Track reconstruction of charged particles using a 4D quantum algorithm

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EPS-HEP 2023, Hamburg
25/8/2023

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Track reconstruction

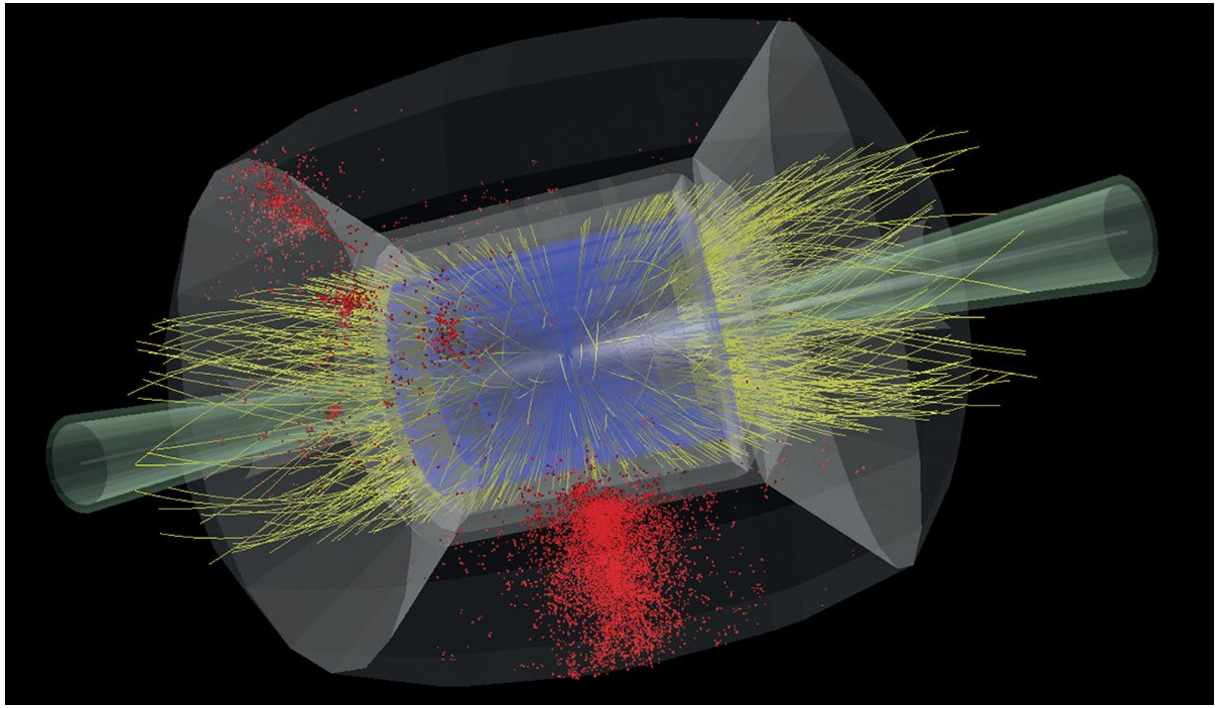
- Tracking can be challenging

  → good detectors and algorithms needed

- Future collider facilities plan to use detectors with timing capability

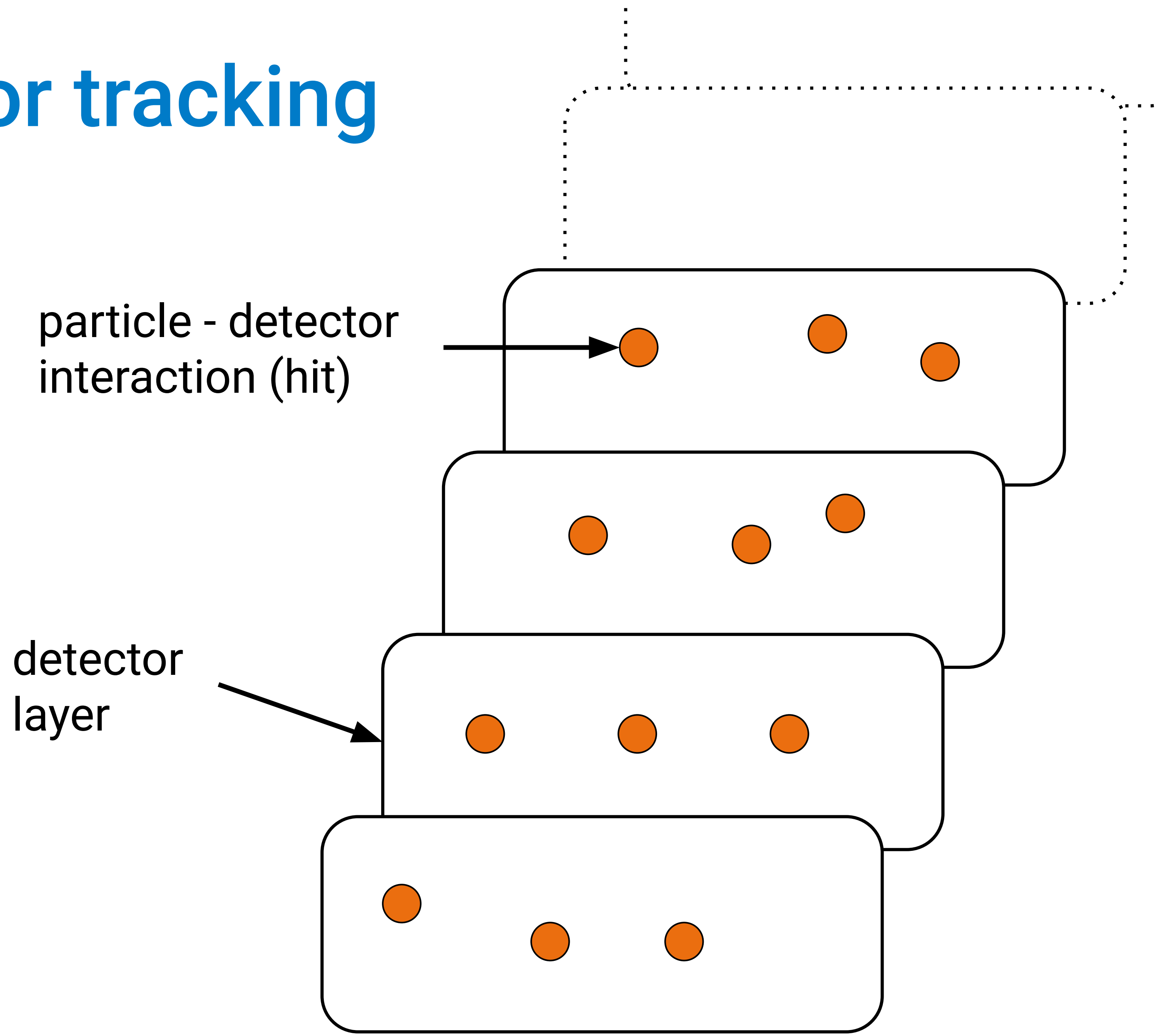
  → 4D algorithms

- This talk:
  - 4D quantum algorithm for pattern recognition of charged particle tracks
  - Results for a Muon Collider detector as an example



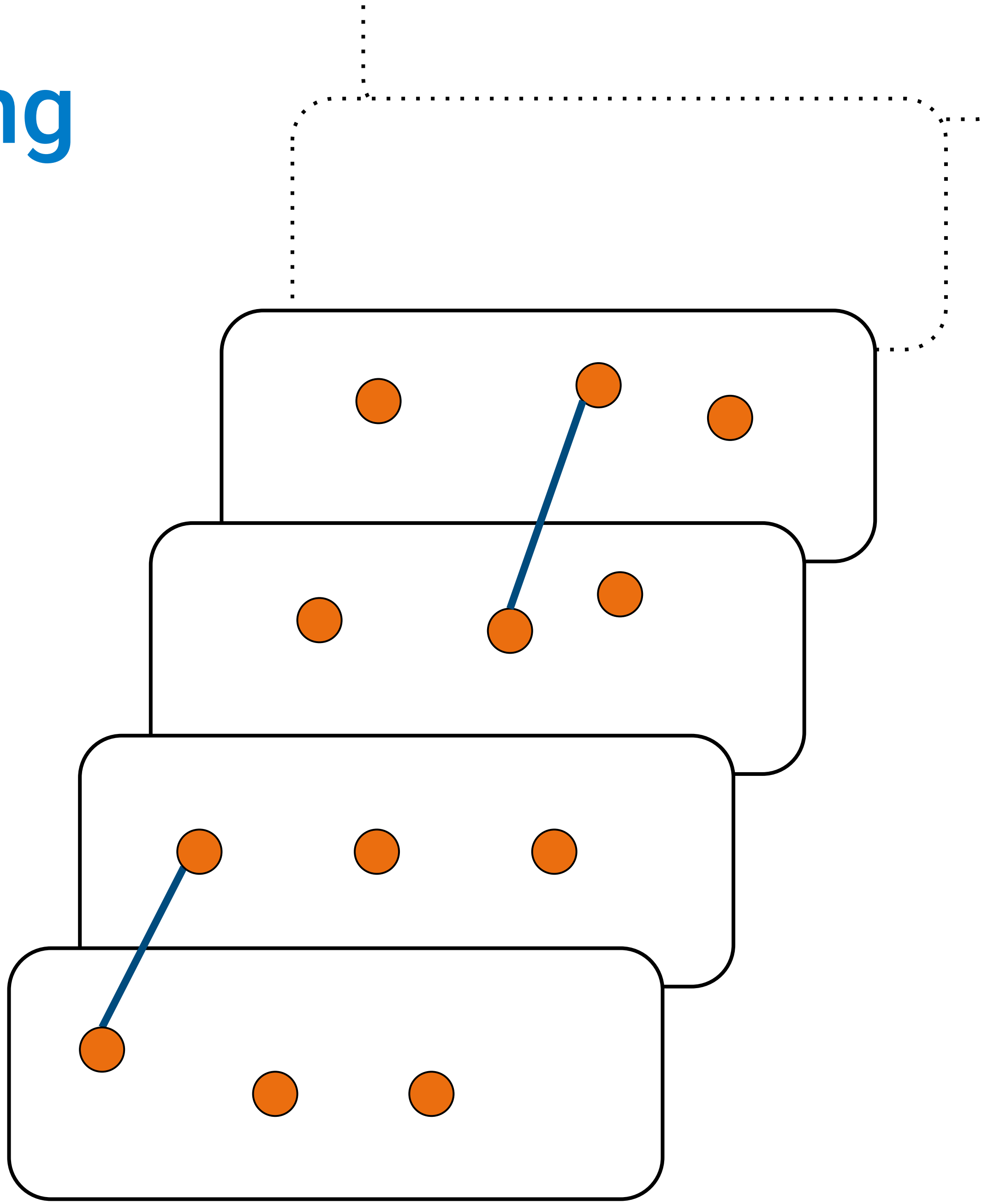
Source: https://home.cern/science/accelerators/muon-collider

Pattern recognition for tracking



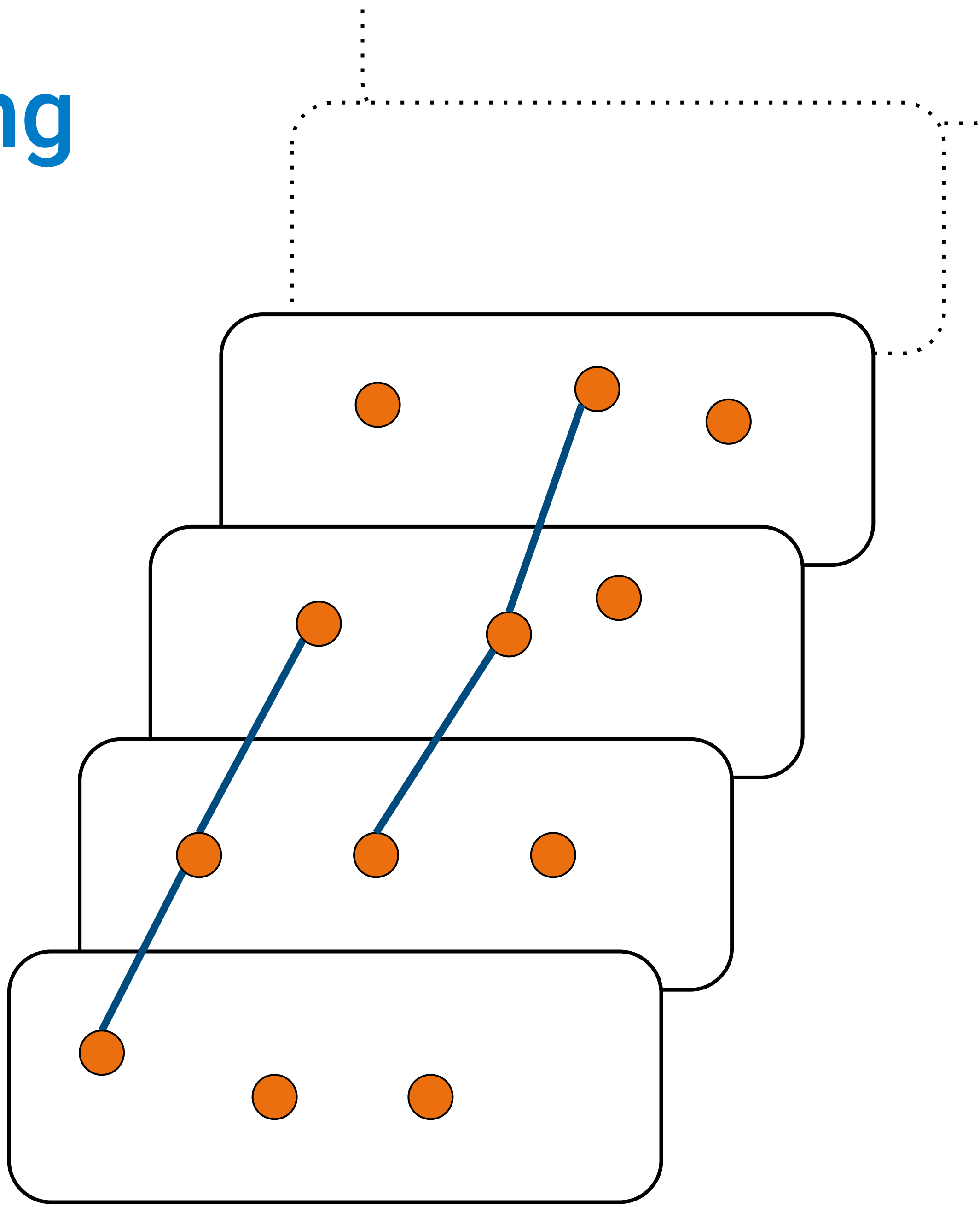
Pattern recognition for tracking

- **Doublets** as elementary patterns



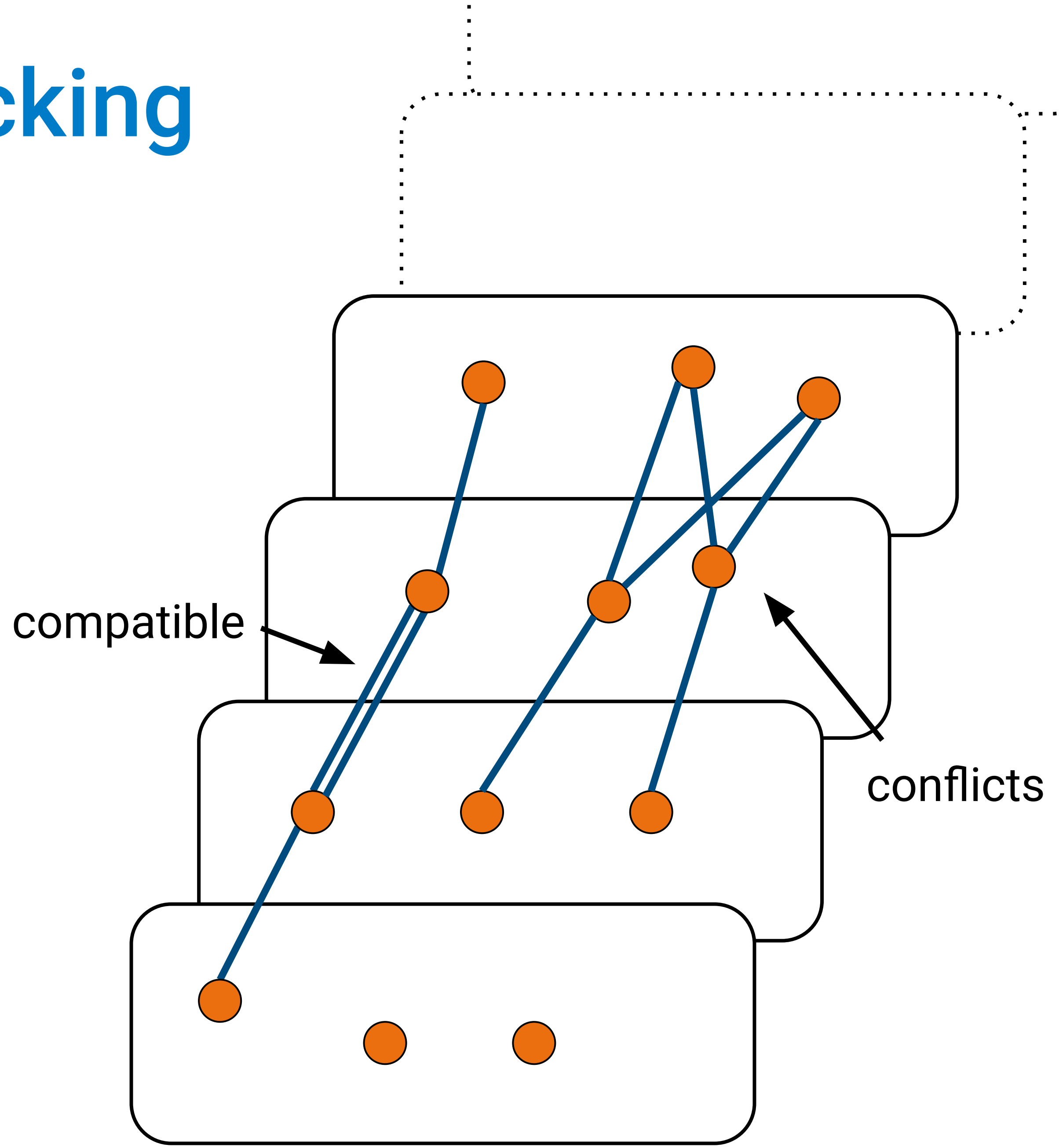
Pattern recognition for tracking

- **Doublets** as elementary patterns
- **Triplets** as “building blocks”



Pattern recognition for tracking

- **Doublets** as elementary patterns
- **Triplets** as “building blocks”
- **Relations** of triplets as key feature



Pattern recognition for tracking

- **Doublets** as elementary patterns
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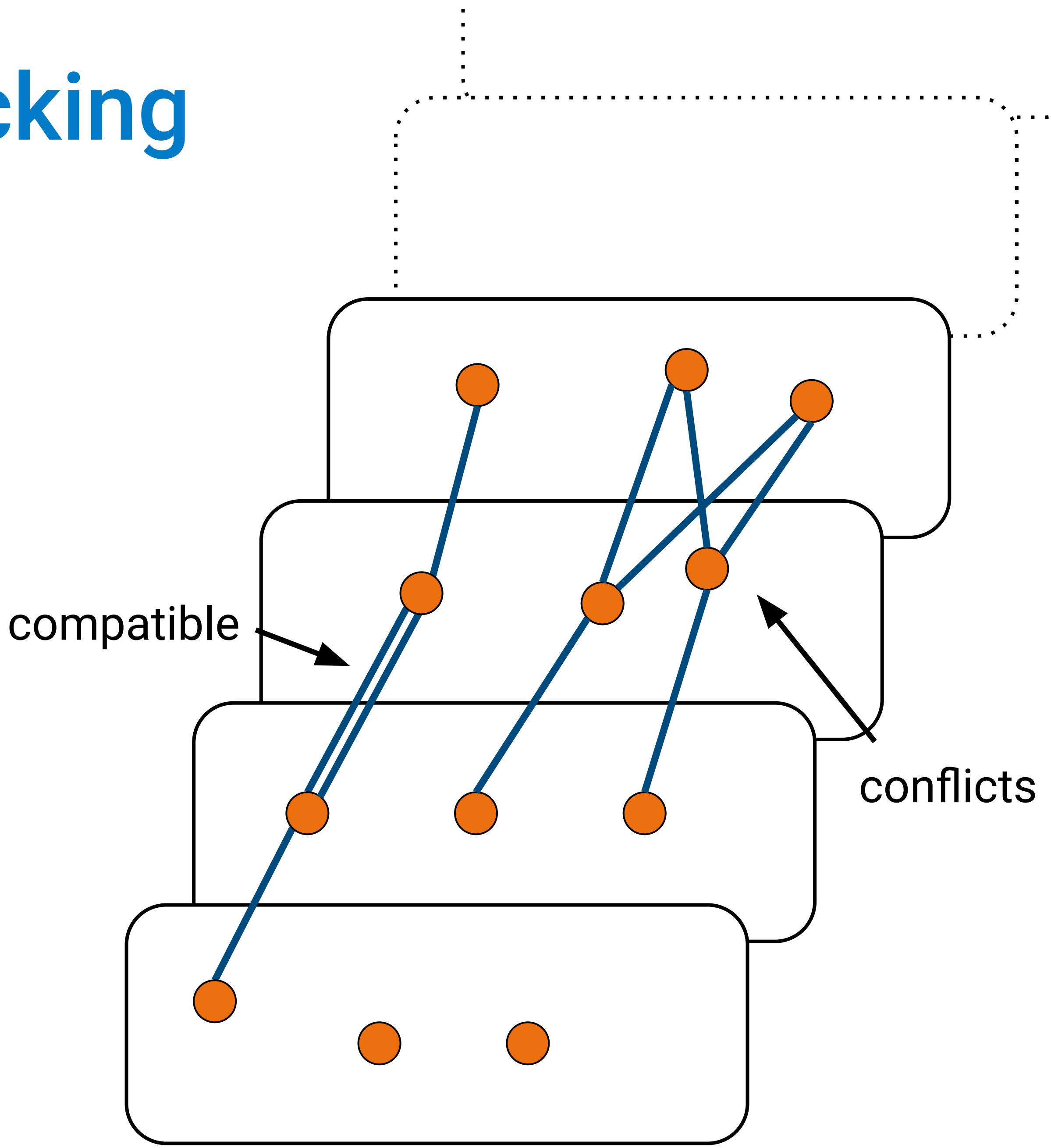
- **Relations** of triplets as key feature



Goal:

Identify triplets stemming from a single particle and combine them to tracks

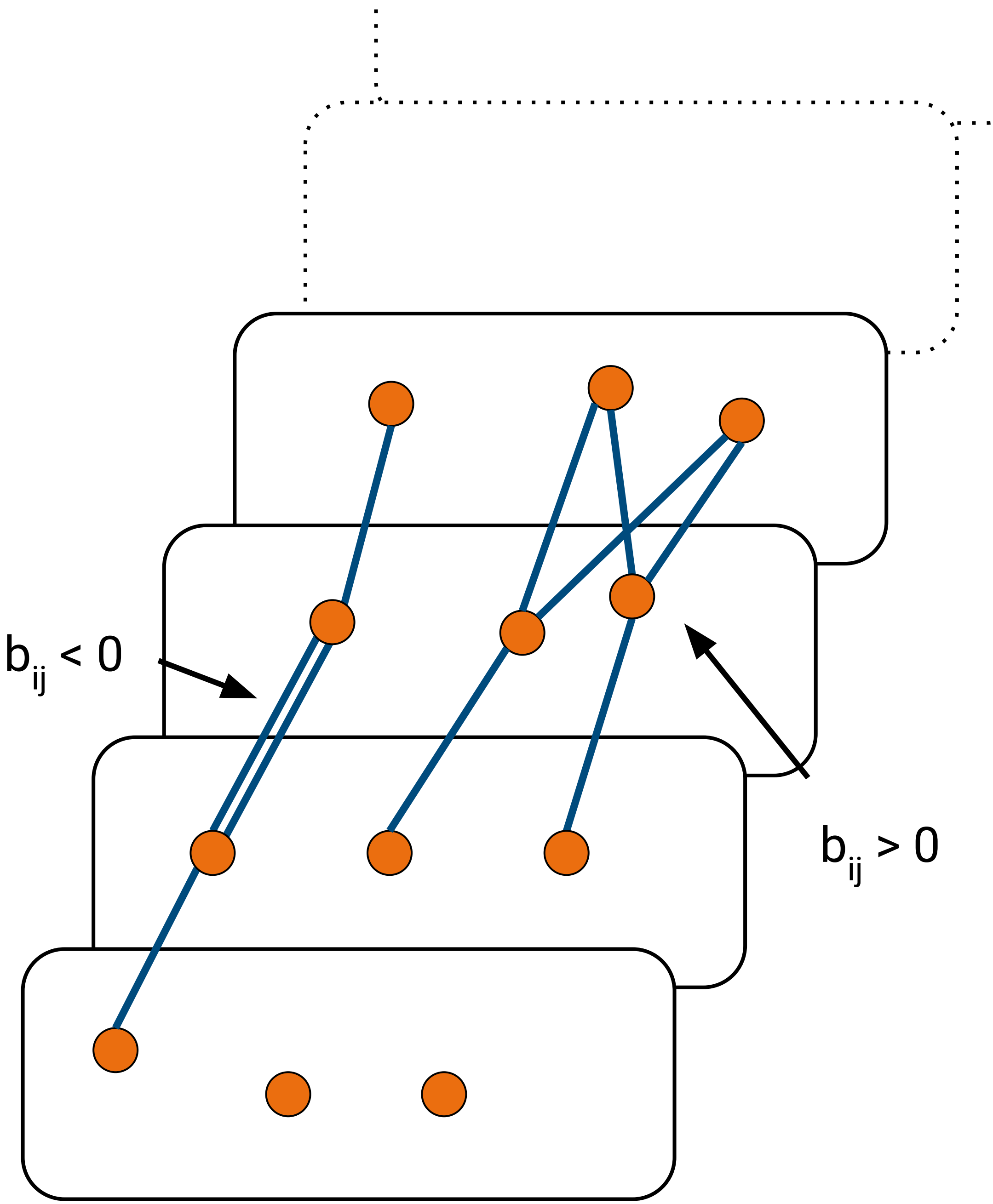
QUBO

Quadratic Unconstrained Binary Optimisation

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

- $\mathbf{T_i} \in \{0, 1\}$
- $\mathbf{b_{ij}}$: interaction
- $\mathbf{a_i}$: quality

Coefficients can be set by using **spatial** and/or **temporal** information of the triplets



QUBO

Quadratic Unconstrained Binary Optimisation

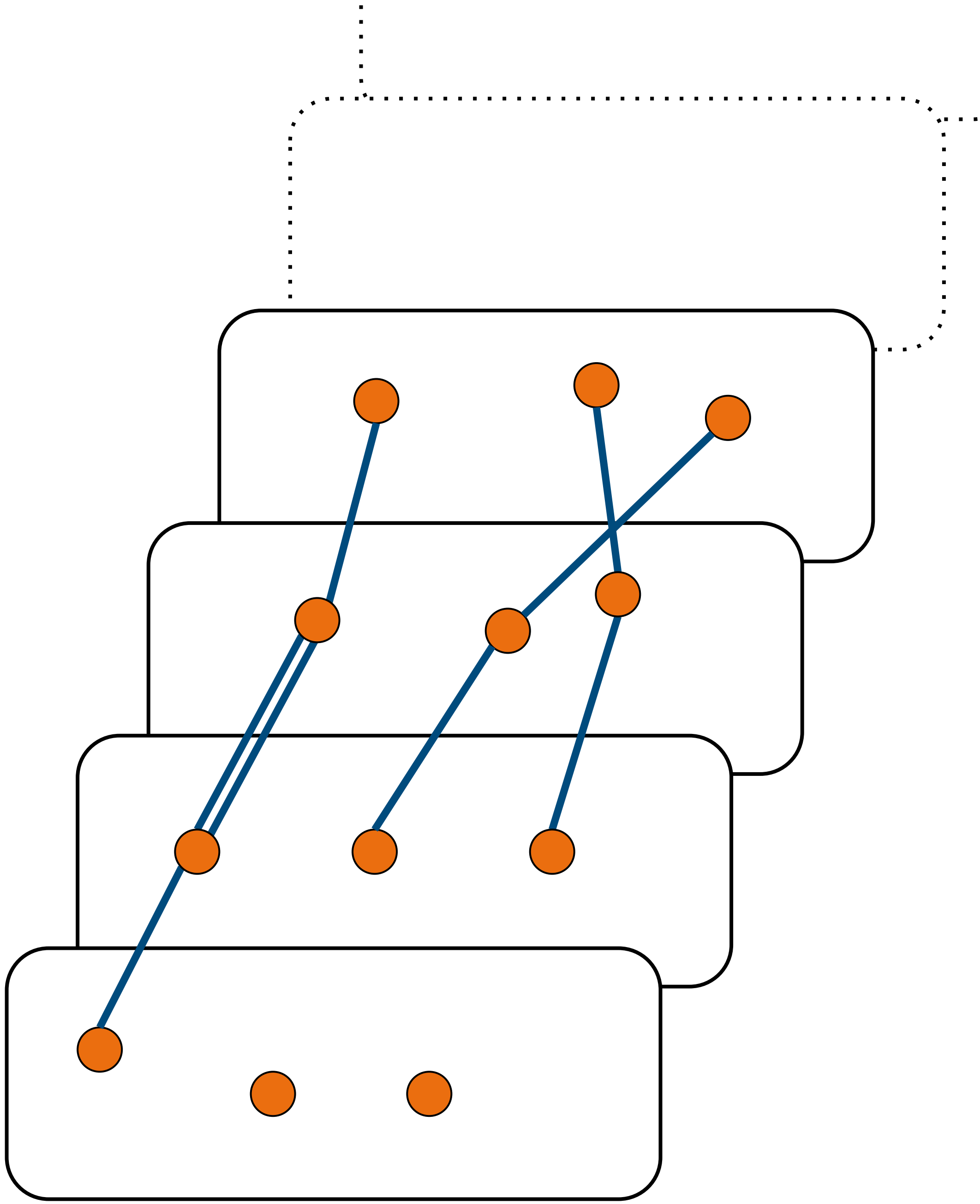
$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

Minimise Hamiltonian cost function:

- Ground state → best set of triplets
- $\mathbf{v}_{\mathbf{binary}}$: $[T_1, T_2, T_3, \dots, T_N] \rightarrow [0, 1, 1, \dots, 0]$ as result

Computation:

- Matrix diagonalisation (analytic solution)
- Hybrid quantum-classical algorithm (VQE)



QUBO

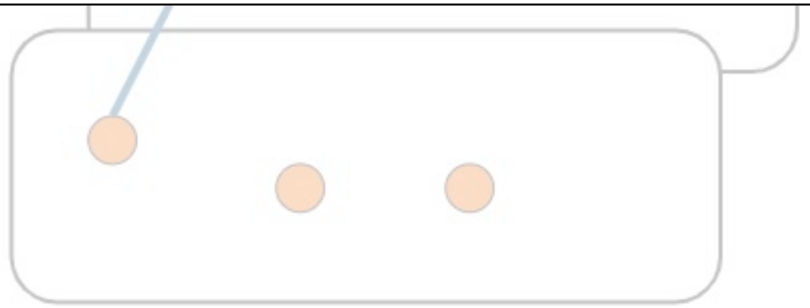
Quadratic Unconstrained Binary Optimisation

- *"Quantum algorithms for charged particle track reconstruction in the LUXE experiment"* (Yee Chinn Yap, Talk)

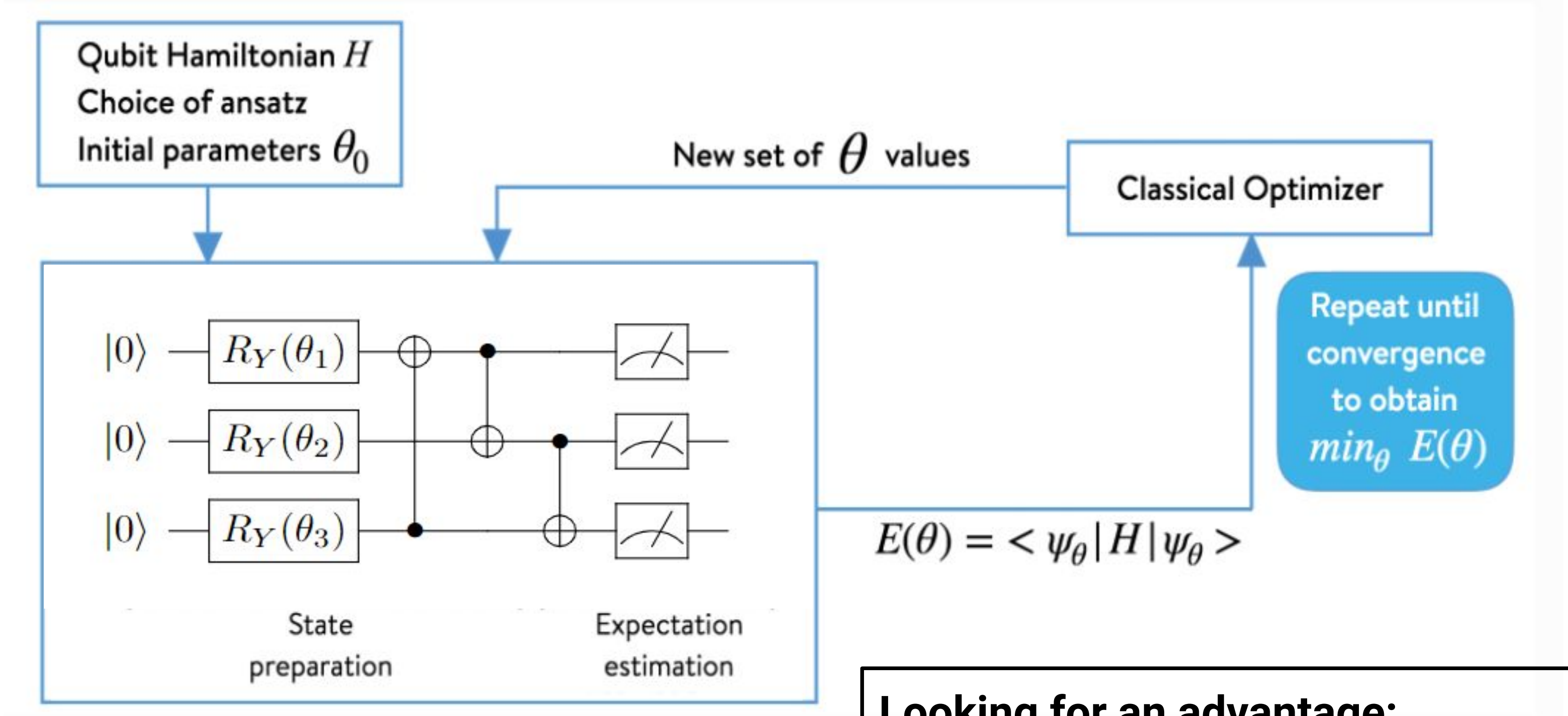
- *"Assessing the potential of quantum annealers for track reconstruction at LUXE"* (Annabel Kropf, Poster)

- See also: arxiv:2304.01690

- Matrix diagonalisation (analytic solution)
- Hybrid quantum-classical algorithm



VQE - Variational Quantum Eigensolver



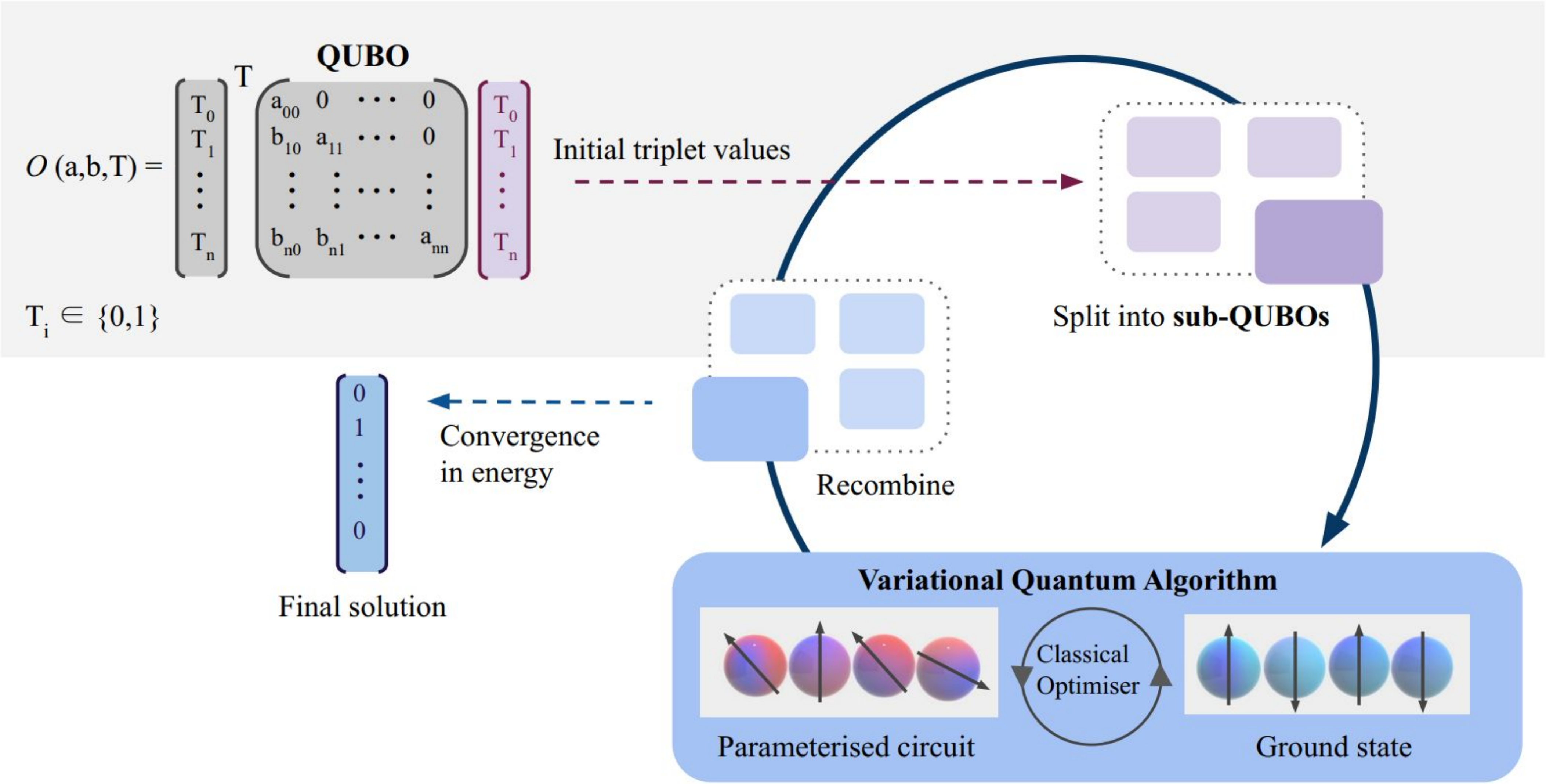
Source: edited from http://openqemist.1qbit.com/

Looking for an advantage:

dim(H) = 2^n x 2^n, then <ψ|H|ψ> needs

- $O(2^{2n})$ operations classically
- Possibly only O(Poly(n)) operations with a quantum computer

Sub-QUBOs

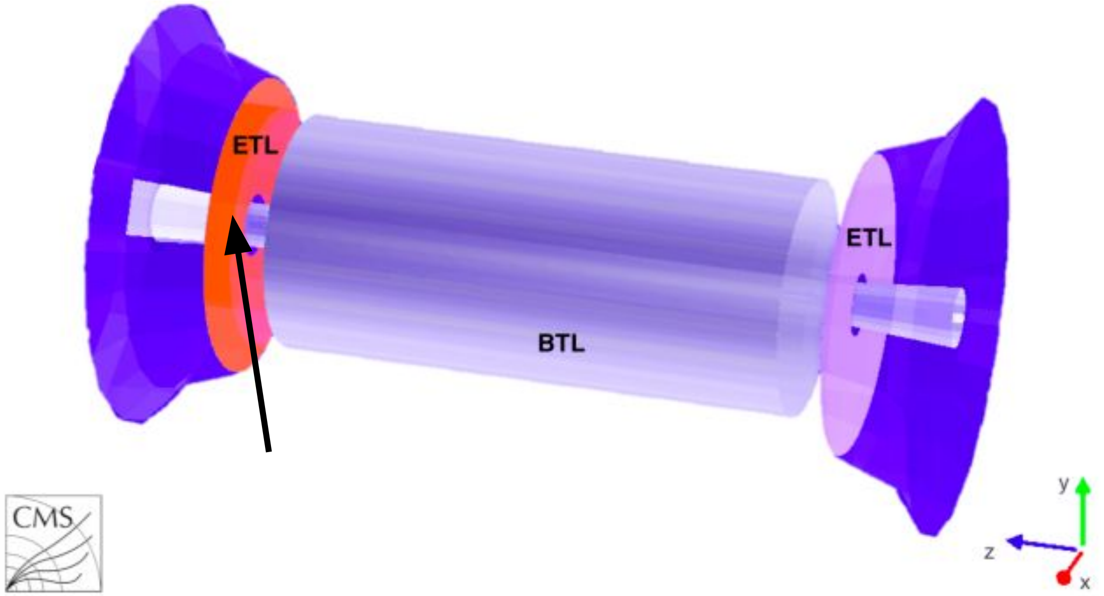


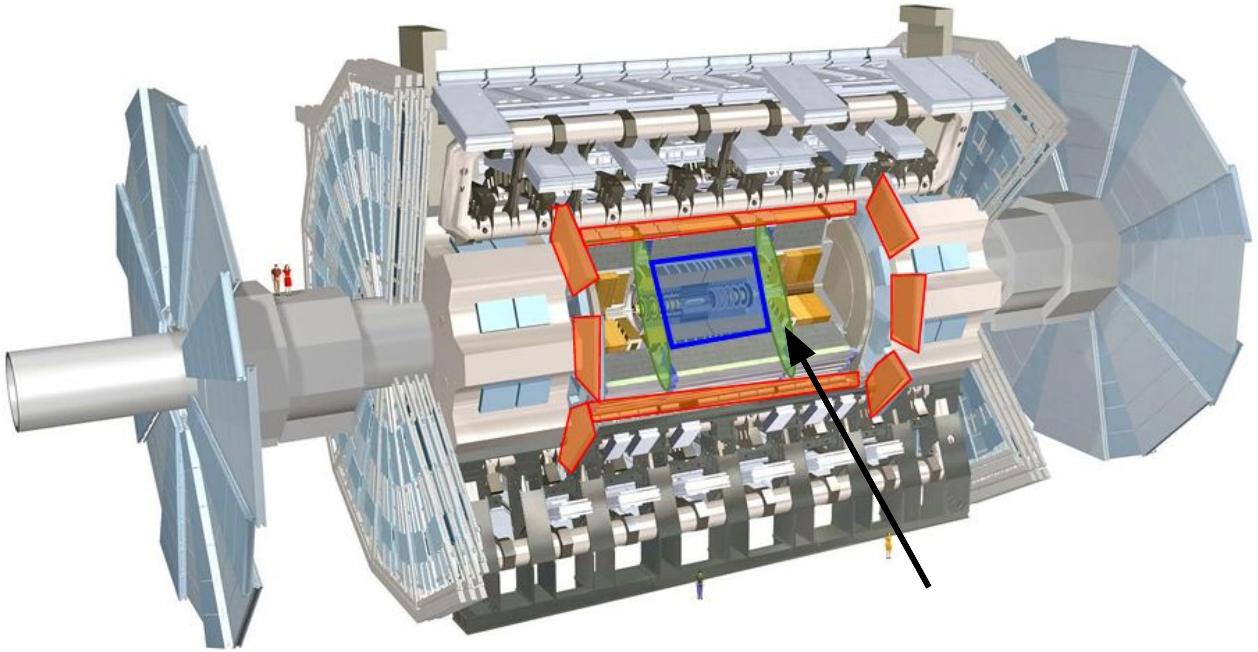
Source: arxiv:2304.01690

Timing information used for particle tracking

- Purpose: Reduce ambiguity and complexity of reconstructing trajectories
- Planned Phase-2 upgrade ATLAS(HGTD)/CMS(MIP):

  Timing layers in forward direction to reduce pileup background



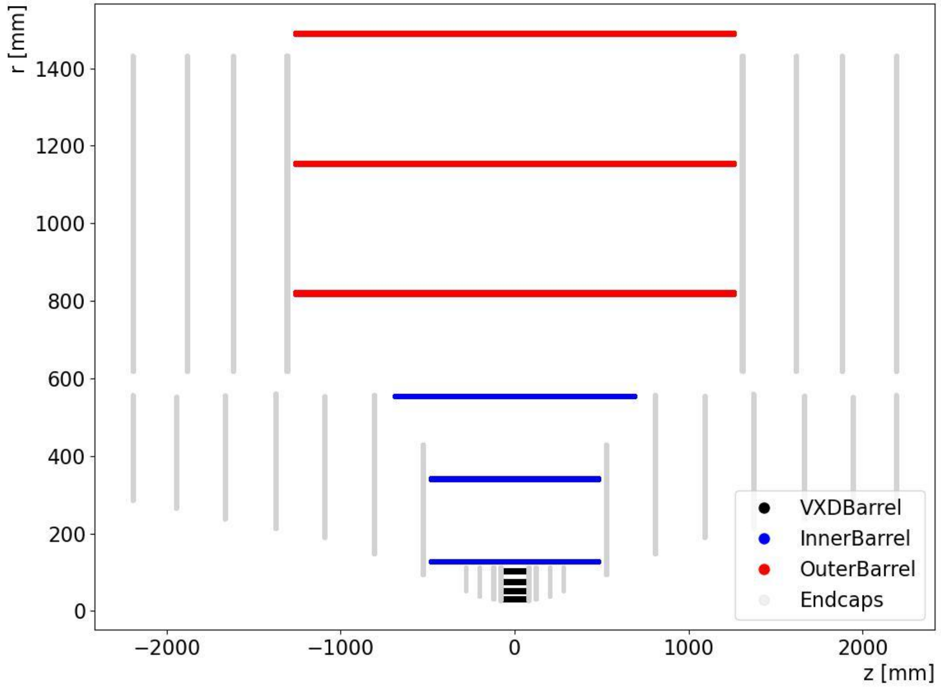


Source: CERN-LHCC-2019-003

Source: CERN-LHCC-2020-007

Tracking at Muon Collider

- Muon Collider detector as an example of a next generation detector with timing layers everywhere
- VXD Tracker: σ_t = 30ps, Inner/Outer Tracker: σ_t= 60ps



Detector geometry described as in arxiv:2303.08533

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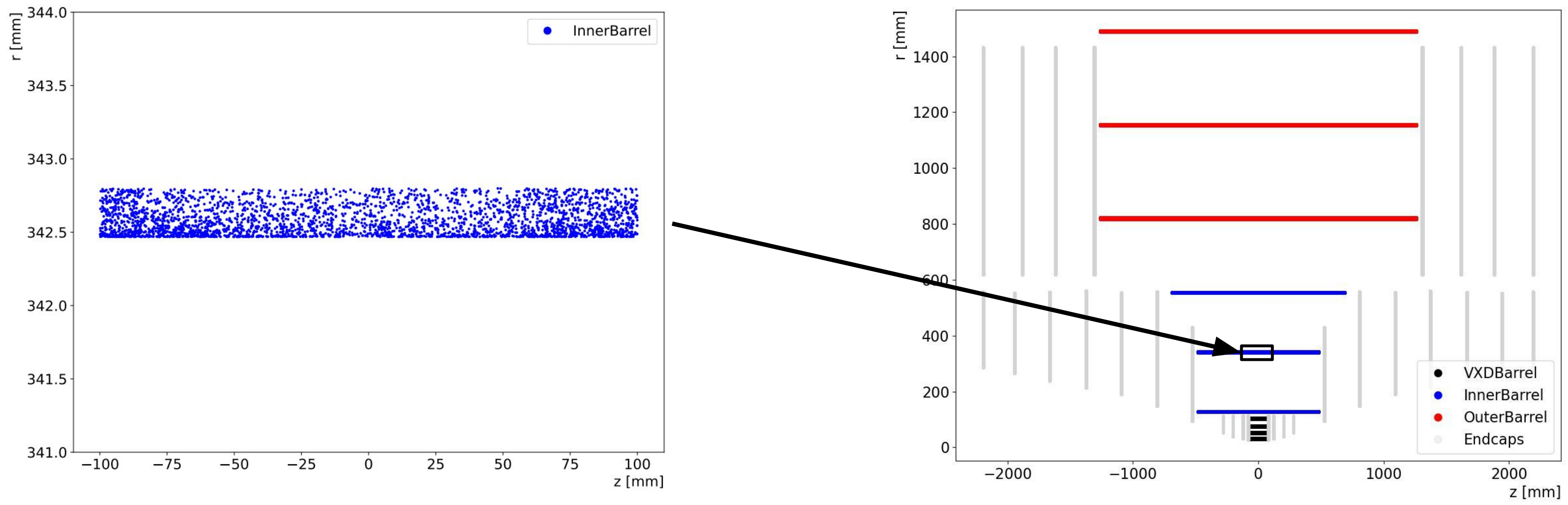

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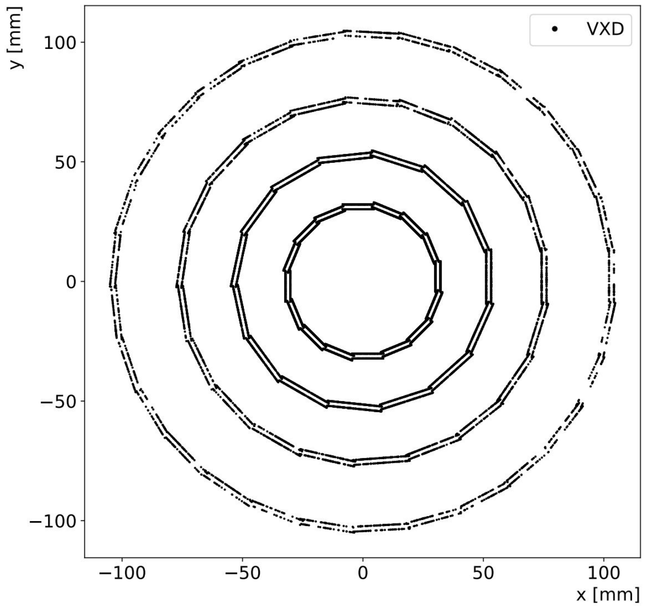

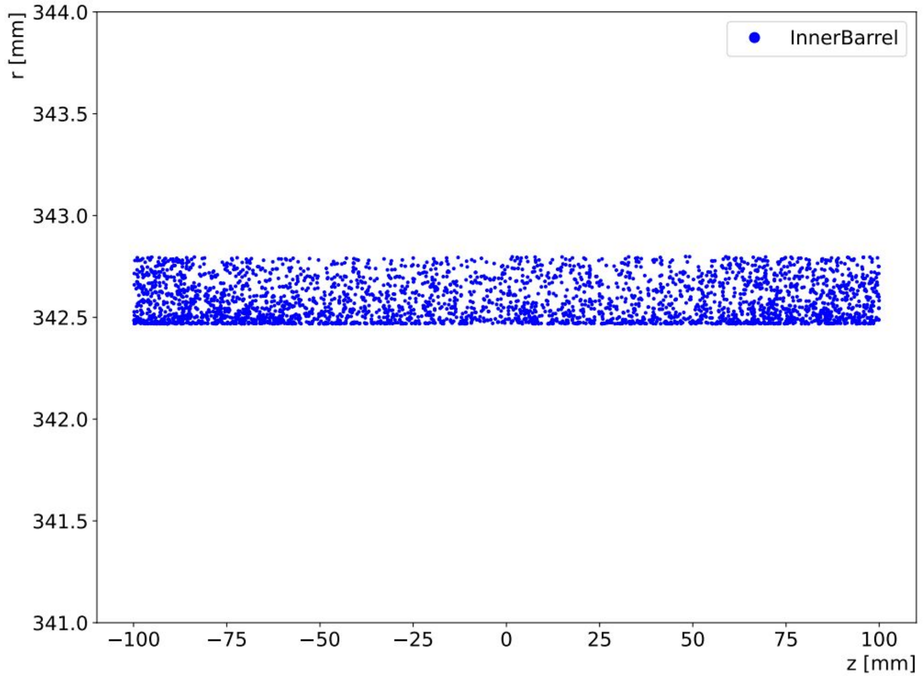

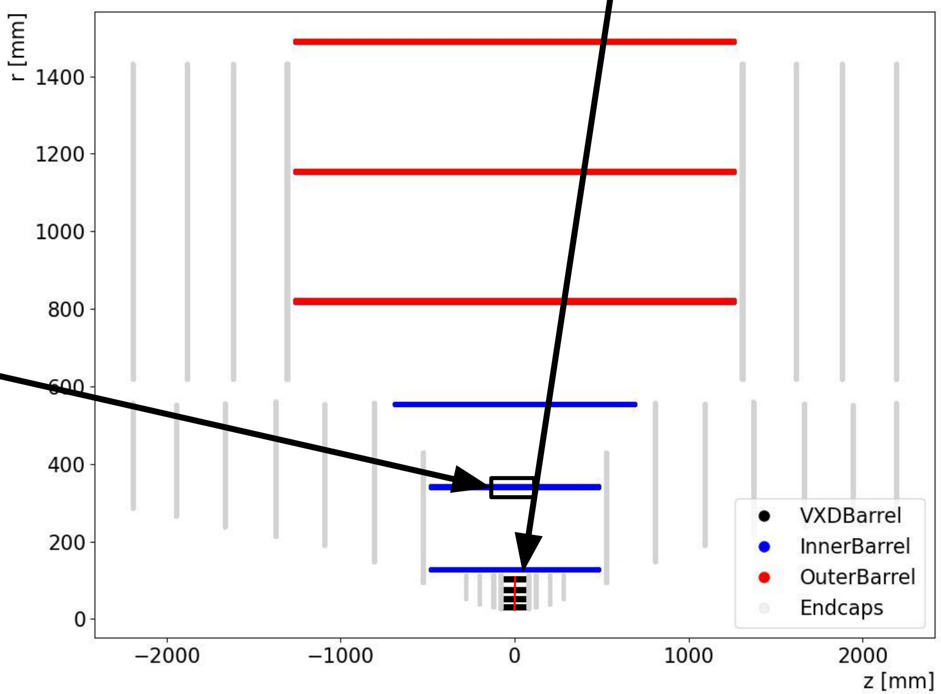

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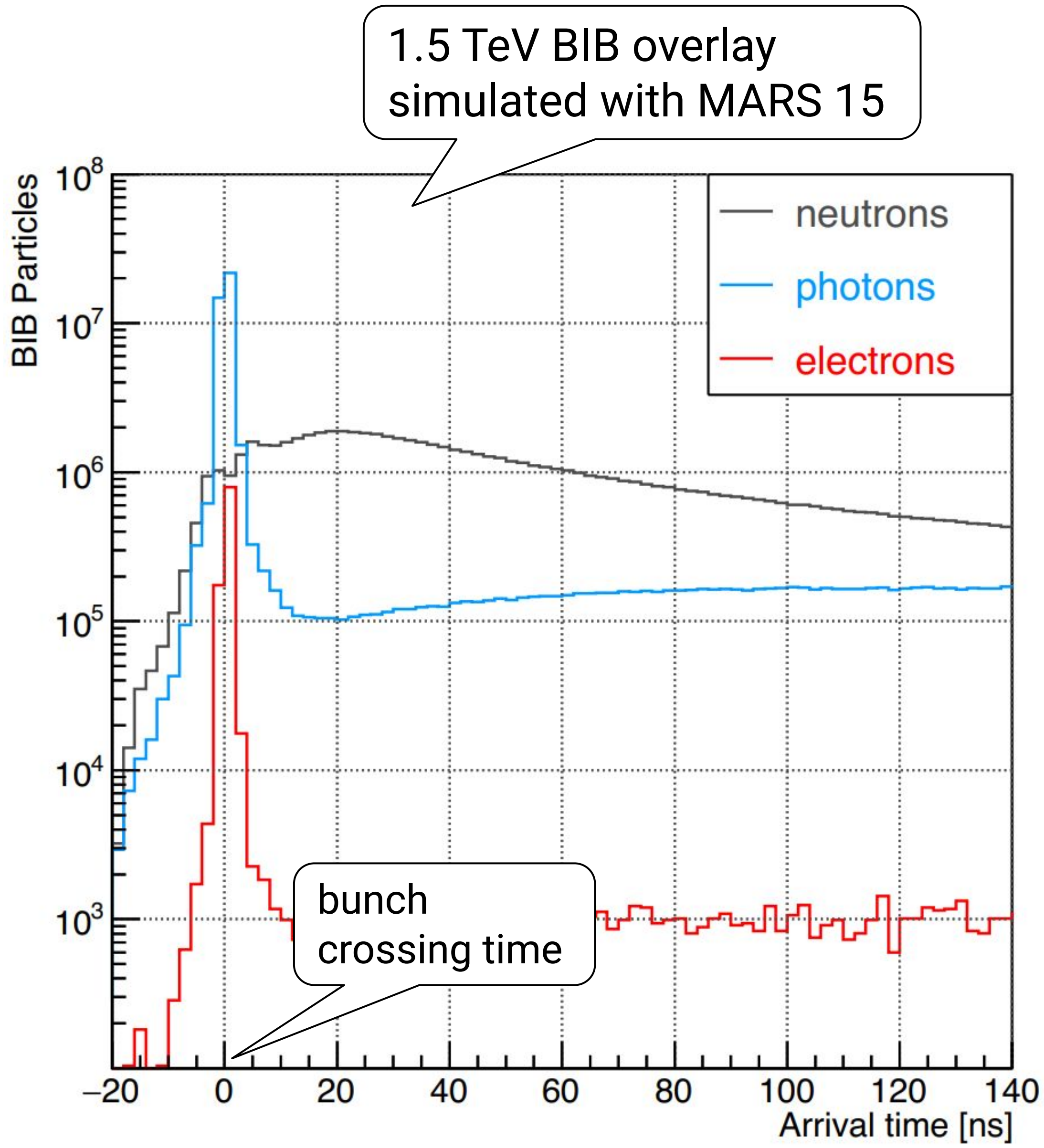
Tracking at Muon Collider

Beam muon decays induce secondary particle showers that reach the detector (**B**eam-**I**nduced-**B**ackground)

Time information as a crucial component to suppress BIB particles

4D tracking with a quantum algorithm:
Time information is directly used for calculating the QUBO parameters



Source: arxiv:2303.08533

4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

quality values a_i are set to 0

Interaction b_{ij}:

- Curvature of triplets
- Scattering, alignment of hits in the r-z plane
- Time compatibility of hits of the interacting triplets

→ **4D modeling of triplet interactions**

Time component of b_{ij}

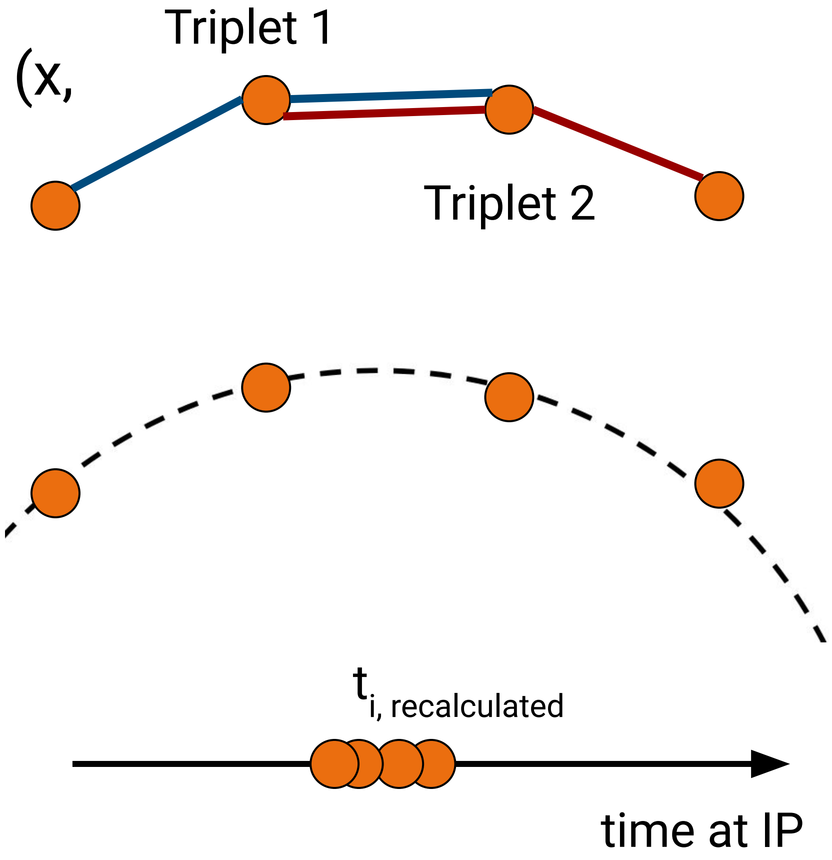
data from detector: (x, y, z, t_{raw})

↓

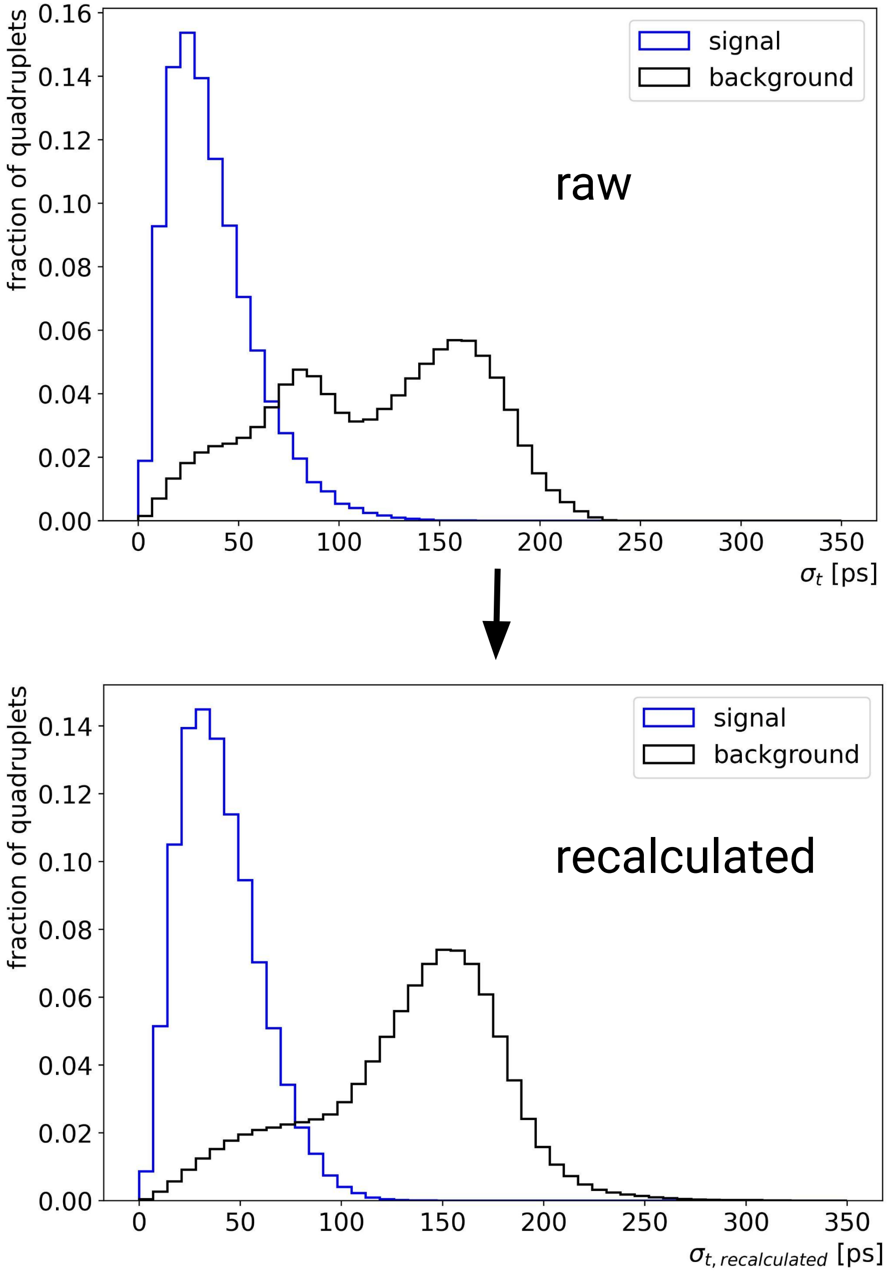
p_T / curvature

↓

$t_{recalculated} = t_{raw} - tof_{curvature}$



$\sigma(t_{recalculated})$ = “time spread of hits”

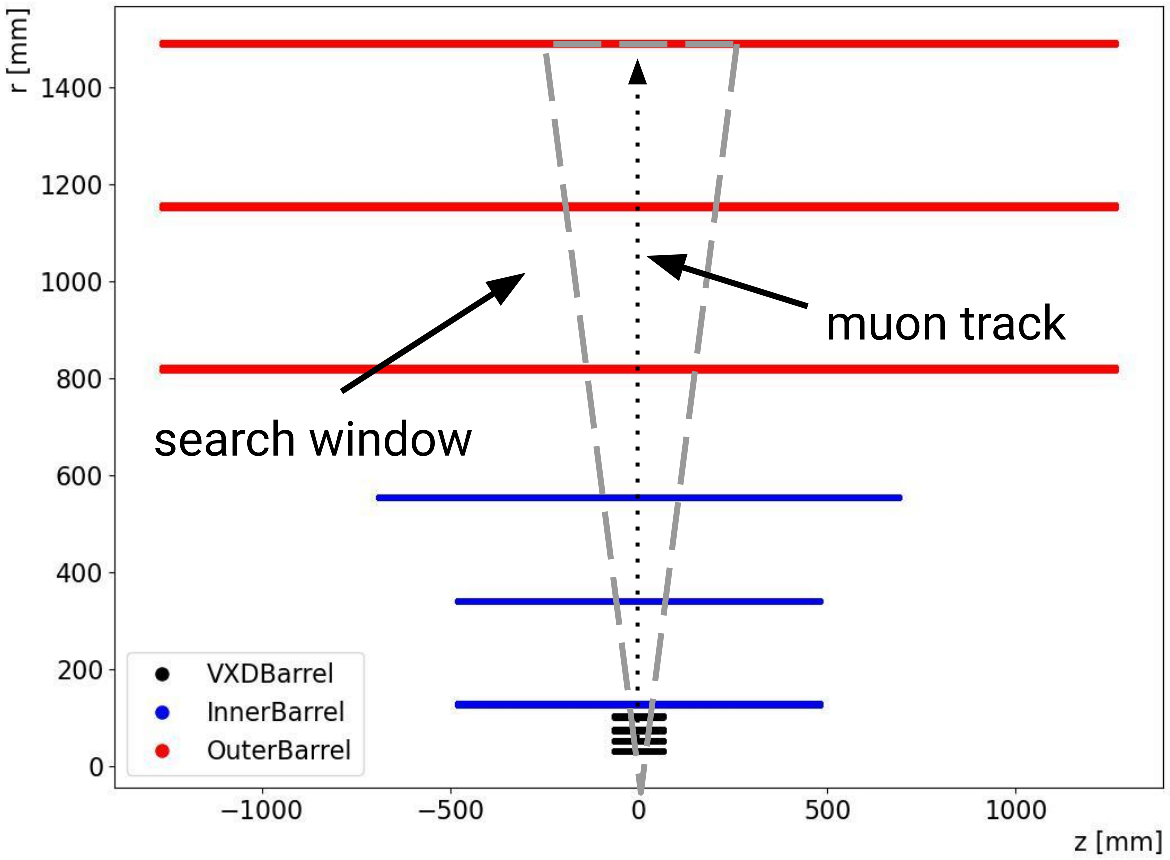


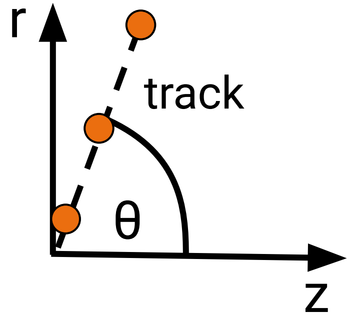
Reconstruct a single muon track within a large BIB

Setup

- Single muon events overlaid with BIB
- 0.5 GeV < p_T < 5.0 GeV
- θ = 90°
- Only hits in a small volume around track are considered

Goal: Reconstruct a single muon track within a large BIB





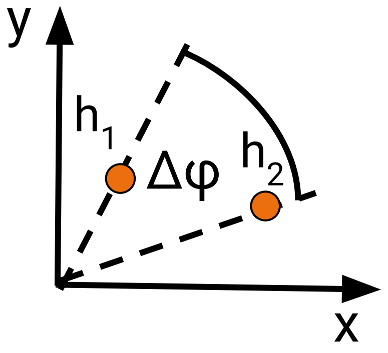
Pre-selection

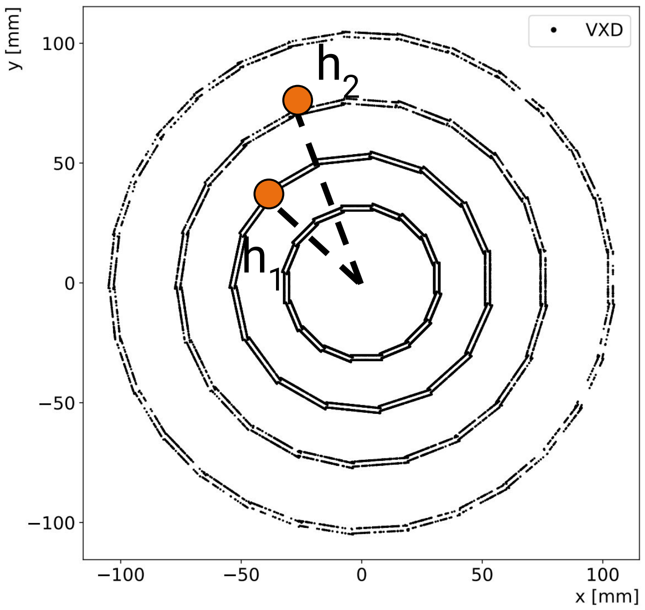
$h_1: (x_1, y_1, z_1)$ $\Delta\varphi = \varphi(h_2) - \varphi(h_1)$

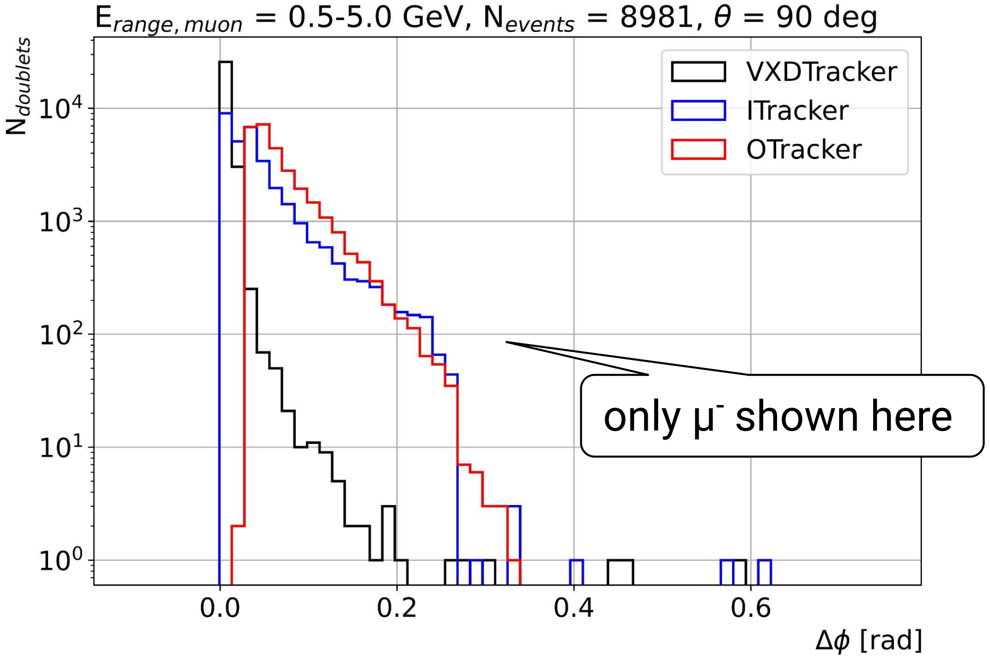
$h_2: (x_2, y_2, z_2)$ $\Delta\theta = \theta(h_2) - \theta(h_1)$

$r = \text{sqrt}(x^2 + y^2)$

- **Doublets: Δθ** and **Δφ**







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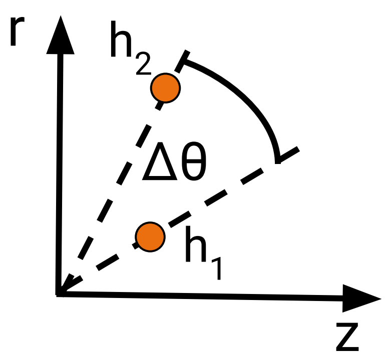
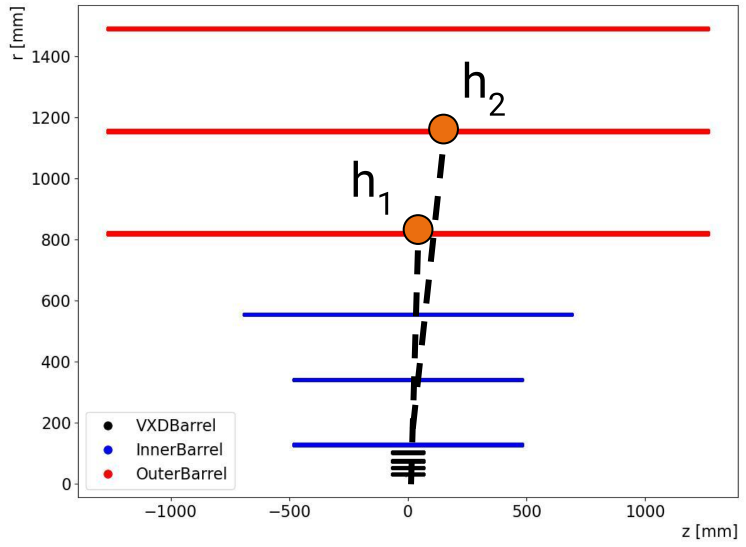
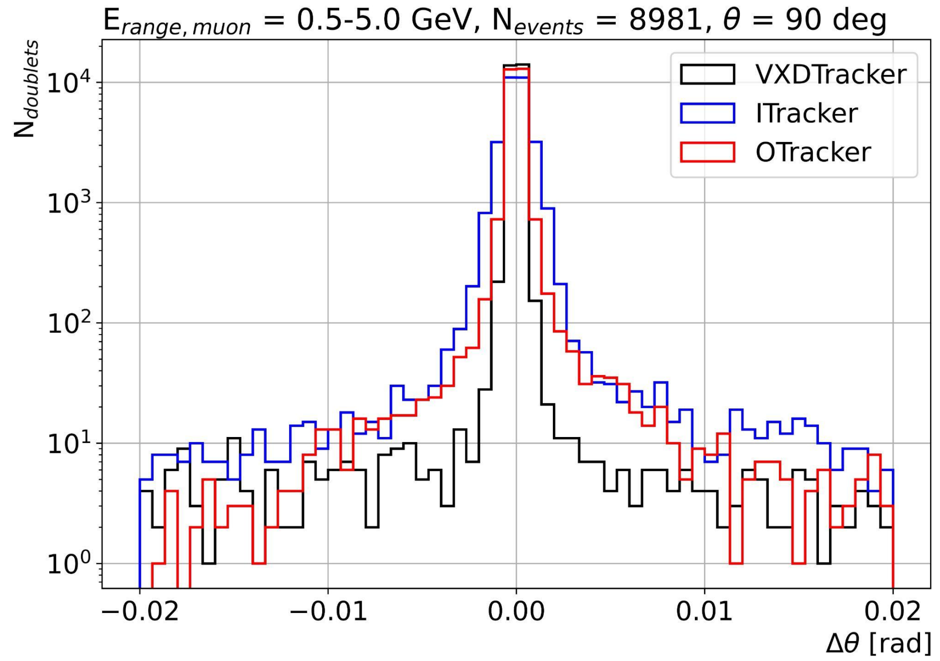
- **Doublets: Δθ** and **Δφ**



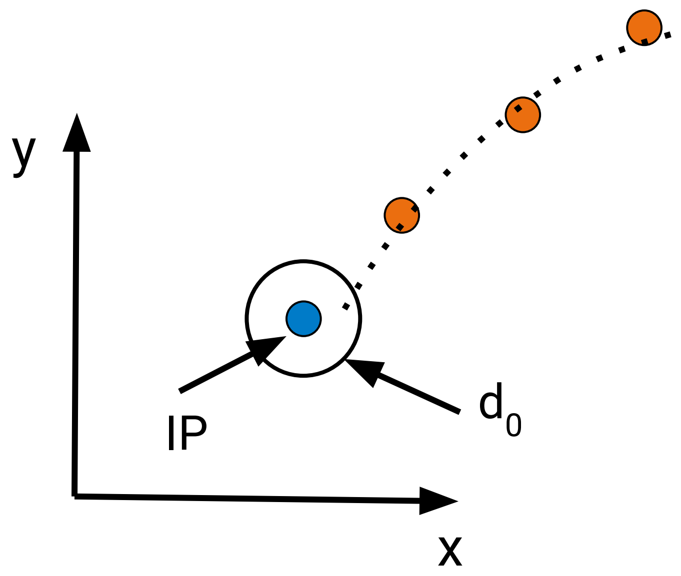
“scattering”

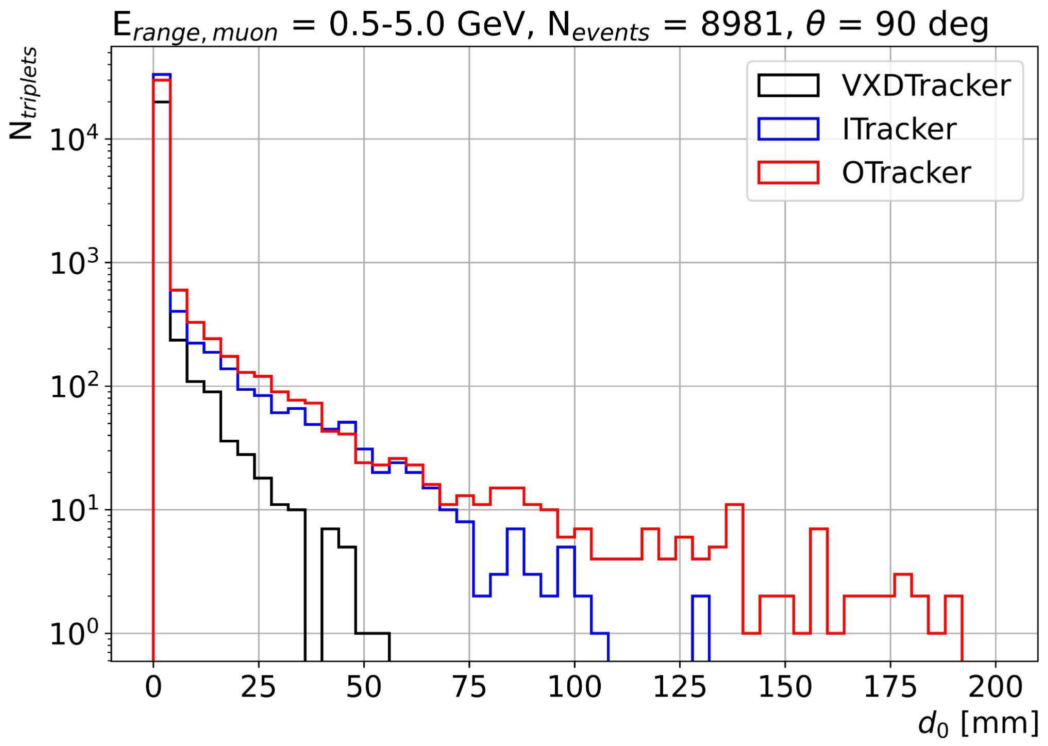




Pre-selection

- **Doublets: Δθ** and **Δφ**
- **Triplets:** transverse impact parameter $\mathbf{d_0}$





Results

Optimisation

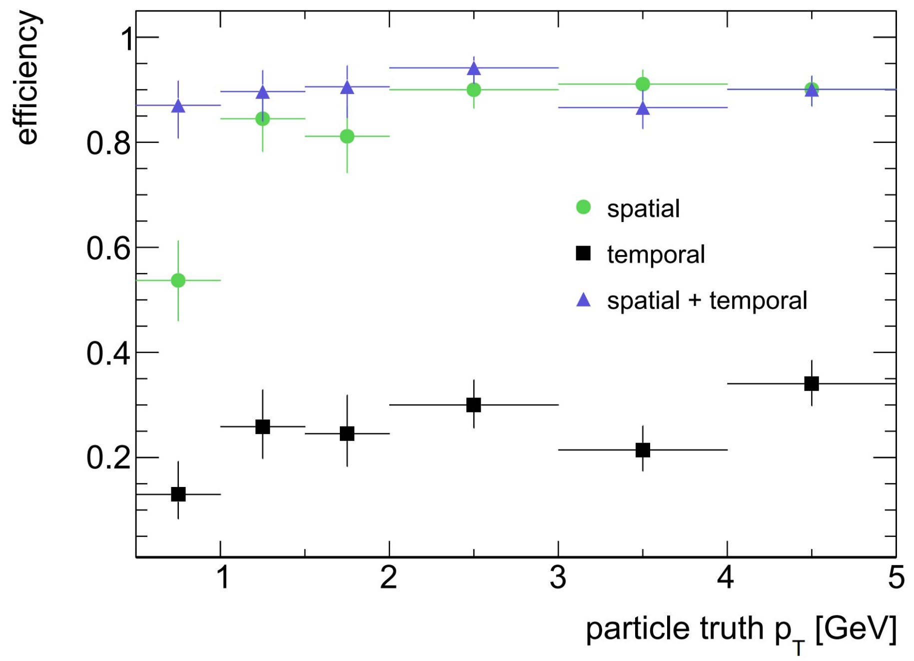
- Matrix diagonalisation, sub-QUBO size =18

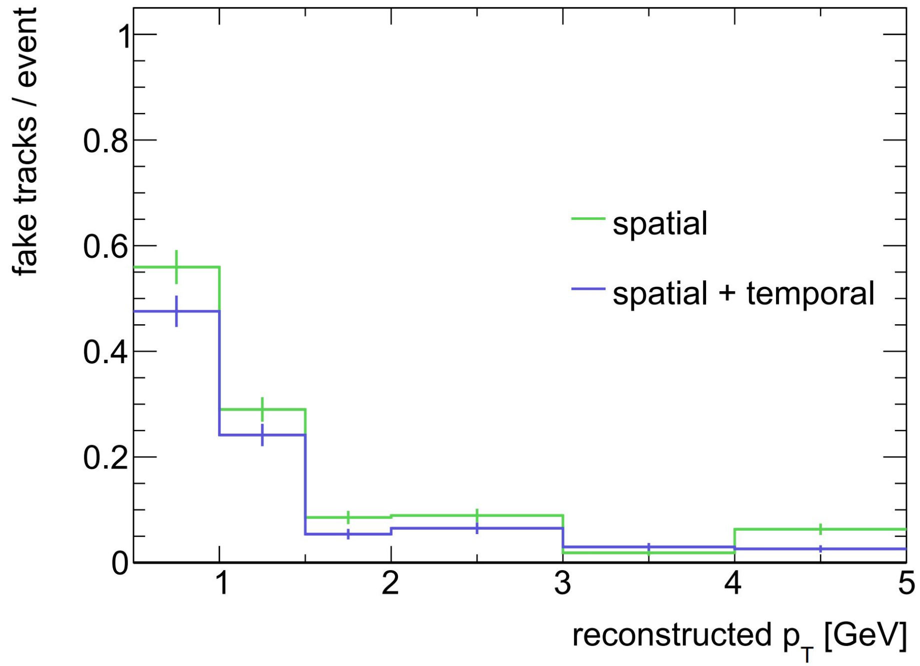
Criteria

- Track selected if ≥ 6 hits
- Matched if majority of hits stem from signal, else fake

Fake rate

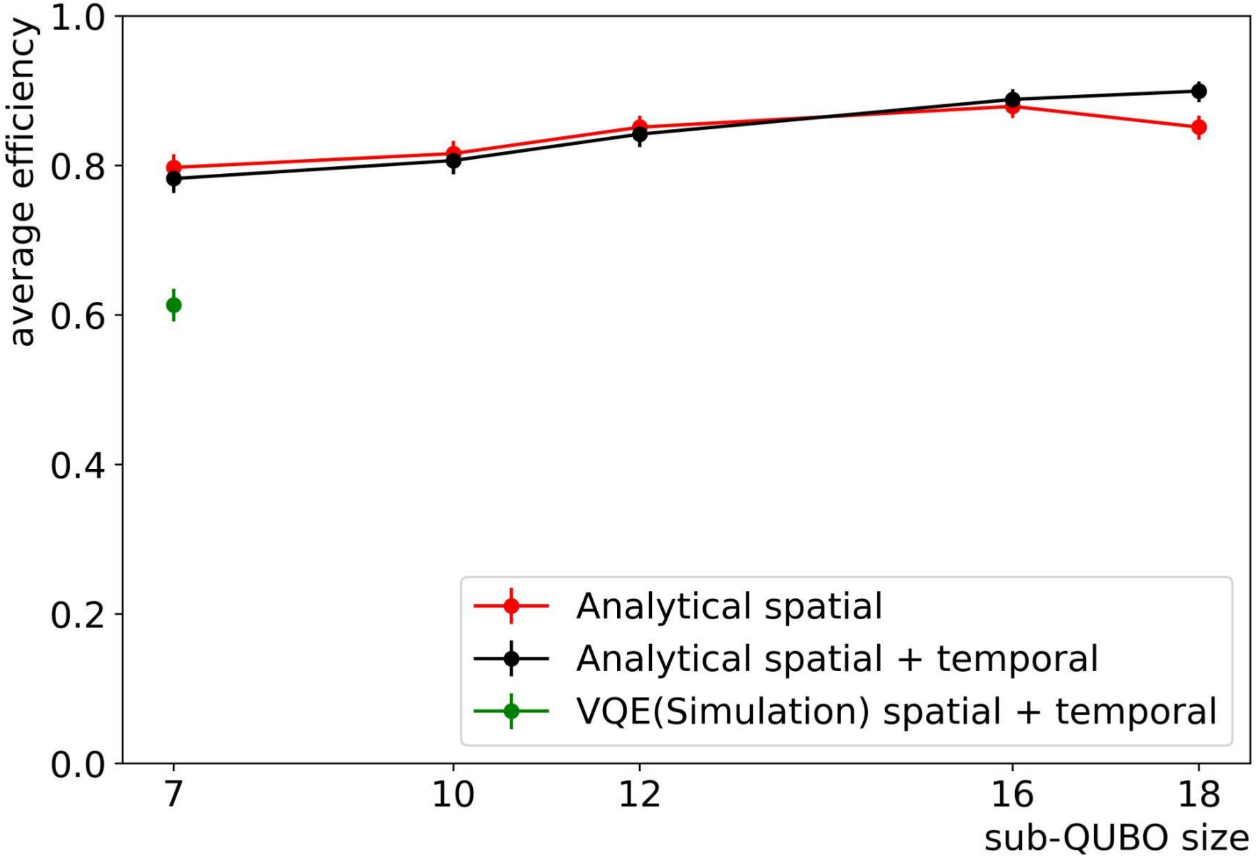
- Fake tracks in search window
- O(1) fake track per event





Results

- Increasing the sub-QUBO size leads to a higher efficiency
- VQE requires optimisation to match the matrix diagonalisation (analytical) result



Summary

- A 4D quantum tracking approach using a QUBO formulation was presented
  → QUBO parameters consist of a combination of spatial and temporal information of the detector hits

- Adding time information to the QUBO parameters b_{ij} improves the track reconstruction efficiency, especially at low p_T

- Example test application shown for a future high-energy muon collider

Thank You!

Appendix: QUBO parameter settings

Trivial

- -1 if connection possible

compatible curvature: c=1
contrary curvature: c=2

Spatial

- $f(q/p_T) : 0.5 \cdot (c - \min([pT_{triplet\ 1}\ pT_{triplet\ 2}]) / \max([pT_{triplet\ 1}\ pT_{triplet\ 2}])) \rightarrow$ **[0, 1]**
- $\max(\Delta\theta / 0.01) \rightarrow$ **[0, 1]**
- average of both is spatial value

Temporal

- $\min(\sigma(t_{hits}) / 250\ [ps]\ , 1) \rightarrow$ **[0, 1]**

Connections are rescaled to be inside **[-1.0, -0.9]**

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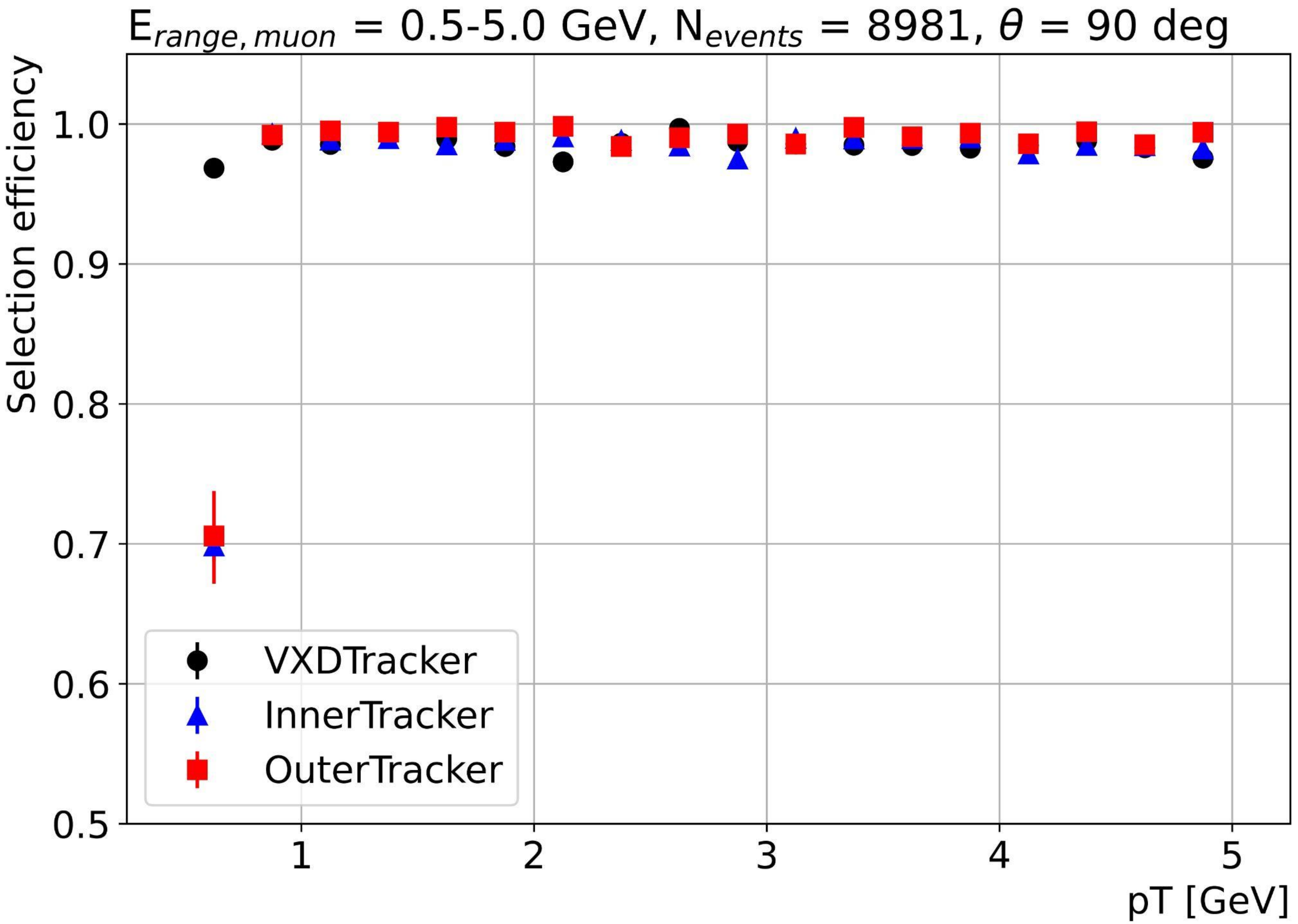
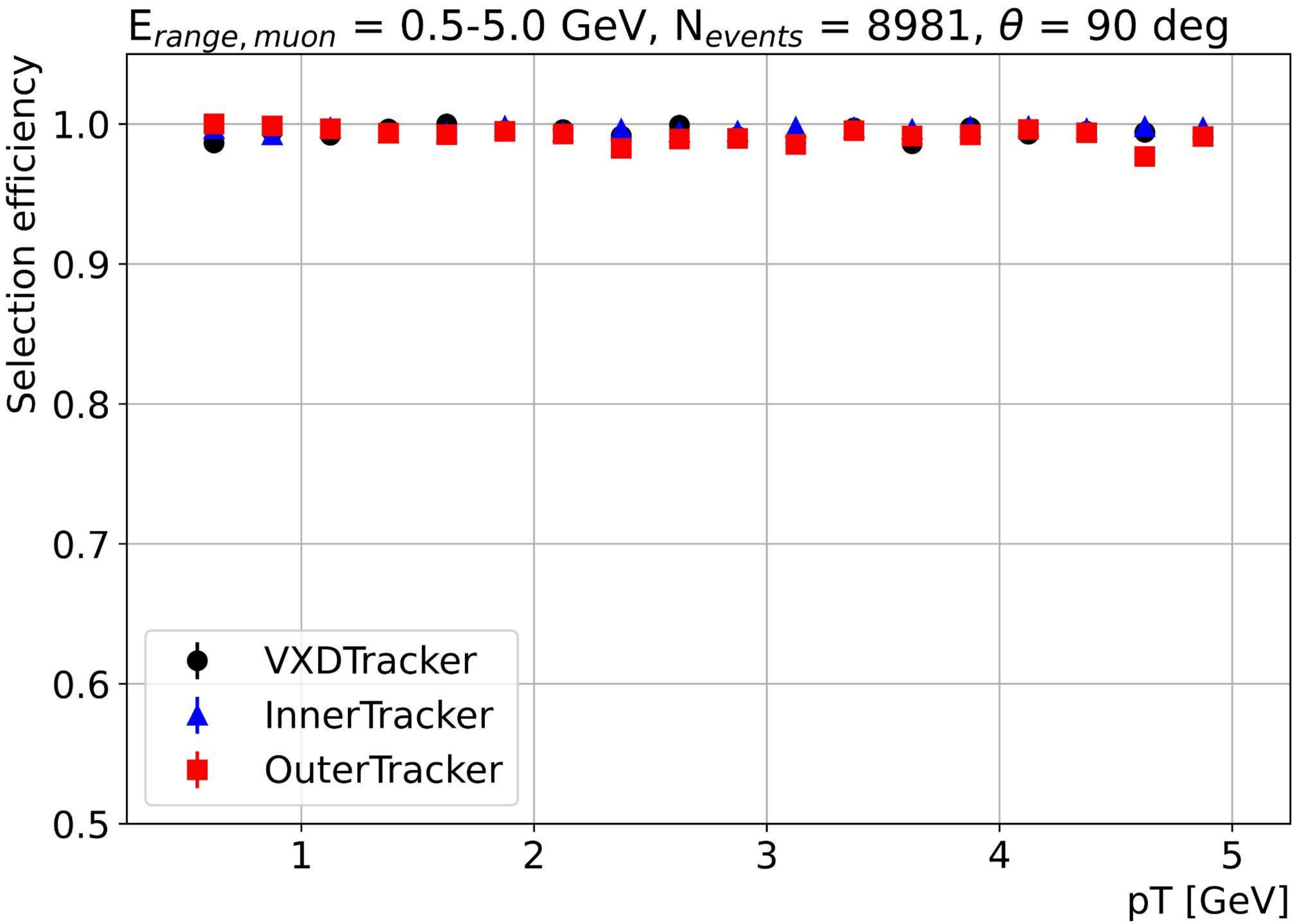
Appendix: Signal only - doublet pre-selection



Appendix: Signal only - triplet pre-selection



Appendix: Preselection values

VXD:	$\Delta\varphi = 0.05$	$\Delta\theta = 0.01$	d0 = 15mm
ITracker:	$\Delta\varphi = 0.2$	$\Delta\theta = 0.01$	d0 = 50mm
OTracker:	$\Delta\varphi = 0.25$	$\Delta\theta = 0.005$	d0 = 50mm

Appendix: Triplet statistics

N_{events} = 538,

$N_{triplets}$ = 15087781 → 28044 / event

$N_{triplets, signal}$	= 5048	→ 9	/ event
$N_{triplets, majority signal}$	= 40137	→ 75	/ event
$N_{triplets, majority background}$	= 278561	→ 518	/ event
$N_{background}$	= 14764025	→ 27442	/ event