COBI

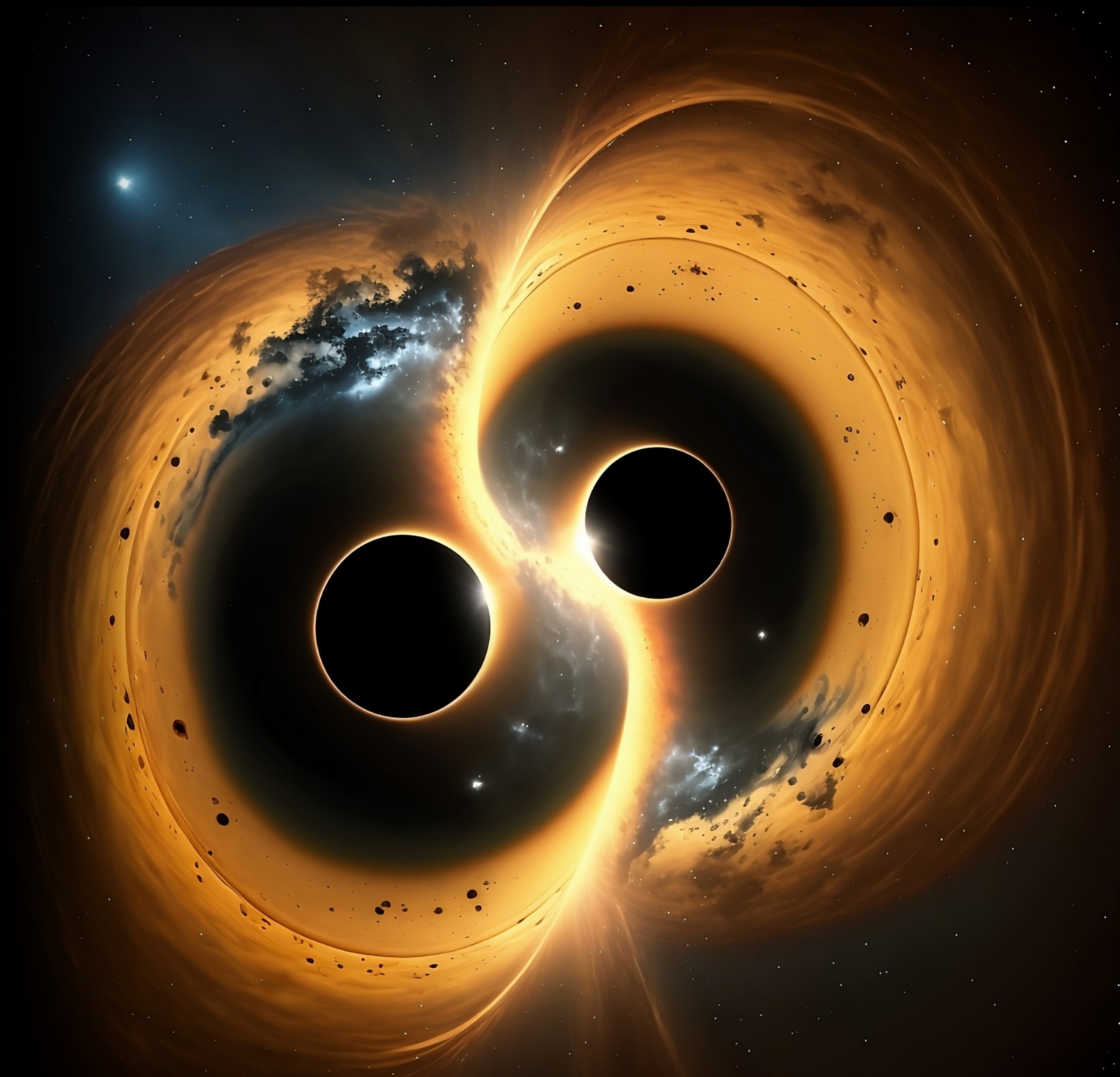
Detecting Continuous GWs from Inspiraling Light PBHs

M. Andrés-Carcasona[1], O. J. Piccinni[1]

1 Institut de Física d'Altes Energies (IFAE), Barcelona Institute of Science and Technology, E-08193 Barcelona, Spain

mandres@ifae.es

22/08/2023



OUTLINE

1 – INTRODUCTION
Explanation of the problem at hand

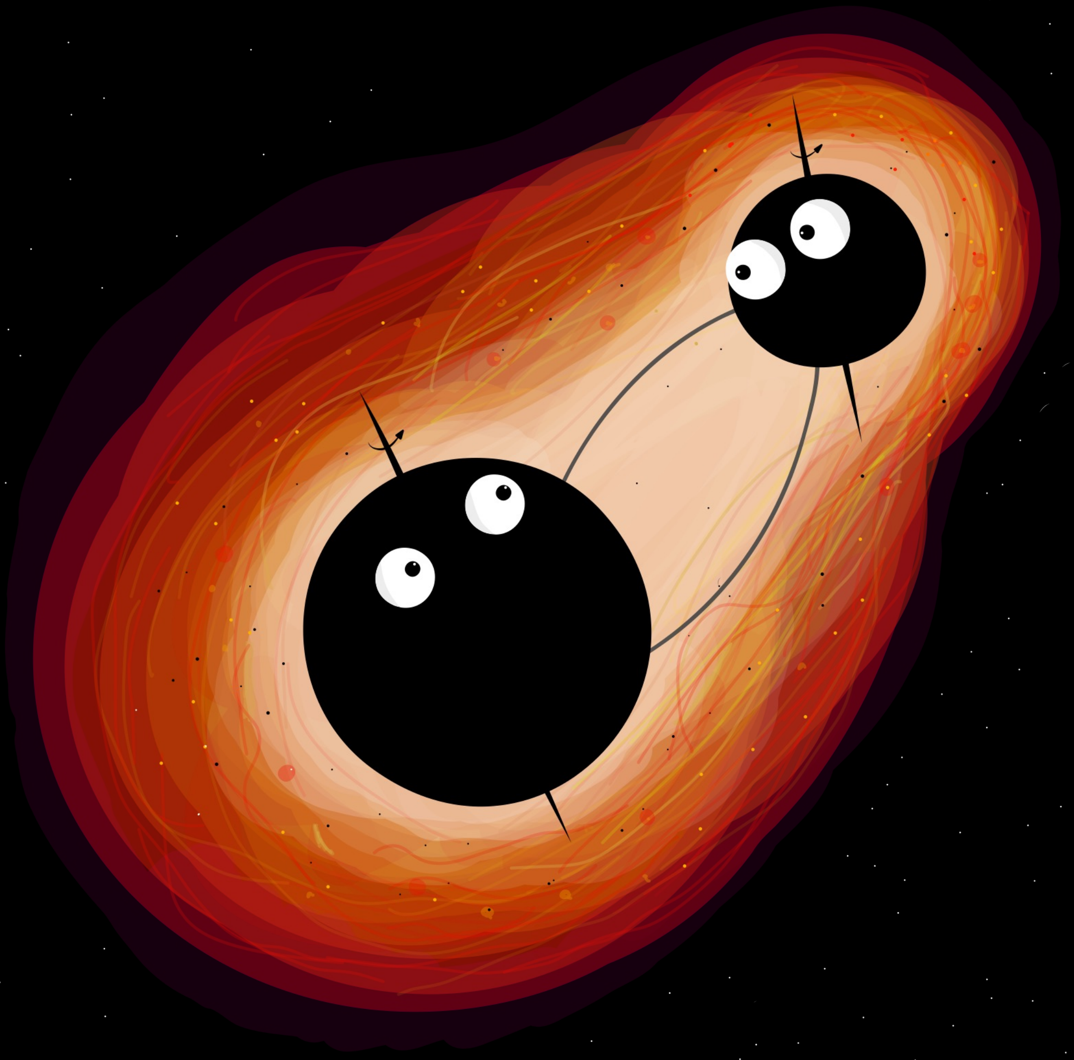
2 – THE NEW METHOD
The new proposed method is described

3 – SENSITIVITY ESTIMATION
With the new method an analytical sensitivity (understood as the maximum distance where signals can be detected) is computed

4 – CONSTRUCTION OF THE SEARCH GRID
The way to construct the grid where the parameter space is going to be explored is explained

5 – CONCLUSIONS AND FUTURE WORK
A small recap of everything and the things still pending to do are shown

1 – INTRODUCTION



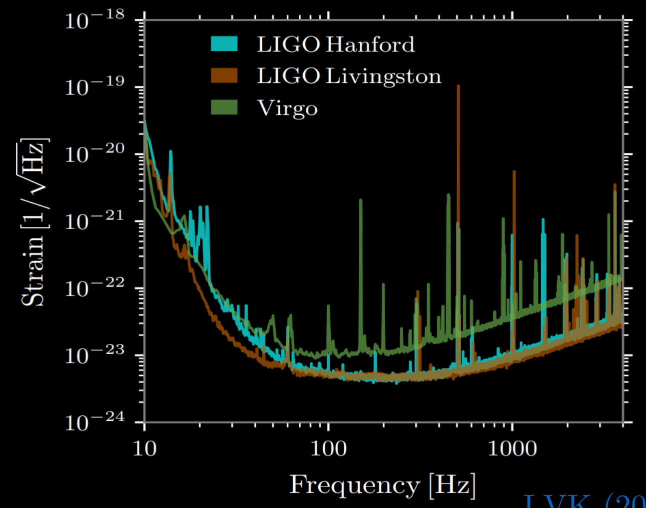
We restrict ourselves to the cases fulfilling the following conditions:

1. Binary systems
2. Long inspiral phase (>10 h)
3. High SNR to be detected

1 – INTRODUCTION

THE DATA

The data measured in one detector can be assumed to be $h(t) = s(t) + n(t)$, where $n(t)$ represents the noise and is characterized by a power spectral density (PSD) and the signal can be modeled by $s(t) = A(t)e^{i\Phi(t)}$.



LVK (2021)

The traditional problem of continuous gravitational waves (CW) is that the signal is extremely faint but lasts for very long in the detector. A method that can be employed is that of performing a heterodyne correction to then obtain a peakmap. This peakmap is a collection of peaks in time and frequency, created by computing the periodogram of a set of fast Fourier transforms (FFTs) of the data, normalized by an average spectrum, and selecting only the peaks above a given threshold. P. Astone et al. (2014)

THE CORRECTION

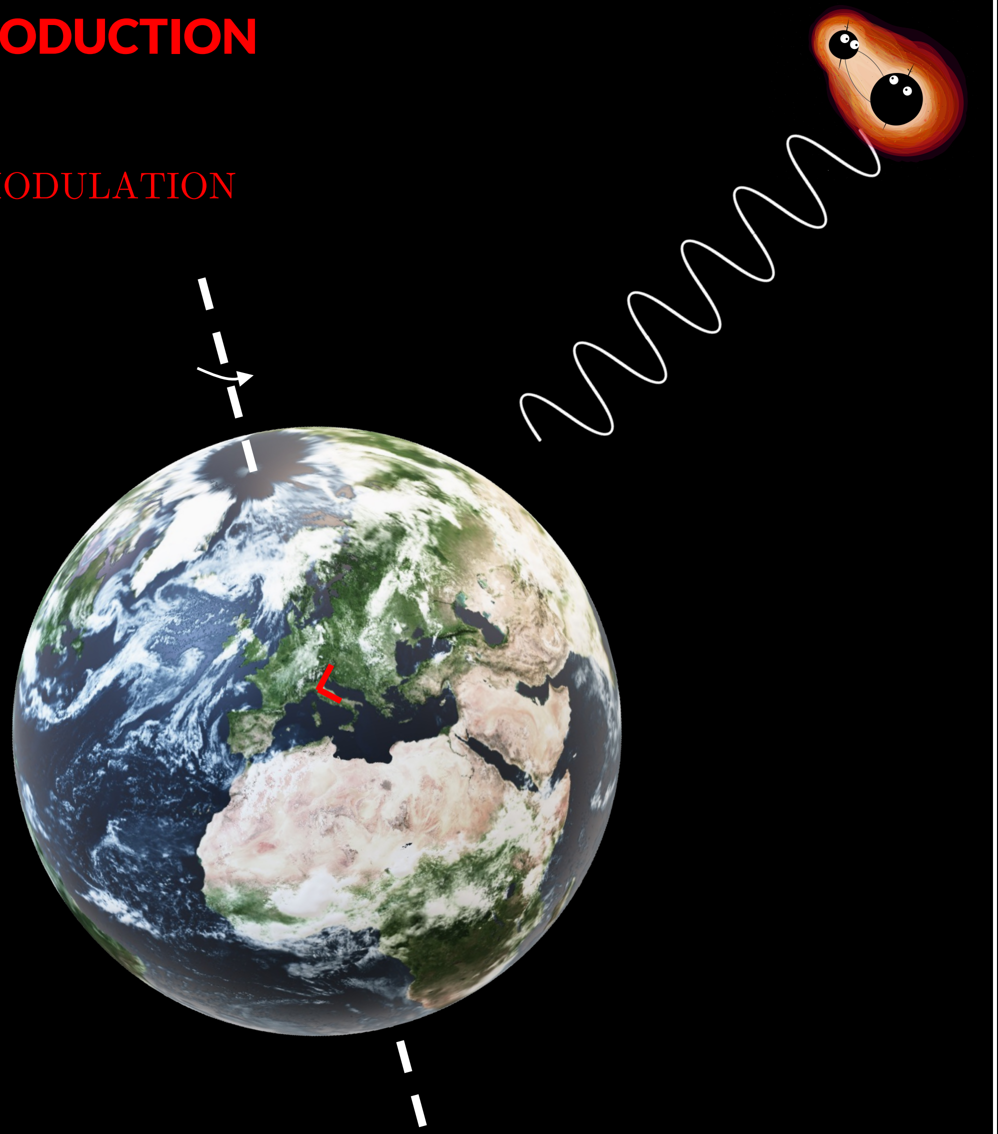
The heterodyne correction is based on applying the following operation on the data:

$$h_{corr}(t) = [s(t) + n(t)]e^{-i\Phi_{corr}(t)}$$

O.J. Piccinni et al. (2018)

1 – INTRODUCTION

DOPPLER MODULATION



$$f_{det}(t) = f(t)\left(1 + \frac{\vec{v}\cdot\hat{n}}{c}\right)$$

The velocity vector considers the Earth orbit around the sun and its own rotation

P. Astone et al. (2014)

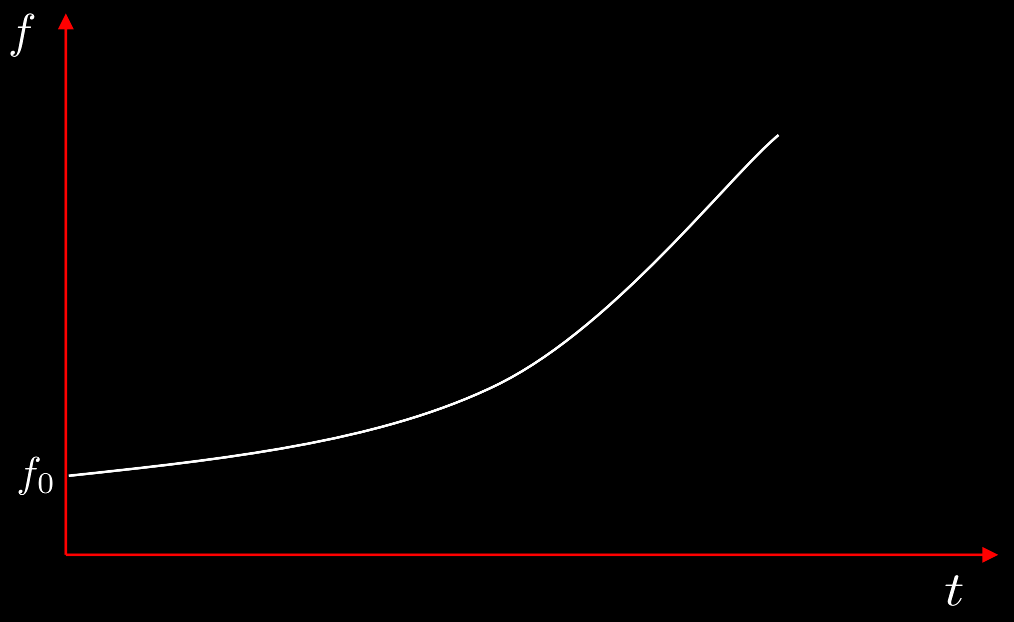
SIGNAL SPIN-UP

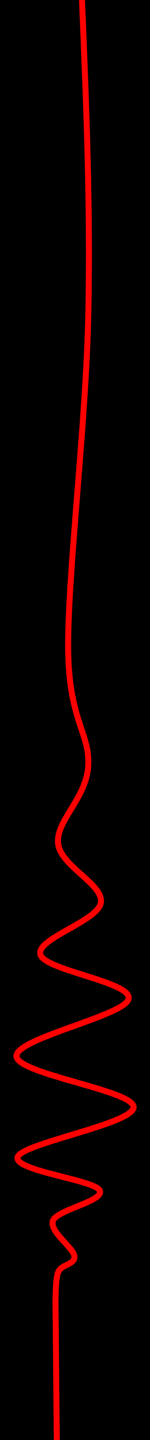
Assuming the small mass approximation and taking only the 0PN order, the spin-up of the signal can be modelled as:

$$\frac{df}{dt} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}_c}{c^3}\right)^{5/3} f^{11/3} = kf^n$$

Integrating it leads to an expression of the form of a power law

$$f(t) = f_0\left(1 + \frac{t}{\tau}\right)^{\alpha}$$

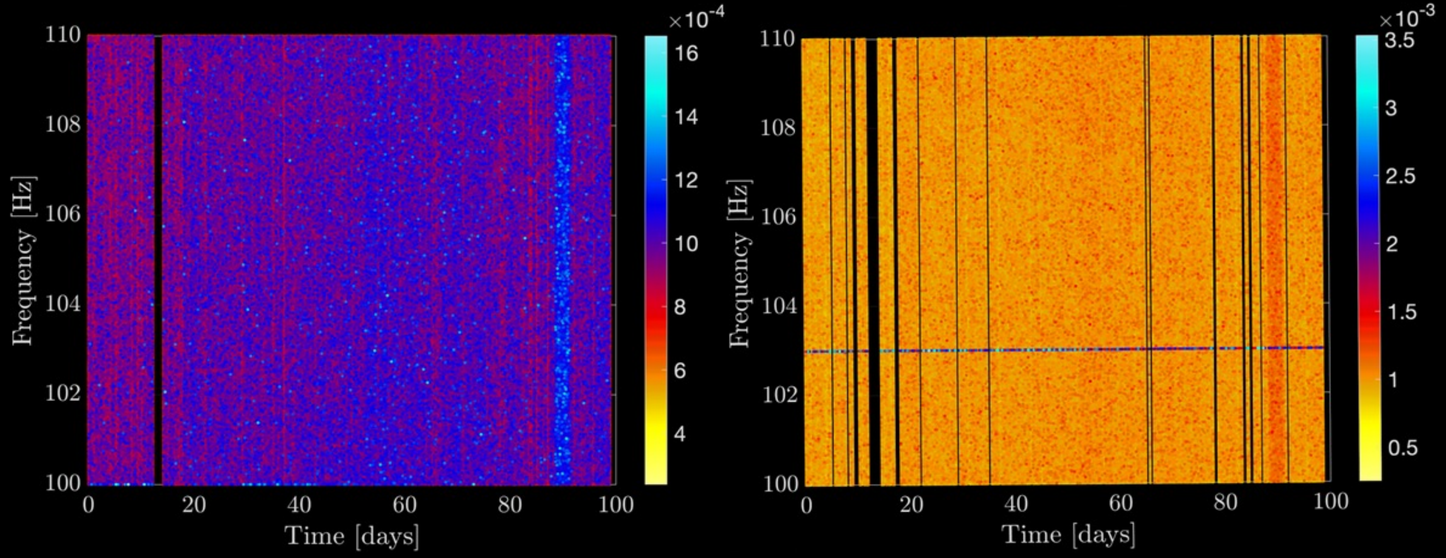




1 – INTRODUCTION

With all this information we can compute the correction phase as $\Phi_{corr}(t) = \Phi_{dopp}(t) + \Phi_{sig}(t)$ where each individual phase can be computed from the relation

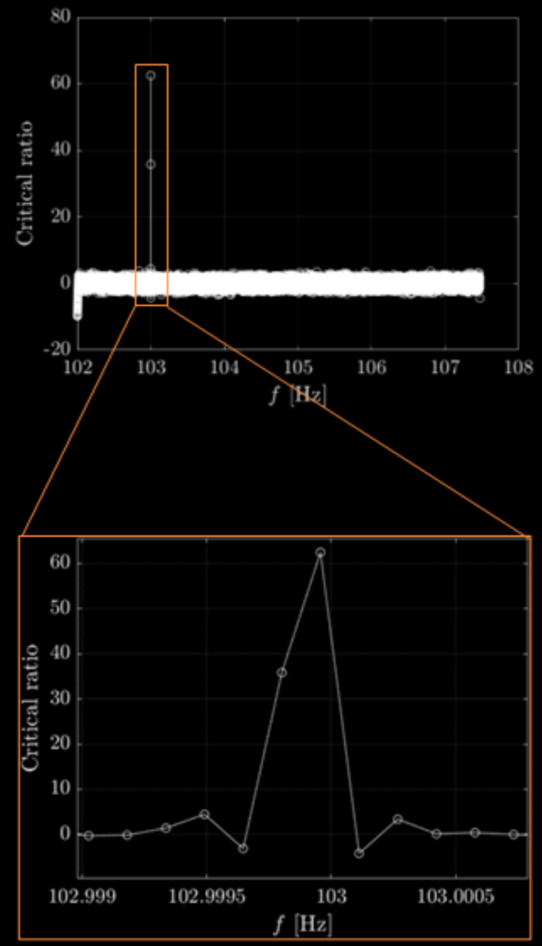
$$f(t) = \frac{1}{2\pi}\frac{d\Phi(t)}{dt}$$



Peakmap of an injected signal with a chirp mass of 10^{-4} Ms at 1 kpc and with a reference frequency of 103 Hz with no correction (left) and after the heterodyne correction (right).

Project into the frequency axis and compute the critical ratio

$$CR = \frac{n-\mu}{\sigma}$$



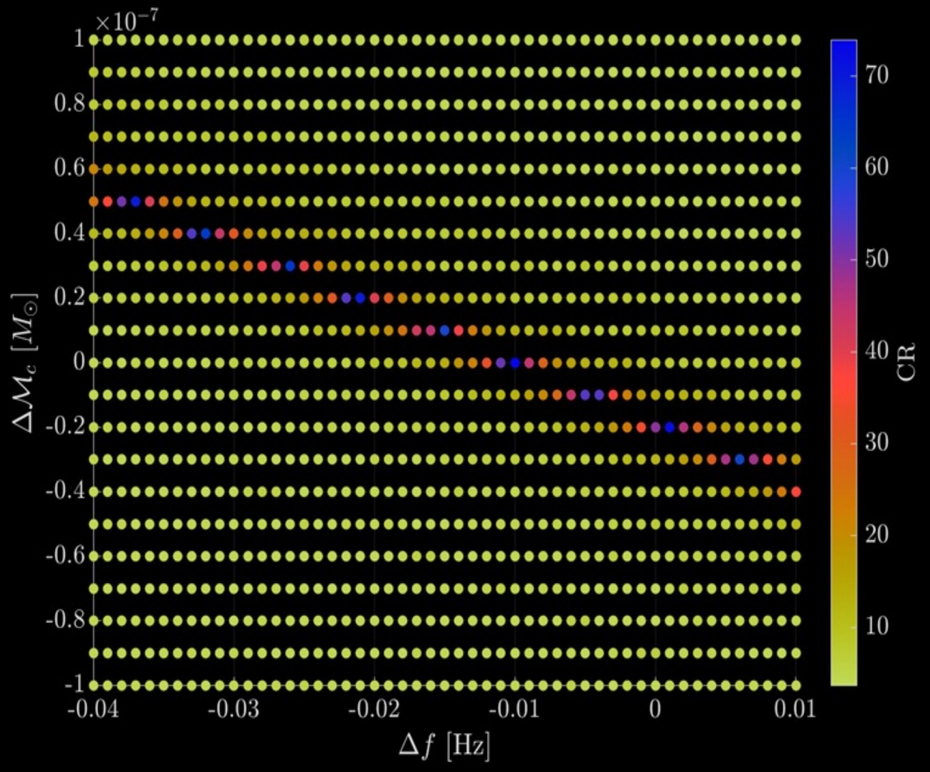
PROBLEM: The parameter space grid is two-dimensional and, therefore, computationally expensive

2 – THE NEW METHOD

THE IDEA IN A NUTSHELL

Exploiting the fact that there is a degeneracy of the two parameters we can reduce the computing cost by searching in a one-dimensional grid instead of a two-dimensional one.

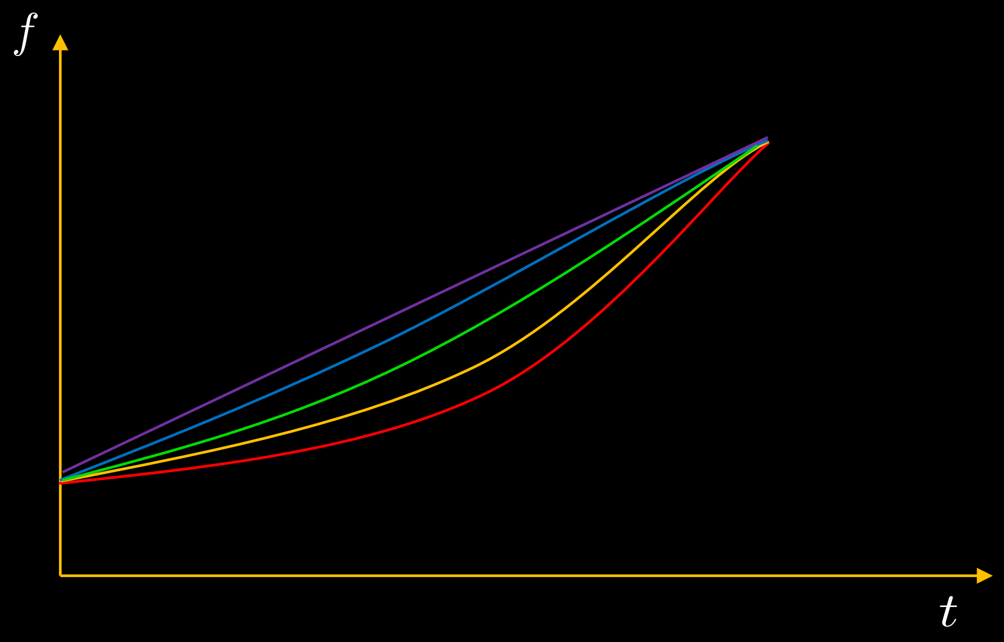
MOTIVATION

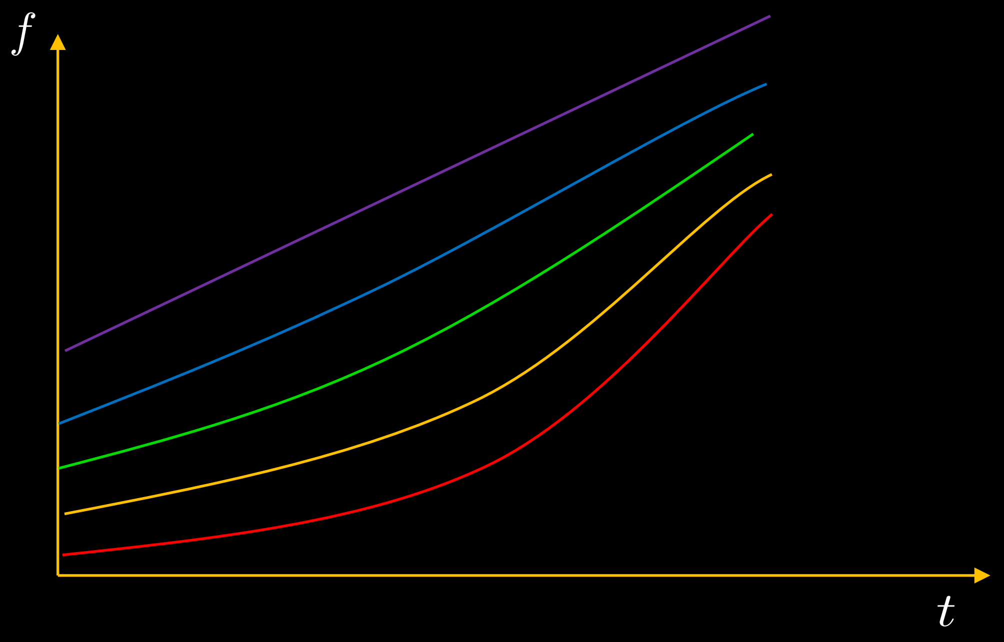


Performing a wrongful correction, we still recovered the signal!!!

2 – THE NEW METHOD

Which are all the signals that have an equal frequency increment during the observing time?



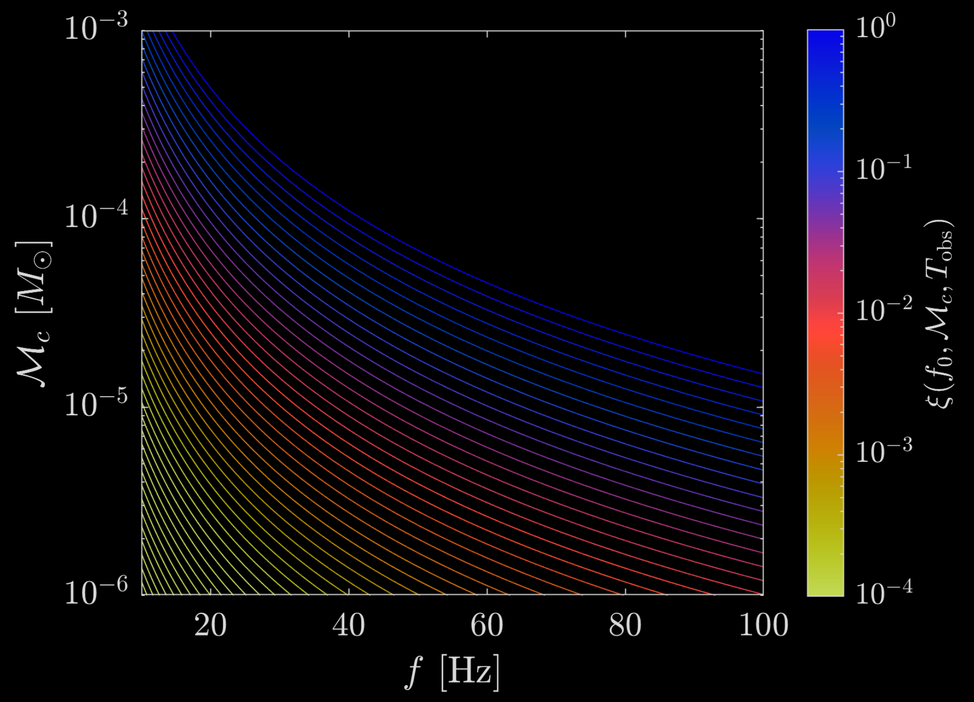


ONE VARIABLE TO RULE THEM ALL

$$\xi(f_0, \mathcal{M}_c, T_{obs}) = \left[f_0^{1/\alpha} + (1-n)k(\mathcal{M}_c)T_{obs} \right]^{\alpha} - f_0$$

2 – THE NEW METHOD

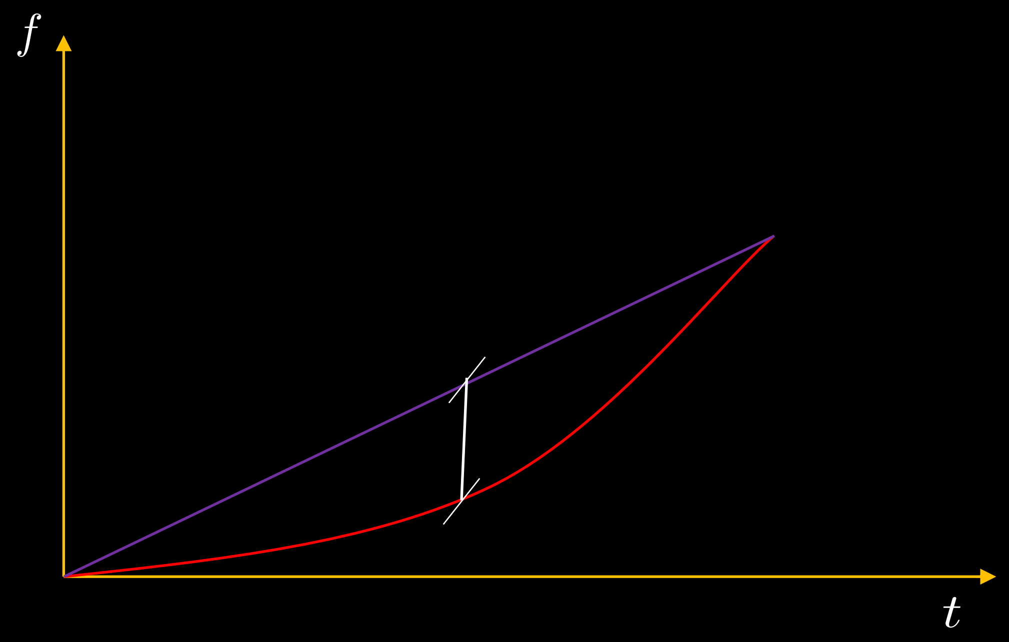
$$\xi(f_0, \mathcal{M}_c, T_{obs}) = \left[f_0^{1/\alpha} + (1-n)k(\mathcal{M}_c)T_{obs}\right]^{\alpha} - f_0$$



2 – THE NEW METHOD

Now, for each value of ξ we can compute the allowed T_{fft} as the maximum possible difference between two signals contained in the parameter space with the same ξ.

$$\max_t |f(t) - f'(t) - f_0 + f'_0| \leq \frac{1}{T_{fft}} \longrightarrow t_m = \frac{f_0^{1/\alpha} - \beta f_0'^{1/\alpha}}{(1-n)(\beta k' - k)} \qquad \beta = \left(\frac{k'}{k}\right)^{\frac{1}{\alpha n}}$$



3 – SENSITIVITY ESTIMATION

The minimum detectable strain at a given confidence level (denoted by the gamma) can be computed as a function of the noise, the length of FFT, the observing time and some factor dependent on the antenna. P. Astone et al. (2014), LVK (2022)

$$h_{min}(f) = \frac{\mathcal{B}}{(T_{obs}/T_{fft})^{1/4}} \sqrt{\frac{S_n(f)}{T_{fft}}} \sqrt{\rho_{CR} - \sqrt{2}\text{erfc}^{-1}(2\Gamma)}$$

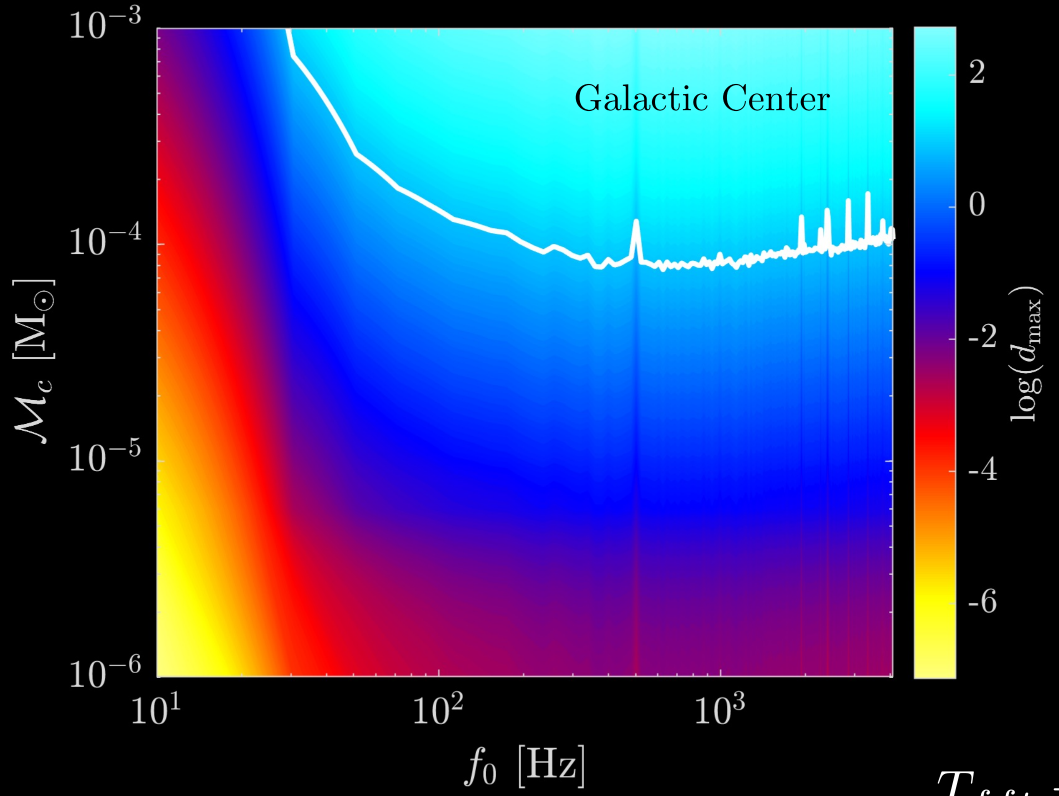
At the same time, an estimation of the strain of the signal can be obtained by evaluating the strain at the initial time. This is,

$$h = \frac{4}{d}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_0}{c}\right)^{3/2}$$

which leads to a minimum reachable distance of

$$d_{min} = \frac{4}{\mathcal{B}}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_0}{c}\right)^{3/2}\left(\frac{T_{obs}}{T_{fft}}\right)^{1/4}\sqrt{\frac{T_{fft}}{S_n(f)}}\left[\rho_{CR} - \sqrt{2}\text{erfc}^{-1}(2\Gamma)\right]^{-1/2}$$

3 – SENSITIVITY ESTIMATION



$T_{fft} = 10^4\ s$

4 – CONSTRUCTION OF THE SEARCH GRID

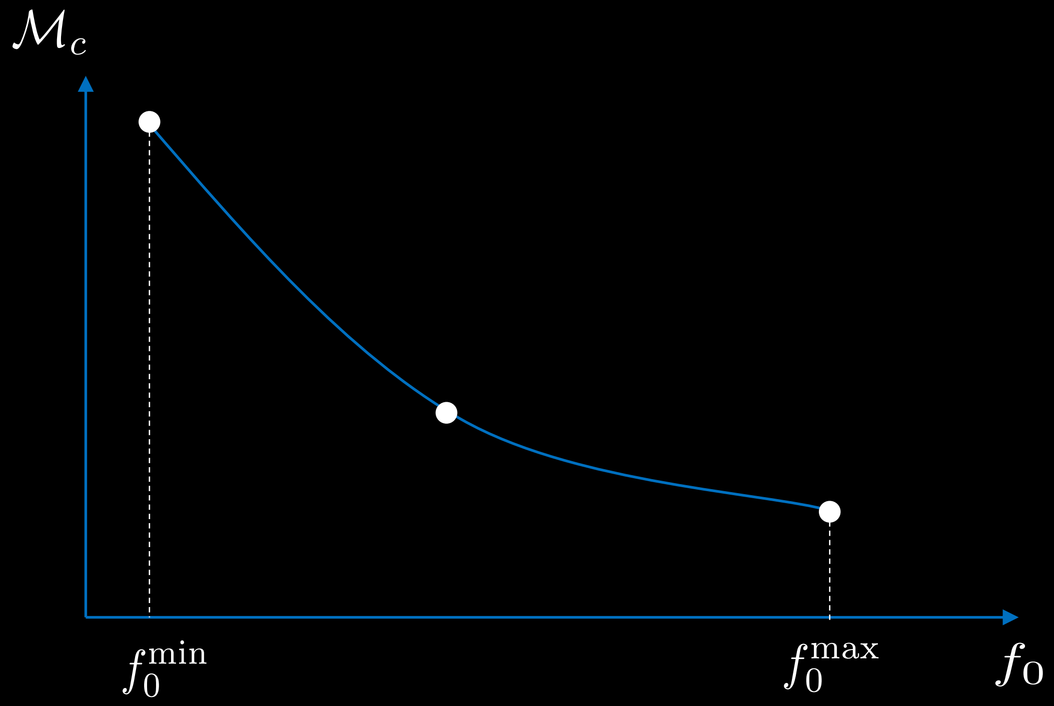
$$\mathcal{G}(\xi, f_0, f_0') = \max_t |f(t) - f'(t) - f_0 + f_0'|$$

1. Define the limits $\xi_{\min}, \xi_{\max}$, $f_0^{\min}$, $f_0^{\max}$. Set $\xi_0 = \xi_{\min}$.
2. Solve the following optimization problem for a given ξ_i

$$\min_{f_0} \quad \max(\mathcal{G}(\xi, f_0, f_0^{\min}), \mathcal{G}(\xi, f_0, f_0^{\max}))$$

$$\text{s.t.} f_0 \in [f_0^{\min}, f_0^{\max}]$$

3. Set $T_{fft} = 1/\mathcal{G}_{\max}$
4. Set $i = i + 1$ and $\xi_i = \xi_{i-1} + 1/T_{fft}$.
5. If $\xi_i < \xi_{\max}$ go to step 2.



4 – CONSTRUCTION OF THE SEARCH GRID

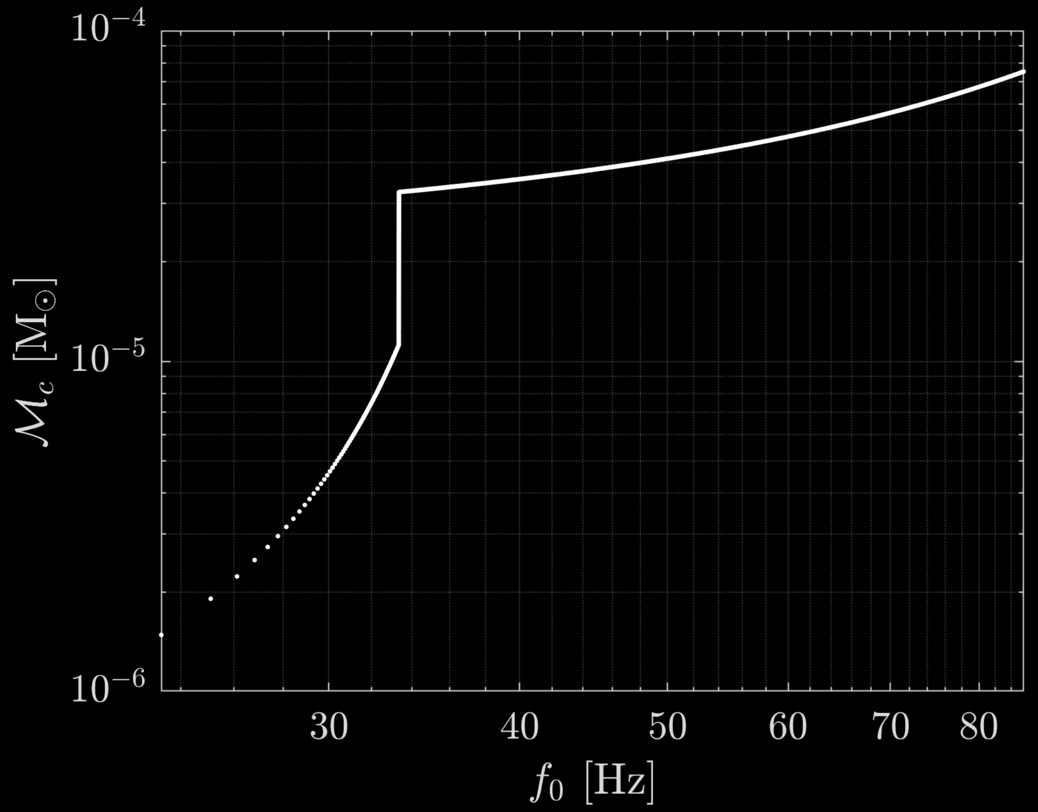
$$\boxed{\mathcal{G}(\xi, f_0, f_0') = \max_t |f(t) - f'(t) - f_0 + f_0'|}$$

1. Define the limits $\xi_{\min}, \xi_{\max}$, $f_0^{\min}$, $f_0^{\max}$. Set $\xi_0 = \xi_{\min}$.
2. Solve the following optimization problem for a given ξ_i

$$\min_{f_0} \quad \max(\mathcal{G}(\xi, f_0, f_0^{\min}), \mathcal{G}(\xi, f_0, f_0^{\max}))$$

$$\text{s.t.} f_0 \in [f_0^{\min}, f_0^{\max}]$$

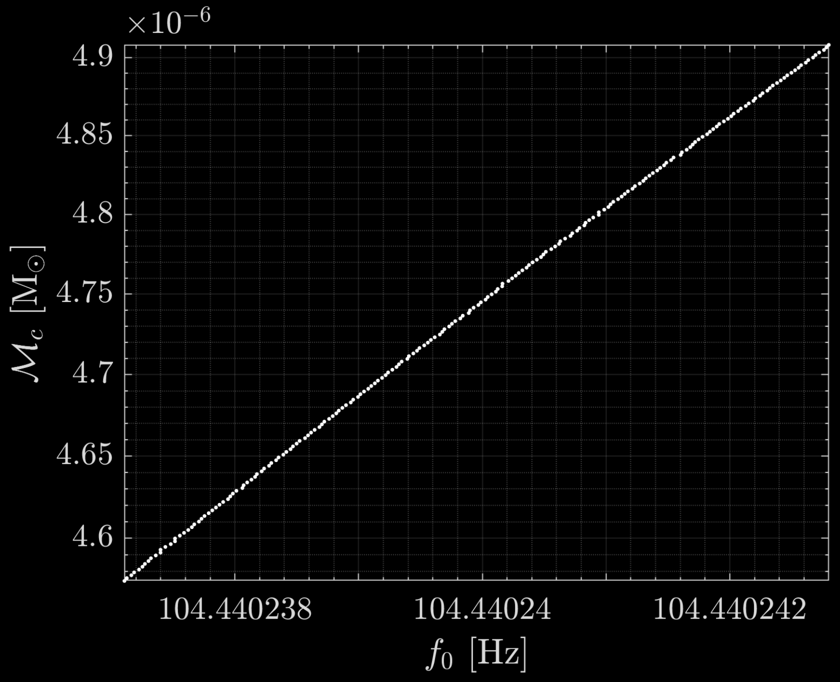
3. Set $T_{fft} = 1/\mathcal{G}_{\max}$
4. Set $i = i + 1$ and $\xi_i = \xi_{i-1} + 1/T_{fft}$.
5. If $\xi_i < \xi_{\max}$ go to step 2.

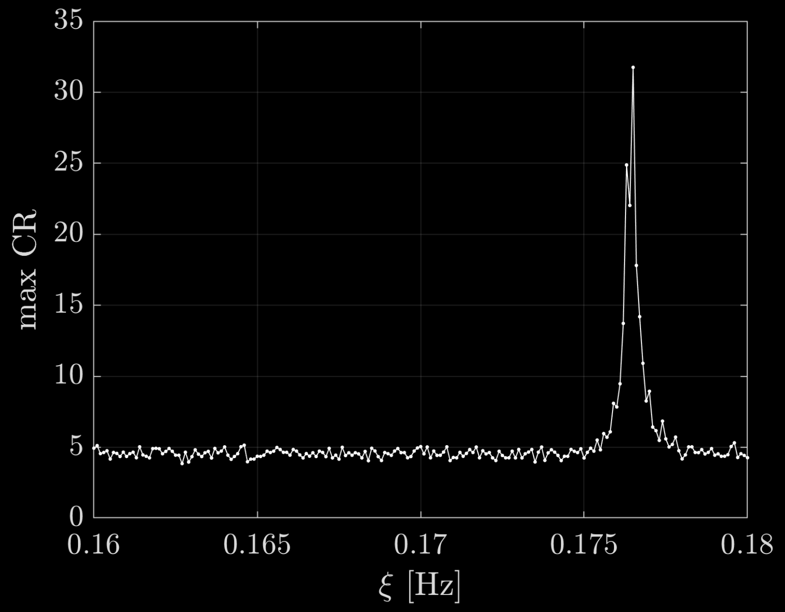


4 – CONSTRUCTION OF THE SEARCH GRID

EXAMPLE OF AN INJECTION AND RECOVERY

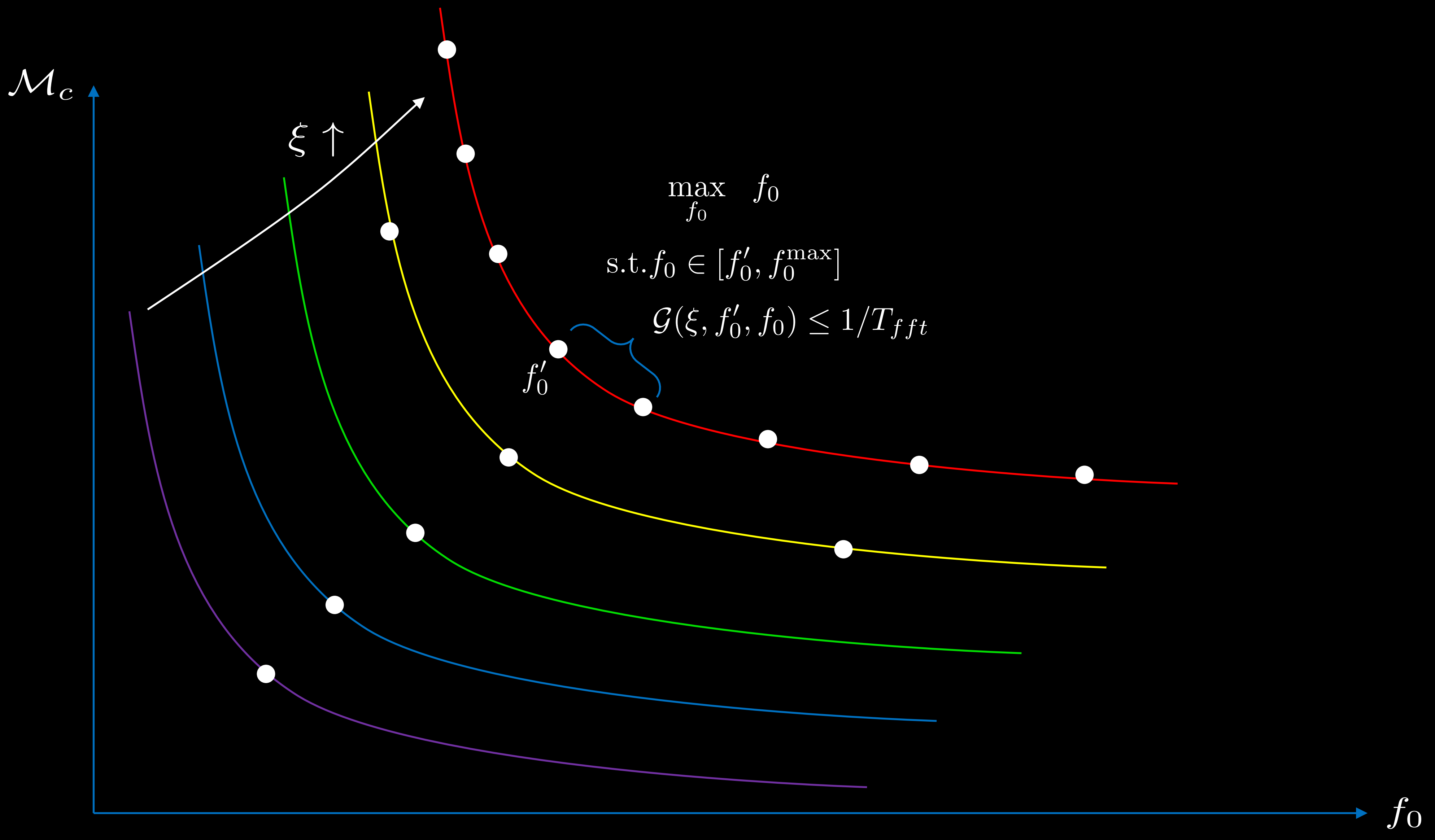
We inject a signal with $f_0 = 103$ Hz and $\mathcal{M}_c = 5 \times 10^{-6}\ M_\odot$ at 0.02 kpc





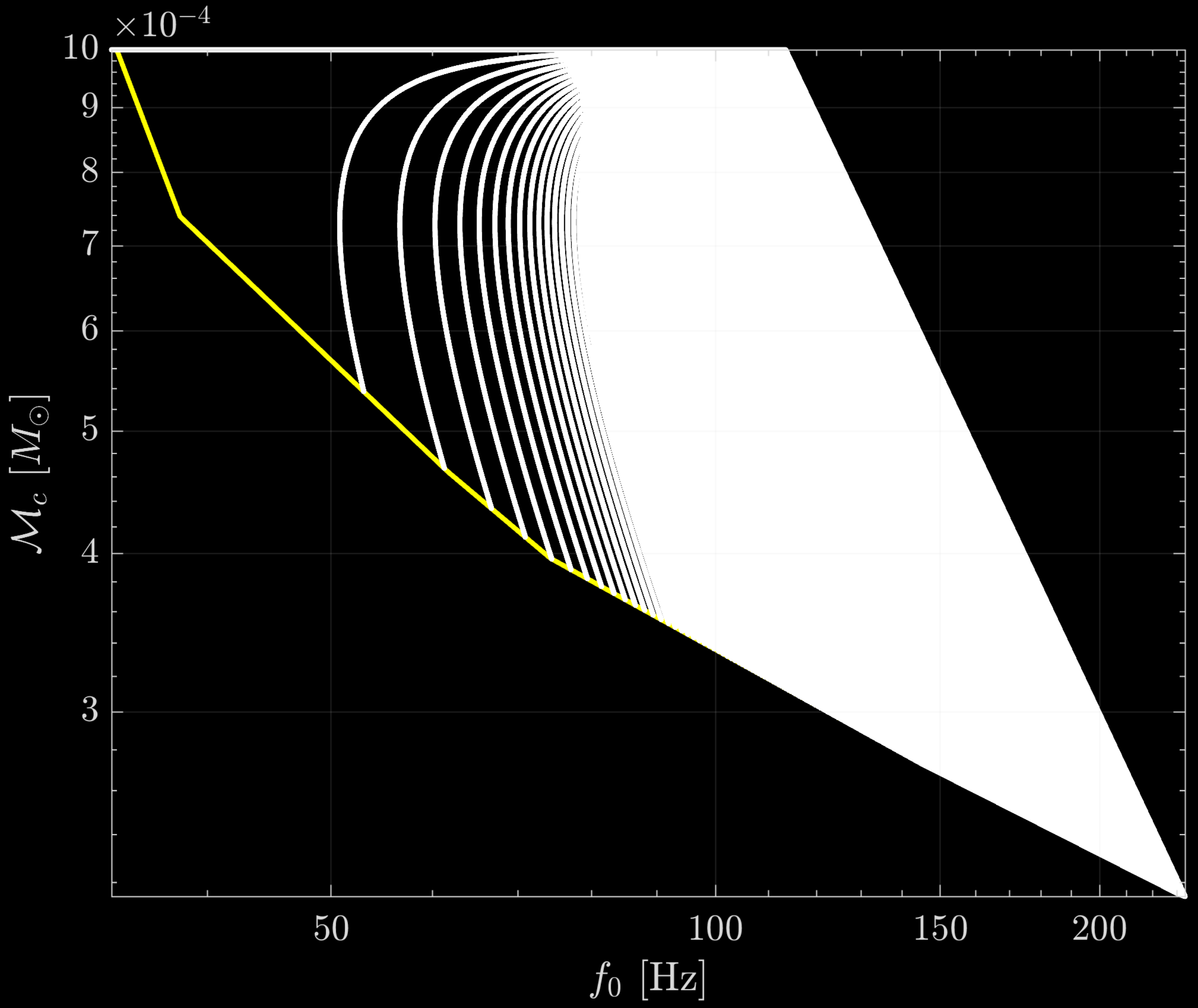
4 – CONSTRUCTION OF THE SEARCH GRID

What if we want to fix the Tfft?



4 – CONSTRUCTION OF THE SEARCH GRID

As shown before with a constant FFT length of 10.000 s the GC can be probed for a region of the parameter space. We can follow the idea of the previous slide to construct a grid only covering where we can reach the GC. The entire grid has of the around 69M points.



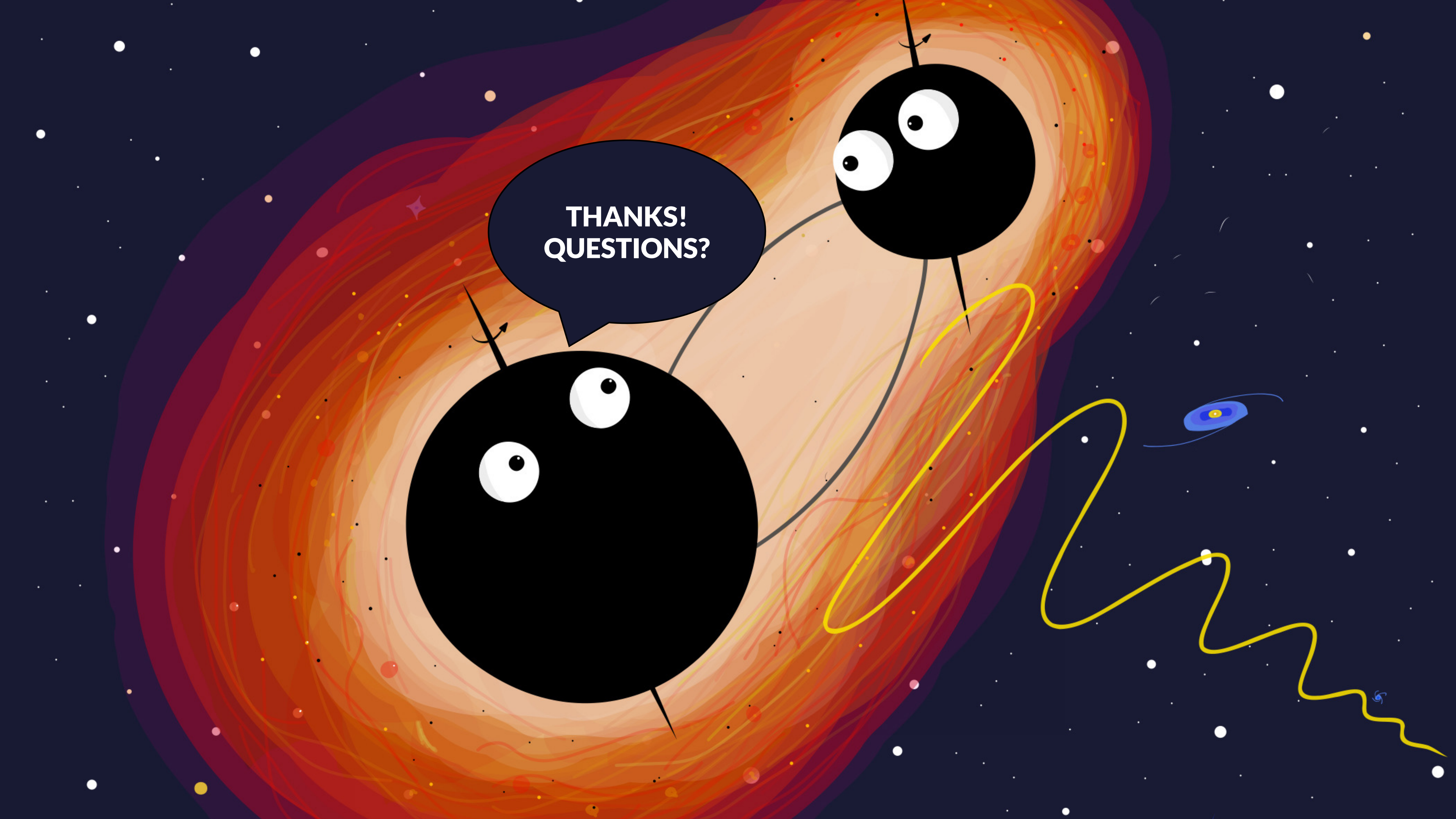
5 – CONCLUSIONS AND FUTURE WORK

We have reviewed the basic theory of a CW method based on the heterodyne correction applied to inspiraling PBHs

We have explained a new method that may reduce the complexity of the problem

We have shown that the method sensitivity would be enough to reach the galactic center

Still there are details to be taken care of and finish the code to run the actual search


THANKS!
QUESTIONS?