

# Full NNLO QCD corrections to diphoton production

**Federico Coro**

IFIC - Universitat de Valencia



VNIVERSITAT  
DE VALÈNCIA





Based on: [arXiv:2308.10885](https://arxiv.org/abs/2308.10885)




In collaboration with M. Becchetti, R. Bonciani, L. Cieri and F. Ripani

# Outline of the talk




## Introduction

-  Motivations
-  State of the art

## Double Virtual Contribution

-  Form factors
-  Master Integrals
-  Hard Function

## Final Results

-  Double Virtual
-  Double Real
-  Real-Virtual

## Conclusions

# Motivations

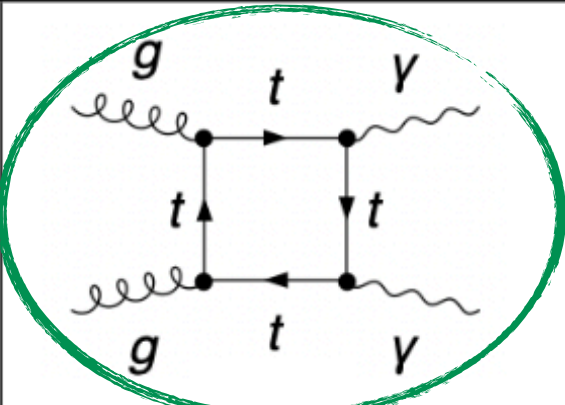

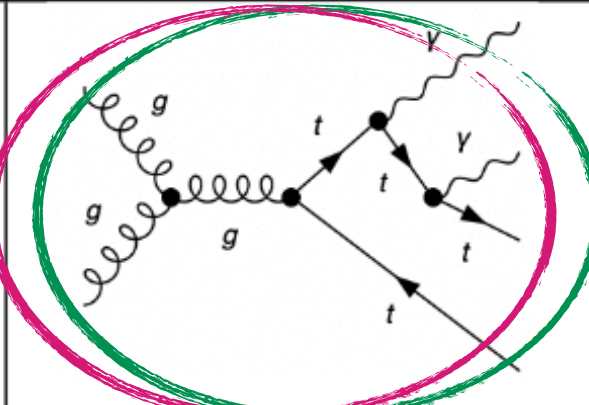
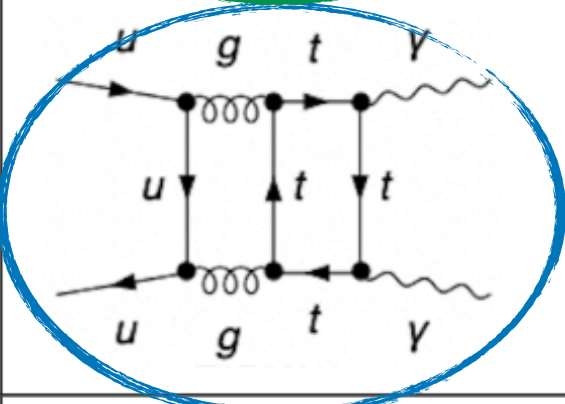
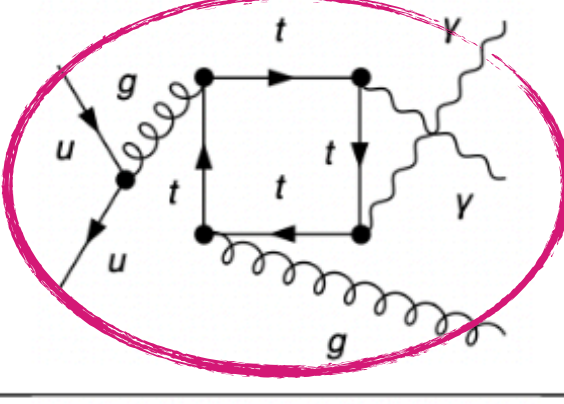
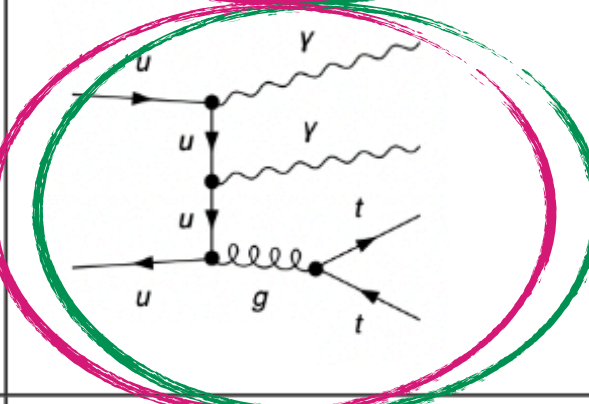

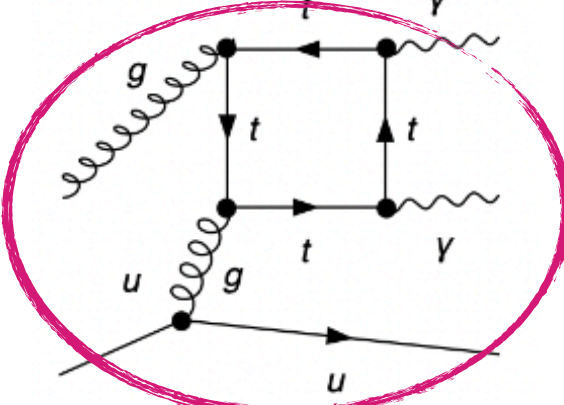

- ❖ Diphoton is an experimentally clean final state
- ❖ QCD background for Higgs
- ❖ Important to measure the fundamental parameters within the Standard Model
- ❖ Search for new physics

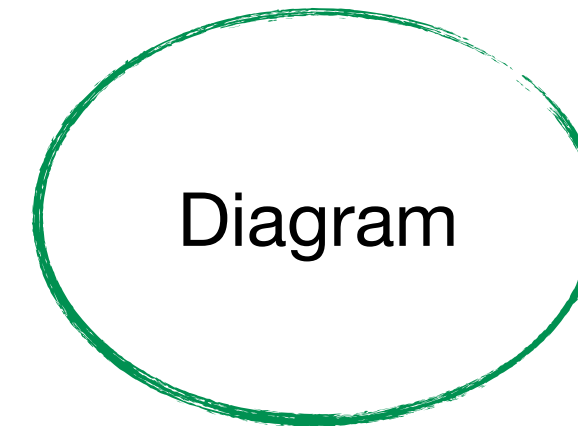
# State of the art

- ❖ Full NLO
- ❖ QCD NNLO
- ❖  $\gamma\gamma + jets$
- ❖ Form factors up to 3 loops

# Massive Corrections

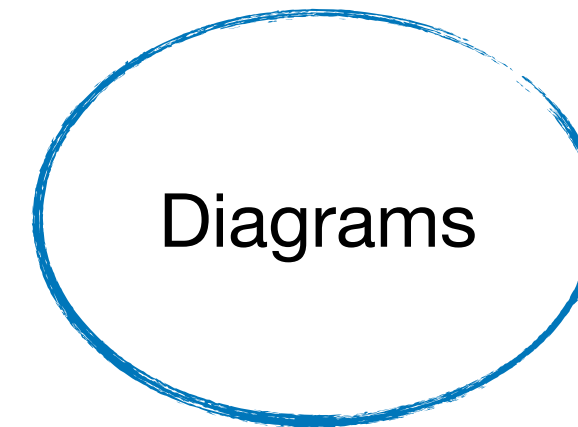
Massive corrections  $\mathcal{O}(\alpha_s^2)$

| Channels   | $\gamma\gamma$  | $\gamma\gamma j$   | $\gamma\gamma jj$   |
|------------|---|--|---|
| $gg$       |    |    |    |
| $q\bar{q}$ |   |   |   |
| $qg$       |  |  |  |

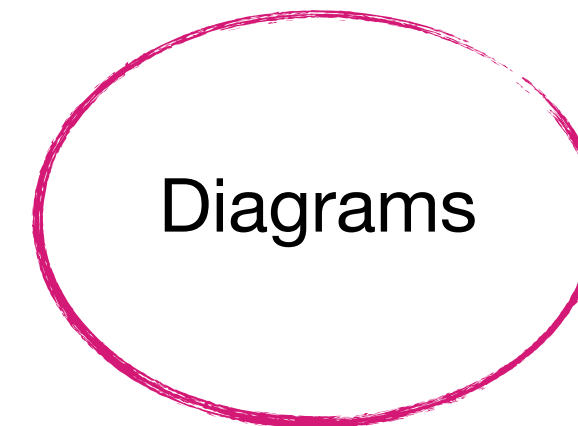


[J.M.Campbell,R.K.Ellis,Y.Li,C.Williams]

[F.Buccioni,J-N.Lang,J.M.Lindert,P.Maierhofer,  
S.Pozzorini,H.Zhang,M.Zoller]

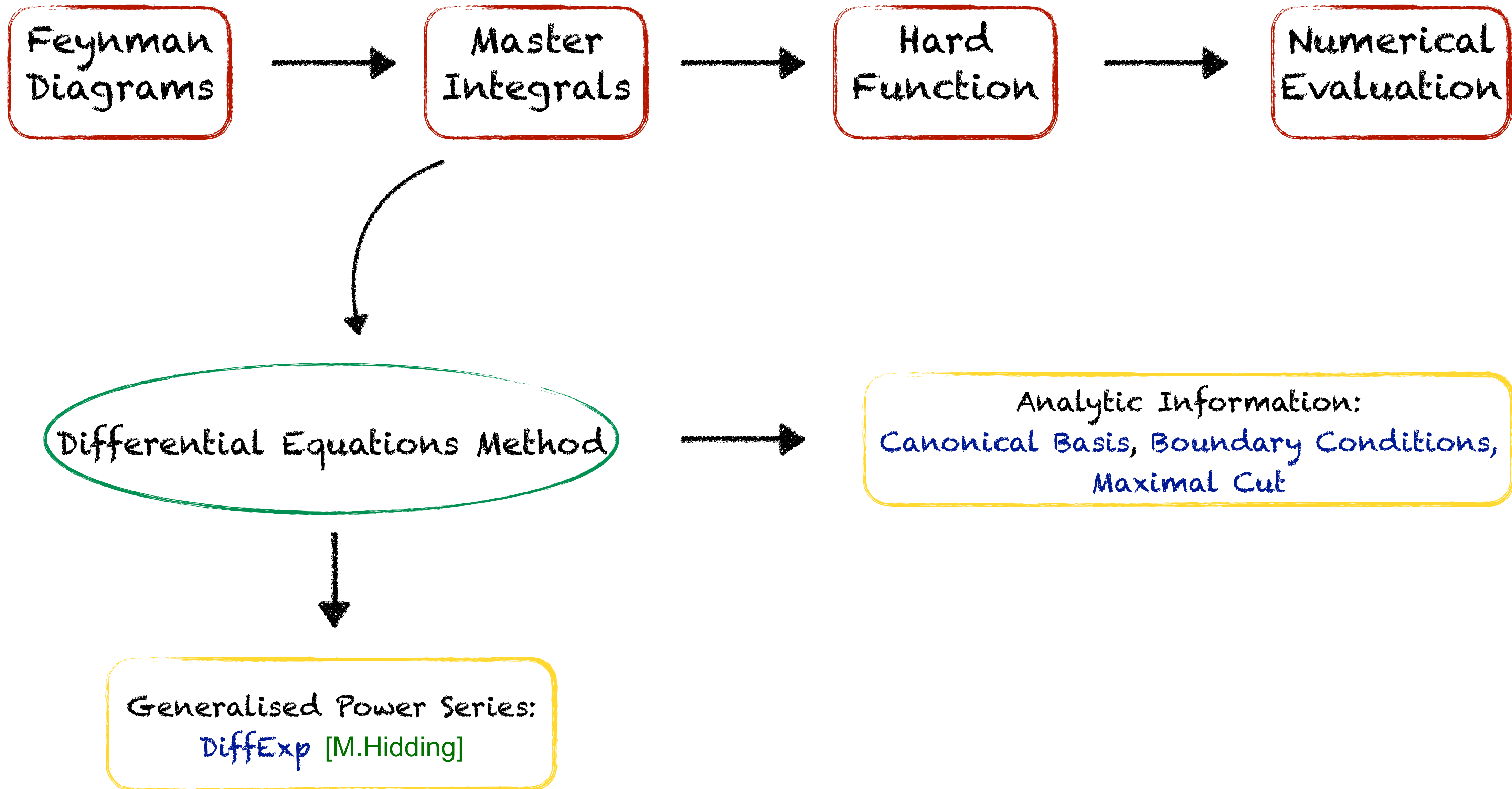


Original results and main focus of the talk



Evaluated for the final result

# Computational pipeline



# Form factors

At any order in QCD perturbation theory, the amplitude can be decomposed as:

$$\mathcal{A}_{q\bar{q},\gamma\gamma}(s, t, m_t^2) = \sum_{i=1}^4 \mathcal{F}_i(s, t, m_t^2) \bar{v}(p_2) \Gamma_i^{\mu\nu} u(p_1) \epsilon_{3,\mu} \epsilon_{4,\nu}$$

In dimensional regularisation:

$$\Gamma_1^{\mu\nu} = \gamma^\mu p_2^\nu, \quad \Gamma_2^{\mu\nu} = \gamma^\nu p_1^\mu, \quad \Gamma_3^{\mu\nu} = p_{3,\rho} \gamma^\rho p_1^\mu p_2^\nu, \quad \Gamma_4^{\mu\nu} = p_{3,\rho} g^{\mu\nu}$$

[F.Caola,A.Von Manteuffel,L.Tancredi]

The form factors admits a perturbative expansion:

$$\mathcal{F}_i = \mathcal{F}_i^{(0)} + \left(\frac{\alpha_s^B}{\pi}\right) \mathcal{F}_i^{(1)} + \left(\frac{\alpha_s^B}{\pi}\right)^2 \mathcal{F}_i^{(2)} + \dots$$



Massive contribution appears at  $\mathcal{O}(\alpha_s^2)$  :

$$\mathcal{F}_i^{(2)} = \delta_{kl} C_F (4\pi\alpha_{em}) \left[ Q_q^2 \mathcal{F}_{i;0}^{(2)} + Q_t^2 \mathcal{F}_{i;2}^{(2)} \right]$$

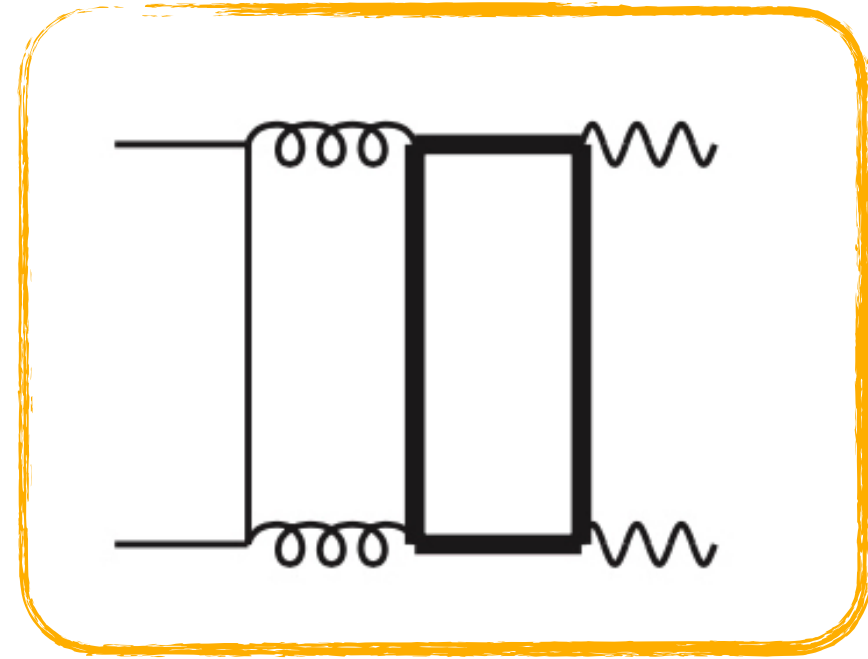
$Q_q$  is the charge of light quark  
 $Q_t$  is the charge of heavy quark

# Two-loop Feynman diagrams

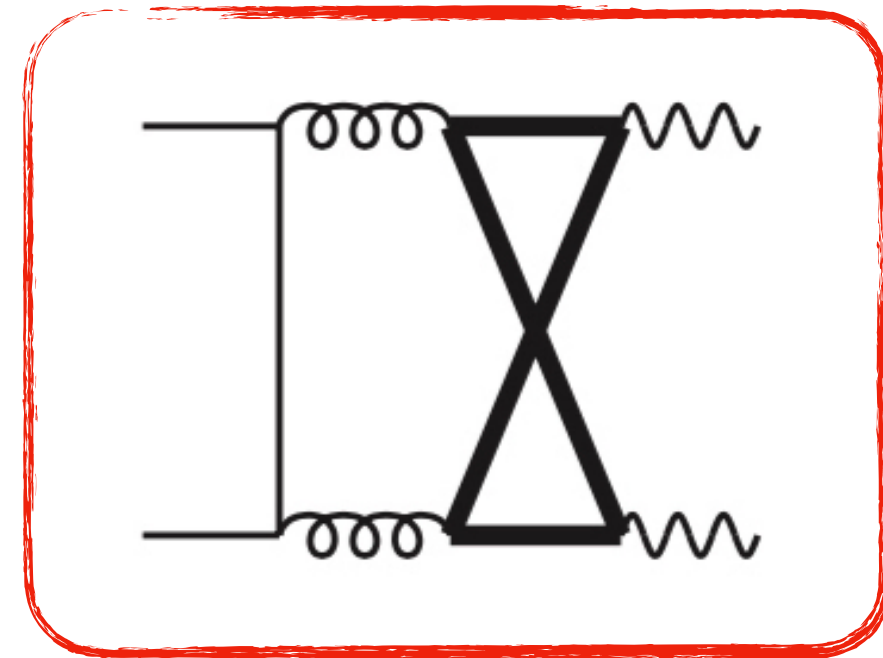
At partonic level the scattering process is:  $q(p_1) + \bar{q}(p_2) \rightarrow \gamma(p_3) + \gamma(p_4)$

External particles on-shell and the top quark running in the loop

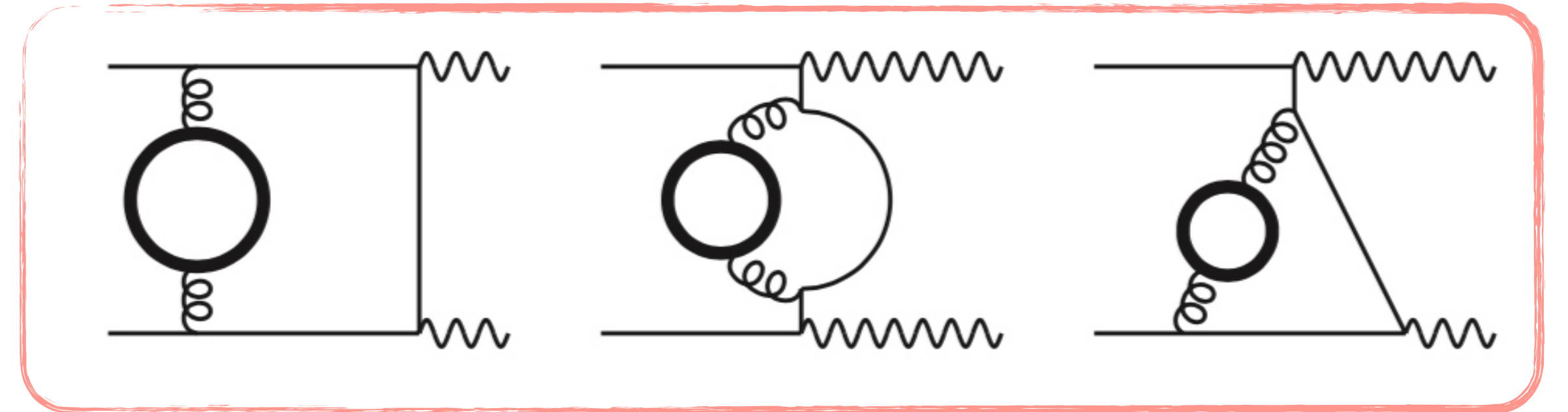
Feynman diagrams generated with **FeynArts** [T.Hahn]



PLA



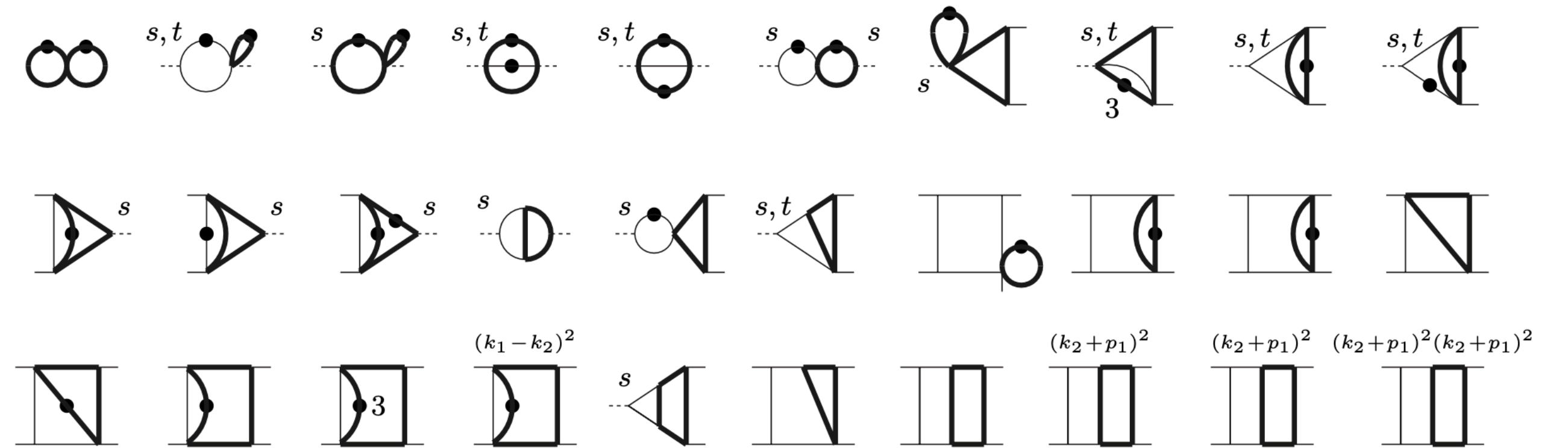
NPL



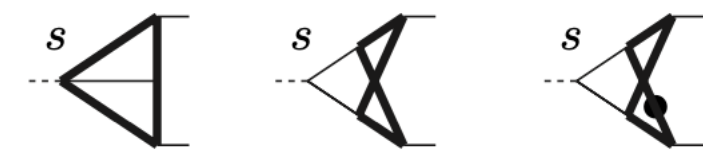
PLB

# Master Integrals

## PLA and PLB Master Integrals

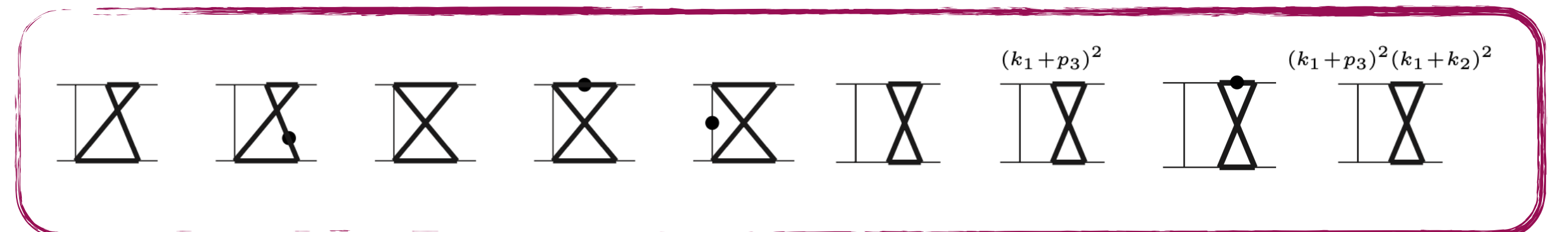


[M.Becchetti,R.Bonciani]



[A.Von Manteuffel,L.Tancredi]

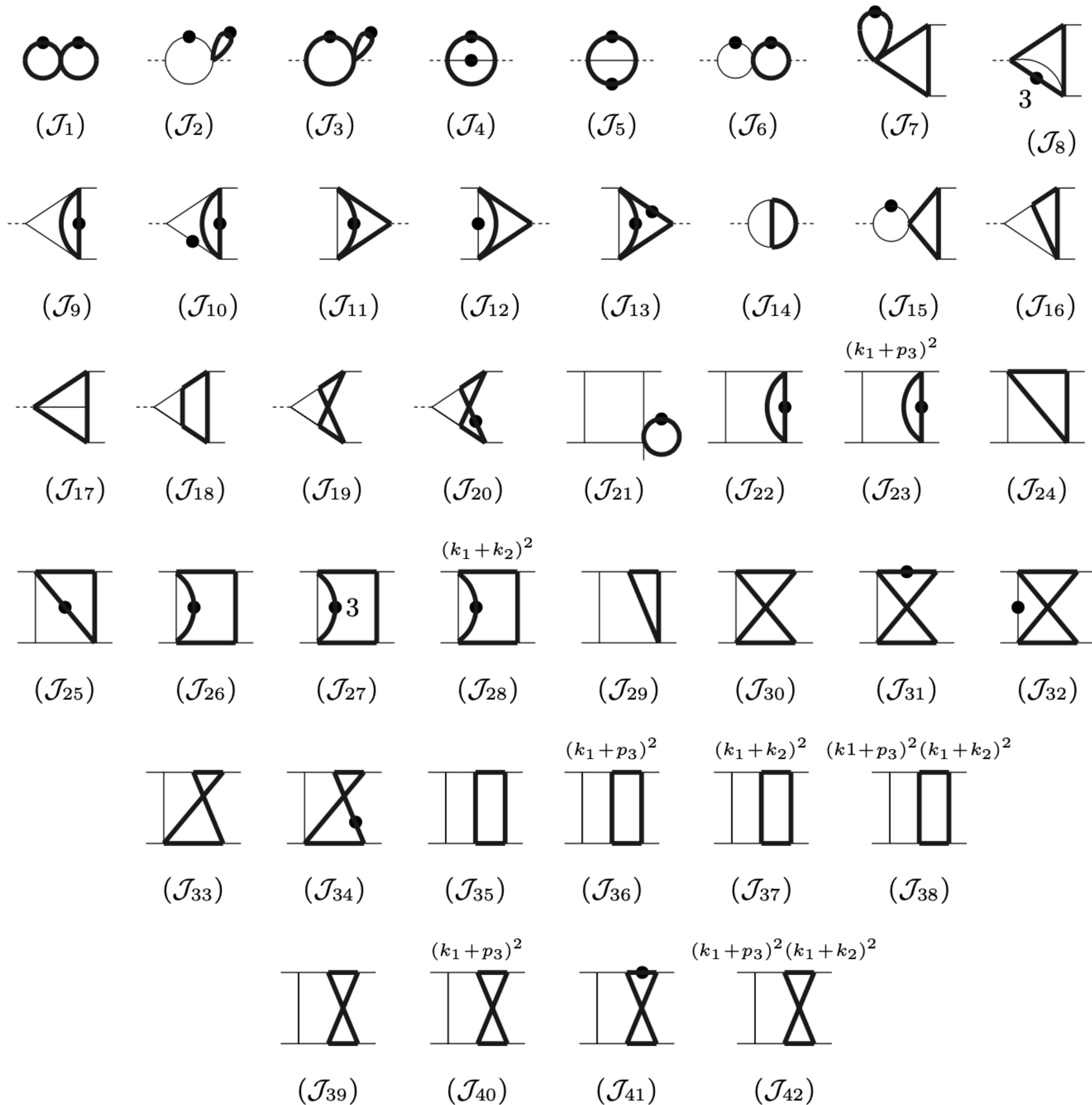
## NPL Master Integrals



Original MIs



# Master Integrals



Now we have  
42 MIs  
for all the  
process!

# Evaluation of the Master Integrals

The MIs are computed through the differential equations method:

**PLA family:**

$$df(\underline{x}, \epsilon) = \epsilon dA(\underline{x})f(\underline{x}, \epsilon)$$

Canonical logarithmic form!

[J.M.Henn]

with respect to the kinematic invariants:  $\underline{x} = \{y, z\}$ ,  $y = \frac{s}{m_t^2}$ ,  $z = \frac{t}{m_t^2}$

Boundary Conditions:  $\underline{x} = 0$

❖ Non linearizable square roots

Five different square roots in the letters

❖ Non trivial solution!

❖ Big expressions!

**PLB family:**

This topology contains only one different MIs from the other two topologies, which was computed analytically

# Evaluation of the Master Integrals

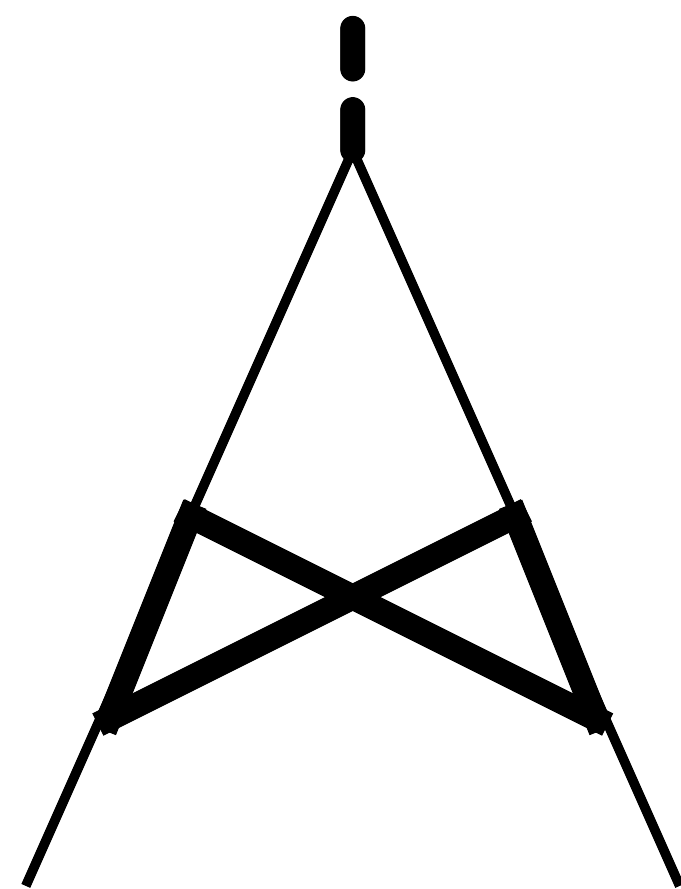
NPL family:

$$df(\underline{x}, \epsilon) = \epsilon dA(\underline{x})f(\underline{x}, \epsilon) + d\tilde{A}(\underline{x}, \epsilon)f(\underline{x}, \epsilon)$$

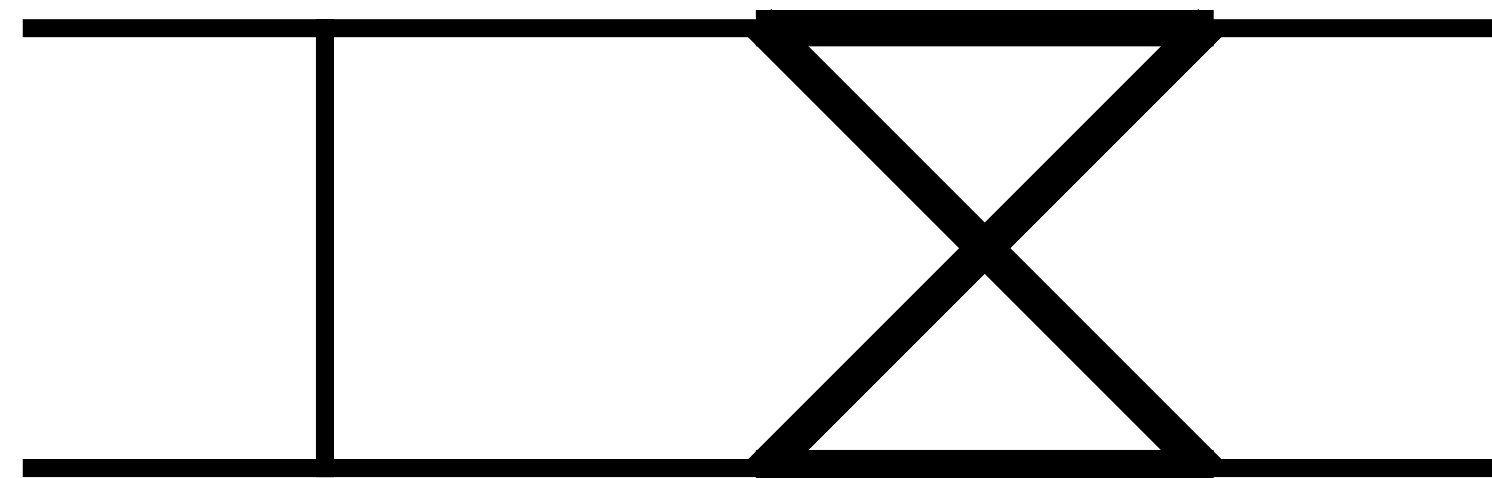
Two different subsets

Canonical  
Logarithmic

ELLIPTIC  
Sectors



eMPLS

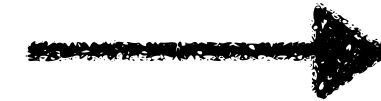


- ❖ Non trivial solution!
- ❖ Nine square roots in the alphabet
- ❖ Integrals involving eMPLs kernels

[A.Von Manteuffel, L.Tancredi]

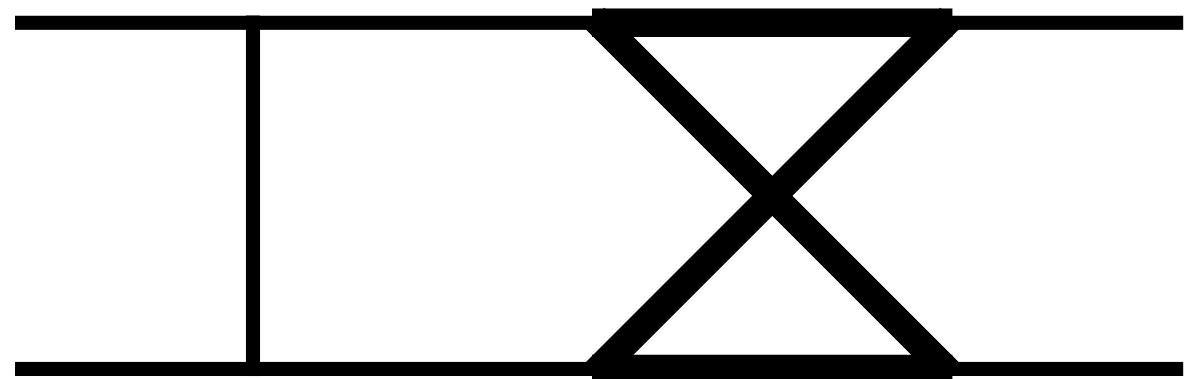
# Maximal Cut

The homogeneous part of the DEs contains elliptic functions

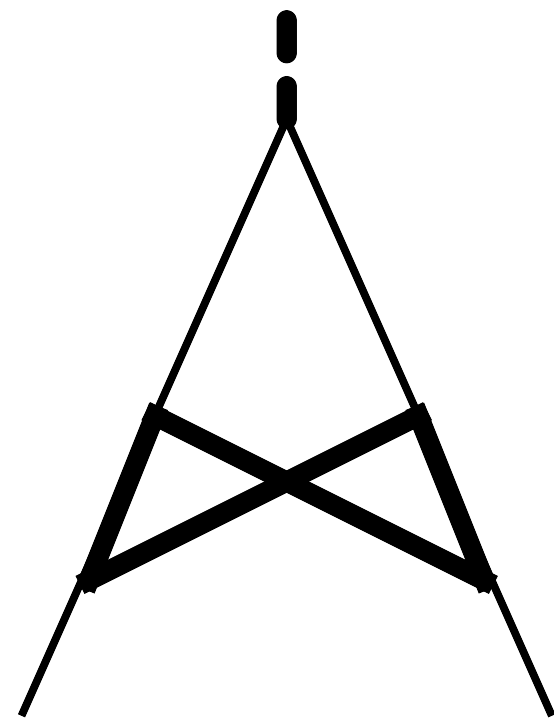


This is verified by the **Maximal Cut**

Genus one



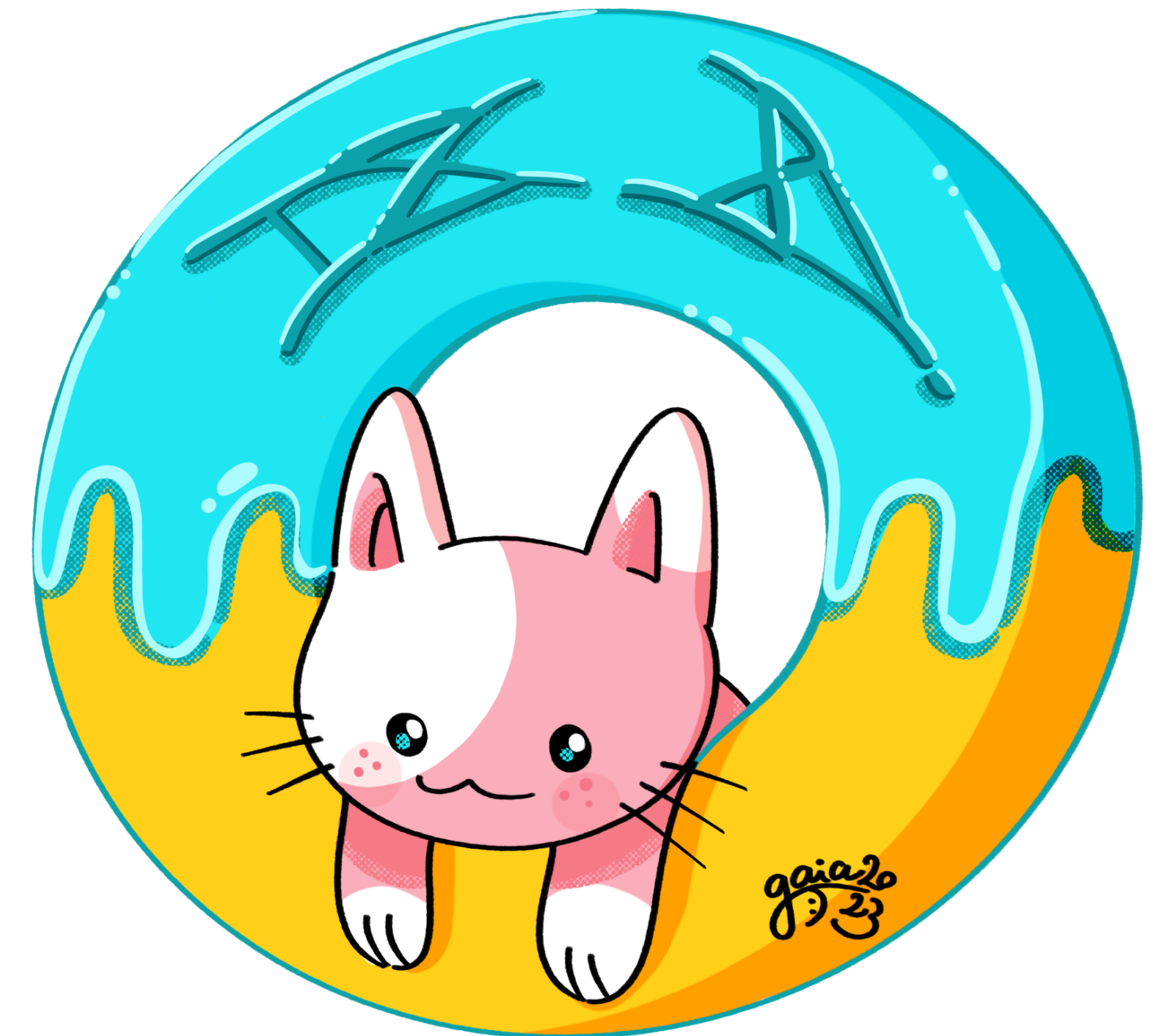
$$y_c^2 = (z_8 + t)(z_8 + s + t)(z_8 - z_+)(z_8 - z_-)$$



$$y^2 = \bar{x}_2(\bar{x}_2 - 1)(\bar{x}_2 - b_+)(\bar{x}_2 - b_-)$$

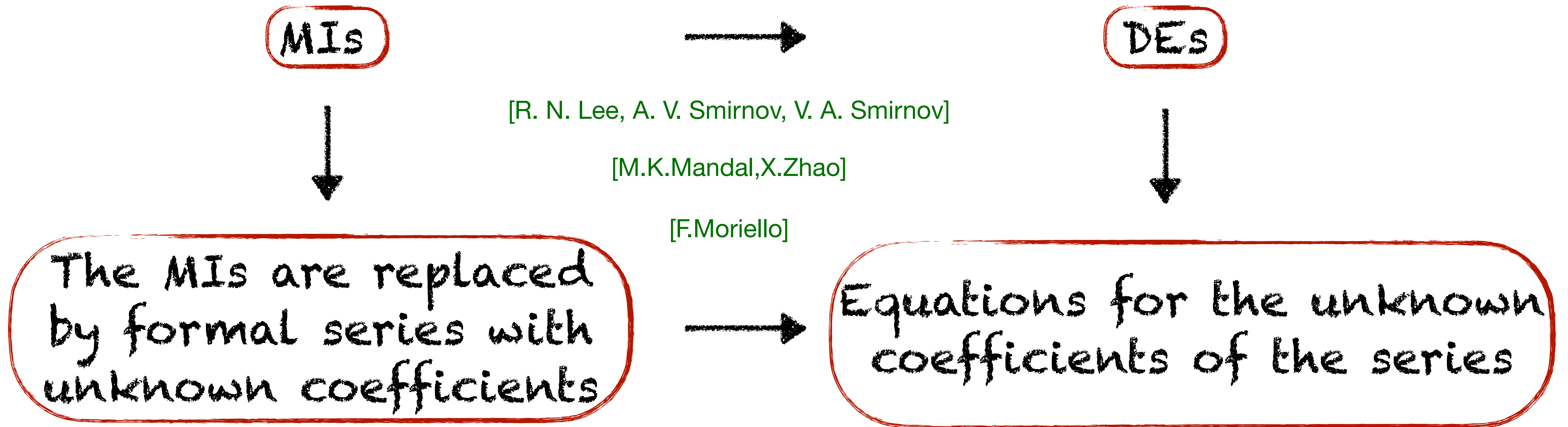
[J.Broedel,C.Duhr,F.Dulat,B.Penante,L.Tancredi]

The elliptic curve  $y_c^2$  degenerates to  $y$  in the forward limit  $t = 0$



[G.Fontana]

# Generalised power series approach



## Pros:

- ❖ It doesn't depend on the function space, so it allows us to avoid elliptic integrals
- ❖ Values at arbitrary phase-space points
- ❖ Can be used to perform phenomenological studies

# Numerical evaluation of the Master Integrals

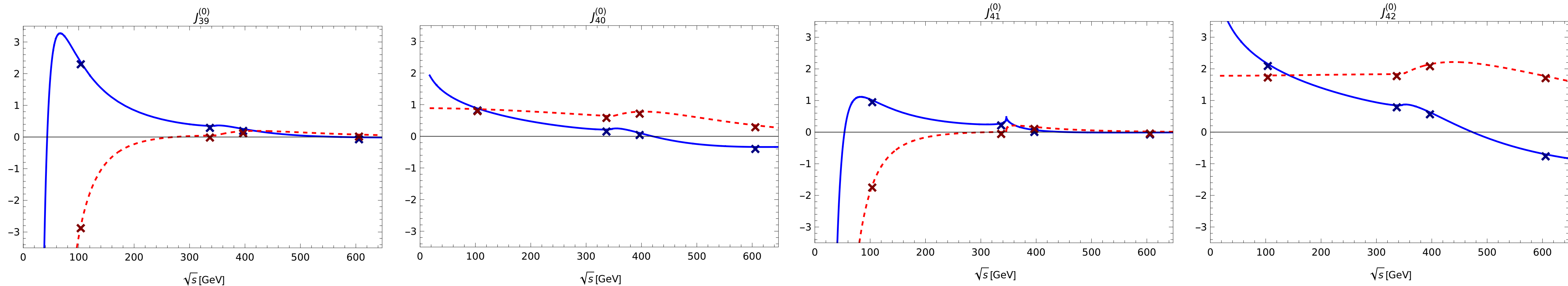
The numerical evaluation of the Master Integrals has been made with DiffExp [M.Hidding]

Several check for the numerical evaluation with AMFLow [X.Liu,Y.Ma]

Correspondance  
up to 200 digits!

----- Imaginary Part  
————— Real Part

For the four elliptic boxes in NPL Topology, at a fixed angle:



# Hard Function

$\mathcal{F}_i^{(2)}$  does not have IR poles!

After remove the UV poles, we can compute the NNLO Hard Function

In  $q_T$  - subtraction scheme:

$$d\sigma_{NNLO}^{\gamma\gamma} = \mathcal{H}_{NNLO}^{\gamma\gamma} \otimes d\sigma_{LO}^{\gamma\gamma} + [d\sigma_{NLO}^{\gamma\gamma+jets} - d\sigma_{NLO}^{CT}]$$

The diagram shows the equation above with four arrows pointing from the terms to explanatory text below:

- An arrow from  $\mathcal{H}_{NNLO}^{\gamma\gamma}$  points to the text "Contains our massive contribution" (written in yellow).
- An arrow from  $d\sigma_{LO}^{\gamma\gamma}$  points to the text "LO cross section" (written in green).
- An arrow from  $d\sigma_{NLO}^{\gamma\gamma+jets}$  points to the text "NLO cross section for  $\gamma\gamma + jet$ " (written in purple).
- An arrow from  $d\sigma_{NLO}^{CT}$  points to the text "CT needed to cancel the IR singularities" (written in red).

The Hard function admit a perturbative expansion:

$$\mathcal{H}^{\gamma\gamma} = 1 + \frac{\alpha_S}{\pi} \mathcal{H}_{NLO}^{\gamma\gamma} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_{NNLO}^{\gamma\gamma} + \dots$$

# Numerical evaluation of the Hard Function

A numerical grid has been prepared for all the MIs of the PLA and NPL, covering the  $2 \rightarrow 2$  physical space:

$$s > 0, \quad t = -\frac{s}{2}(1 - \cos(\theta)), \quad -s < t < 0$$

$-0.99 < \cos(\theta) < +0.99$       24 different values

$8 \text{ GeV} < \sqrt{s} < 2.2 \text{ TeV}$       573 different values

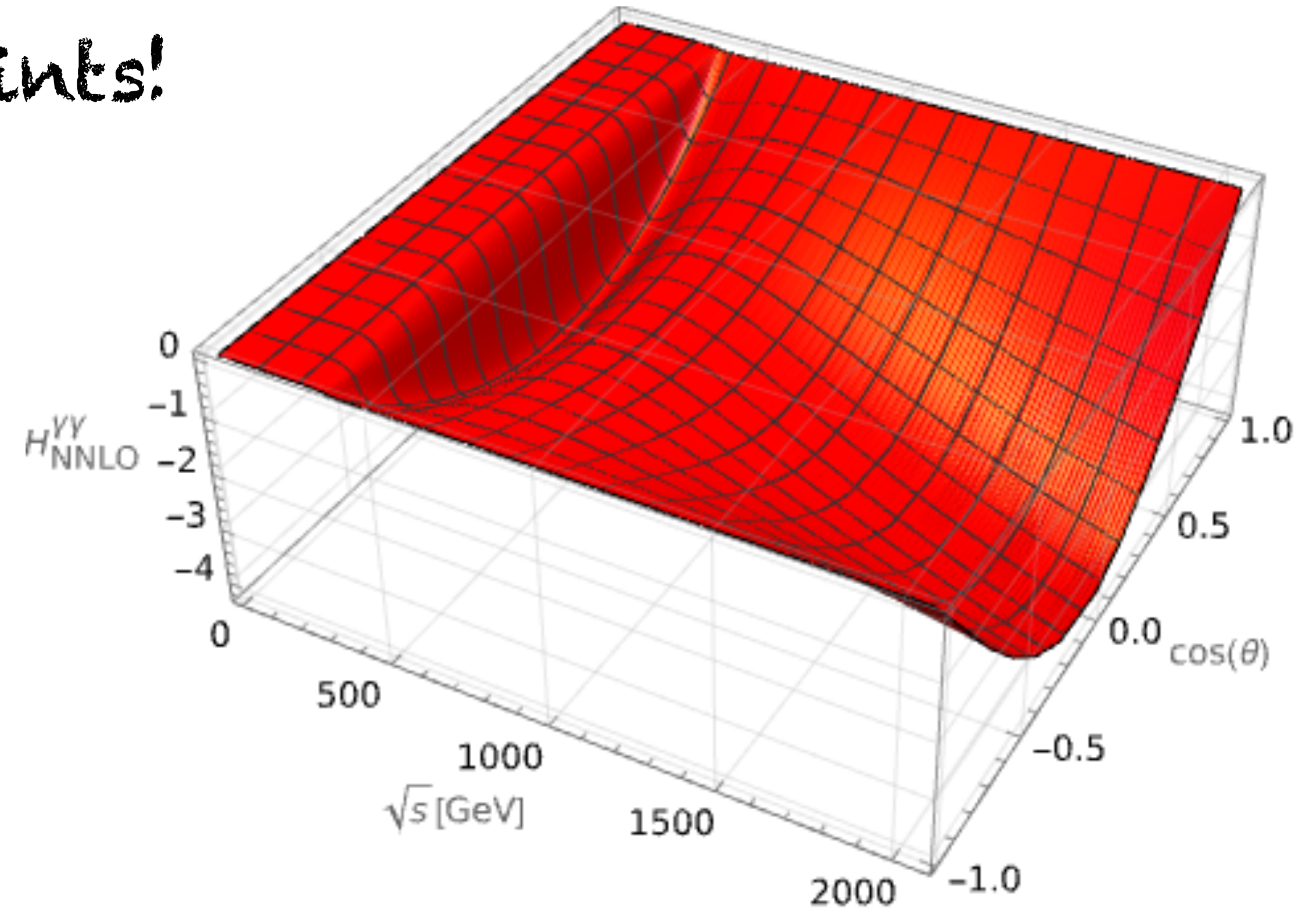
**13752 points!**

DiffExp time for the  $H_{NNLO}^{\gamma\gamma}$  MIs evaluation:

PLA Topology: 32 MIs in  $\mathcal{O}(2.5h)$

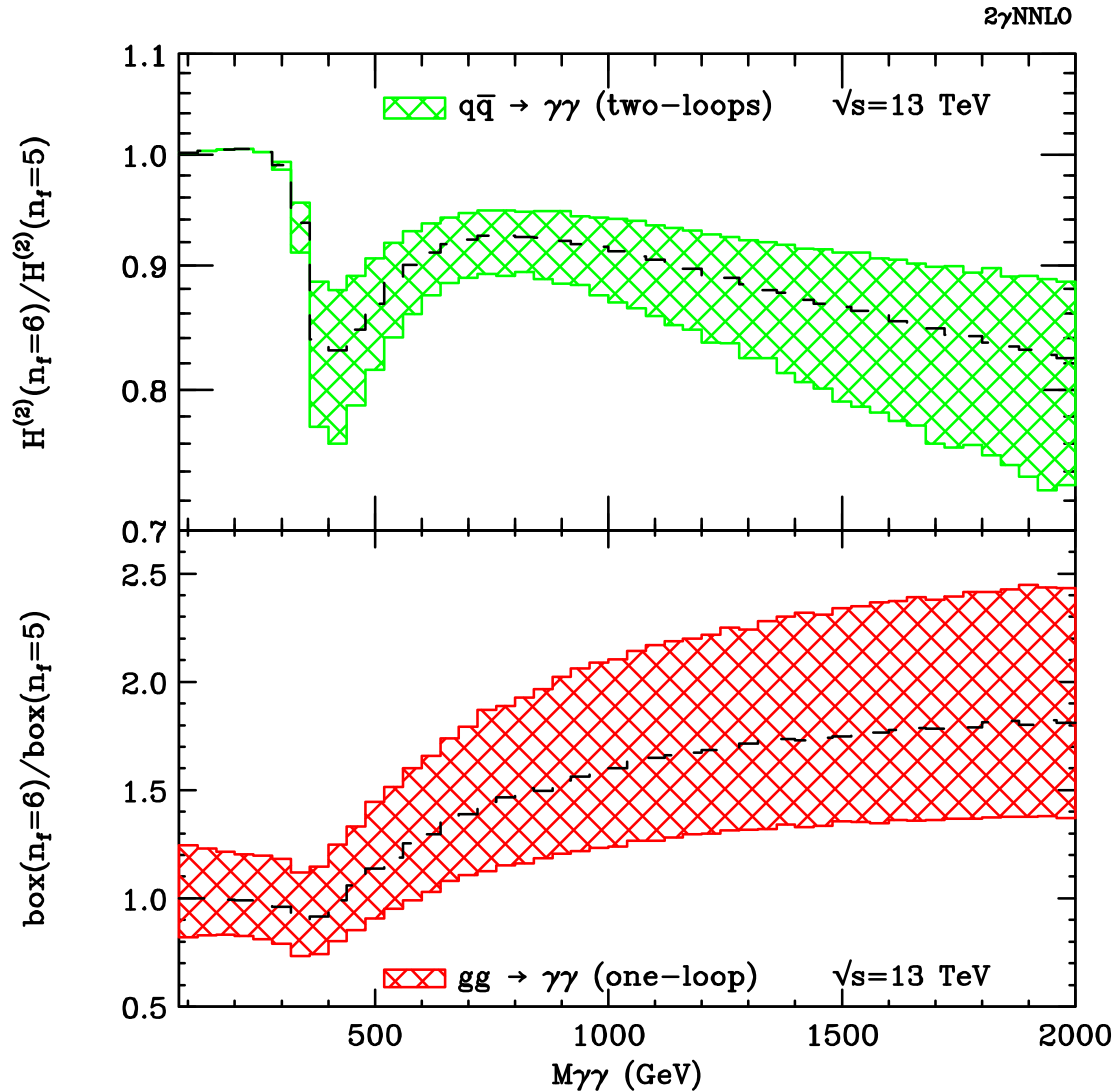
NPL Topology: 36 MIs in  $\mathcal{O}(10.5h)$

**On a single core!**





# Final Results



Fiducial cuts

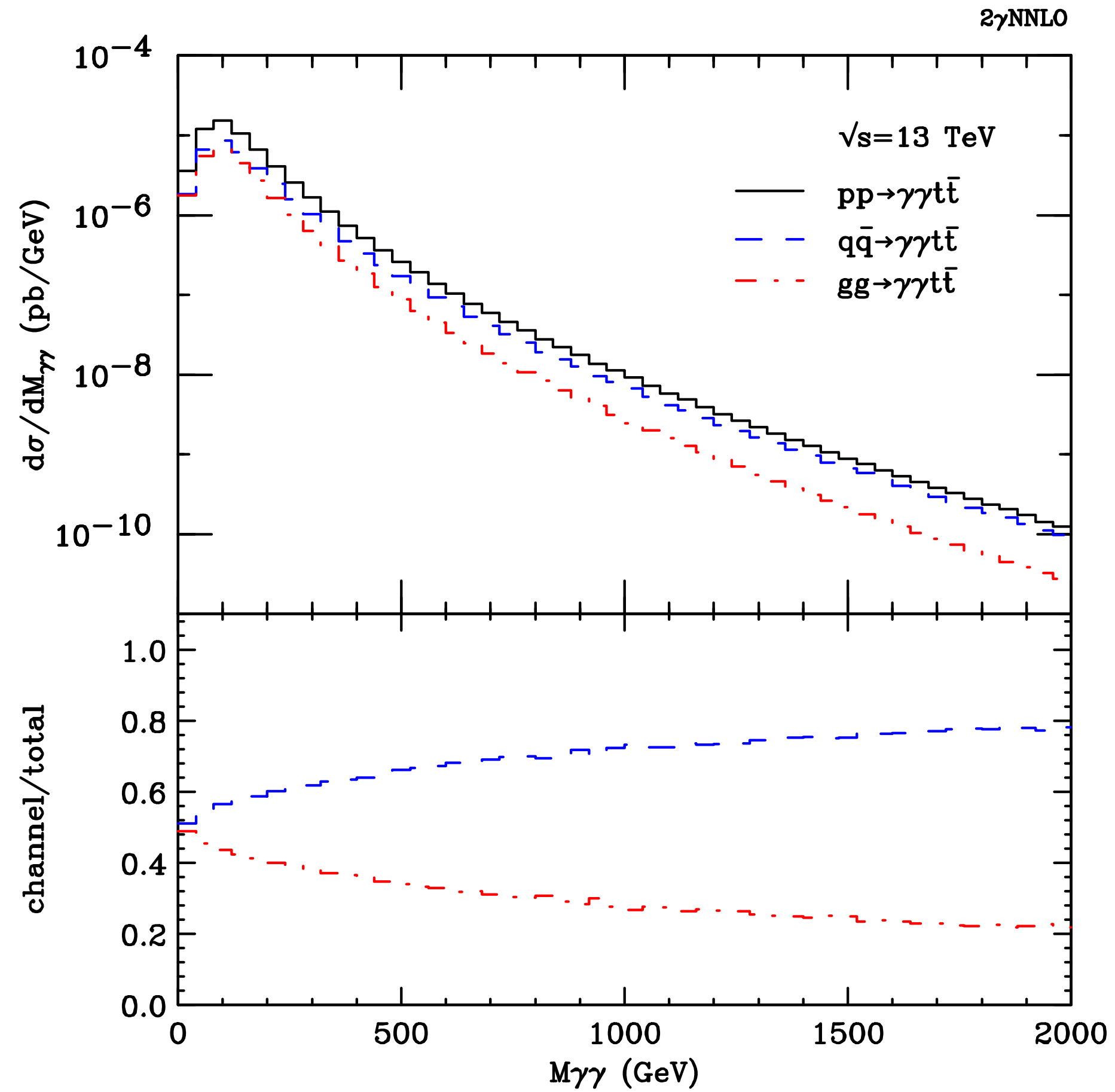
- ❖  $\sqrt{s} = 13$  TeV
- ❖  $p_{T_\gamma}^{\text{Hard}} \geq 40$  GeV
- ❖  $p_{T_\gamma}^{\text{Soft}} \geq 30$  GeV
- ❖  $|y_\gamma| < 2.37$  Excluding  $1.37 < |y_\gamma| < 1.52$

[The ATLAS Collaboration]

Smooth isolation cone

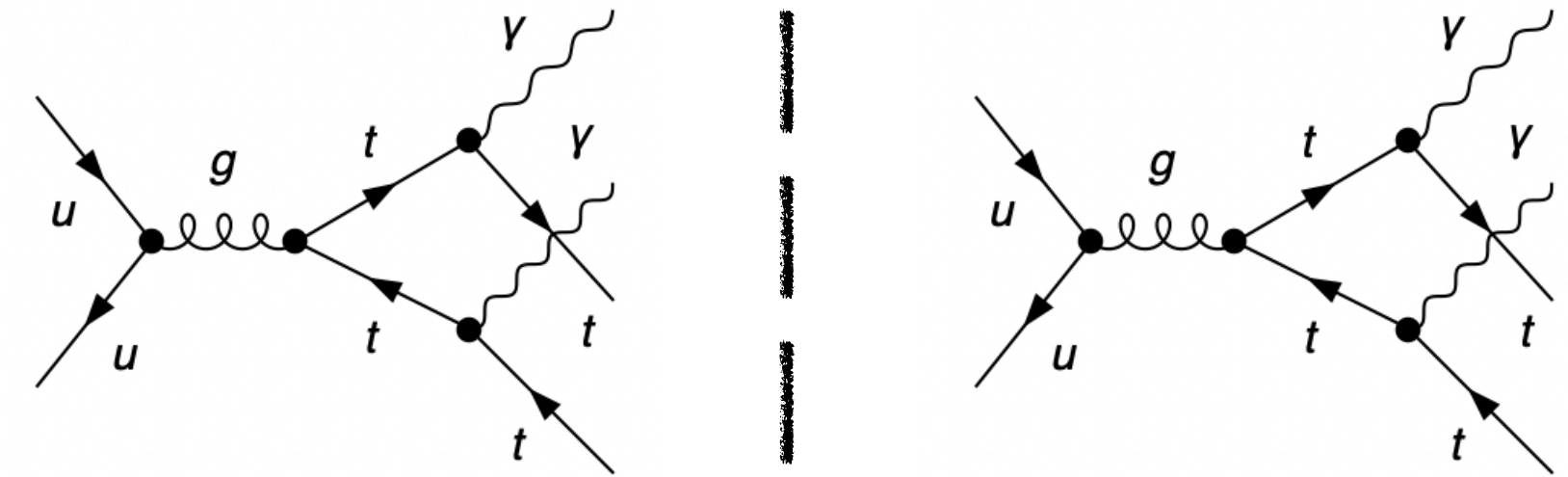
- ❖  $E_T^{\text{had}}(r) \leq \epsilon p_{T_\gamma} \chi(r; R)$
- ❖  $\chi(r; R) = \left(\frac{r}{R}\right)^{2n}$
- ❖  $R = 0.4$
- ❖  $\epsilon = 0.09$
- ❖  $n = 1$

# Final Results

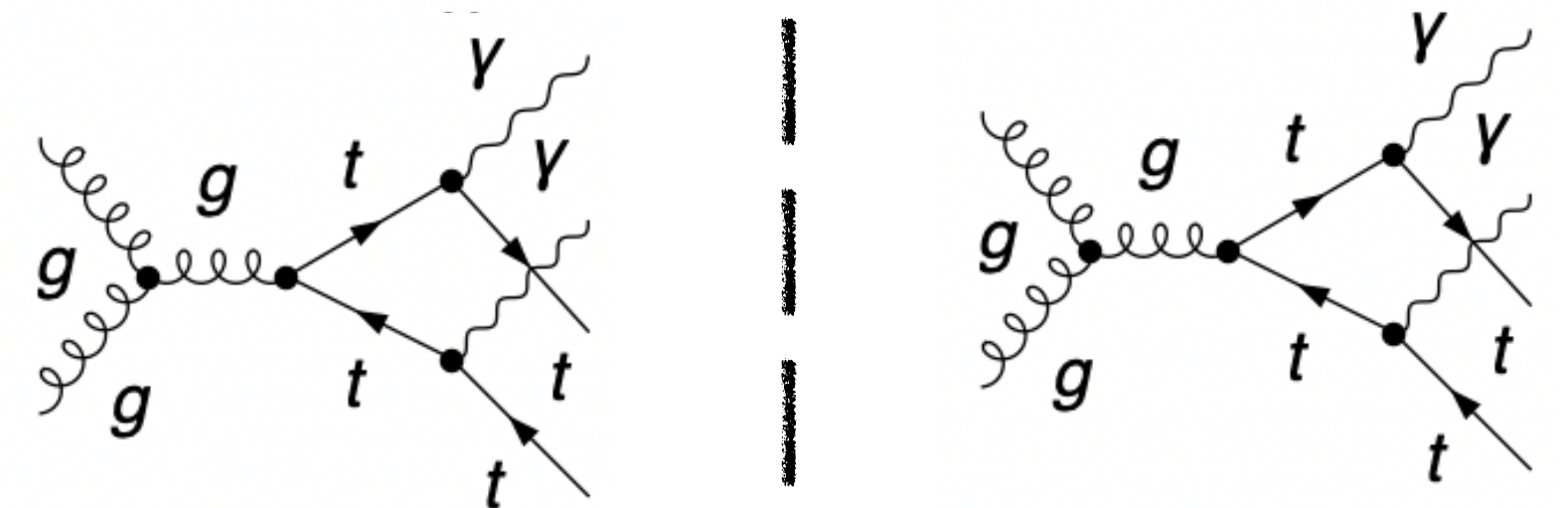


Invariant mass distribution of the Double-Real contribution to the NNLO fully massive result

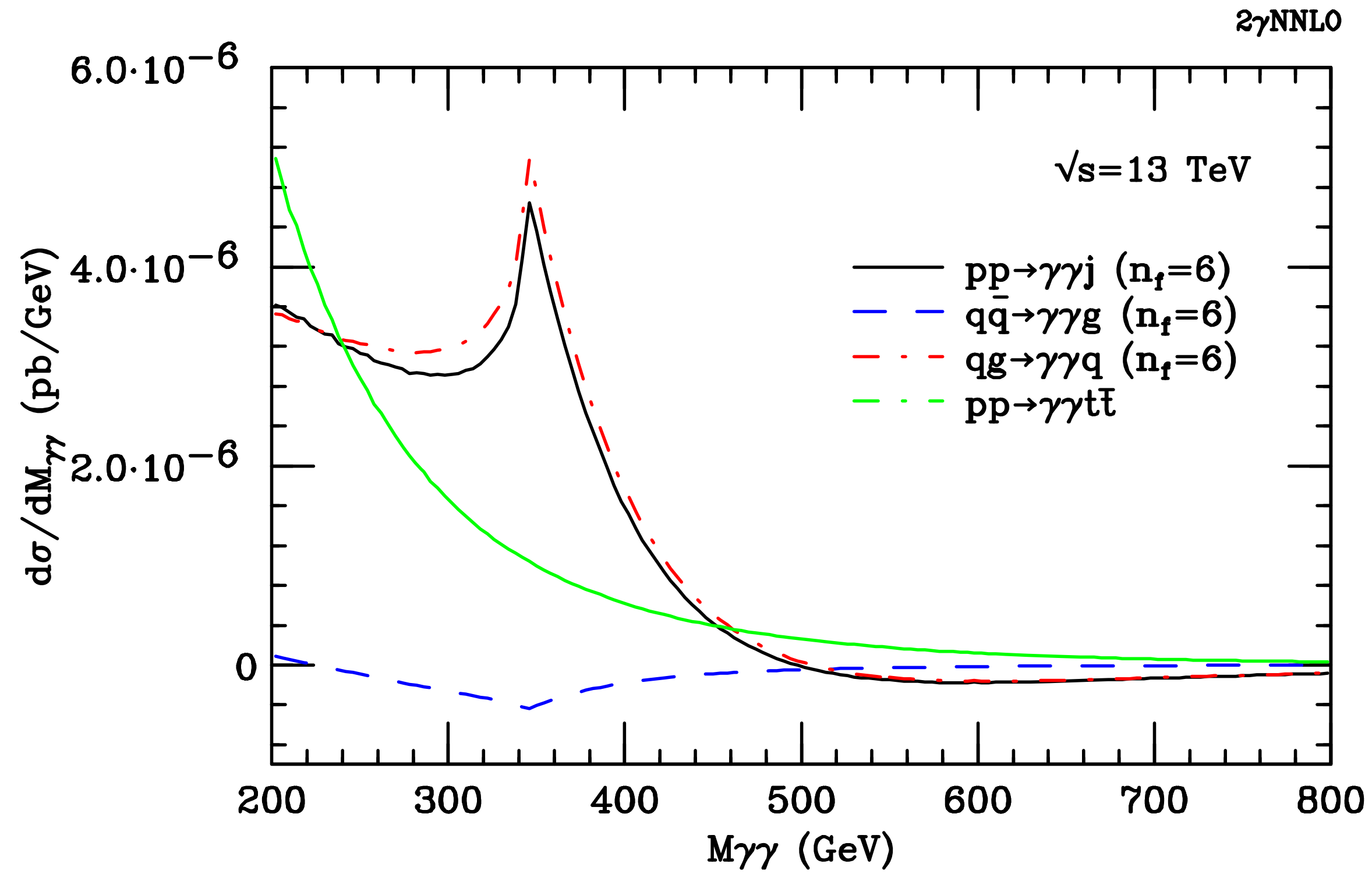
$$q\bar{q} \rightarrow t\bar{t}\gamma\gamma$$



$$gg \rightarrow t\bar{t}\gamma\gamma$$

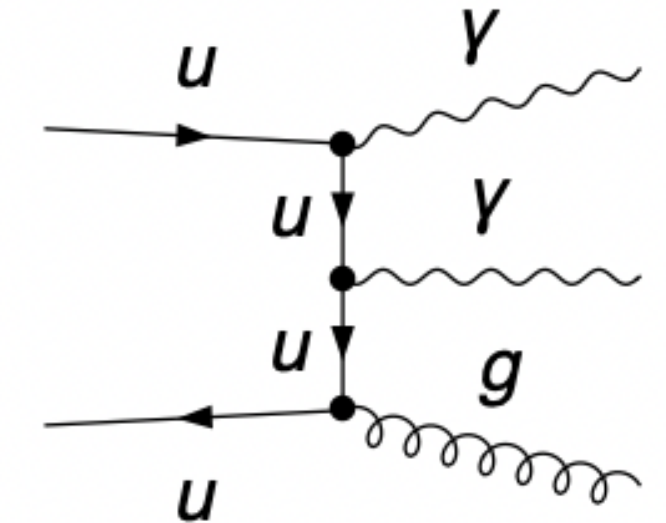
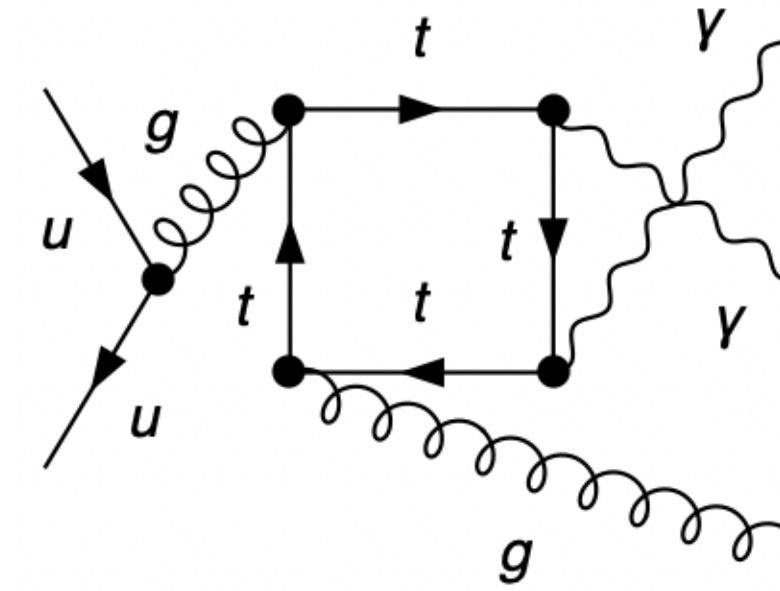


# Final Results

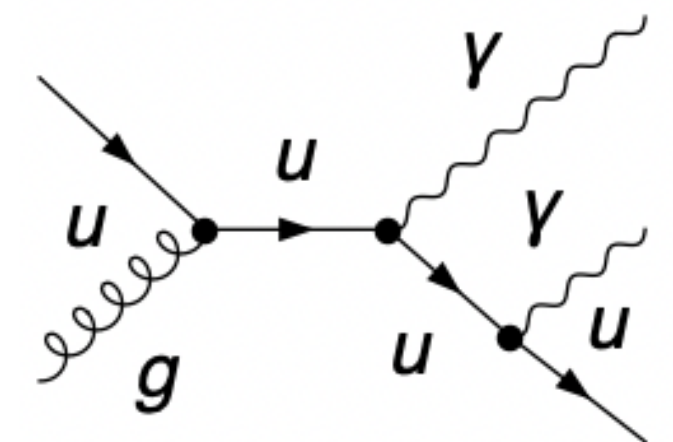
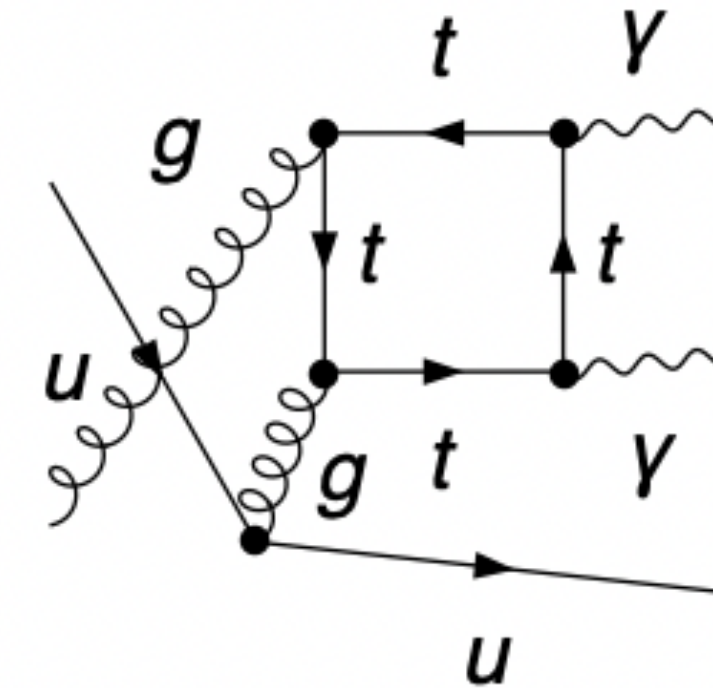


Invariant mass  
distribution of the  
one-loop massive  
contribution at NNLO

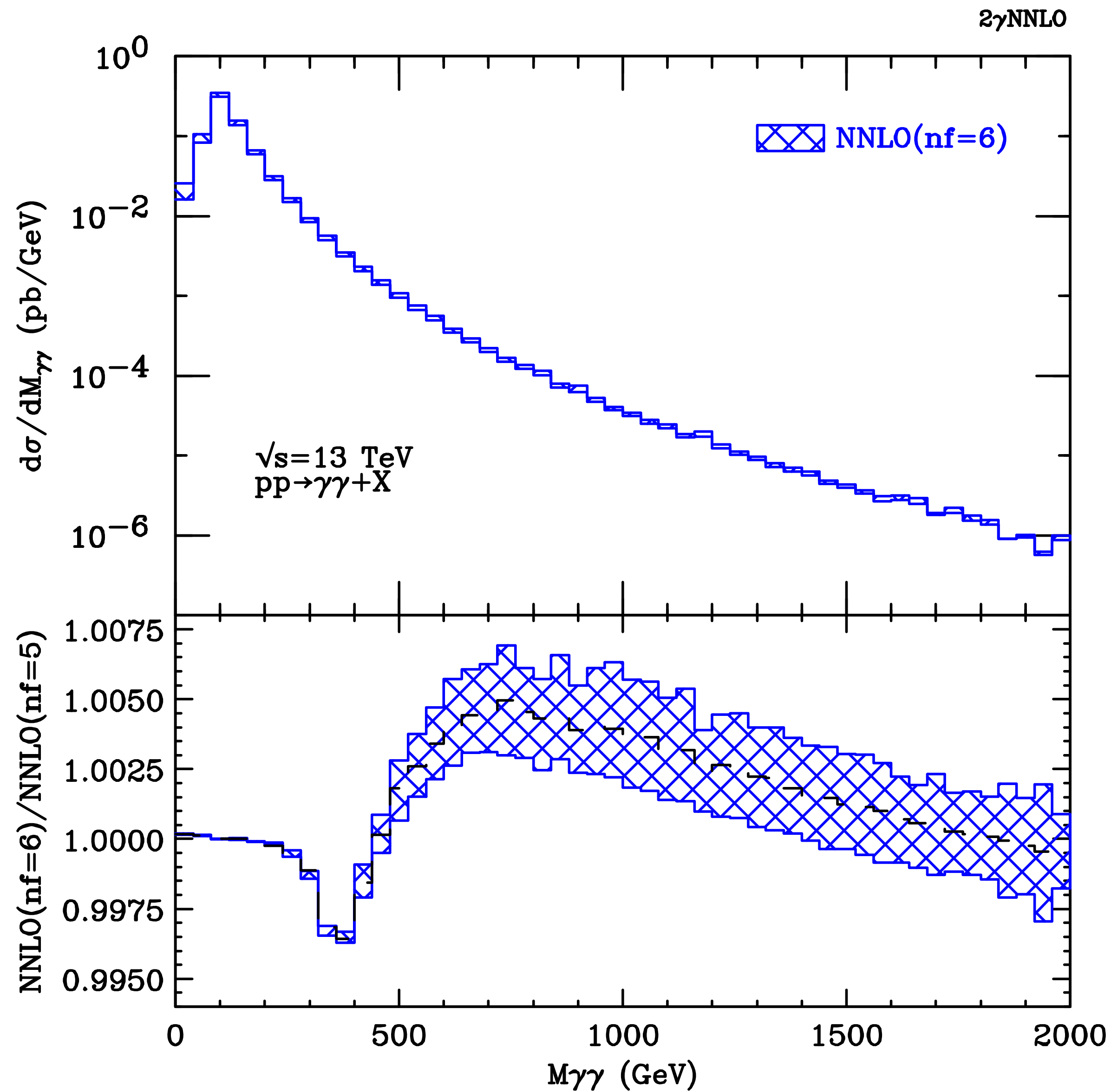
$q\bar{q} \rightarrow \gamma\gamma g$



$qg \rightarrow \gamma\gamma q$



# Final Results



NNLO invariant mass distribution with full top quark mass dependence

# Conclusions

- ❖ We computed the massive two-loop form factors
- ❖ The MIs were evaluated using the generalised power series method
- ❖ Computation of the Massive Hard Function NNLO
- ❖ We obtained the first phenomenological results for the full massive NNLO diphoton production

THANKS FOR YOUR  
ATTENTION!