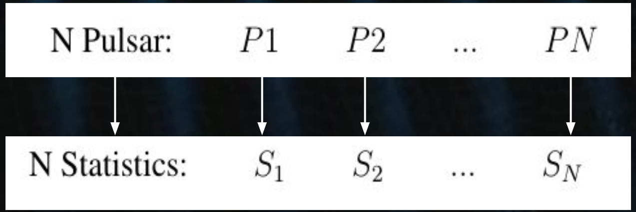
$$S = |\mathbf{A}^+|^4|\hat{H}_+|^2 + |\mathbf{A}^\times|^4|\hat{H}_\times|^2$$

CWs targeted search

NO evidence of CWs signal in the LIGO/Virgo data

O3 LVK Targeted search: *Abbott et al 2022 ApJ 935 1*

- 236 known pulsars
- Three detectors (LIGO and Virgo) : O3 data
- Bayesian analysis
    - F-stat and 5-vector on “high-value” pulsars
- NO CW detection →upper limits
    - on the amplitude/ellipticity



Ensemble search

How to improve the detection probability?

Combining sources to weak signals detection!

Statistically

- Sum of F-stats
  Chen et al 2016 Phys.Rev.D94
- Hierarchical Bayesian method
  Pitkin et al 2018 Phys.Rev.D98
- 5n-vector ensemble method
  D'Onofrio et al 2021 CQG 38 13502

Stochastic method

- Giazotto et al. 1997 Phys.Rev.D 55
- Stochastic Targeted search
  De Lillo et al 2022 MNRS 513
  Deepali et al Phys. Rev. D 106, 043019

In this presentation, results on O3 data

The ensemble statistic T(k)

- Simplest way to define an ensemble statistic : take the sum of the statistics S_i
  - with ~200 pulsars, how many signals can be near the detection thr?
- To optimize the det. prob. we need to estimate the signals “strength”
  - Rank pulsars for increasing p-values (≡ decreasing S_i)

$$\overline{S}_{(1)} < \overline{S}_{(2)} < \ldots < \overline{S}_{(N)}$$

- Construct the ensemble statistic T(k) as the partial sum:

$$\boxed{\overline{T}(k) = \sum_{i=N-k+1}^{N} \overline{S}_{(i)}} \longrightarrow$$

Partial sum of order statistics

P-value of ensemble

To reconstruct T(k) noise distribution:

1) ***Gaussian noise case*** →Monte Carlo procedure →Sensitivity test
    a) Improvement in det. prob. depends on the power of single tests and on N

P-value of ensemble

To reconstruct T(k) noise distribution:

1) ***Gaussian noise case*** →Monte Carlo procedure →Sensitivity test

    a) Improvement in det. prob. depends on the power of single tests and on N

2) ***Real case*** →S distribution from off-source frequencies in a band (tenth of Hz) near the GW frequency

    a) BSD framework (*Piccinni et al 2019 CQG. 36 015008)*

    b) Generalize the Monte Carlo procedure starting from the experimental S distribution for each pulsar

The 5n-vector ensemble method

Fix pulsars to analyze

J0835-4510 → $\bar{S}_1$

J0534+2200 → $\bar{S}_2$

J1813-1749 → $\bar{S}_3$

J2229+6114 → $\bar{S}_4$

Measure detection statistics S

The 5n-vector ensemble method

Fix pulsars to analyze

J0835-4510 | J0534+2200 | J1813-1749 | J2229+6114

Measure detection statistics S

$\bar{S}_1$ | $\bar{S}_2$ | $\bar{S}_3$ | $\bar{S}_4$

Order for decreasing values

$\bar{S}_{(1)}$ | $\bar{S}_{(2)}$ | $\bar{S}_{(3)}$ | $\bar{S}_{(4)}$

Measure ensemble statistic T(k)

$$\overline{T}(k) = \sum_{i=N-k+1}^{N} \overline{S}_{(i)}$$

$$\bar{T}(2) = \bar{S}_{(4)} + \bar{S}_{(3)}$$

$$\bar{T}(1) = \bar{S}_{(4)}$$

P-value of ensemble

Application to O3 data [1]

- 223 pulsars used in O3 targeted search
  - considering also pulsar in **binary systems (168 out of 223)**
- O3 data for LIGO and Virgo detectors
  - **“weighted” 5n-vector**

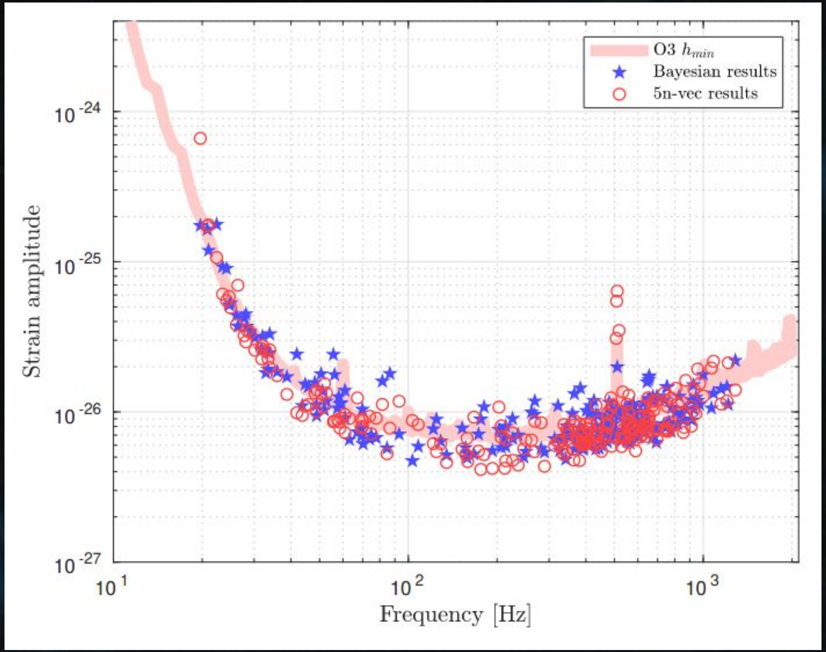
1. Single pulsar analysis
   a. *single harmonic search*
2. Ensemble analysis —> 2 ensembles :
   a. all pulsars, all detectors (N = 223)
   b. millisecond pulsars, LIGO det. (N = 165)
3. Upper limit

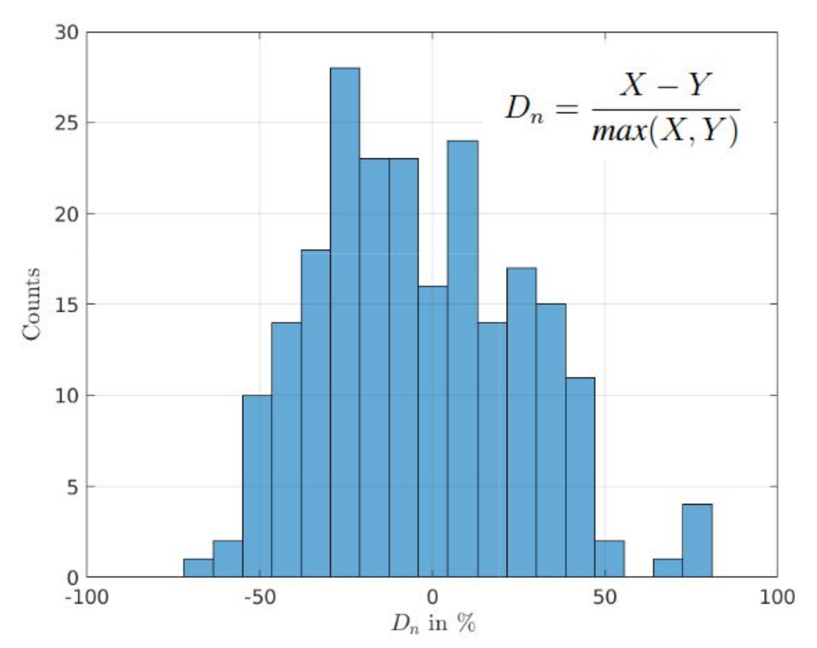
[1] “Search for gravitational wave signals from known pulsars in LIGO-Virgo O3 data using the 5n-vector ensemble method”, submitted to PRD

Single pulsar analysis

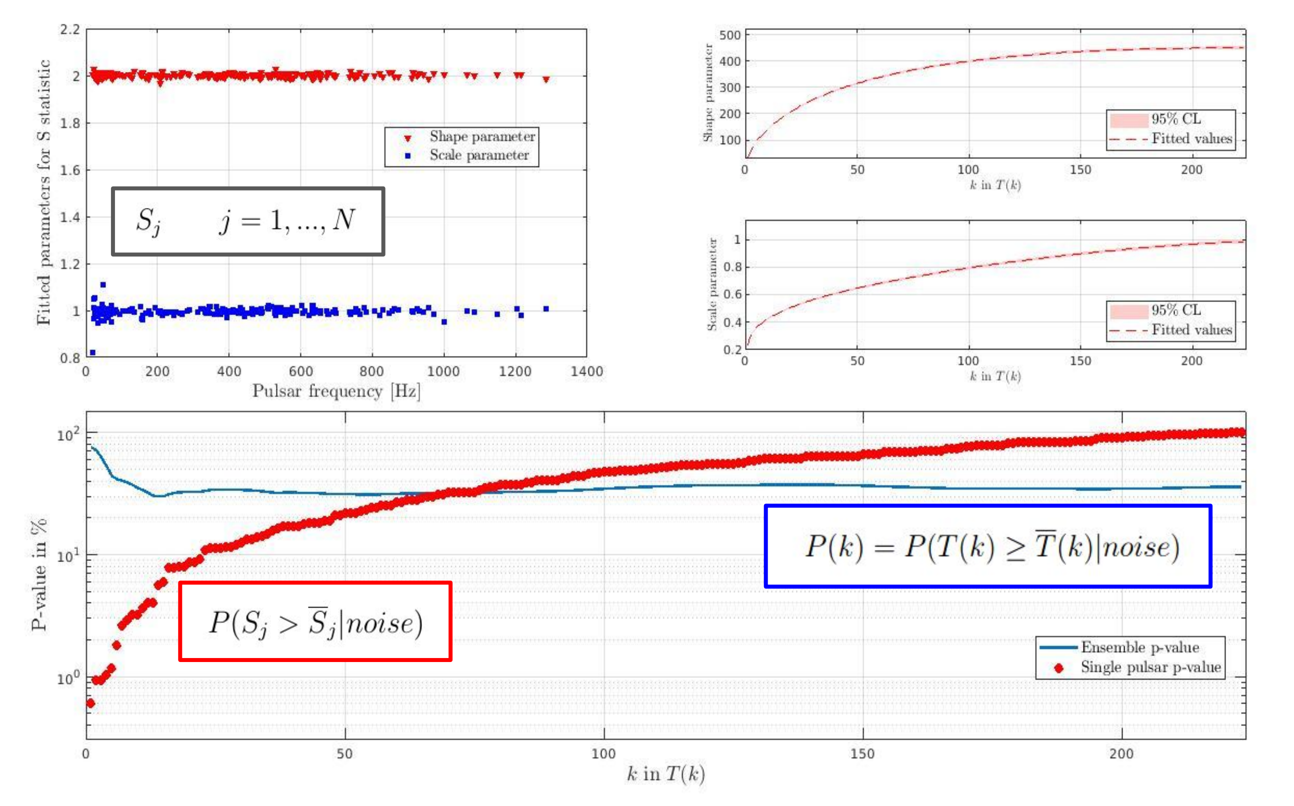
223 pulsars used in O3 targeted search (*R. Abbott et al. 2021*)

First results on binary systems for the 5n-vector method!





LIGO-Virgo O3 data, 223 pulsars



Ensemble upper limits: going hierarchical

Hypothesis: we will assume a common exponential distribution for the ellipticities

Constraint on the mean ellipticity using two independent procedures :

- $P(\mu_\epsilon|\overline{T}(N)) \propto \left(\int L(\overline{T}(N)|\Lambda)\Pi(\Lambda|\mu_\epsilon)d\Lambda \right) \Pi(\mu_\epsilon)$
- $P(\mu_\epsilon|\overline{S}_1, .., \overline{S}_N) \propto \left(\prod_{i=1}^{N} \int L(\overline{S}_i|\epsilon_i)\Pi(\epsilon_i|\mu_\epsilon)d\epsilon_i \right) \Pi(\mu_\epsilon)$

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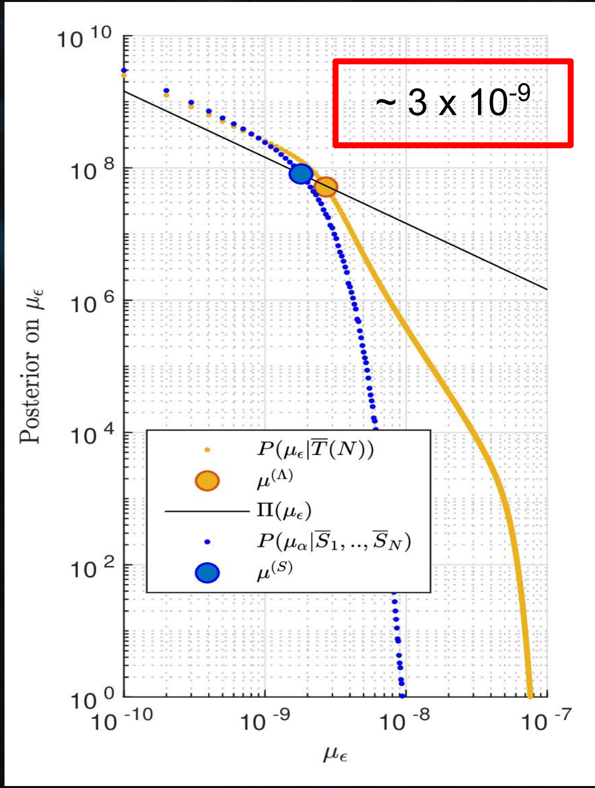
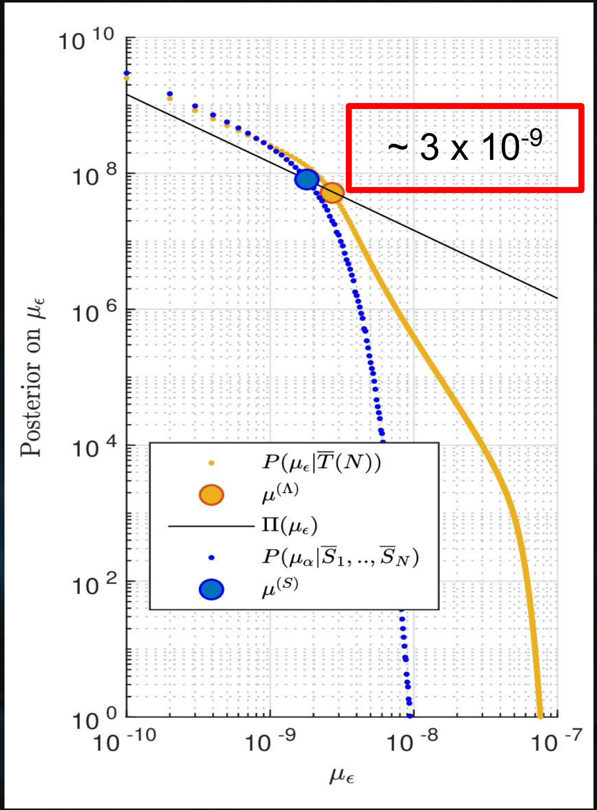
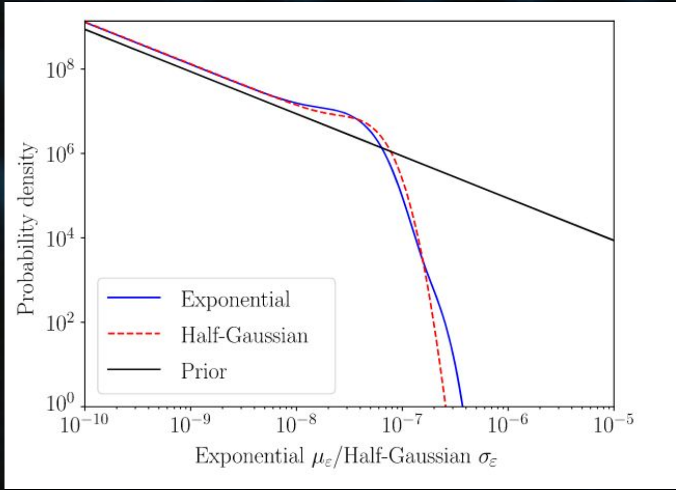
$$P(\mu_\epsilon|\overline{S}_1,..,\overline{S}_N) \propto \left(\prod_{i=1}^{N}\int L(\overline{S}_i|\epsilon_i)\Pi(\epsilon_i|\mu_\epsilon)d\epsilon_i\right)\Pi(\mu_\epsilon)$$



LIGO-Virgo O3 data, 223 pulsars



Results in *Pitkin et al 2018 Phys.Rev.D 98*



92 pulsars the LIGO S6 science run
~ 3.8 × 10^{-8}

Summary

- Pulsars are promising targets for the first CW detection
- Ensemble procedures improve the detection probability for the targeted search
  - 5n-vector ensemble method
- Application to O3 data considering 223 known pulsars
  - First application to binary systems for the 5n-vectors
  - No evidence of CW signals from the ensembles
  - Upper limits on the mean ellipticity of $\sim 3 \times 10^{-9}$

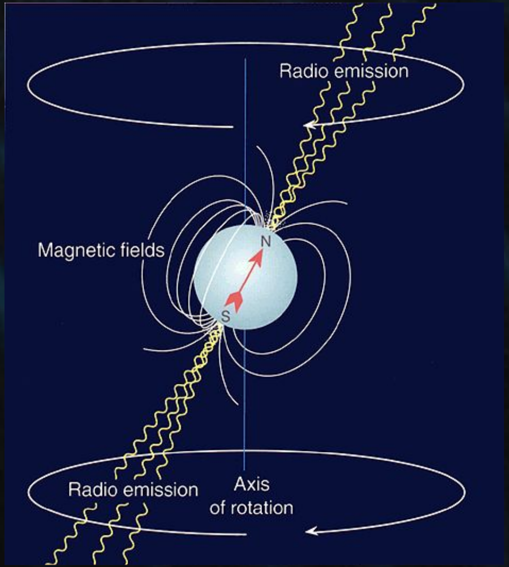
    Theoretical minimum limit for millisecond pulsars of $\approx 10^{-9}$ supposed in *Astrophysical Journal L. 863, L40 (2018)*

This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation.


Thank you!

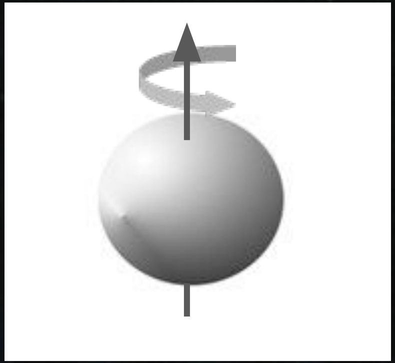
Targeted search
Multi-messenger approach

- CW searches have a strong multi-messenger approach
- EM information constraints extrinsic parameters
- Pulsar observed in radio, X-ray, Gamma-ray band
  - ~ 3000 known* pulsars ($10^{8\div9}$ expected NSs)
- Targeted search for known pulsars:
  - full coherent analysis
  - LVK: 3 pipelines (Bayesian, F-stat, 5n-vec method)
- CW detection can return information about the physics of neutron stars (EOS, superfluidity, superconductivity, solid core..) depending on the emission scenarios



**http://www.atnf.csiro.au/research/pulsar/psrcat/*

CWs emission

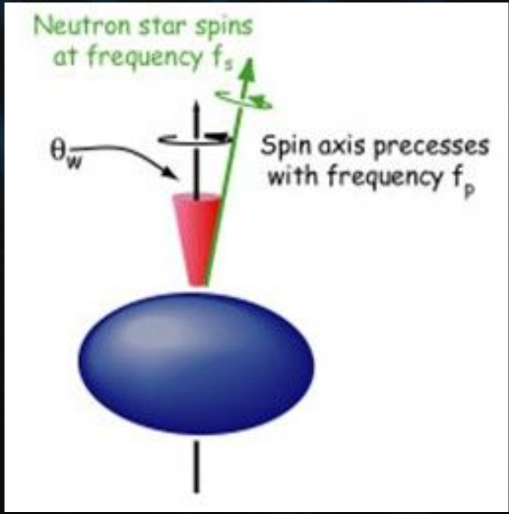


- “bumpy” neutron star [1]

$$\boxed{f_{gw} = 2f_{rot}}$$

- “wobble” radiation [1]
- superfluid component [2]

$$\boxed{f_{gw} = f_{rot} \text{ and } 2f_{rot}}$$



- R-modes [3]

$$\boxed{f_{gw} \approx \frac{3}{4} f_{rot}}$$

[1] Jones, arXiv:2111.08561 (2021)
[2] Jones, Monthly Notices of the Royal Astronomical Society, 402 4 (2010)
[3] Idrisy et al, Phys. Rev. D 91, 024001 (2015)

Tools

- **5-vector method**, matched filter in frequency domain

$$x(t) = h(t) + n(t)$$

$$h(t) = H_0(H_+A^+ + H_\times A^\times)e^{\jmath\omega_0 t+\gamma_0}$$

$$H_+ = \frac{\cos 2\psi - \jmath\eta\sin 2\psi}{\sqrt{1+\eta^2}} \qquad H_\times = \frac{\sin 2\psi + \jmath\eta\cos 2\psi}{\sqrt{1+\eta^2}}$$

$$A_+ = a_0 + a_{1c}\cos\Omega t + a_{1s}\sin\Omega t + a_{2c}\cos 2\Omega t + a_{2s}\sin 2\Omega t$$

$$A_\times = b_{1c}\cos\Omega t + b_{1s}\sin\Omega t + b_{2c}\cos 2\Omega t + b_{2s}\sin 2\Omega t$$

It can be rewritten in terms of Signal 5-VECs $\mathbf{A}^+ \quad \mathbf{A}^\times$

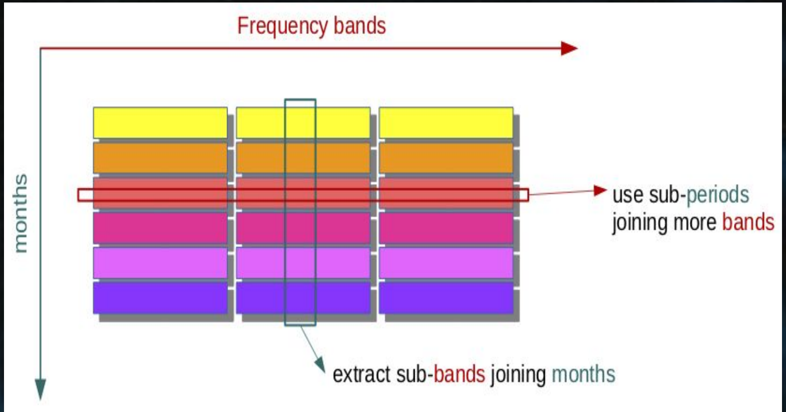
$$\hat{H}_{+/x} = \frac{\mathbf{X}\cdot\mathbf{A}^{+/x}}{|\mathbf{A}^{+/x}|^2} \rightarrow H_0 e^{i\gamma}H_{+/x}$$

- **5n-vector method,** extension to a network of n detectors

$$\mathbf{X} = [\mathbf{X}_L, \mathbf{X}_H] \qquad \mathbf{A}^+ = [\mathbf{A}^+_L, \mathbf{A}^+_H] \qquad \mathbf{A}^\times = [\mathbf{A}^\times_L, \mathbf{A}^\times_H]$$

$$S = |\mathbf{A}^+|^4|\hat{H}_+|^2 + |\mathbf{A}^\times|^4|\hat{H}_\times|^2 \longrightarrow \text{5n-vec definition}$$

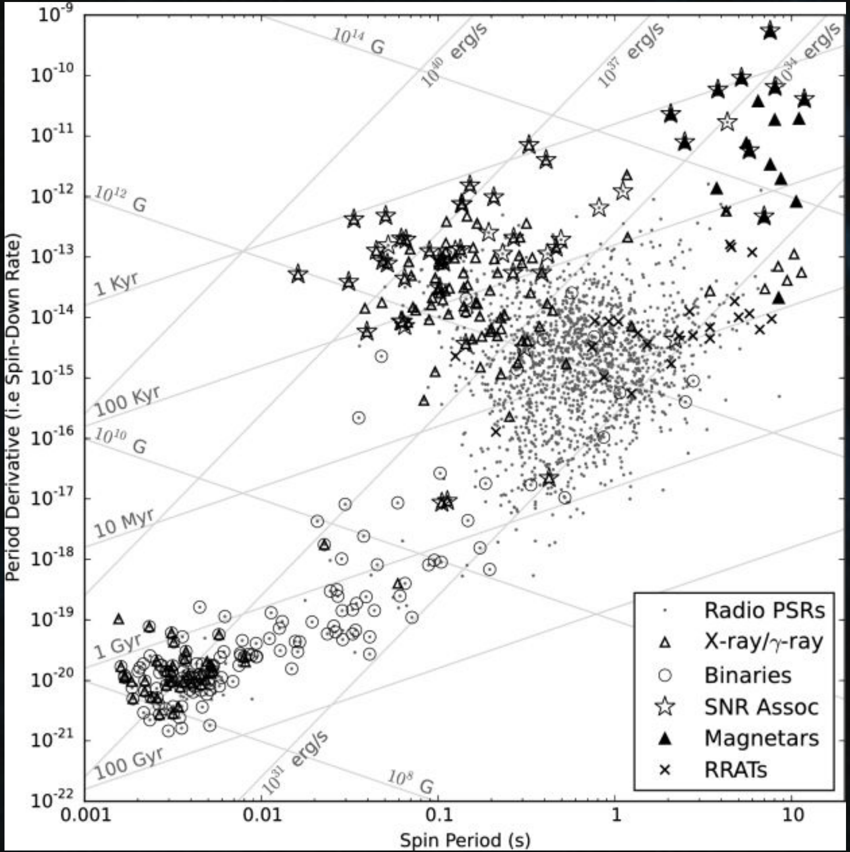
Band Sample Data (BSD)*



- Database of sub-databases that reduces computational cost (narrow frequency band required)
- “BSD-file” is a complex time series that covers 10 Hz/1 month of original data
- We can extract sub-bands joining months (for targeted search, 1 Hz frequency band)

**Piccinni et al 2019 Class. Quantum Grav. 36 015008*

P-Pdot diagram



Credit to:
Condon and Ransom, "Essential Radio Astronomy" (2016)

"High accuracy"

Sky position

$$\Delta\theta < 0.1\,\mathrm{arcsec}\left(\frac{10^7\,\mathrm{s}}{T}\right)^2\left(\frac{1\,\mathrm{kHz}}{f_0}\right)$$

Spin-down frequency

$$\frac{1}{1\,\mathrm{yr}} \approx 10^{-7}\,\mathrm{Hz}$$

$$\dot{f}\cdot 1\,\mathrm{yr} < \frac{1}{1\,\mathrm{yr}} \quad \text{or } \dot{f} < 10^{-15}\,\mathrm{Hz/s}$$

See *Maggiore, "Gravitational waves : part I"* for more details

Targeted Search : O3 results

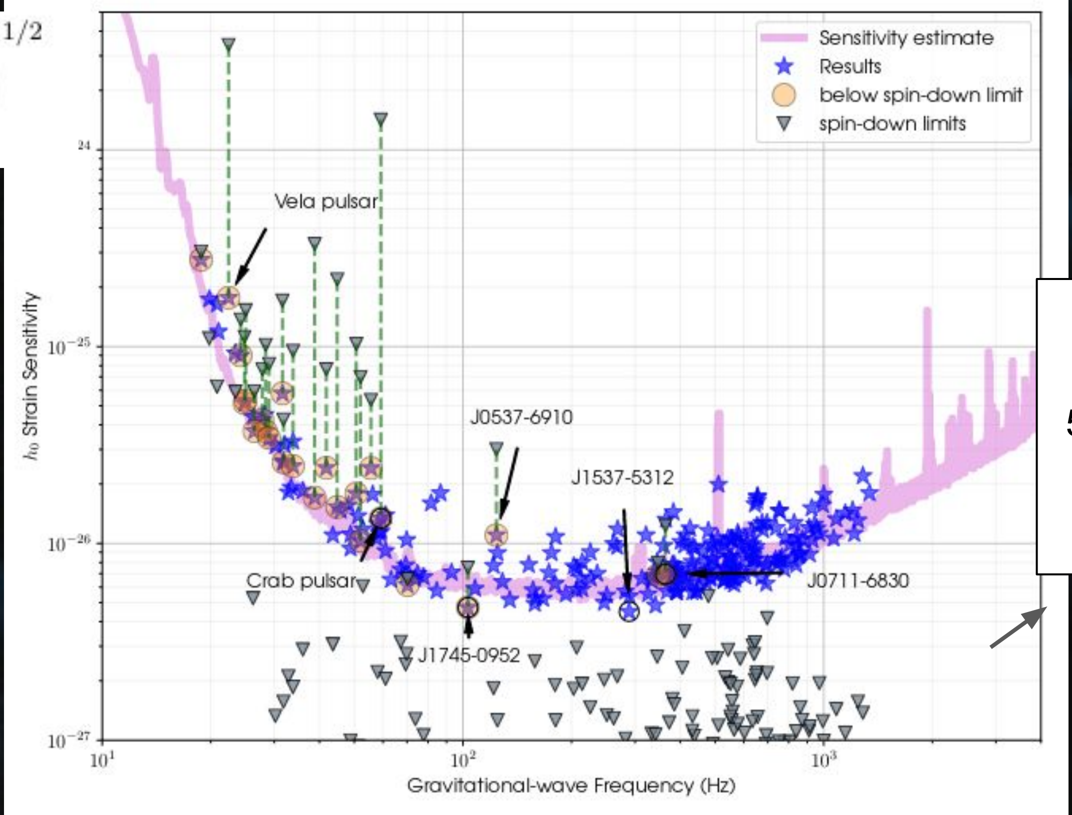
LVK+, ApJ 935 1 (2022)

- 236 known pulsars
  - 74 not in previous searches
  - 161 millisecond pulsars
- Three detectors (LIGO and Virgo) : O3 data combined with O2 data
- Single-harmonic search $f_{gw} = 2f_{rot}$ and Dual-harmonic search $f_{gw} = f_{rot} \text{ and } 2f_{rot}$
- Bayesian analysis
  - F-statistic and 5-vector analysis on high value pulsars (~20 out of 236)
- NO CW detection →upper limits
  - on the amplitude
  - on the ellipticity

O3 results : Bayesian method

$$h_0^{SD} = \frac{1}{d}\left(\frac{5}{2}\frac{GI_z}{c^3}\frac{\dot{f}_{rot}}{f_{rot}}\right)^{1/2}$$

$$h_{min} \approx 11\sqrt{\frac{S_h(f)}{T_{obs}}}$$



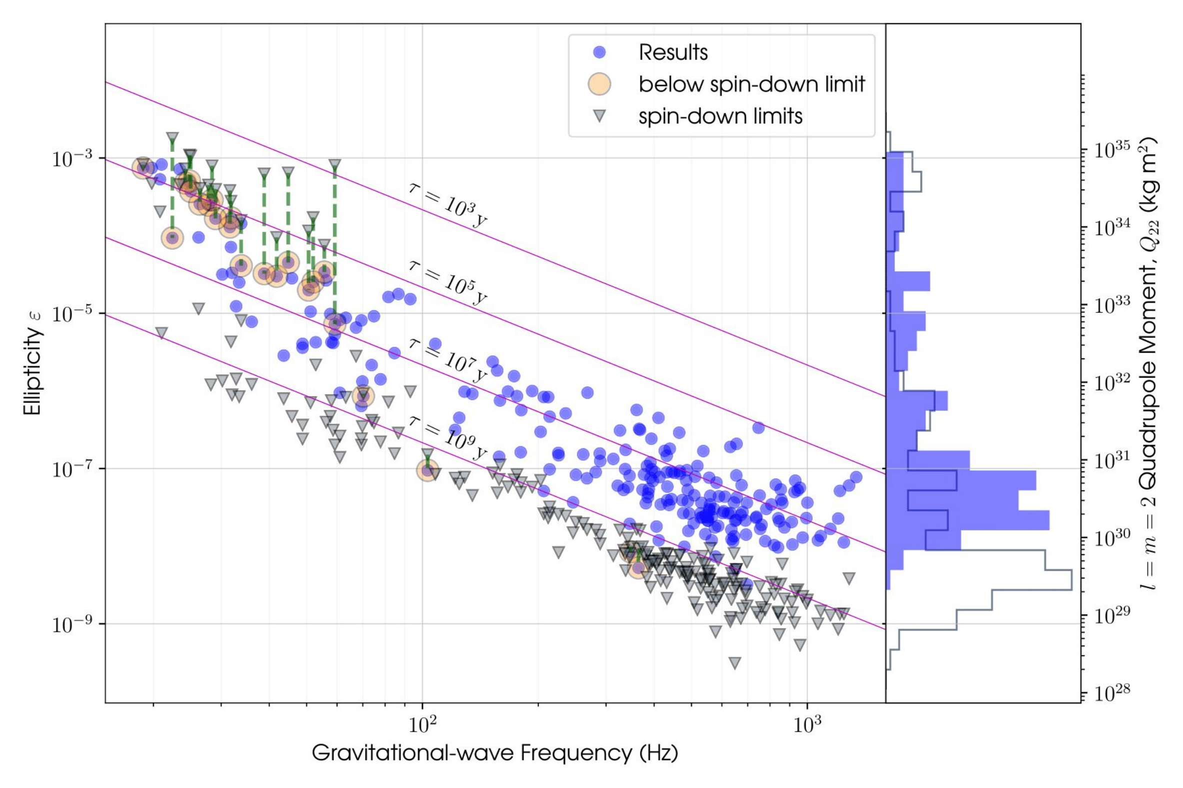
Best limit on ellipticity:
J0711−6830 with
5.26×10^{-9} (at a distance
of 0.1 kpc)
“mountain” < 30 μm

O3 results : ellipticity

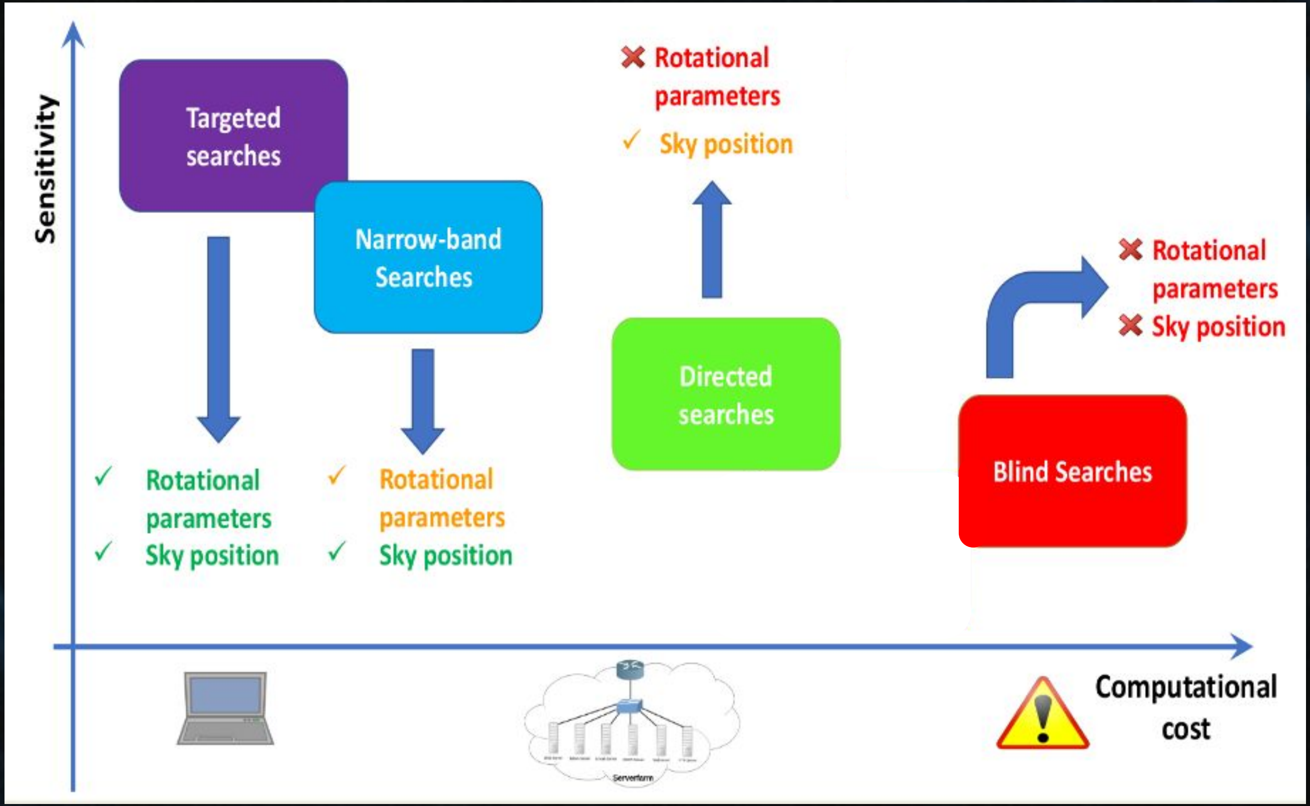
- Best limit on ellipticity was J0711−6830 with 5.26×10^{-9} (at a distance of 0.1 kpc)

- Best overall ellipticity was J0636+5129 with 3.2×10^{-9}

[1] LVK+, ApJ 935 1 (2022)

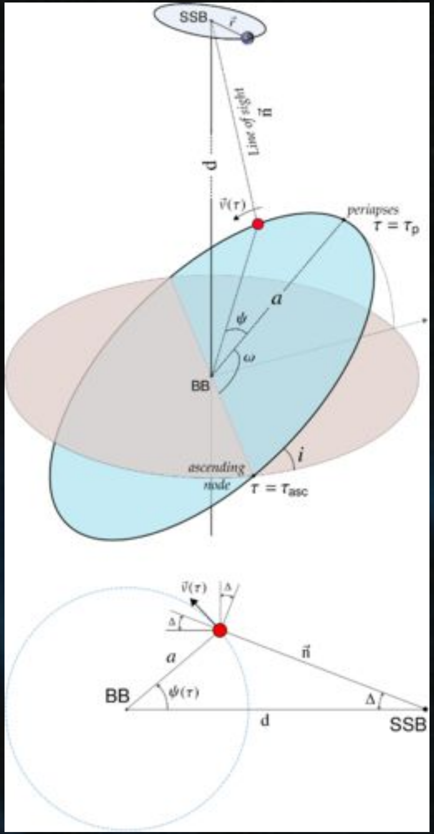


CW searches



Credit to:
[C. Palomba]

Heterodyne correction



Heterodyne method : data are multiplied by a complex exponential function that removes the phase modulation. The starting data is complex, already filtered and sub-sampled before applying the heterodyne: **BSD files** (*Piccinni et al 2019 Class. Quantum Grav. 36 015008*).

$$x(t) = [h(t) + n(t)]e^{-i\Phi_{corr}(t)}$$

$$\Phi_{corr}(t) = \Phi_{sd}(t) + \Phi_d(t) + \Phi_{bin}$$

$$\Phi_d = 2\pi \int_{\tau_{ref}}^{t} f_0(t')\frac{\vec{r}\cdot\hat{n}}{c}dt' \approx \frac{2\pi}{c}p_{\hat{n}}(t)f(t)$$

$$\Phi_{sd} = 2\pi \int_{\tau_{ref}}^{t} \left[\dot{f}(t' - \tau_{ref}) + \frac{1}{2}\ddot{f}_0(t' - \tau_{ref})^2 + \dots\right] dt'$$

$$\Phi_{bin} = a_p \left[\sin\omega\,(\cos E - e) + \cos\omega \sin(E\sqrt{1-e^2})\right]$$

Credit to: Phys. Rev. D 100 (2 2019)

Binary parameters uncertainties

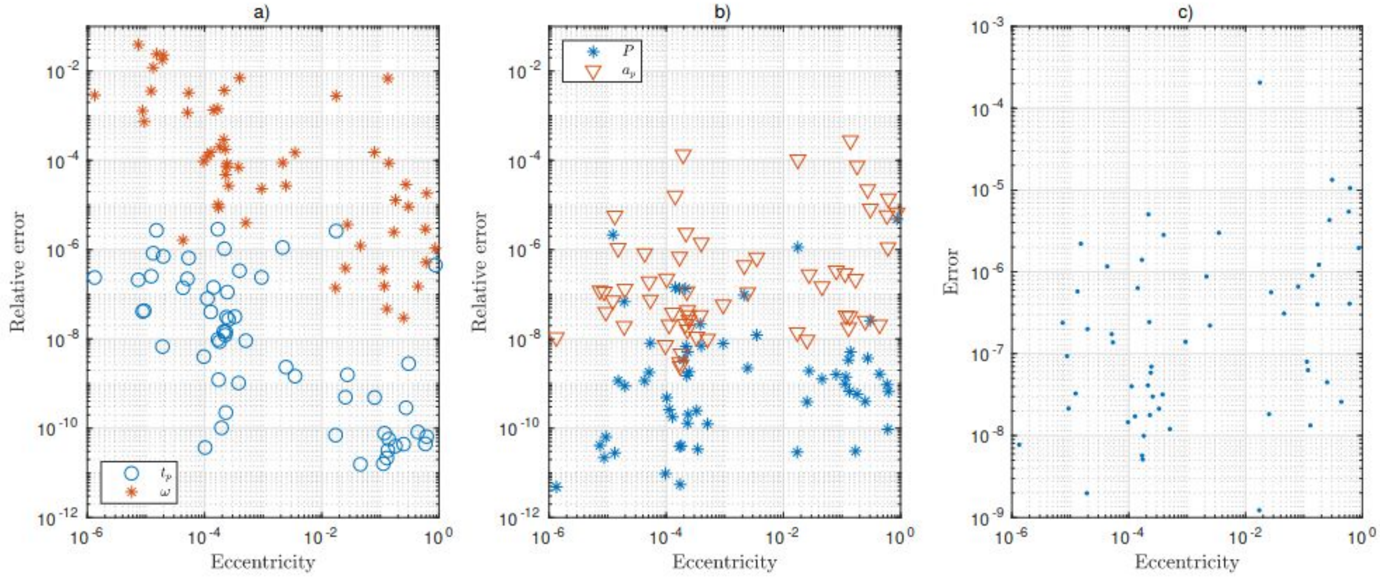


Figure 1: Uncertainties for the orbital parameters provided by the T2 model. *a)* Relative errors on t_p and ω as a function of the eccentricity. *b)* Relative errors on P and p as a function of the eccentricity. *a)* Absolute errors on the eccentricity as a function of the eccentricity values.

Multiple testing

Single testing

S_i → Test statistics

$\overline{S}_i$ → Experimental results

Null hypothesis

$$H_i \; : \; \theta_i = 0$$

Alternative hypothesis

$$K_i \; : \; \theta_i > 0$$

Multiple testing

N experiments

Null hypothesis

$$H = H_1 \cap ... \cap H_N \; : \; \theta_1 = ... = \theta_N = 0$$

Alternative hypothesis

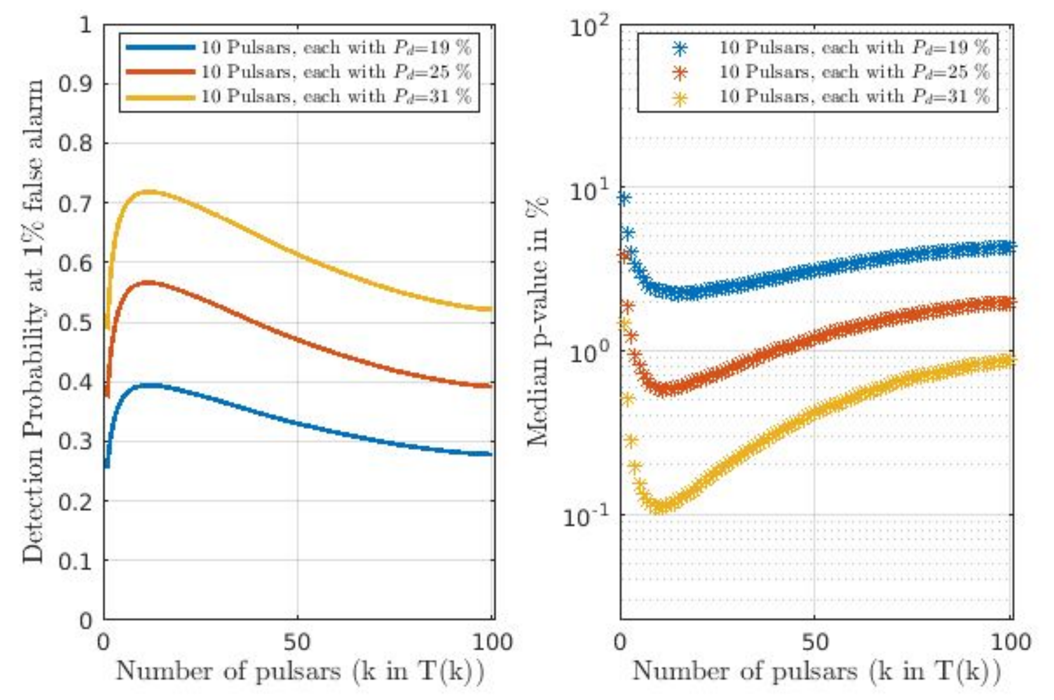
$$K = K_1 \cup ... \cup K_N \; : \; \text{at least one } \theta_i > 0$$

Example : Fisher Test

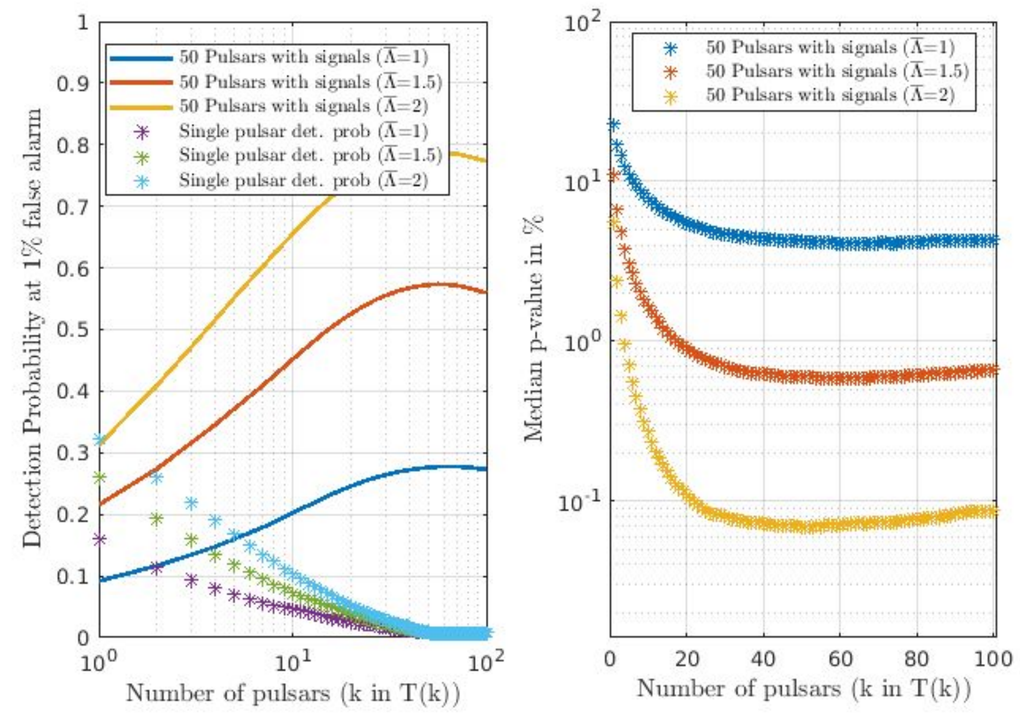
combining p-values to define a new test statistics *F*

$$F = -2 \sum_{i=1}^{N} \log P_i \sim \chi^2(x; 2N)$$

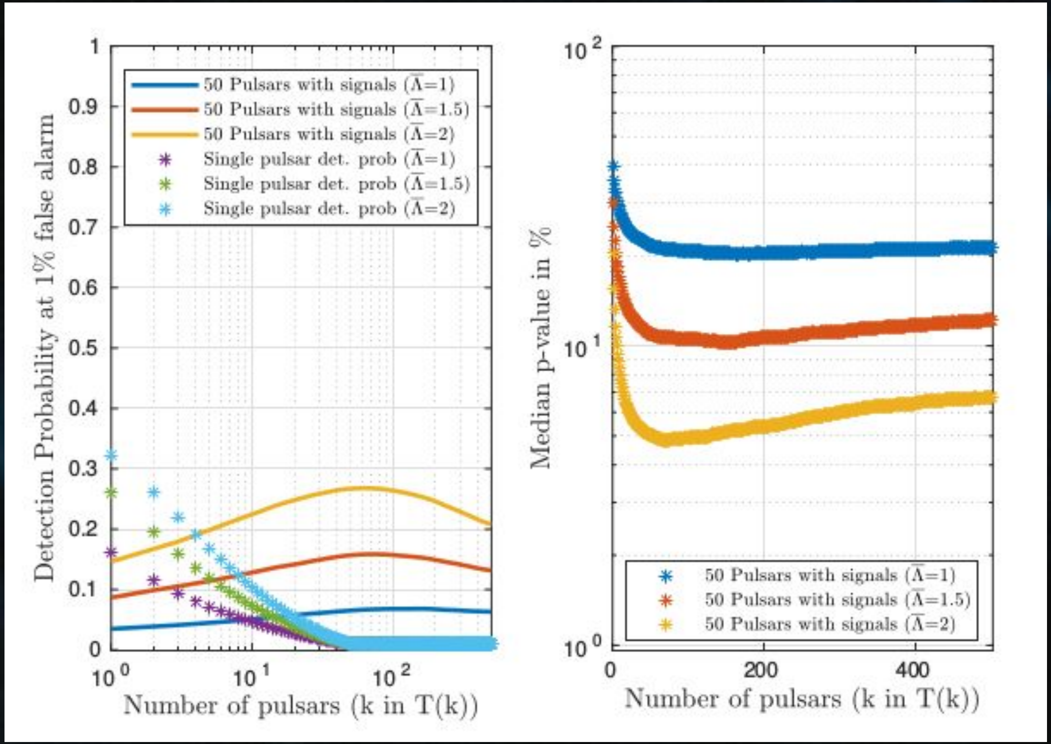
Theoretical test



Theoretical test



Dependence on N



Test with Hardware Injections

HI_3 for LLO O3 data

smallest largest

Fix days to analyze

15/08 –> 16/08 | 16/08 –> 17/08 | 17/08 –> 18/08 | 18/08 –> 19/08

($H/h_{min} \sim 0.3$)

Compute detection statistics

$\bar{S}_1$ | $\bar{S}_2$ | $\bar{S}_3$ | $\bar{S}_4$

Order for decreasing values

$\bar{S}_{(1)}$ | $\bar{S}_{(2)}$ | $\bar{S}_{(3)}$ | $\bar{S}_{(4)}$

Measured ensemble statistic

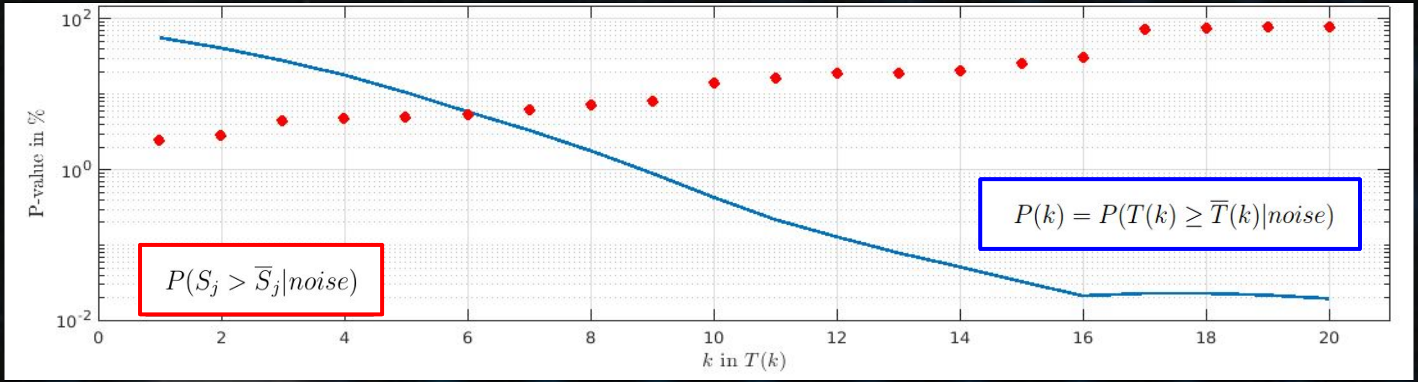
$$\bar{T}(k) = \sum_{i=N-k+1}^{N} \bar{S}_{(i)}$$

$\bar{T}(2) = \bar{S}_{(4)} + \bar{S}_{(3)}$

$\bar{T}(1) = \bar{S}_{(4)}$

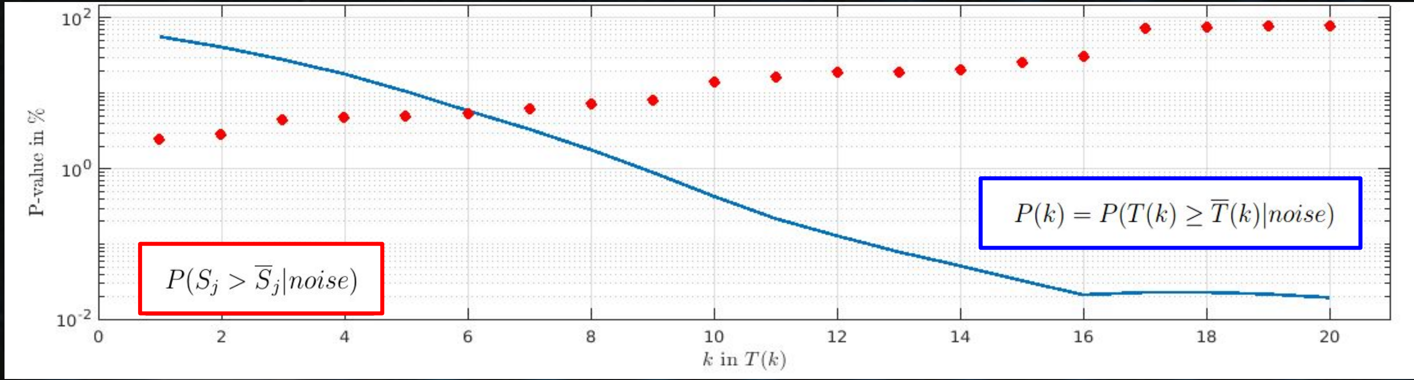
Test with Hardware Injections

LLO, 1 day-long datasets from 15/08/2019, HI_3

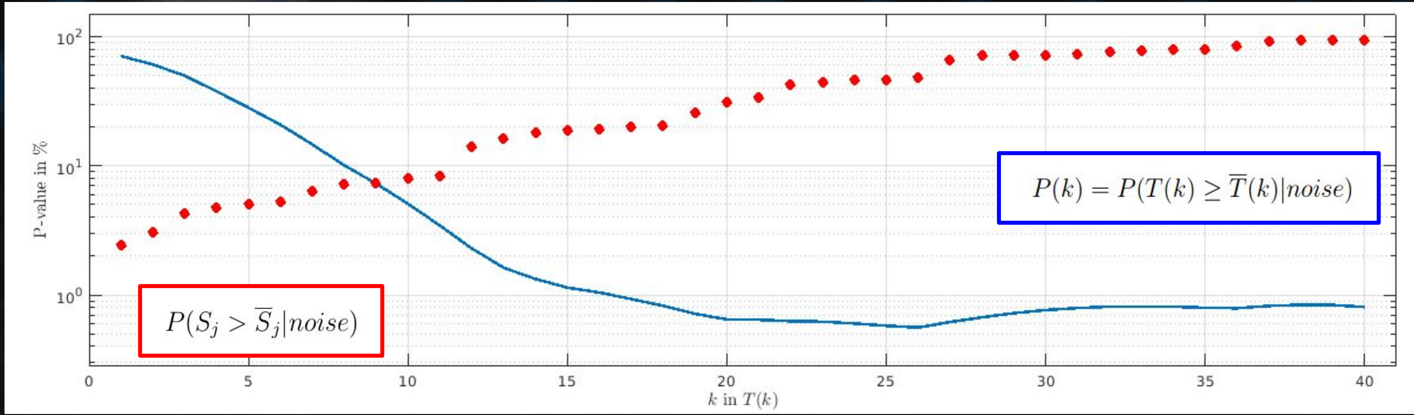


Test with Hardware Injections

LLO, 1 day-long datasets from 15/08/2019, HI_3

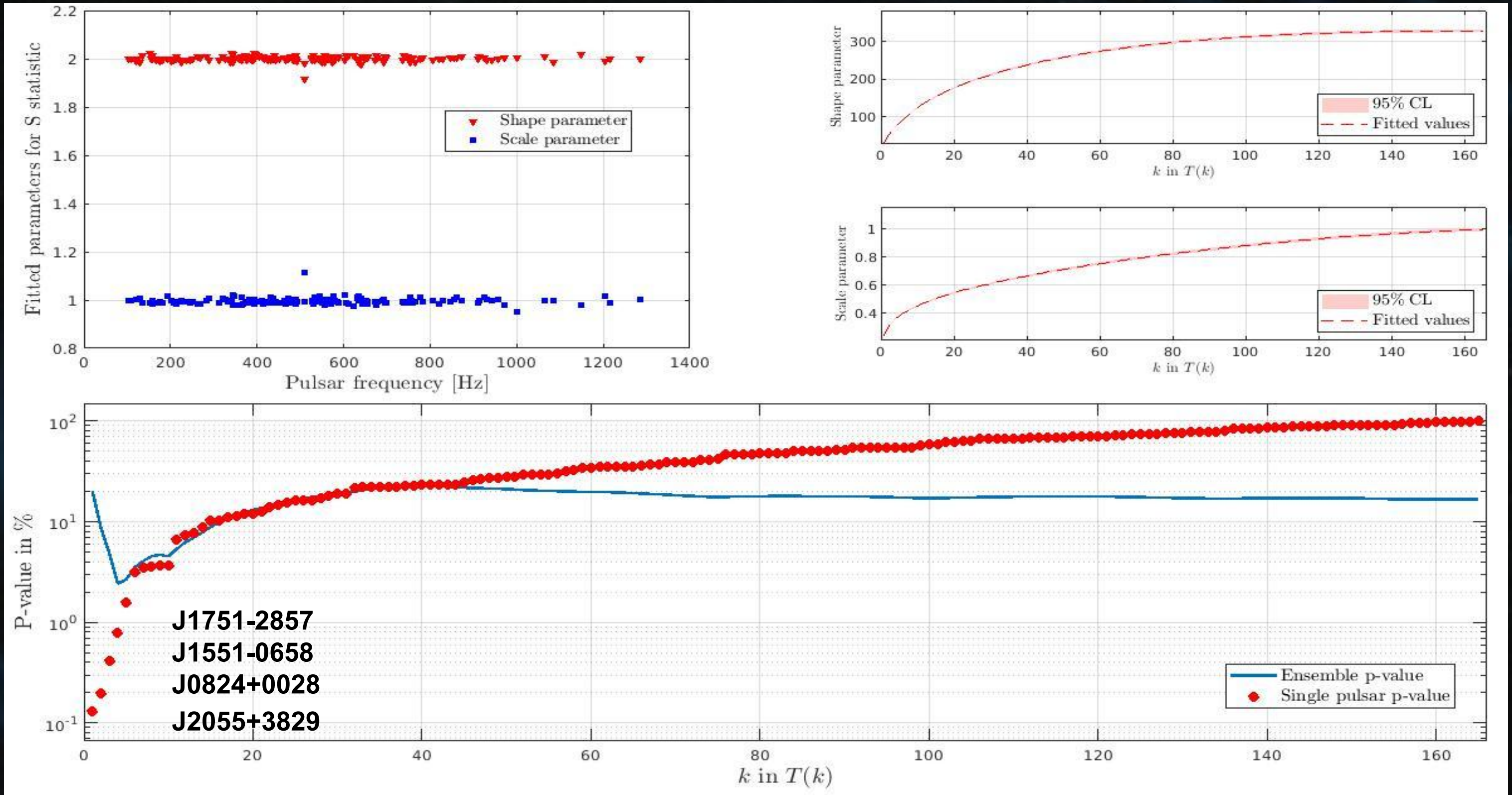


20 days with signals over the 40 considered days



Ensemble analysis

Millisecond pulsars, LIGO detectors



Discussion

- The lowest limit on the ellipticity from the targeted search is 3.2×10^{-9} for J0636+5129 while from the all-sky search is 1.4×10^{-9} for a neutron star at 10 pc and 2047.5 Hz.
- Theoretical minimum limit on the ellipticity for millisecond pulsars of $\approx 10^{-9}$ supposed in [1]
- The upper limits in [2,3] on average ellipticity for the neutron star population are $O(10^{-8})$ from cross-correlation based searches of a stochastic signal
- The upper limits on the hyperparameter for the exponential distribution do not consider the uncertainties on the distance.
- The hierarchical procedures assume that all the analyzed pulsar ellipticities are drawn from a common distribution that can be too simplicistic to describe the true ε distribution.

[1] Astrophysical Journal L. 863, L40 (2018); [2] Phys. Rev. D 106, 043019 (2022); [3] Mon. Not. Roy. Astron. Soc. 513, 1105 (2022)