Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Research Training Group Physics of the Heaviest Particles at the LHC

Funded by Deutsche Forschungsgemeinschaft German Research Foundation

AutoEFT: Constructing and Exploring On-Shell Bases of Effective Field Theories

Magnus Schaaf

in collaboration with R. Harlander, J. Linder, M. Rzehak

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University

24 August 2023 — European Physical Society Conference on High Energy Physics

Outline

Effective Field Theories

Permutation Symmetries

AutoEFT

The Standard Model as EFT (SMEFT)

Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i C_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \dots$$

$\mathcal{O}$s invariant under Lorentz and gauge symmetry

Operators

$$\mathcal{O}^{(5)} = (L_\alpha H)(L^\alpha H)$$

$$\mathcal{O}^{(6)} = (H^\dagger H)\, G^A_{\mu\nu} G^{A\,\mu\nu},\; (H^\dagger \tau^I H)\, W^I_{\mu\nu} B^{\mu\nu},\; \dots$$

$$\vdots$$

A complete set of *independent* operators is called a **basis**.

The Standard Model as EFT (SMEFT)

Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i C_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \dots$$

$\mathcal{O}$s invariant under Lorentz and gauge symmetry

Operators

$$\mathcal{O}^{(5)} = (L_\alpha H)(L^\alpha H)$$

$$\mathcal{O}^{(6)} = (H^\dagger H)\, G^A_{\mu\nu} G^{A\,\mu\nu},\ (H^\dagger \tau^I H)\, W^I_{\mu\nu} B^{\mu\nu},\ \dots$$

$$\vdots$$

A complete set of *independent* operators is called a **basis**.

The Standard Model as EFT (SMEFT)

Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i C_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)}\mathcal{O}_i^{(6)} + \dots$$

$\mathcal{O}$s invariant under Lorentz and gauge symmetry

Operators

$$\mathcal{O}^{(5)} = (L_\alpha H)(L^\alpha H)$$

$$\mathcal{O}^{(6)} = (H^\dagger H)\, G^A_{\mu\nu} G^{A\,\mu\nu},\; (H^\dagger \tau^I H)\, W^I_{\mu\nu} B^{\mu\nu},\; \dots$$

$$\vdots$$

A complete set of *independent* operators is called a **basis**.

Redundancies

1. **Equations of motion:** $\mathcal{O} \sim \mathcal{O}' + \frac{\delta S}{\delta \Phi_i} \mathcal{O}''$

2. **Integration by parts:** $\mathcal{O} \sim \mathcal{O}' + \partial \mathcal{O}''$

3. **Permutation symmetries:** $\mathcal{O} \sim \pi_\Phi \circ \mathcal{O}'$

4. **Fierz identities:** $g_{\mu\nu} \sigma^\mu_{\alpha\dot{\alpha}} \sigma^\nu_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta} \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$

Brief History of SMEFT

5

1979

[Weinberg]

Baryon- and Lepton-Nonconserving Processes

Steven Weinberg
Phys. Rev. Lett. **43**, 1566 – Published 19 November 1979

$$f_{abmn}\, \overline{\ell^{C}_{iaL}}\, \ell_{jbL}\, \phi_k^{(m)}\, \phi_\ell^{(n)}\, \epsilon_{ik}\, \epsilon_{j\ell}\,.$$

Only a single *complex* term at $d = 5$

Brief History of SMEFT

5 6

1979 *1986*

[Weinberg] [Buchmüller,Wyler]

Effective lagrangian analysis of new interactions and flavour conservation

W. Buchmüller, D. Wyler

The effective Lagrangian contains 1 dimension-five operator, which violates lepton number, and 80 baryon- and lepton-number conserving dimension-six operators.

22 terms redundant, 1 missing

Brief History of SMEFT

5	6	6
1979	*1986*	*2010*
[Weinberg]	[Buchmüller,Wyler]	[Grzadkowski *et al.*]

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

$$59 + 5 \text{ terms } (n_f = 1)$$

Brief History of SMEFT

5 — *1979* — [Weinberg]

6 — *1986* — [Buchmüller,Wyler]

6 — *2010* — [Grzadkowski *et al.*]

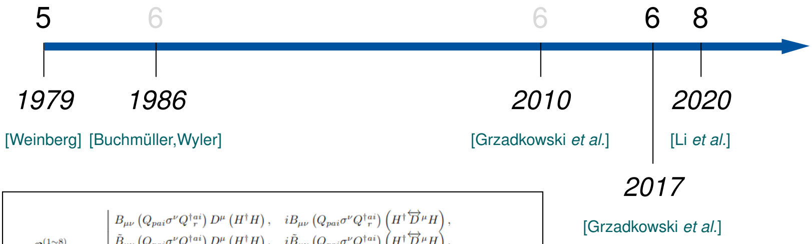
6 — *2017* — [Grzadkowski *et al.*]

B-violating	
Q_{duq}	
Q_{qqu}	
$Q_{qqq}^{(1)}$	
$Q_{qqq}^{(3)}$	
Q_{duu}	

B-violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

$59 + 4$ terms ($n_f = 1$)

Brief History of SMEFT



$$
\begin{array}{l|l}
\mathcal{O}^{(1\sim8)}_{BQQ^\dagger HH^\dagger D} &
\begin{array}{l}
B_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger ai}_{r}\right)D^\mu\left(H^\dagger H\right), \quad iB_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger ai}_{r}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right),\\
\tilde{B}_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger ai}_{r}\right)D^\mu\left(H^\dagger H\right), \quad i\tilde{B}_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger ai}_{r}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right),\\
B_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger aj}_{r}\right)D^\mu\left(H^{\dagger i} H_j\right), \quad iB_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger aj}_{r}\right)\left(H^{\dagger i} \overleftrightarrow{D}^\mu H_j\right),\\
\tilde{B}_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger aj}_{r}\right)D^\mu\left(H^{\dagger i} H_j\right), \quad i\tilde{B}_{\mu\nu}\left(Q_{pai}\sigma^\nu Q^{\dagger aj}_{r}\right)\left(H^{\dagger i} \overleftrightarrow{D}^\mu H_j\right)
\end{array}\\
\mathcal{O}^{(1\sim4)}_{Gu_{\mathbb{C}}u^\dagger_{\mathbb{C}}HH^\dagger D} &
\begin{array}{l}
G^A_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu\left(\lambda^A\right)^b_a u^\dagger_{\mathbb{C}rb}\right)D^\mu\left(H^\dagger H\right), \quad iG^A_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu\left(\lambda^A\right)^b_a u^\dagger_{\mathbb{C}rb}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right),\\
\tilde{G}^A_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu\left(\lambda^A\right)^b_a u^\dagger_{\mathbb{C}rb}\right)D^\mu\left(H^\dagger H\right), \quad i\tilde{G}^A_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu\left(\lambda^A\right)^b_a u^\dagger_{\mathbb{C}rb}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right)
\end{array}\\
\mathcal{O}^{(1\sim4)}_{Wu_{\mathbb{C}}u^\dagger_{\mathbb{C}}HH^\dagger D} &
\begin{array}{l}
W^I_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)D^\mu\left(H^\dagger\tau^I H\right), \quad iW^I_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)\left(H^\dagger\tau^I \overleftrightarrow{D}^\mu H\right),\\
\tilde{W}^I_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)D^\mu\left(H^\dagger\tau^I H\right), \quad i\tilde{W}^I_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)\left(H^\dagger\tau^I \overleftrightarrow{D}^\mu H\right)
\end{array}\\
\mathcal{O}^{(1\sim4)}_{Bu_{\mathbb{C}}u^\dagger_{\mathbb{C}}HH^\dagger D} &
\begin{array}{l}
B_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)D^\mu\left(H^\dagger H\right), \quad iB_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right),\\
\tilde{B}_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)D^\mu\left(H^\dagger H\right), \quad i\tilde{B}_{\mu\nu}\left(u_{\mathbb{C}p}{}^{a}\sigma^\nu u^\dagger_{\mathbb{C}ra}\right)\left(H^\dagger \overleftrightarrow{D}^\mu H\right)
\end{array}
\end{array}
$$

44.807 operators ($n_f = 3$) also by [Murphy; 2020]

Brief History of SMEFT

5	6	6	6	8	12
1979	*1986*	*2010*	*2017*	*2020*	*2023*
[Weinberg]	[Buchmüller,Wyler]	[Grzadkowski *et al.*]	[Grzadkowski *et al.*]	[Li *et al.*]	[Harlander,Kempkens,MS]


```
# '1LL_3QL.yml' generated by AutoEFT 1.0.0
version: 1.0.0
type:
- {LL: 1, QL: 3}
- complex
generations: {LL: 3, QL: 3}
n_terms: 3
n_operators: 57
invariants:
  Lorentz:
    O(Lorentz,1): +eps(1_1,3_1)*eps(2_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
    O(Lorentz,2): +eps(1_1,2_1)*eps(3_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
  SU3:
    O(SU3,1): +eps(2_1,3_1,4_1) * LL*QL(2_1)*QL(3_1)*QL(4_1)
  SU2:
    O(SU2,1): +eps(1_1,3_1)*eps(2_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
    O(SU2,2): +eps(1_1,2_1)*eps(3_1,4_1) * LL(1_1)*QL(2_1)*QL(3_1)*QL(4_1)
permutation_symmetries:
- vector: Lorentz * SU3 * SU2
- symmetry: {LL: [1], QL: [1, 1, 1]}
  n_terms: 1
  n_operators: 3
```


75.577.476 operators ($n_f = 3$)

Outline

Effective Field Theories

Permutation Symmetries

AutoEFT

Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

Permutation Symmetries

Warsaw Basis *v1*

$$Q_{qqq}^{(1)} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$

$$Q_{qqq}^{(3)} = \epsilon^{\alpha\beta\gamma}(\tau^I\epsilon_{jk})(\tau^I\epsilon_{mn})[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$

$$p, r, s, t = 1, 2, 3 \quad \Rightarrow \quad 2 \cdot 3^4 = 162 \text{ operators}$$

Warsaw Basis *v3*

$$Q_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$

$$p, r, s, t = 1, 2, 3 \quad \Rightarrow \quad 3^4 = 81 \text{ operators}$$

Institute for Theoretical Particle Physics and Cosmology | RWTH AACHEN UNIVERSITY

Permutation Symmetries

Warsaw Basis *v1*

$$Q_{qqq}^{(1)} = \epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$
$$Q_{qqq}^{(3)} = \epsilon^{\alpha\beta\gamma}(\tau^I\epsilon_{jk})(\tau^I\epsilon_{mn})[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$

$$p, r, s, t = 1, 2, 3 \quad \Rightarrow \quad 2 \cdot 3^4 = 162 \text{ operators}$$

Warsaw Basis *v3*

$$Q_{qqq} = \epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}[(q_p^{\alpha j})^T C\, q_r^{\beta k}][(q_s^{\gamma m})^T C\, l_t^n]$$

$$p, r, s, t = 1, 2, 3 \quad \Rightarrow \quad 3^4 = 81 \text{ operators}$$

Permutation Symmetries Decomposed

Warsaw Basis *v1*

$$Q^{(1)}_{qqq} = \frac{1}{3}\mathcal{O}_{[2,1]} - \frac{1}{3}\mathcal{O}_{[3]}$$

$$Q^{(3)}_{qqq} = -\mathcal{O}_{[1,1,1]} - \frac{1}{3}\mathcal{O}_{[2,1]} - \frac{2}{3}\mathcal{O}'_{[2,1]}$$

$\Rightarrow \quad 3 \cdot (8 + 10) + 3 \cdot (1 + 8 + 8) = 105$ operators

Warsaw Basis *v3*

$$Q_{qqq} = \frac{1}{2}\mathcal{O}_{[1,1,1]} + \frac{1}{6}\mathcal{O}_{[3]} + \frac{1}{3}\mathcal{O}'_{[2,1]}$$

$\Rightarrow \quad 3 \cdot (1 + 10 + 8) = 57$ operators ✓

Permutation Symmetries Decomposed

Warsaw Basis *v1*

$$Q_{qqq}^{(1)} = \frac{1}{3}\mathcal{O}_{[2,1]} - \frac{1}{3}\mathcal{O}_{[3]}$$

$$Q_{qqq}^{(3)} = -\mathcal{O}_{[1,1,1]} - \frac{1}{3}\mathcal{O}_{[2,1]} - \frac{2}{3}\mathcal{O}'_{[2,1]}$$

$$\Rightarrow \quad 3 \cdot (8 + 10) + 3 \cdot (1 + 8 + 8) = 105 \text{ operators}$$

Warsaw Basis *v3*

$$Q_{qqq} = \frac{1}{2}\mathcal{O}_{[1,1,1]} + \frac{1}{6}\mathcal{O}_{[3]} + \frac{1}{3}\mathcal{O}'_{[2,1]}$$

$$\Rightarrow \quad 3 \cdot (1 + 10 + 8) = 57 \text{ operators } \checkmark$$

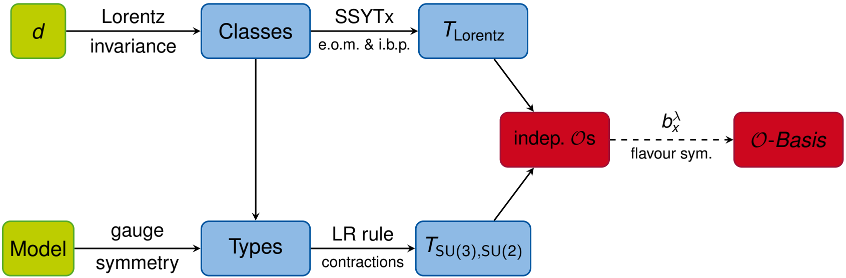
Outline

Effective Field Theories

Permutation Symmetries

AutoEFT

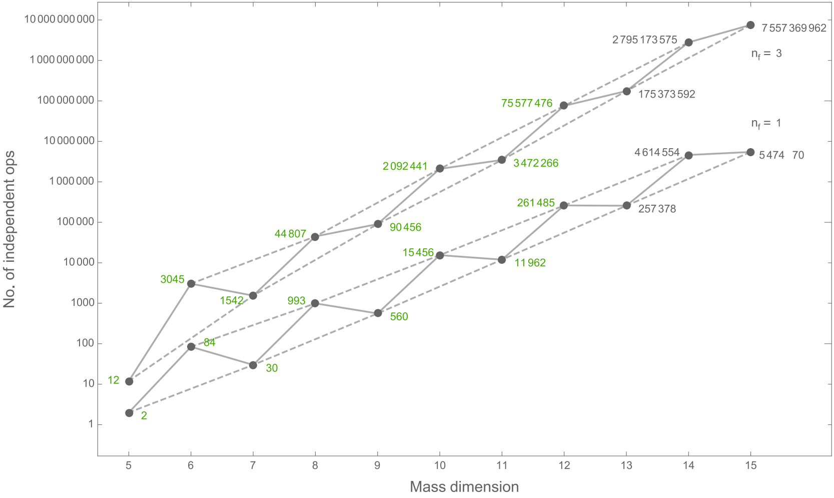
Automated Construction



implemented in `Python` using `SageMath & FORM`

Comparison to Hilbert Series
SMEFT

[Henning,Lu,Melia,Murayama; 2017]



Custom Models

```yaml
name: SMEFT

# arbitrary SU(N) and
# U(1) groups

symmetries:
  sun_groups:
    SU3_C:
      N: 3
    SU2_W:
      N: 2
  u1_groups:
    U1_Y: {}
```

```yaml
fields:

  GL:
    representations:
      Lorentz: -1
      SU3_C: [2,1]
  uC:
    representations:
      Lorentz: -1/2
      SU3_C: [1,1]
      U1_Y: -2/3
    generations: 3


  ...
```

Custom Models

```yaml
name: GRSMEFT

# arbitrary SU(N) and
# U(1) groups

symmetries:
  sun_groups:
    SU3_C:
      N: 3
    SU2_W:
      N: 2
  u1_groups:
    U1_Y: {}
```

```yaml
fields:

  GL:
    representations:
      Lorentz: -1
      SU3_C: [2,1]
  uC:
    representations:
      Lorentz: -1/2
      SU3_C: [1,1]
      U1_Y: -2/3
    generations: 3

  CL:
    representations:
      Lorentz: -2
```

Comparison to Hilbert Series
GRSMEFT

Gravity

$$C_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - \left(g_{\mu[\rho} R_{\sigma]\nu} - g_{\nu[\rho} R_{\sigma]\mu}\right) + \frac{1}{3} g_{\mu[\rho} g_{\sigma]\nu} R$$

$$C_{\mathrm{L}\,\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\, \sigma^{\mu\nu}_{\alpha\beta}\, \sigma^{\rho\sigma}_{\gamma\delta} \in (2, 0)$$

[Marinissen,Rahn,Waalewijn; 2020]

Dimension	5	6	7	8	9	10	11	12	13	14	15
One generation	2	94	30	1096	580	17797	12936	314650	291702	5812440	6518462
Three generations	12	3055	1542	45816	91284	2160964	3567228	79514441	182542620	2995340275	8023911776

TABLE III. Number of operators in the GRSMEFT of a given dimension with 1 or 3 generations.

Summary

AutoEFT

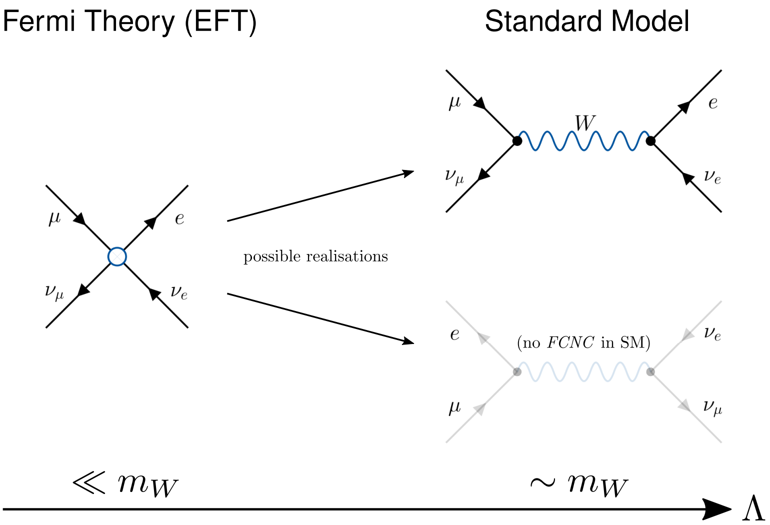
- Automated EFT construction
- Custom low energy models
- Arbitrary mass dimensions d
- Validated up to $d = 12$

Work in progress

- Translate operators
- Operator projection
- Basis transformation

Backup

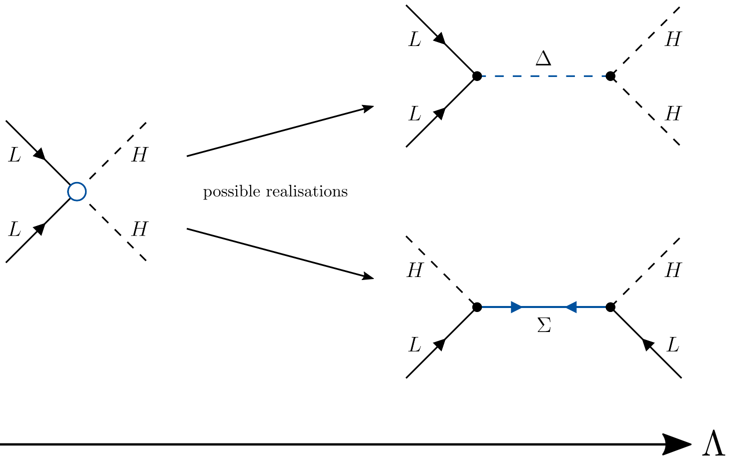
New Physics in Effective Field Theories



New Physics in Effective Field Theories

SMEFT

High Energy Theory ?



Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY

Redundancies

Fierz identities

$$g_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = 2\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}$$
$$\epsilon^{\alpha\beta}\delta^{\gamma}_{\kappa} + \epsilon^{\beta\gamma}\delta^{\alpha}_{\kappa} + \epsilon^{\gamma\alpha}\delta^{\beta}_{\kappa} = 0$$
$$\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta^{\dot{\kappa}}_{\dot{\gamma}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta^{\dot{\kappa}}_{\dot{\alpha}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta^{\dot{\kappa}}_{\dot{\beta}} = 0$$

Field strength tensor

$$[D_\mu, D_\nu] = -iF_{\mu\nu}$$

Redundancies

Equations of motion

$$
\begin{aligned}
D^2\phi + J_\phi &= 0 \\
i\not{D}\psi + J_\psi &= 0 \\
D_\mu F^{\mu\nu} + J_A^\nu &= 0
\end{aligned}
$$

Integration by parts

$$D_\mu(XY) \sim 0 \implies X\, D_\mu Y \sim -D_\mu X\, Y$$

Building Blocks

Lorentz group: $SL(2,\mathbb{C}) \simeq SU(2)_l \times SU(2)_r$

$$\phi \in (0,0)$$

$$\psi_\alpha \in (1/2,0) \qquad \psi^{\dagger\,\dot\alpha} \in (0,1/2)$$

$$F_{\mathrm{L}\,\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0) \qquad F_{\mathrm{R}}^{\dot\alpha\dot\beta} = -\frac{i}{2} F^{\mu\nu}\bar\sigma_{\mu\nu}^{\dot\alpha\dot\beta} \in (0,1)$$

$$D_\alpha^{\dot\alpha} = D_\mu \sigma^{\mu}{}_{\alpha}^{\dot\alpha} \in (1/2,1/2)$$

Gauge group: $SU(3)_C \times SU(2)_W \times U(1)_Y$

$$G_{abc} = \epsilon_{acd}(\lambda^A)^d_b G^A \qquad W_{ij} = \epsilon_{jk}(\tau^I)^k_i W^I$$

Operator Structure

Explicit index notation

$$\epsilon^{abc}\,(L^i\,Q_{j,a})\,(Q_{i,b}\,Q^j_c) = \epsilon^{abc}\,L^i_\alpha\,Q^\alpha_{j,a}\,Q^\beta_{i,b}\,Q^j_{c,\beta}$$

$$= \underbrace{\epsilon^{abc}}_{T_{\mathrm{SU(3)}}}\;\underbrace{\epsilon^{ik}\epsilon^{jl}}_{T_{\mathrm{SU(2)}}}\;\underbrace{\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}}_{T_{\mathrm{Lorentz}}}\;\underbrace{L^\alpha_i\,Q^\beta_{j,a}\,Q^\gamma_{k,b}\,Q^\delta_{l,c}}_{\prod_n \Phi_n}$$

operators of same *type* only differ in choice of T
e.g.: $T_{\mathrm{SU(2)}} = \epsilon^{ij}\epsilon^{kl}$

$\Rightarrow T_{\mathrm{SU(3)}}, T_{\mathrm{SU(2)}}$ & T_{Lorentz} govern the symmetry

SMEFT Operators

General operator

$$\mathcal{O} = T^{\{g\}}_{\mathrm{SU(3)}} T^{\{h\}}_{\mathrm{SU(2)}} T^{\{l\}}_{\mathrm{Lorentz}} \times \prod_{i=1}^{N} (D^{n_i} \Phi_i)_{\{g\},\{h\},\{l\}}$$

Invariant tensors

$$T_{\mathrm{SU(3)}} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)^b_a, \epsilon_{abc}, \epsilon^{abc} \rangle$$
$$T_{\mathrm{SU(2)}} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)^j_i, \epsilon_{ij}, \epsilon^{ij} \rangle$$
$$T_{\mathrm{Lorentz}} \in \langle \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \bar{\sigma}^{\mu\,\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

SMEFT Operators

General operator

$$\mathcal{O} = T^{\{g\}}_{\mathrm{SU}(3)} T^{\{h\}}_{\mathrm{SU}(2)} T^{\{l\}}_{\mathrm{Lorentz}} \times \prod_{i=1}^{N} (D^{n_i}\Phi_i)_{\{g\},\{h\},\{l\}}$$

Invariant tensors

$$T_{\mathrm{SU}(3)} \in \langle f^{ABC}, d^{ABC}, \delta^{AB}, (\lambda^A)_a^b, \epsilon_{abc}, \epsilon^{abc} \rangle$$

$$T_{\mathrm{SU}(2)} \in \langle \epsilon^{IJK}, \delta^{IJ}, (\tau^I)_i^j, \epsilon_{ij}, \epsilon^{ij} \rangle$$

$$T_{\mathrm{Lorentz}} \in \langle \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \bar{\sigma}^{\mu\,\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}} \rangle$$

Irreducible Representations

Lorentz group: $\mathrm{SL}(2,\mathbb{C}) \simeq \mathrm{SU}(2)_l \times \mathrm{SU}(2)_r$

$$\phi \in (0,0) \qquad \psi_\mathrm{L} \in (1/2,0) \qquad F_\mathrm{L} \in (1,0)$$

Equations of motion

$$\partial^2\phi \in \underbrace{(0,0)}_{\sim\phi} \oplus \underbrace{(1,0) \oplus (0,1)}_{\sim F} \oplus (1,1)$$

$$\partial\psi_\mathrm{L} \in \underbrace{(0,1/2)}_{\sim\psi_\mathrm{R}} \oplus (1,1/2)$$

$$\partial F_\mathrm{L} \in \underbrace{(1/2,1/2)}_{\sim A} \oplus (3/2,1/2)$$

Irreducible Representations

Lorentz group: $SL(2,\mathbb{C}) \simeq SU(2)_l \times SU(2)_r$

$$\phi \in (0,0) \qquad \psi_\mathrm{L} \in (1/2,0) \qquad F_\mathrm{L} \in (1,0)$$

Equations of motion

$$\partial^2\phi \in \underbrace{(0,0)}_{\sim\phi} \oplus \underbrace{(1,0)\oplus(0,1)}_{\sim F} \oplus (1,1)$$

$$\partial\psi_\mathrm{L} \in \underbrace{(0,1/2)}_{\sim\psi_\mathrm{R}} \oplus (1,1/2)$$

$$\partial F_\mathrm{L} \in \underbrace{(1/2,1/2)}_{\sim A} \oplus (3/2,1/2)$$

$\Rightarrow$ **only keep symmetric combinations**

Equations of Motion

$$(D^{r-|h|}\Phi)_{\alpha^{(1)}\ldots\alpha^{(r-h)}\dot{\alpha}^{(1)}\ldots\dot{\alpha}^{(r+h)}} \equiv \begin{cases} D_{\alpha^{(1)}\dot{\alpha}^{(1)}}\ldots D_{\alpha^{(r+h)}\dot{\alpha}^{(r+h)}}\Phi_{\alpha^{(r+h+1)}\ldots\alpha^{(r-h)}}\,, & \text{h<0} \\ D_{\alpha^{(1)}\dot{\alpha}^{(1)}}\ldots D_{\alpha^{(r-h)}\dot{\alpha}^{(r-h)}}\Phi_{\dot{\alpha}^{(r-h+1)}\ldots\dot{\alpha}^{(r+h)}}\,, & \text{h>0} \end{cases}$$

$$\begin{aligned} D_{[\alpha\dot{\alpha}}D_{\beta]\dot{\beta}}\phi &= -\epsilon_{\alpha\beta}\tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}D^2\phi + \frac{i}{2}\epsilon_{\alpha\beta}\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}[D_\mu, D_\nu]\phi \\ D_{[\alpha\dot{\alpha}}\psi_{\beta]} &= -\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot{\alpha}} \\ D_{[\alpha\dot{\alpha}}F_{L\,\beta]\gamma} &= 2\epsilon_{\alpha\beta}\sigma^{\nu}_{\gamma\dot{\alpha}}D^\mu F_{\mu\nu} \end{aligned}$$

$$(D^{r-|h|}\Phi)^{(\dot{\alpha}^{(1)}\ldots\dot{\alpha}^{(r+h)})}_{(\alpha^{(1)}\ldots\alpha^{(r-h)})} \equiv (D^{r-|h|}\Phi)^{\dot{\alpha}^{r+h}}_{\alpha^{r-h}} \in \left(\frac{r-h}{2}, \frac{r+h}{2}\right)$$

Irreducible Representations

Integration by parts

$$-D\phi_1\,\phi_2\phi_3 D\phi_4 = \phi_1 D\phi_2\,\phi_3 D\phi_4 + \phi_1\phi_2 D\phi_3\,D\phi_4$$

Irreducible representation of $\mathrm{SU}(N)$

$$-\ \begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 & 4 \\ \hline\end{array}\ =\ -\ \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array}\ +\ \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}$$

Irreducible Representations

Integration by parts

$$-D\phi_1\,\phi_2\phi_3 D\phi_4 = \phi_1 D\phi_2\,\phi_3 D\phi_4 + \phi_1\phi_2 D\phi_3\,D\phi_4$$

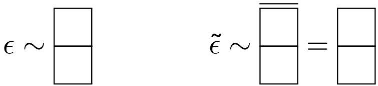
Irreducible representation of $\mathrm{SU}(N)$

$$-\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 & 4 \\ \hline \end{array} = -\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} + \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}$$

$\Rightarrow$ **only keep Semi-Standard Young Tableaux**

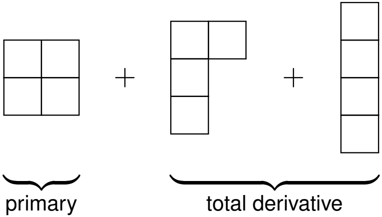
Lorentz Structure as SU(N) State

Young diagrams for $N = 4$ fields



$$\Downarrow$$

$$T_{\text{Lorentz}} = (\tilde{\epsilon}_{\dot{\alpha}_k \dot{\alpha}_l})^{\otimes \tilde{n}} (\epsilon^{\alpha_i \alpha_j})^{\otimes n} \overset{n=\tilde{n}=1}{\sim}$$



Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY

Irreducible Representations

Gauge group: $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$

$$\mathrm{SU}(3): \quad \mathbf{3} \sim \yng(1) \quad \bar{\mathbf{3}} \sim \yng(1,1) \quad \mathbf{8} \sim \yng(2,1)$$

$$\mathrm{SU}(2): \quad \mathbf{2} \sim \yng(1) \quad \mathbf{3} \sim \yng(2)$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\boxed{i} \rightarrow \begin{array}{|c|c|}\hline i & j \\ \hline\end{array} \rightarrow \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1}\end{array} \rightarrow \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline\end{array} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \rightarrow \begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \rightarrow \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1}\end{array} \rightarrow \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline\end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\ytableaushort{i} \rightarrow \ytableaushort{ij} \rightarrow \ytableaushort{ij,k} \rightarrow \ytableaushort{ij,kl} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\ytableaushort{i} \rightarrow \ytableaushort{i,j} \rightarrow \ytableaushort{ik,j} \rightarrow \ytableaushort{ik,jl} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\begin{array}{|c|}\hline i \\ \hline\end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline\end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1}\end{array} \to \begin{array}{|c|c|}\hline \color{red}{i} & j \\ \hline \color{red}{k} & l \\ \hline\end{array} \sim \epsilon^{\color{red}{ik}}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\begin{array}{|c|}\hline i \\ \hline\end{array} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1}\end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline\end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\boxed{i} \to \begin{array}{|c|c|}\hline i & j \\ \hline\end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1}\end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline\end{array} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1}\end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline\end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\boxed{i} \to \begin{array}{|c|c|}\hline i & j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

Irreducible Representations

Invariant contractions

$$L_i \sim \boxed{i} \qquad Q_j \sim \boxed{j} \qquad Q_k \sim \boxed{k} \qquad Q_l \sim \boxed{l}$$

$$\boxed{i} \to \begin{array}{|c|c|}\hline i & j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} \sim \epsilon^{ik}\epsilon^{jl} L_i Q_j Q_k Q_l$$

$$\boxed{i} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j \\ \cline{1-1} \end{array} \to \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array} \sim \epsilon^{ij}\epsilon^{kl} L_i Q_j Q_k Q_l$$

$\Rightarrow$ **tensors according to Littlewood-Richardson rule**

Permutation Symmetries

LQ^3

2 Lorentz $\times$ 2 SU(2) invariant contractions:

$$\mathcal{O}_1 = \mathcal{O}_1^{\text{Lorentz}} \otimes \mathcal{O}_1^{\text{SU}(2)} \qquad \mathcal{O}_2 = \mathcal{O}_1^{\text{Lorentz}} \otimes \mathcal{O}_2^{\text{SU}(2)}$$

$$\mathcal{O}_3 = \mathcal{O}_2^{\text{Lorentz}} \otimes \mathcal{O}_1^{\text{SU}(2)} \qquad \mathcal{O}_4 = \mathcal{O}_2^{\text{Lorentz}} \otimes \mathcal{O}_2^{\text{SU}(2)}$$

Change of basis

$$\begin{pmatrix} \mathcal{O}_{[3]} \\ \mathcal{O}_{[2,1],1} \\ \mathcal{O}_{[2,1],2} \\ \mathcal{O}_{[1,1,1]} \end{pmatrix} = \begin{pmatrix} 2 & -1 & -1 & 2 \\ -1 & 2 & 2 & -1 \\ -1 & -1 & -1 & 2 \\ 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{O}_1 \\ \mathcal{O}_2 \\ \mathcal{O}_3 \\ \mathcal{O}_4 \end{pmatrix}$$

Permutation Symmetries

Adding flavor: $LQ^{f_1}Q^{f_2}Q^{f_3}$

$$\kappa^{f_1 f_2 f_3}_{[3]}\mathcal{O}^{f_1 f_2 f_3}_{[3]} + (\kappa^{f_1 f_2 f_3}_{[2,1]} + \hat{\kappa}^{f_1 f_3 f_2}_{[2,1]})\mathcal{O}^{f_1 f_2 f_3}_{[2,1],1} + \kappa^{f_1 f_2 f_3}_{[1,1,1]}\mathcal{O}^{f_1 f_2 f_3}_{[1,1,1]}$$

$$= \kappa^{f_1 f_2 f_3}(\mathcal{O}^{f_1 f_2 f_3}_{[3]} + \mathcal{O}^{f_1 f_2 f_3}_{[2,1],1} + \mathcal{O}^{f_1 f_2 f_3}_{[1,1,1]}) \equiv \kappa^{f_1 f_2 f_3}\mathcal{O}^{f_1 f_2 f_3}_{LQ^3}$$

Single independent term remains

$$\mathcal{O}_{LQ^3} = \mathcal{O}_1 + 2\mathcal{O}_3 + \mathcal{O}_4$$

$\rightarrow 3 \cdot (10 + 8 + 1) = 57$ independent components of $\kappa^{f_1 f_2 f_3}$

Field Definitions

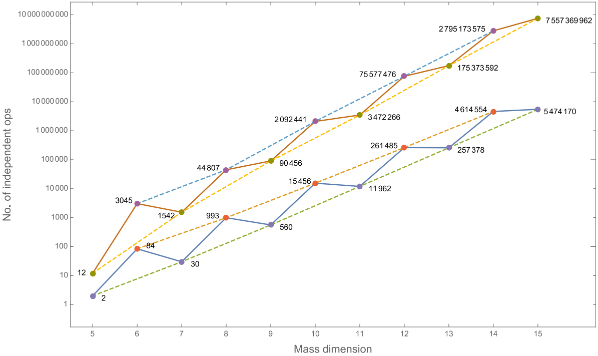
$$q_{\mathrm{L}} = \begin{pmatrix} Q \\ 0 \end{pmatrix} \quad u_{\mathrm{R}} = \begin{pmatrix} 0 \\ u_{\mathbb{C}}^{\dagger} \end{pmatrix} \quad d_{\mathrm{R}} = \begin{pmatrix} 0 \\ d_{\mathbb{C}}^{\dagger} \end{pmatrix} \quad l_{\mathrm{L}} = \begin{pmatrix} L \\ 0 \end{pmatrix} \quad e_{\mathrm{R}} = \begin{pmatrix} 0 \\ e_{\mathbb{C}}^{\dagger} \end{pmatrix}$$

$$F_{\mathrm{L/R}} = \frac{1}{2}\left(F \pm i\tilde{F}\right) \qquad C_{\mathrm{L/R}} = \frac{1}{2}\left(C \pm i\tilde{C}\right)$$

Hilbert Series

$$H(p, \phi_1, \ldots, \phi_n) = \int \mathrm{d}\mu_{\text{conf.}}\, \mathrm{d}\mu_{\text{gauge}} \sum_{i=1}^{\infty} p^i \chi^*_{[i;0]} \prod_j \mathrm{PE}[\phi_j \chi_j(q, \alpha, \beta)]$$

[Henning,Lu,Melia,Murayama; 2017]



Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

Research Training Group Physics of the Heaviest Particles at the LHC
RWTH AACHEN UNIVERSITY

Funded by
DFG Deutsche Forschungsgemeinschaft
German Research Foundation

Magnus Schaaf

Institute for Theoretical Particle Physics and Cosmology
RWTH Aachen University
Sommerfeldstr. 16
52074 Aachen, Germany

www.particle-theory.rwth-aachen.de

TTK Institute for Theoretical Particle Physics and Cosmology
RWTH AACHEN UNIVERSITY