Precise predictions for the trilinear Higgs self-coupling in the Standard Model and beyond

predicting κ_λ in *any* model at the one-loop order [2305.03015]

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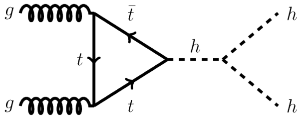
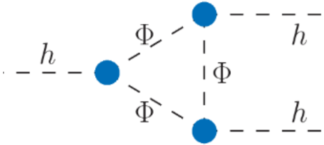
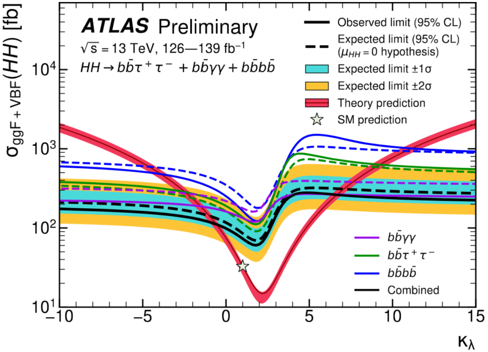
Why the trilinear self-coupling?

$$V_{\rm SM} \supset \frac{m_h^2}{2} h^2 + \frac{3m_h^2}{v}\kappa_\lambda h^3 + \frac{3m_h^2}{v^2}\kappa_{2\lambda} h^4$$

- probes electroweak symmetry breaking mechanism
- influences thermal evolution if $V_{\rm SM}$
- important input for electroweak phase transition
- important input for di-Higgs production
- very sensitive to BSM







[Phys. Lett. B 843 (2023) 137745]

The SM scalar sector

- V_{SM} fixed at tree-level by $m_h \approx 125\,\mathrm{GeV}$ and $v = (\sqrt{2}G_F)^{-1/2} \approx 246\,\mathrm{GeV}$:

$$V_{\mathrm{SM}}(h) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}h^3 + 3\frac{m_h^2}{v^2}h^4$$
$$= \frac{m_h^2}{2}h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

- However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.

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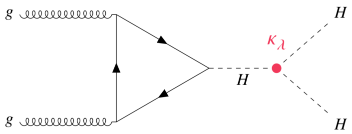
- However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.
- BSM: deformation of the scalar potential possible (at tree-level or via quantum corrections)!

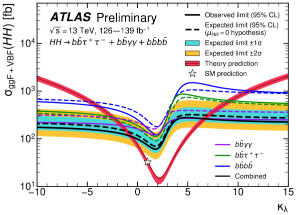
$$V_{\text{BSM}}(h, h_{\text{BSM}}, \dots) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}\kappa_\lambda h^3 + 3\frac{m_h^2}{v^2}\kappa_{2\lambda} h^4 + \dots$$

κ_λ: describes deviation from SM: $\kappa_\lambda = \frac{\lambda_{hhh}^{\text{BSM}}}{\lambda_{hhh}^{\text{SM}}}$ in model "BSM".

Constraining λ_{hhh} / κ_λ using Higgs pair production

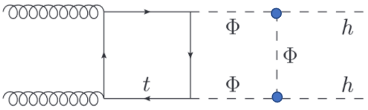
Leading-order parametrization used by experiment:



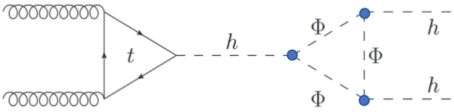


$pp \to hh$: $-0.4 < \kappa_\lambda < 6.3$ [Phys. Lett. B 843 (2023) 137745]

Possible next-to-leading order BSM contributions in concrete models:



$\propto \mathcal{O}(y_t^2 g_{h\Phi\Phi}^2)$ (not included)



$\propto \mathcal{O}(y_t g_{h\Phi\Phi}^3)$ (included)

When to apply the κ_λ-constraint to BSM models?

- only κ_λ is *significantly* modified by BSM physics
- no additional resonance in *s*-channel
- all other couplings SM-like

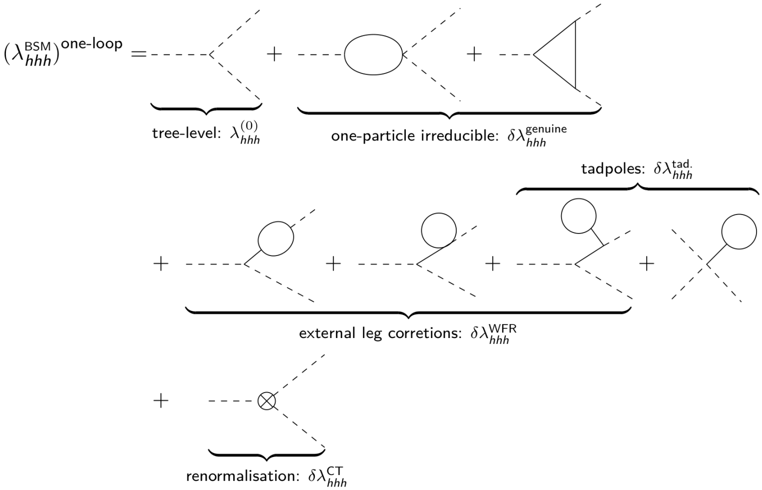
→ a scenario often enforced by experimental constraints

λ_{hhh} beyond the SM

- Many studies for λ_{hhh} already exist
  - additional doublets [Kanemura et al. '04][Basler et al. '17][Braathen et al. '19],
  - singlets [Kanemura et al. '16][Basler et al. '19],
  - triplets [Aoki et al. '18][Chiang et al. '18],
  - SUSY [Hollik et al. '02][Brucherseifer et al. '13][Dao et al. '13][Dao et al. '15][Borschensky et al. '22]
- Higher-order corrections can be significant [talk by Johannes Braathen]
- Many more models to explore!

$\rightarrow$ `anyH3` [Bahl, Braathen, MG, Weiglein '23]:
automated tool to calculate λ_{hhh} in *any* model

Higher-order corrections to λ_{hhh} in any renormalisable theory



> Solid lines:
  - scalars,
  - fermions,
  - gauge/vector bosons,
  - ghosts

> possibility to exclude/restrict certain particles and/or topologies

> automatic non-trivial renormalization
  - OS or $\overline{\text{MS}}$ masses
  - $\sim$ size of two-loop

Feature list (so far) of anyH3

- import/convert arbitrary UFO models
- definition of renormalisation schemes

```
# schemes.yml
renormalization_schemes:
  OS:
      mass_counterterms:
        h1: OS
        h2: OS
      VEV_counterterm: OS
  MS:
      mass_counterterms:
        h1: MS
        h2: MS
      VEV_counterterm: MS
```

- optional: full p^2 dependence
- numerical / analytical / LaTeX outputs
- ...
- Python-library with command-line- and Mathematica-interface

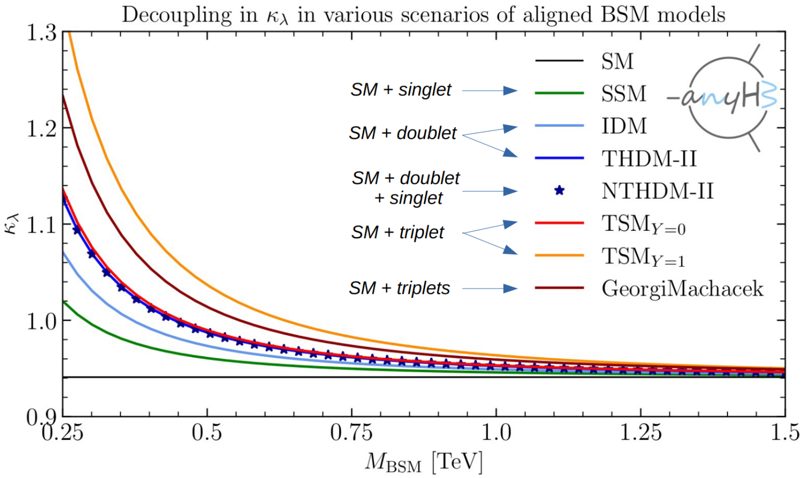
pip install anyBSM

```
from anyBSM import anyH3
myfancymodel = anyH3('path/to/UFO/model')
result = myfancymodel.lambdahhh()
```

- more examples at anybsm.gitlab.io

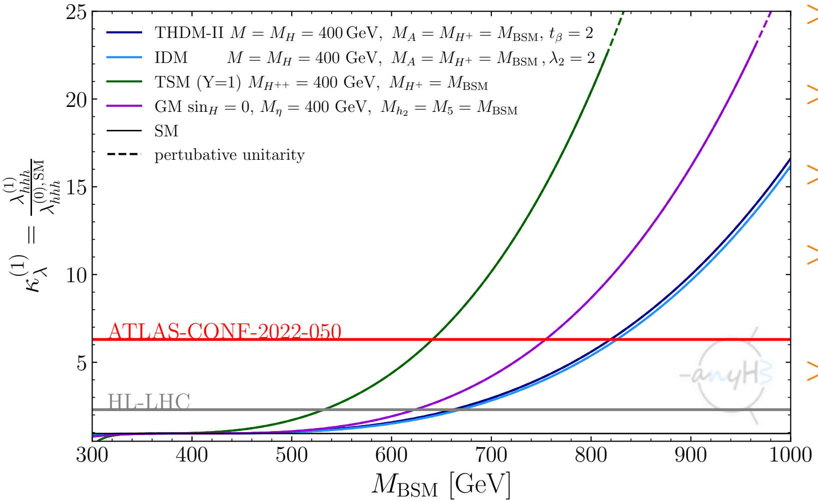
Results

Cross-check: Decoupling w/ alignment



- many models built-in and cross-checked
- ensure *appropriate* decoupling behaviour
- recover SM result for $M_{\text{BSM}} \to \infty$
- easy to implement new models (`UFO`)
- further checks
  - literature (if available, e.g. MSSM)
  - UV-finiteness
  - FeynArts/FormCalc

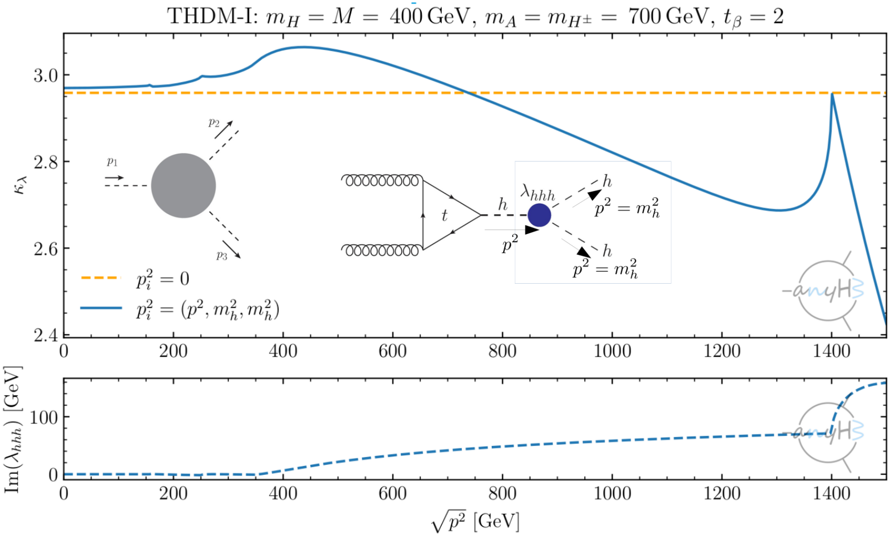
Alignment w/o decoupling



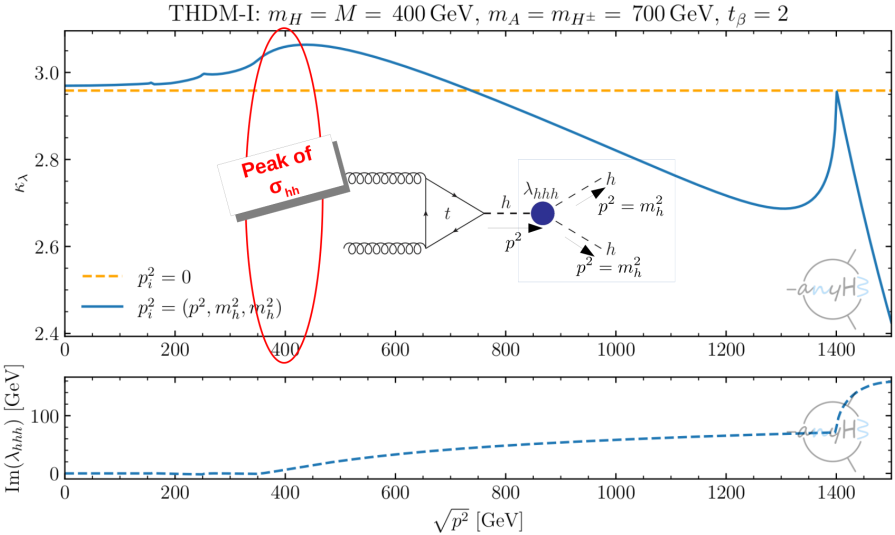
- alignment means $\kappa^{\text{tree-level}}_{h x_{\text{SM}} x_{\text{SM}}} = 1$
- mass splitting within the same multiplet
- induces large couplings for $M_{\text{BSM}} \to \infty$
- corrections large-enough to exclude parameter space
- see [Bahl, Braathen, Weiglein '22] for two-loop results in the THDM [Talk by J.Braathen]

Simplest case: $V_{H\Phi_{\text{BSM}}} \propto \lambda_{H\Phi_{\text{BSM}}} H^2 \Phi^2_{\text{BSM}} + \mu^2 \Phi^2_{\text{BSM}} \quad \Rightarrow \quad \lambda_{H\Phi_{\text{BSM}}} \propto (M^2_{\text{BSM}} - \mu^2)/v^2$

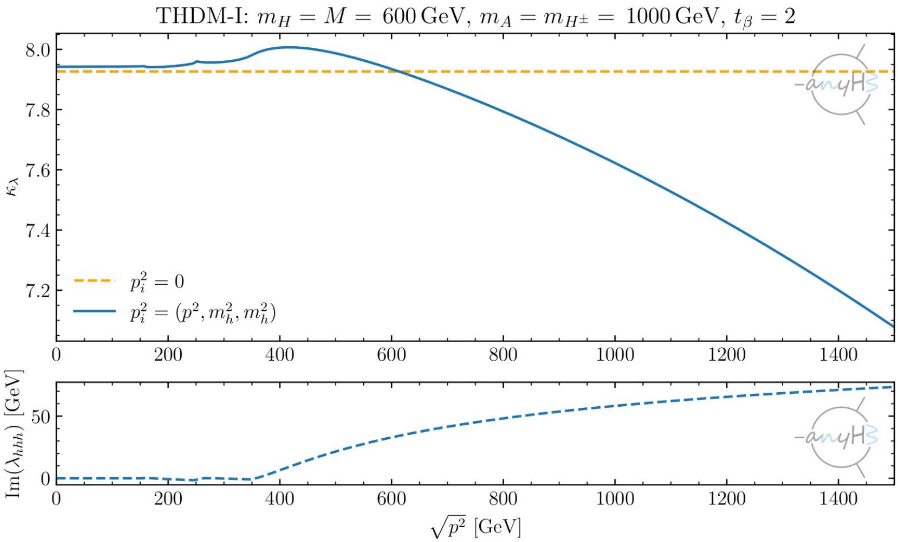
Momentum dependence in the THDM



Momentum dependence in the THDM

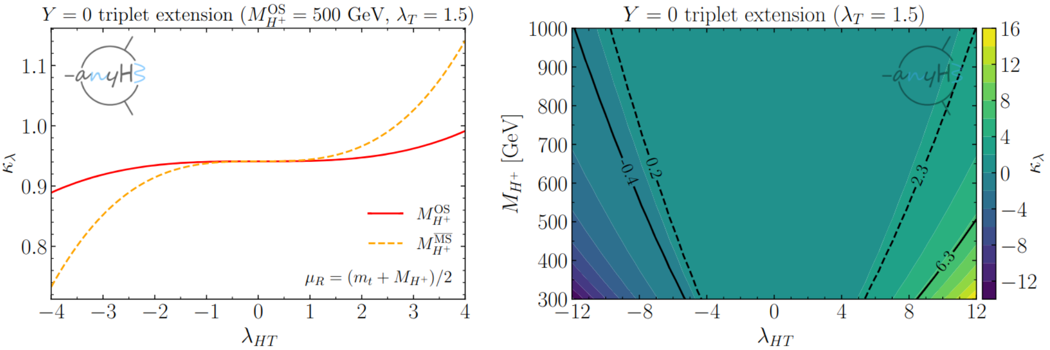


Momentum dependence in the THDM



Renormalization scheme comparison for a real Triplet

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2, \ \langle T\rangle = 0, \langle\Phi\rangle = v_{SM}$$



estimate of two-loop corrections generated by triplet self-coupling

Outlook / Summary

> Outlook:
- non-SM self-couplings (e.g. $\kappa_{\lambda_{hhH}}$)
- influence on σ_{hh} (esp. momentum effects)
- more models
- go beyond one-loop
- κ_t and κ_{tt}

> Summary:
- `anyH3`: λ_{hhh} in arbitrary ren. QFTs
  - at the full one-loop order
  - optional momentum dependence
  - with arbitrary choice of renormalization schemes
- uses `UFO` input
- analytical results; fast numerical results $\mathcal{O}(\text{ms})$
- many models already implemented:
  SM, SM+**singlets, doublets, triplets**, **SUSY**, ...
- found large mass-splitting effects

More info

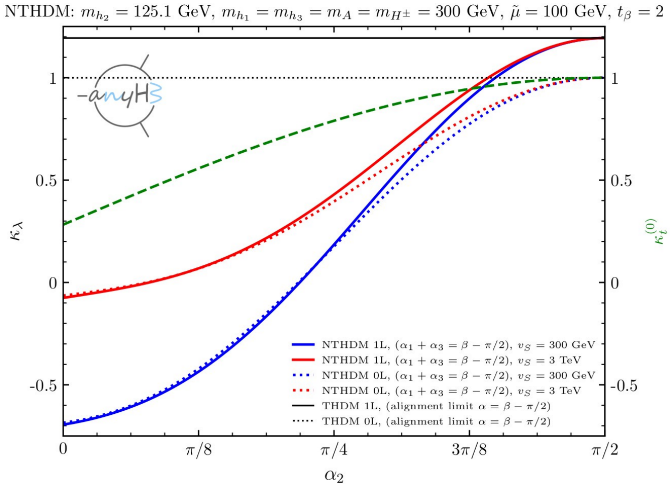
> `pip install anyBSM`

> `anyBSM --help`

> documentation, tutorials and examples: anybsm.gitlab.io

Backup

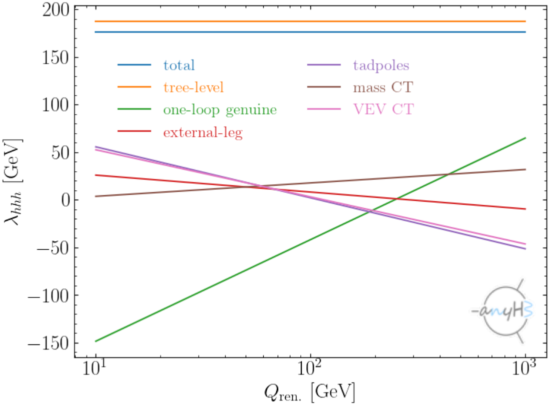
The sign of κ_λ in the NTHDM



- NTHDM=THDM+ real singlet
- 3 CP-even scalars $h_{1,2,3}$, 3 mixing angles $\alpha_{1,2,3}$
- $\alpha_2 \to \pi/2$: decoupling of singlet + alignment
- **attention**: from ggHH we only get $sgn(\kappa_t/\kappa_\lambda)$, the relative sign of top- and Higgs modifiers!
- $\kappa_t = \frac{y_t^{\text{BSM}}}{y_t^{\text{SM}}}$ strongly constrained

Simple cross-check: UV-finiteness in the SM

Numerically: scale independent result

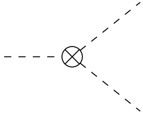


Analytically: cancellaiton of $^1/_\epsilon$ poles

```
<< anyBSM`
LoadModel["SM"]
lam = lambdahhh[];
(lam["total"] - lam["treelevel"] //. UVparts // Simplify) == 0

True
```

Renormalisation of λ_{hhh}

$\delta\lambda^{\mathsf{CT}}_{hhh} =$  $= \quad ?$

- one-loop $\rightarrow$ renormalisation of all parameters entering $\lambda^{(0),\mathsf{BSM}}_{hhh}$ at tree-level
- In the SM $\lambda^{(0),\,\mathsf{SM}}_{hhh} = \frac{3m_h^2}{v}$
- In general:

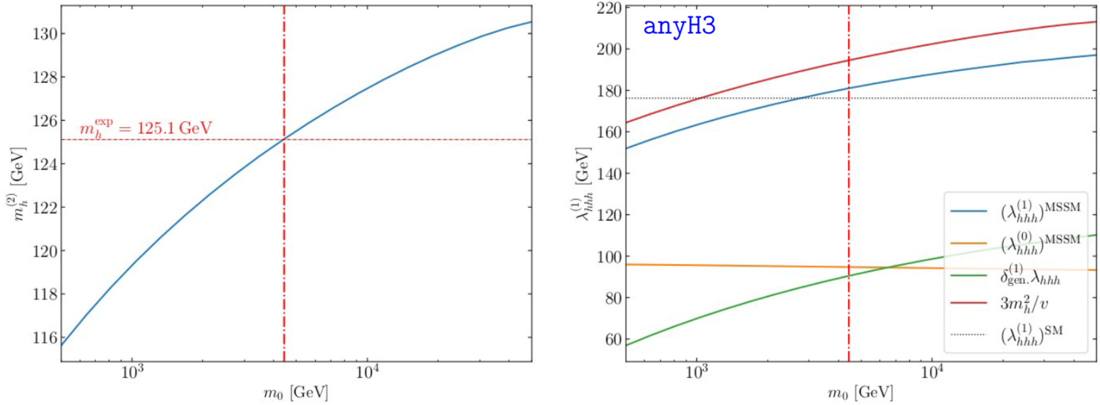
$$\lambda^{(0),\mathsf{BSM}}_{hhh} \equiv \lambda^{(0),\mathsf{BSM}}_{hhh}\Big(\underbrace{v^{\mathsf{SM}},\ m_h^{\mathsf{SM}}}_{\text{SM Higgs sector}},\ \underbrace{m_{X_i}}_{\text{further (OS) masses}},\ \underbrace{v_j}_{\text{BSM VEVs}},\ \underbrace{\alpha_k}_{\text{mixing angles}},\ \underbrace{g_l}_{\text{indep. couplings}}\Big)$$

- user's choice:
  - **SM sector**: fully OS or $\overline{\mathsf{MS}}/\overline{\mathsf{DR}}$
  - **BSM masses** (scalars/vectors/fermions): OS **or** $\overline{\mathsf{MS}}/\overline{\mathsf{DR}}$
  - **Additional couplings/vevs/mixings**: $\overline{\mathsf{MS}}/\overline{\mathsf{DR}}$ by default. **Custom ren. conditions possible!**

$$\delta^{(1)}_{\mathsf{CT}}\lambda_{hhh} = \sum_p \left(\frac{\partial\lambda^{(0),\mathsf{BSM}}_{hhh}}{\partial p}\right)\delta^{\mathsf{CT}}p,\ \text{with } p = \{m_h^{SM}, v^{\mathsf{SM}},\ m_{X_i}, \alpha_j, \dots\}^{\overline{\mathsf{MS}}/\mathsf{OS}/\text{custom}}$$

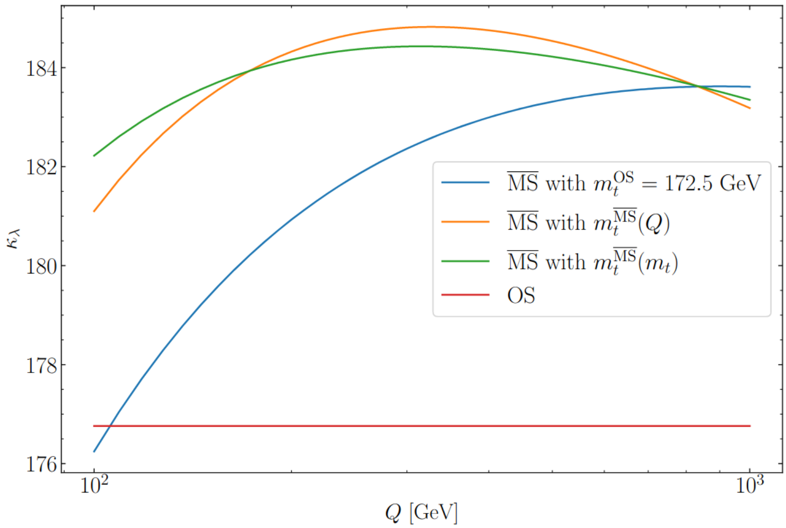
Full MSSM result: interface to SPheno

CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\mathrm{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno



- Example for a very simple version of the constrained MSSM → BSM parameters m_0, $m_{1/2}$, A_0, sgn(μ), tanβ
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3

Scheme- and top-mass- uncertainty



(Default) Renormalization choice of $(v^{\mathrm{SM}})^{\mathrm{OS}}$ and $(m_i^2)^{\mathrm{OS}}$

- $v^{\mathrm{OS}} \equiv \frac{2M_W}{e}\sqrt{1-\frac{M_W^2}{M_Z^2}}$ with (remember: $\lambda_{hhh}^{(0)} \approx 3m_h^2/v$)
  - $\delta^{(1)} M_V^2 = \frac{\mathrm{Re}\Pi_V^{(1),T}}{M_V^{2\;OS}}$, $V = W, Z$
  - $\delta^{(1)} e = \frac{1}{2}\Pi_\gamma + \mathrm{sign}(\sin\theta_W)\frac{\sin\theta_W}{M_Z^2\cos\theta_W}\Pi_{\gamma Z}$
- attention *(i)*: $\rho^{\text{tree-level}} \neq 1 \rightarrow$ further CTs needed (depends on the model)
  $\rightarrow$ ability to define *custom* renormalisation conditions
- scalar masses: $m_i^{\mathrm{OS}} = m_i^{\mathrm{pole}}$
  - $\delta^{\mathrm{OS}} m_i^2 = -\mathrm{Re}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

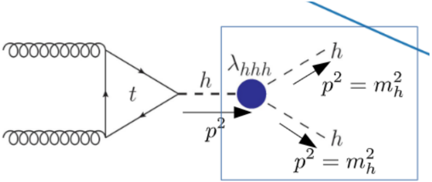
Treatment of external leg corrections

default treatment of external legs:

$$\delta^{(1)\,,\text{ext.-legs}}\lambda_{hhh} = -\sum_i \left[\frac{1}{2}\Sigma'_{hh}(p_i^2)\lambda^{(0)}_{hhh} + \underbrace{\sum_{j,h_j\neq h} \frac{\Sigma_{hh_j}(p_i^2)}{p_i^2 - m^2_{h_j}}\lambda^{(0)}_{h_j hh}}_{=0,\text{ for alignment}} \right]$$

- Attention: insert into di-Higgs production: need one off- and two on-shell Higgses:

$$\delta^{(1),\text{ext.-legs}}\lambda_{hhh} = -\left(\tfrac{1}{2}+\tfrac{1}{2}\right)\Sigma'_{hh}(m_h^2)\lambda^{(0)}_{hhh}$$



- possible to turn-off default behaviour and specify ext.-leg contributions in terms of selfenergies

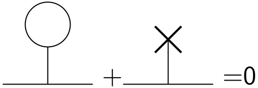
Treatment of tadpoles: many possibilities

At tree-level:

- define $t_h = \frac{\partial V}{\partial h}\Big|_{h=0}$ and $m_h^2 = \frac{\partial^2 V}{\partial h^2}\Big|_{h=0}$
- then $V_{\text{SM}} \supset t_h h + \frac{1}{2} m_h^2 h^2 + \frac{m_h^2 - t_h/v}{2v} h^3 + \frac{m_h^2 - t_h/v}{8v^2} h^4$
- popular choice $t_h = 0$ (but not the only choice!)

At one-loop: in general the renormalized tadpole consists of $\hat{t}_h = t_h + t_h^{(1)} + \delta t_h^{(1)}$

- "OS" tadpoles
  - demand $\hat{t}_h = t_h = 0$ at one-loop such that $t_h^{(1)} = -\delta t_h^{(1)}$
  - effectively no need to "attach" tadpoles to any diagrams



- "Fleischer-Jegerlehner (FJ)" tadpoles
  - demand $t_h = 0$ at one-loop but let $\delta t_h^{(1)}$ cancel only divergent pieces
  - need to consider finite contributions of *all* 1PI diagrams
- "tadpole-free $\overline{\text{MS}}$ scheme"
  - set $\delta t_h^{(1)} = 0$ and demand $\hat{t}_h = 0 \Rightarrow t_h = -t_h^{(1)}$

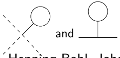
Treatment of tadpole corrections for λ_{hhh}

w/o specifying a concrete scheme, nor the vacuum (in the alignment limit):

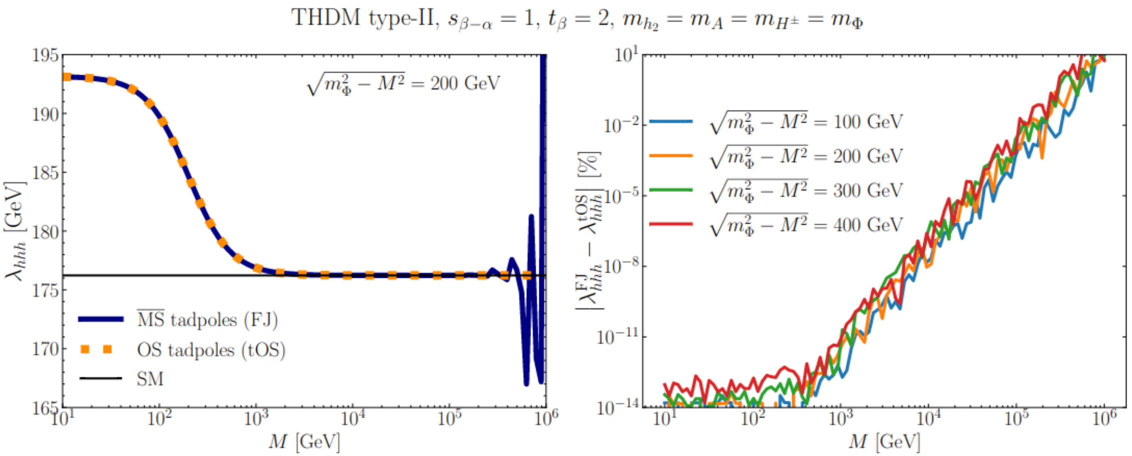
$$\lambda_{hhh}^{\text{tadpoles}} = -\underbrace{\frac{3t_h}{v^2}}_{\text{tree-level}} - \underbrace{\frac{6}{v^2}\delta^{(1)}_{\text{CT}}t_h}_{\text{CT-inserted diagrams}} + \underbrace{\delta^{(1)}_{\text{tadpoles}}\lambda_{hhh}}_{\text{tadpole diagrams}} + \underbrace{\frac{3}{v}\delta^{(1)}_{\text{CT, tadpoles}}m_h^2 - \frac{3m_h^2}{v^2}\delta^{(1)}_{\text{CT, tadpoles}}v}_{\text{tad. contr. to input parameters}}$$

- In the SM (and BSM+alignment): once λ_{hhh} is expressed in terms of *physical* input parameters, its result is independent of the treatment (OS, FJ, ...) of the tadpoles (up to higher orders):

$$\delta^{(1)}\lambda_{hhh} \supset \frac{3}{v^2}\delta^{(1)}t_h\big|_{\text{finite}}$$

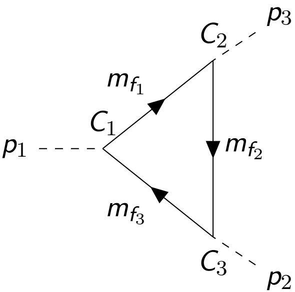
- However: `UFO` models do (often) **not** contain the explicit dependence on the tree-level/one-loop/one-loop-CT tadpoles.
- Thus: we choose the Fleischer-Jegerlehner treatment $t_h^{\text{tree-level}} = 0$ and renormalize $\delta^{(1)}t_h^{\text{CT}}\big|_{\text{finite}} = 0$ in the $\overline{\text{MS}}$ scheme per default (can also turn-off automatic tadpoles and implement own scheme).
- only need to take into account tadpole contributions to all two- and three-point functions: 

OS vs FJ tadpole treatment



Example: generic fermion triangle

Idea: compute *generic* diagrams i.e. assume most generic



- couplings $C_i = P_L C_i^L + P_R C_i^R$, $P_{R/L} = \frac{1 \pm \gamma_5}{2}$
- as well as loop-masses m_{f_i} and
- external momenta p_i, $i = 1, 2, 3$.

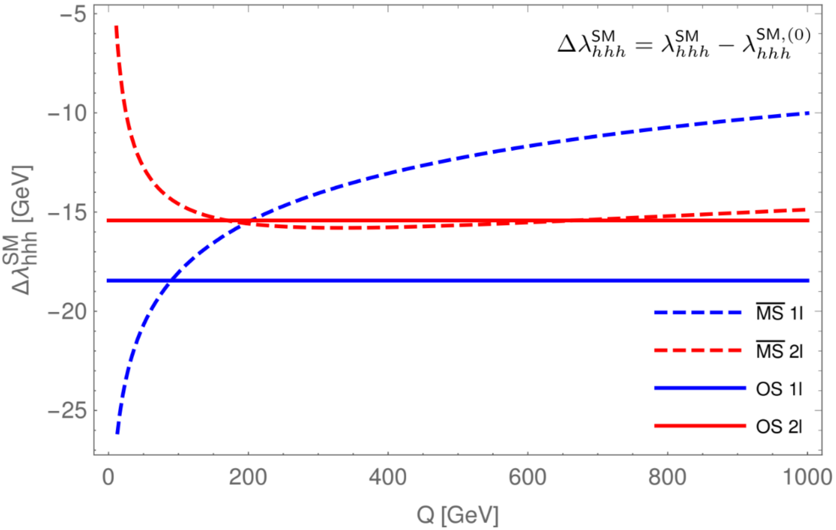
$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

- insert concrete BSM model (`UFO` [Degrande et al. '11])
- evaluate with the help of `(py)COLLIER` [Denner et al. '16]
- `public code anyH3` [Bahl, Braathen, MG, Weiglein '23]

Beyond `anyH3`

Two-loop effects in the SM

[Braathen, Kaneura '19]



W mass prediction

- start with HO corrections to muon decay: $M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F}\left[1+\Delta r\right]$
- and solve for: $M_W^2=M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right]$
- with: $\Delta r^{(1)}=2\delta^{(1)}e+\frac{\Pi_W^{(1),T}(0)-\delta^{(1)}M_W^2}{M_W^2}-\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}+\delta_{\text{vertex+box}}$
- and: $\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}=\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}-\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}\right)$

It's all there but:

- $\delta^{\text{SM}}_{\text{vertex+box}}=-\frac{2\,\text{sign}(\sin\theta_W)}{\cos\theta_W\sin\theta_W M_Z^2}\Pi_{Z\gamma}(p^2=0)+\frac{\alpha_{QED}}{4\pi\sin^2\theta_W}\left(6+\frac{7-4\sin^2\theta_W}{2\sin^2\theta_W}\right)\log(\cos^2\theta_W)$
- $\delta^{\text{BSM}}_{\text{vertex+box}}=$ **needs to be implemented**

However:

- in many models $\Delta r\supset\frac{\delta\sin^2\theta_W}{\sin\theta_W}\approx\delta\rho$ is the dominant effect!

λ_{hhh} in the SM and in SUSY

In the SM at tree-level:

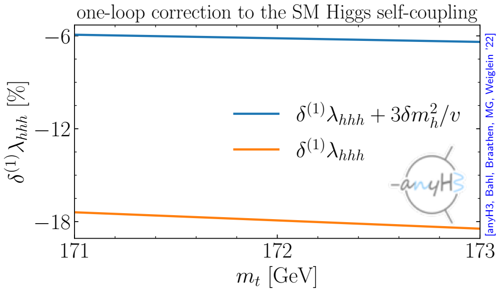
$$V(h) \supset \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \ldots \qquad \Rightarrow \qquad \lambda_{hhh}^{\mathrm{SM}} = \frac{\partial^3 V(h)}{\partial^3 h} = \frac{3 m_h^2}{v}$$

Thus $\lambda_{hhh}^{\mathrm{SM}}$ can be predicted perturbatively as a function of the SM parameters.

- corrections to λ_{hhh} are expected to behave similar to those of the Higgs boson mass
- OS scheme for m_h allows to "absorb" large part of corrections
- in SUSY:
  - $\lambda_{hhh} = 3m_h^2/v$ approximate[Dobado, Herrero, Hollik, Penaranda '02]
  - but m_h not free and $m_h \lesssim m_Z$ at tree-level!

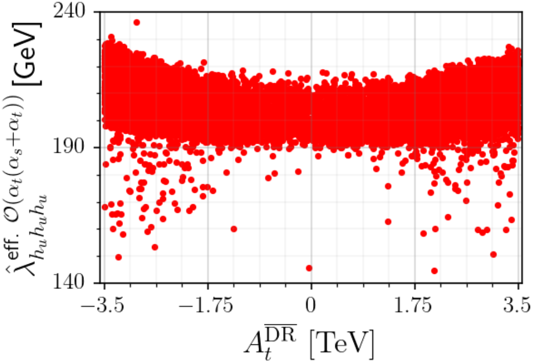
    $\to$ requires loop corrections of about 40 GeV (15-30%)

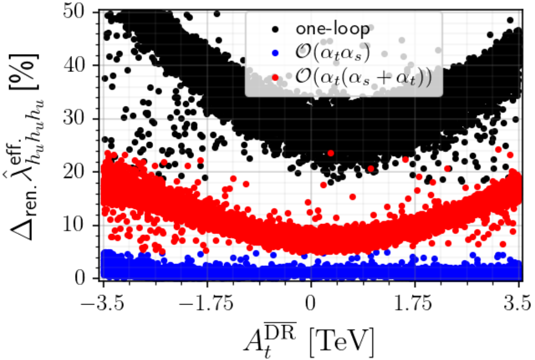
    $\to$ can't stop at one-loop; need higher orders ($\to$KUTS)



$\to$ the precision of λ_{hhh} (order in perturbation theory) should match those of m_h!

Results: λ_{hhh} in the NMSSM at two-loops



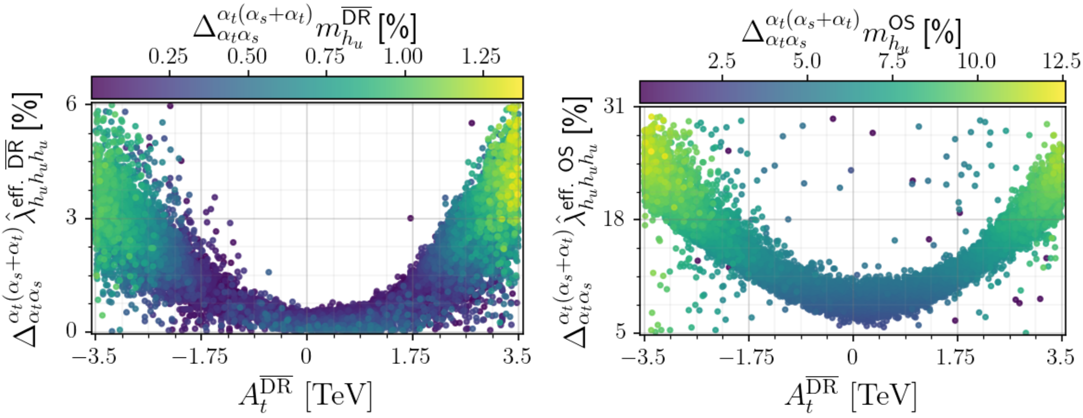


$$\Delta_{\text{ren.}}\lambda_{hhh} = \frac{\lambda_{hhh}(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}}) - \lambda_{hhh}(m_t^{\text{OS}}, A_t^{\text{OS}})}{\lambda_{hhh}(m_t^{\overline{\text{DR}}}, A_t^{\overline{\text{DR}}})}$$ $\sim$ higher-orders $\rightarrow$ estimates theory uncertainty

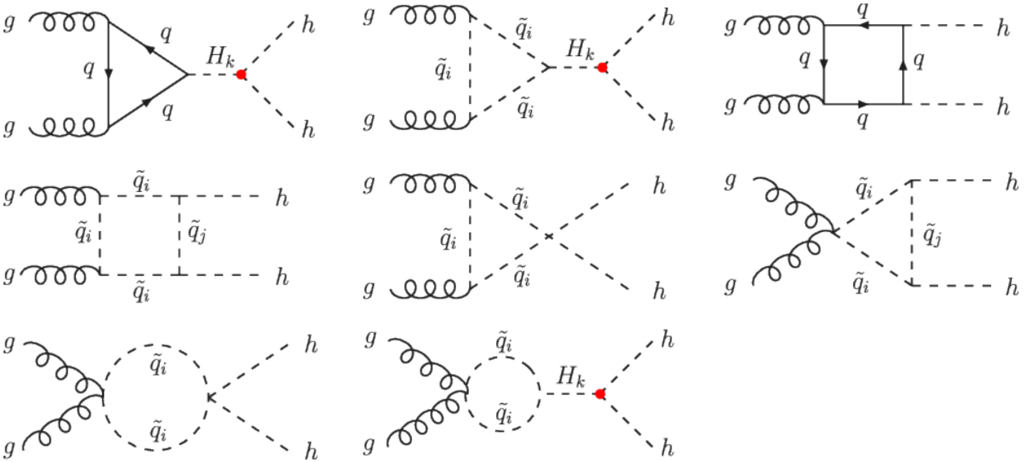
(Points checked against `HiggsSignals 2.6.2` and `HiggsBounds 5.10.2` as well as model-independent constraints on SUSY masses.)

Size of the $\mathcal{O}(\alpha_t^2)$-corrections to λ_{hhh}

...and correlation to $\mathcal{O}(\alpha_t^2)$ m_h-corrections



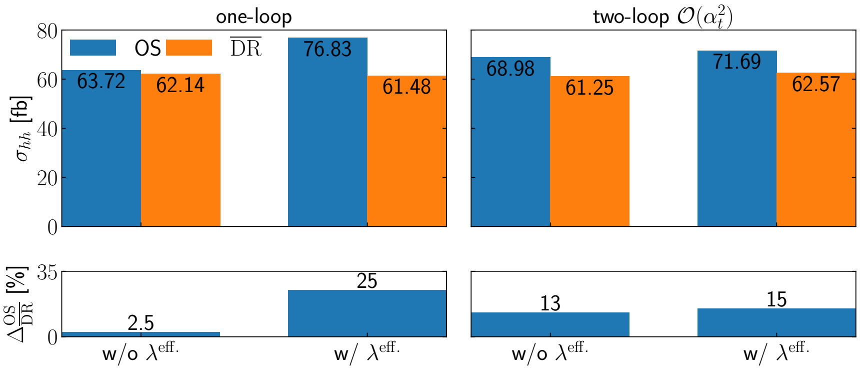
Double Higgs production



Use $\lambda_{hhh}^{\alpha_t^2}$ as input in HPAIR [Spira] to estimate higher-order effects in σ_{hh}.

Double Higgs production

Parameter point with resonant contribution from intermediate BSM Higgs:



- w/o $\lambda^{\text{eff.}}$: loop corrections to masses/mixing angles (and according LSZ-factors)
  $\rightarrow$ corrections to the input parameters
- w/ $\lambda^{\text{eff.}}$: additionally use effective coupling at respective order
  $\rightarrow$ corrections to the di-Higgs process