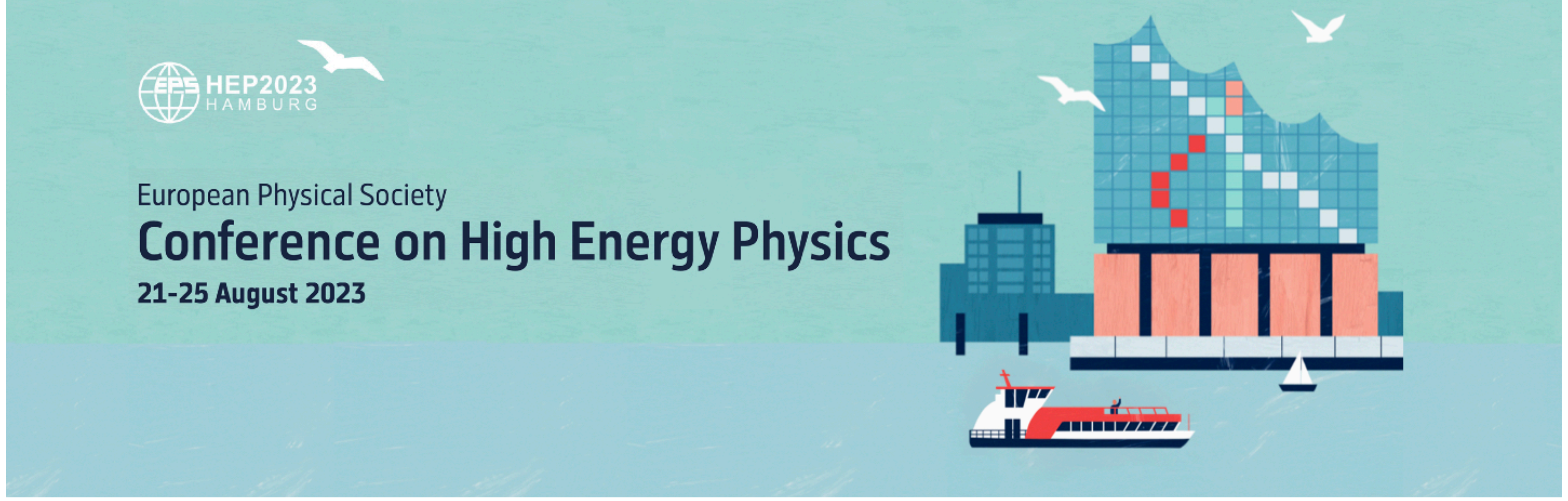

Effective Field Theories for Dark Matter Pairs in the Early Universe

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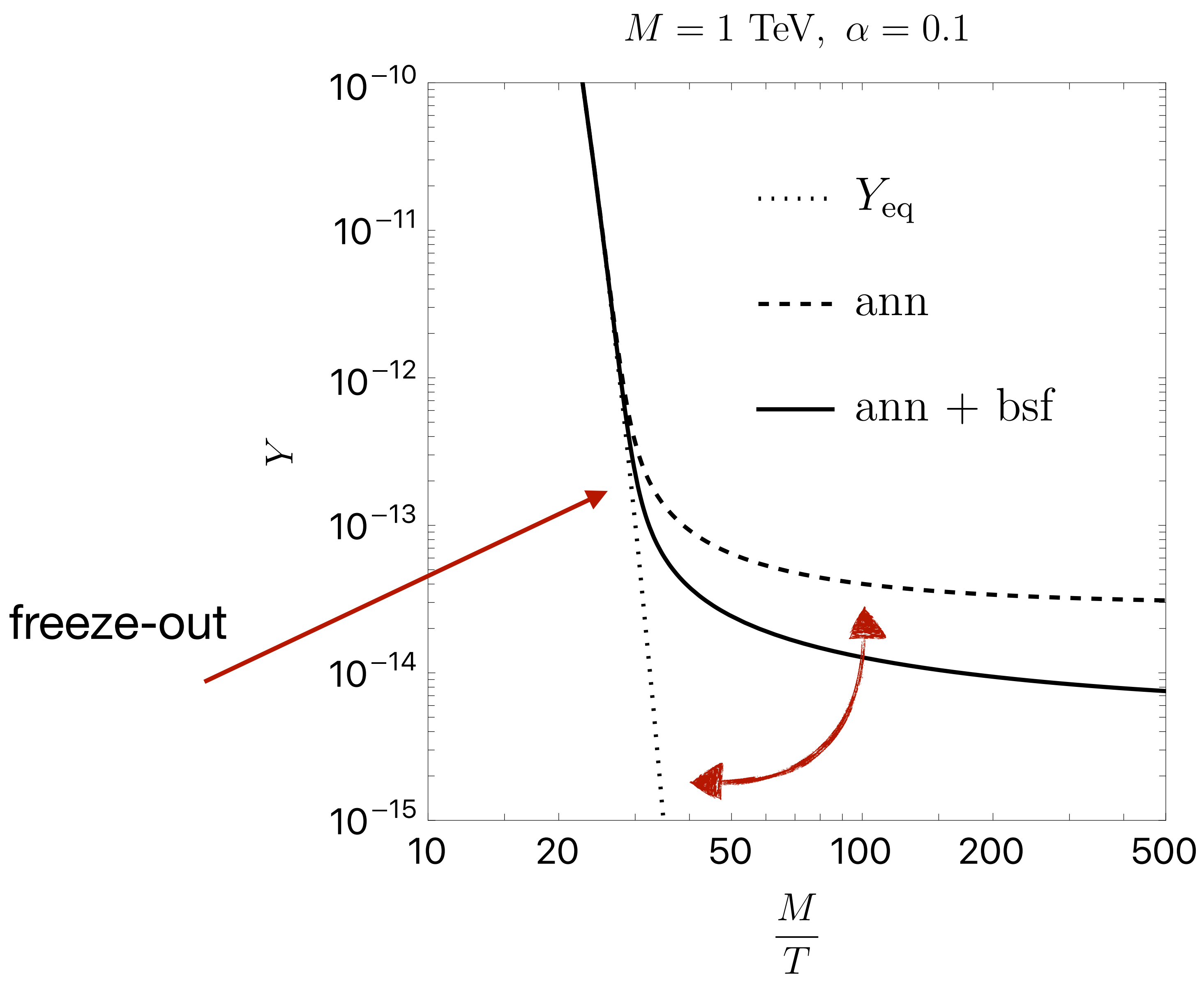
Outline

- Intro: Thermal freeze-out effect
- EFTs for thermal DM pairs
- Hard and near-threshold processes
- DM relic density
- Conclusions

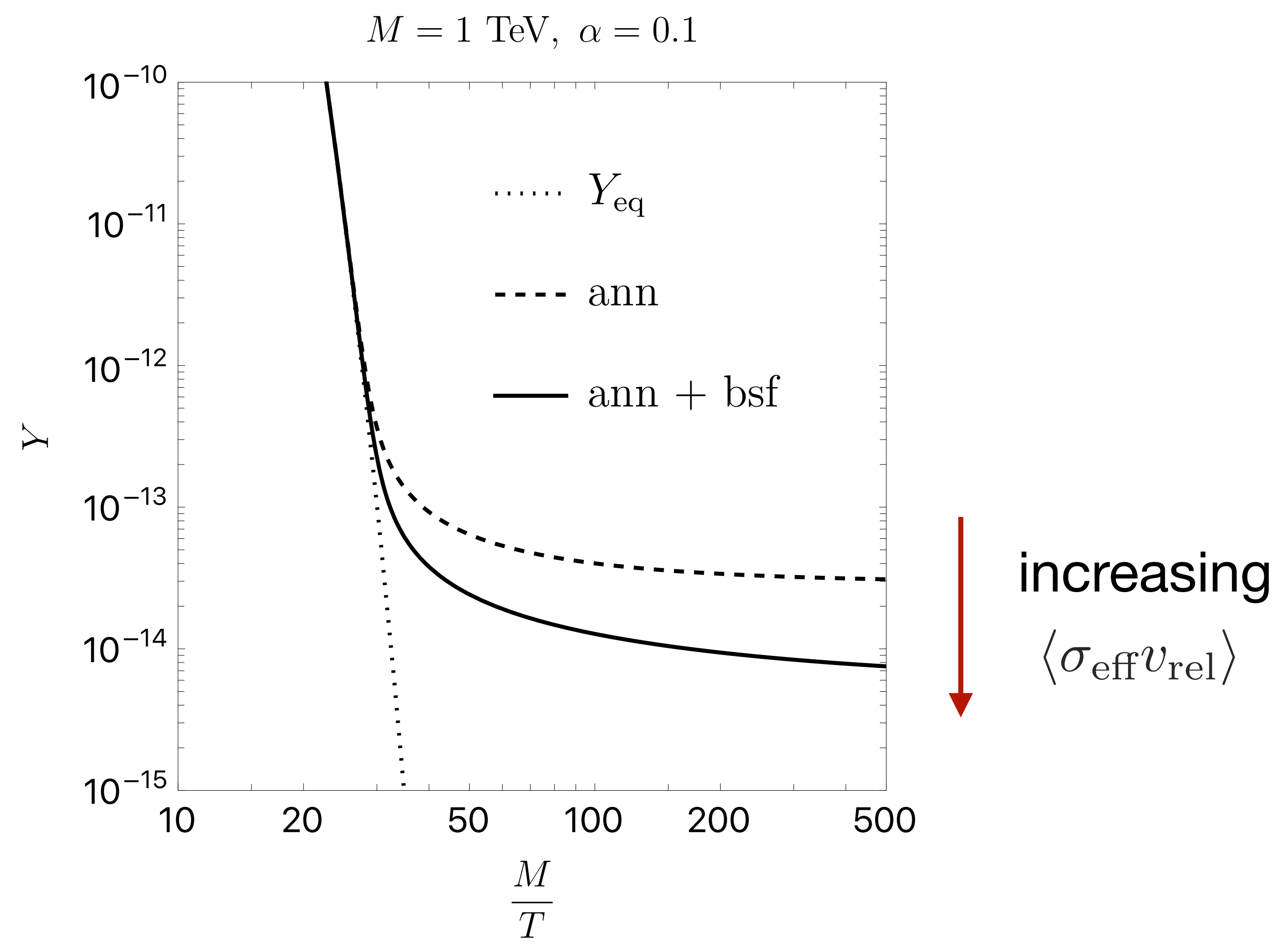
Motivation - Thermal freeze-out of DM

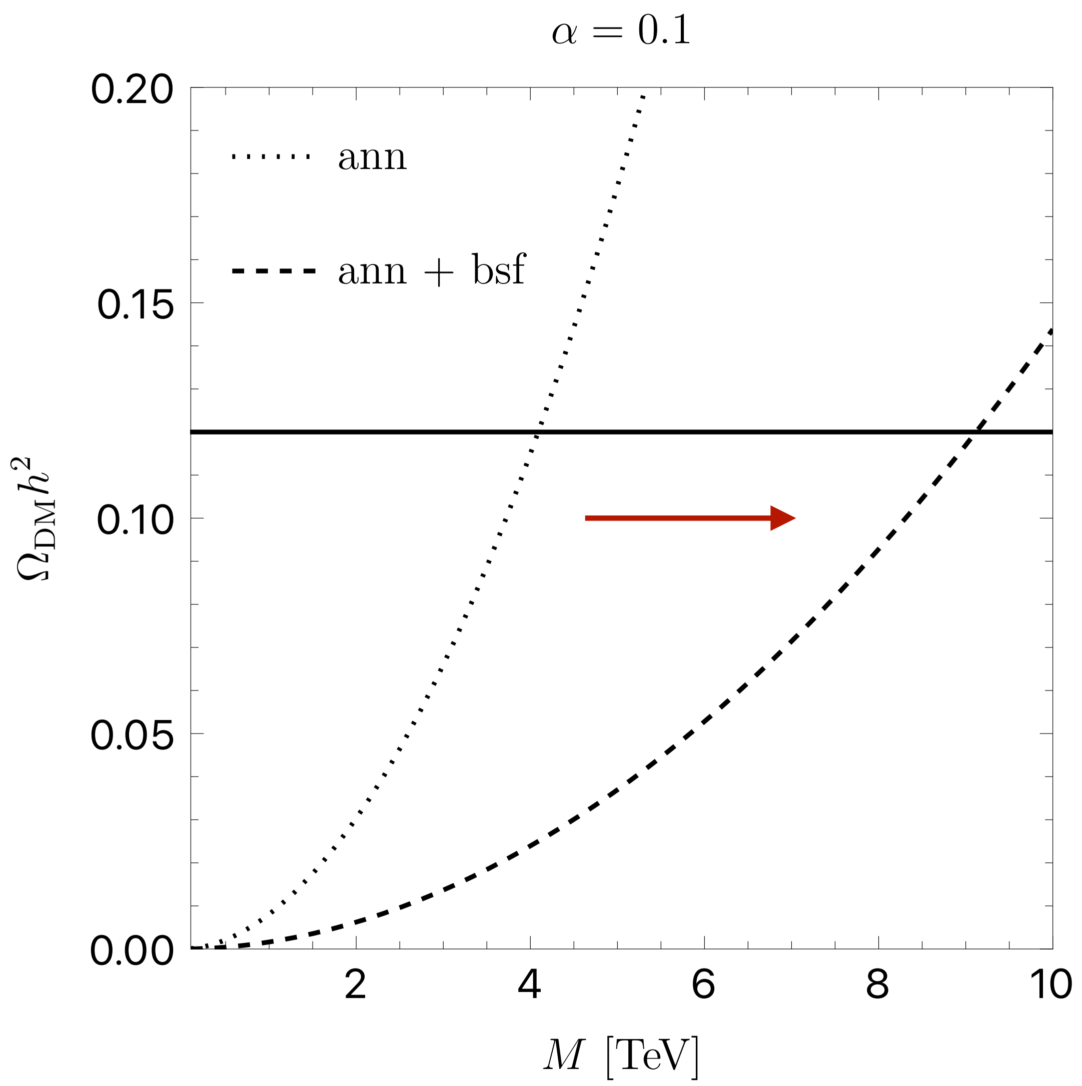
- Particle nature of dark matter (DM) yet unknown
- Present-day DM relic density known: $\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012$
- Plethora of models and extensive literature available
- Categorize DM according to their production mechanism and evolution in early universe
- Prominent scenario: **Thermal freeze-out of DM**
- Time period triggering departure from chemical equilibrium, while kinetic equilibrium maintained
- Freeze-out mechanism widely used for WIMPs, but may be used also to models with stronger self-interactions



Thermal freeze-out of DM

- **Dark sector**: Particle-like DM (mass M) interacting via **long-range mediator**
- Early universe ($T \gtrsim M$): Heavy DM in thermal equilibrium with dark medium
- Expanding universe ($T \lesssim M$): T cools down $\rightarrow$ detailed balance lost
- Evolution equation: $$(\partial_t + 3H)n = -\frac{1}{2}\langle\sigma_{\rm eff} v_{\rm rel}\rangle(n^2 - n_{\rm eq}^2)$$
- Observed DM relic abundance implies heavy DM: $\Omega_{\rm DM}h^2 \sim 3\times 10^{11}\frac{M}{\rm TeV}Y_0 \quad \rightarrow \quad M \sim {\rm TeV}$
- During and after chemical freeze-out, DM is non-relativistic: $H \sim \langle\sigma_{\rm eff} v_{\rm rel}\rangle n_{\rm eq} \quad \rightarrow \quad T \sim M/25$
- During thermal freeze-out, DM being non-relativistic induces a **hierarchy of energy scales**
- Exploit effective field theory (**EFT**) methods at finite T to compute relevant thermal rates

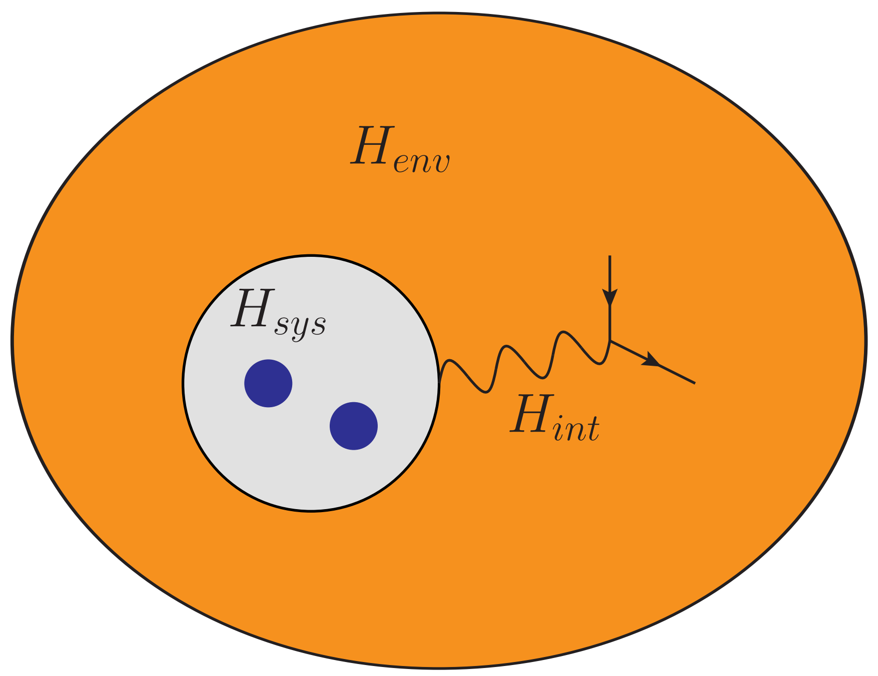




Heavy thermal DM interacting via long-range mediators

- Model for the dark sector: Fermionic DM weakly coupled to dark $\mathrm{U}(1)_{\mathrm{DM}}$

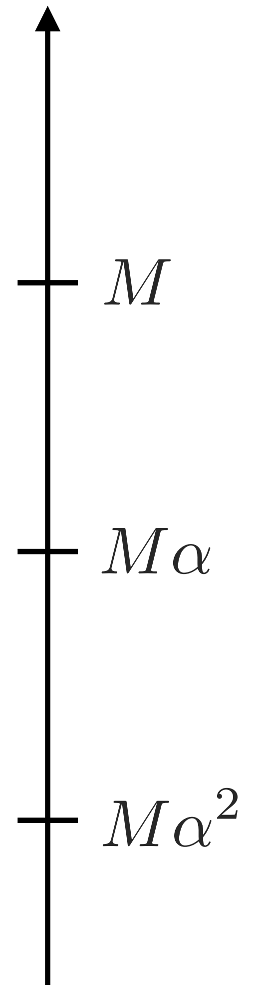
$$\mathcal{L}_{\mathrm{U}(1)} = \bar{X}(i\not{D} - M)X - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\mathrm{portal}}$$



- Thermal medium: Thermal dark gauge fields and SM particles with temperature T

- $\mathcal{L}_{\mathrm{portal}}$: Portal couplings to the SM, additional light d.o.f. in the dark sector, … ⟶ disregard, i.e. purely self-interacting dark sector!

- **Weakly coupled non-relativistic DM**: Separation of energy scales into hard, soft & ultrasoft scales



- Near-threshold contributions at the ultrasoft scale substantially affect the annihilation rate

$$\langle\sigma_{\mathrm{eff}}\, v_{\mathrm{rel}}\rangle = \langle\sigma_{\mathrm{ann}} v_{\mathrm{rel}}\rangle + \sum_n \langle\sigma^n_{\mathrm{bsf}}\, v_{\mathrm{rel}}\rangle\, \frac{\Gamma^n_{\mathrm{ann}}}{\Gamma^n_{\mathrm{ann}} + \Gamma^n_{\mathrm{bsd}}}$$

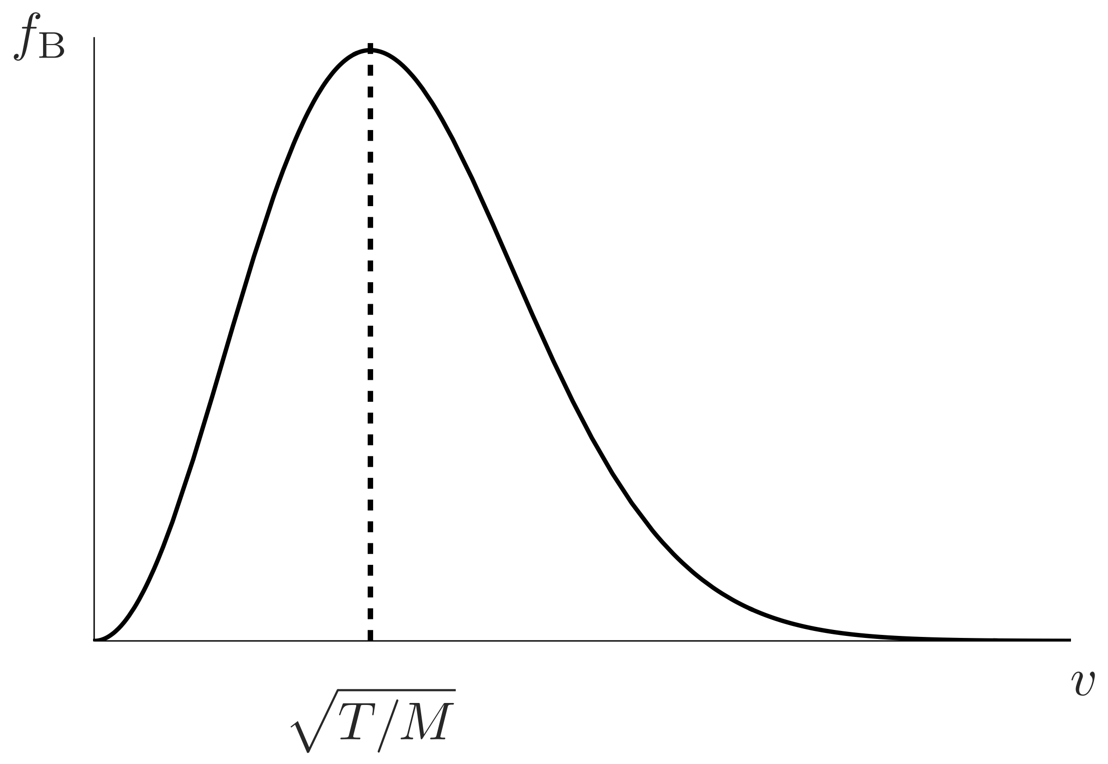
- Repeated soft exchange between non-relativistic particle & antiparticle - **formation of metastable pairs**

Hierarchy of energy scales - Effective d.o.f.

- **Weakly interacting** ($\alpha \ll 1$) **non-relativistic** ($v \ll 1$) **thermal** DM during and after freeze-out

  Momenta follow a Maxwell-Boltzmann distr.

  $$p \sim \sqrt{MT} \gg T$$
  $$v \sim p/M \sim \sqrt{T/M} \ll 1$$



- **Effective d.o.f.** at thermal freeze-out ($T_F \lesssim M/25$) and after freeze-out ($M \gg T$):

  - Non-relativistic, kinetically equilibrated pairs close to threshold with relative velocity $v_{\mathrm{rel}} \sim \sqrt{T/M} \ll 1$

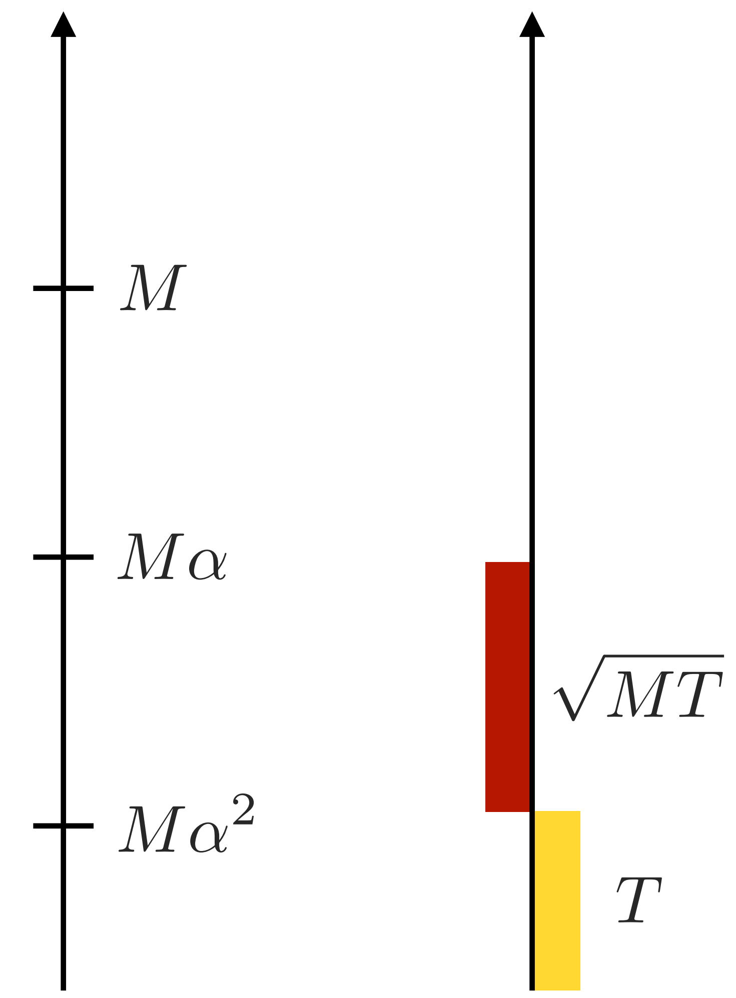
    Hierarchy of energy scales induced by the relative motion: $M \gg M v_{\mathrm{rel}} \gg M v_{\mathrm{rel}}^2$

  - Non-relativistic Coulombic bound-state pairs with relative velocity fixed by the virial theorem $v_{\mathrm{rel}} \sim \alpha \ll 1$

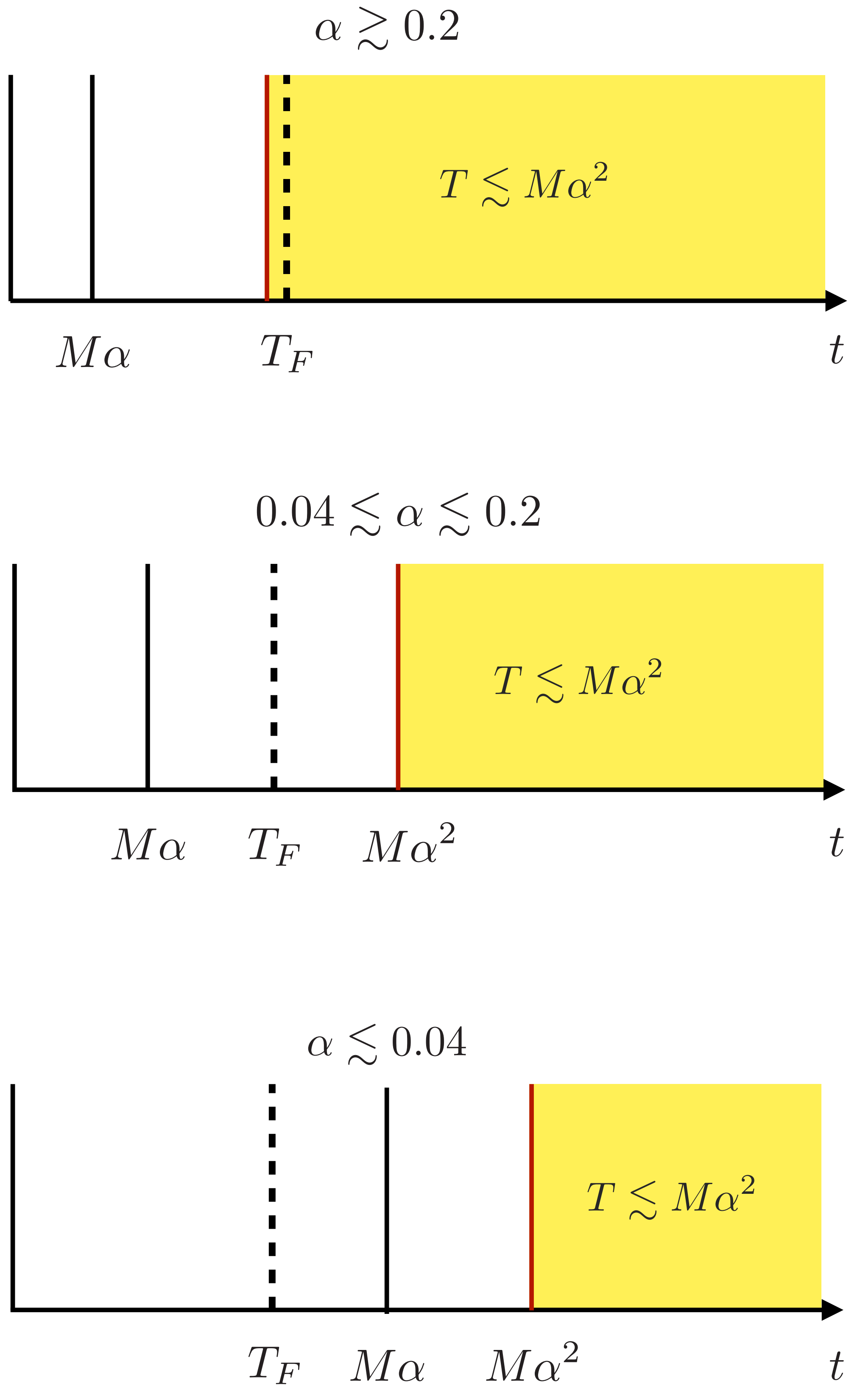
    Hierarchy of energy scales induced by the relative motion: $M \gg M\alpha \gg M\alpha^2$

  - Thermal total momentum due to c.m. motion of the pairs: $P \sim \sqrt{MT} \gg T$

  - Thermal environment: Abelian dark gauge fields with temperature T



Hierarchy of energy scales - EFTs



- Hierarchy of energy scales:

$$M \gg M\alpha \gtrsim \sqrt{MT} \gg M\alpha^2 \gtrsim T$$

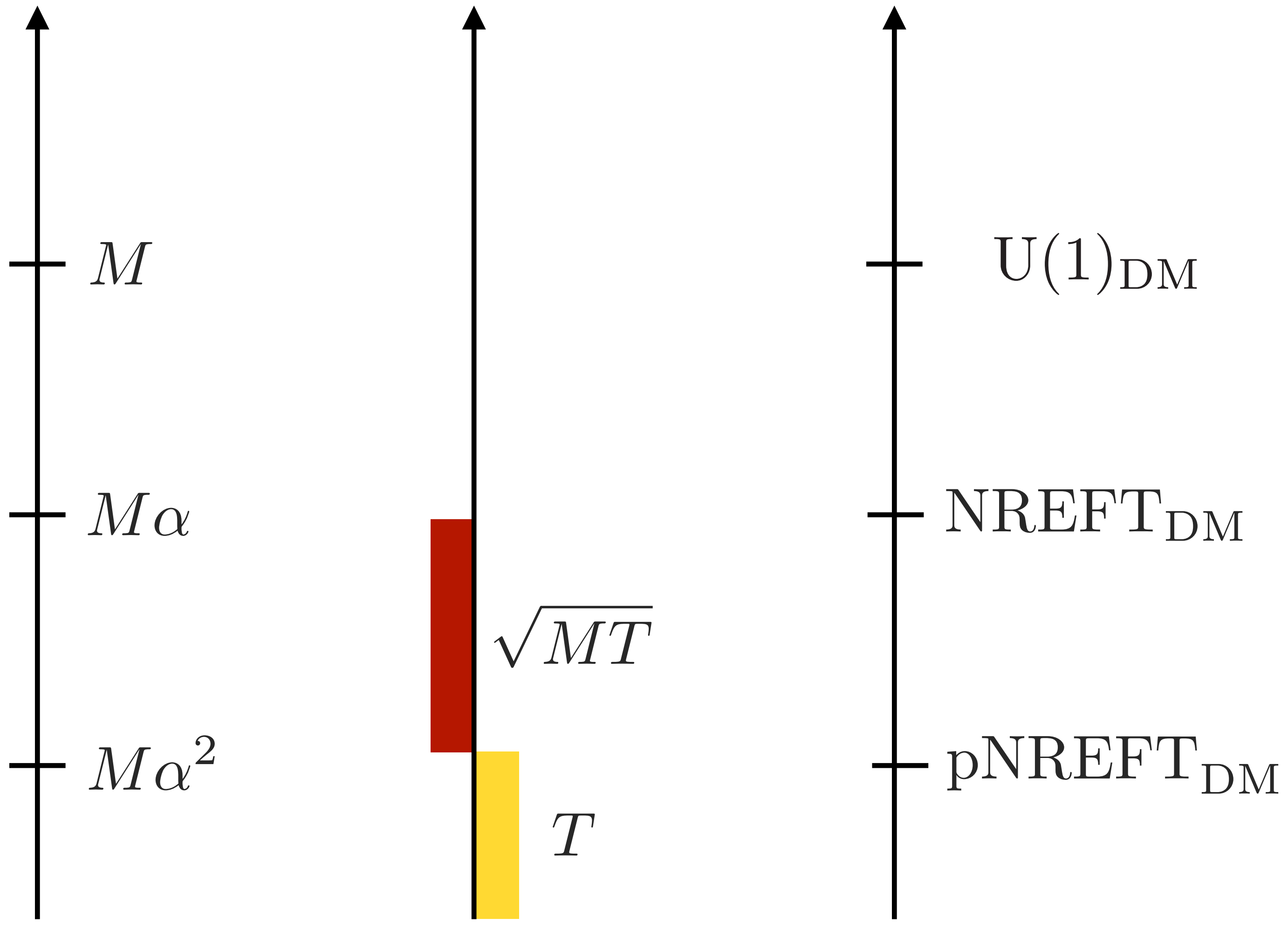
- Condition $T \lesssim M\alpha^2$ fulfilled for most of the time after chemical decoupling $T_F \lesssim M/25$

- Utilize EFT techniques to disentangle the scales at Lagrangian level:

  - Integrate out higher modes subsequently $\longrightarrow$ tower of non-relativistic EFT's, in our case:

    1. First integrate out hard modes of order M $\rightarrow$ $\mathrm{NREFT_{DM}}$
    2. Next integrate out soft modes of order $M\alpha$ $\rightarrow$ $\mathrm{pNREFT_{DM}}$

  - Perform the matching order by order in α $\longrightarrow$ radiative corrections to the matching coefficients



Potential non-relativistic effective field theory (pNREFT)

- Hierarchy of energy scales:

$$M \gg M\alpha \gtrsim \sqrt{MT} \gg M\alpha^2 \gtrsim T$$

- Integrate out energies & momenta of the dark gauge fields of order $M\alpha$, already at the Lagrangian level, organized as a **multipole expansion** in the relative distance $r \ll R \quad \longrightarrow \quad \text{pNREFT}_{\text{DM}}$

- Multipole expansion valid as long as $M\alpha \sim 1/r \gg T, M\alpha^2 \sim 1/R$, and matching can be done as in $T=0$

- For larger $T \gtrsim M\alpha^2$: Possible thermal corrections due to the scale $\sqrt{MT} \gtrsim M\alpha$

Lorentz force:

$$\boldsymbol{F} = g(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$$

- At $\mathcal{O}(\boldsymbol{r})$:

$$\mathcal{L}_{\text{pNRQED}_{\text{DM}}} = \int d^3r\, \phi^\dagger(t,\boldsymbol{r},\boldsymbol{R}) \left[i\partial_0 - H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) + g\,\boldsymbol{r}\cdot\boldsymbol{E}(t,\boldsymbol{R}) + g\,\boldsymbol{r}\cdot\left(\frac{\boldsymbol{P}}{2M} \times \boldsymbol{B}(t,\boldsymbol{R})\right)\right] \phi(t,\boldsymbol{r},\boldsymbol{R})$$

- $$H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = 2M + \underbrace{\frac{\boldsymbol{p}^2_{\text{rel}}}{M} - \frac{\alpha}{r}}_{\sim M\alpha^2,\, T} + \underbrace{\frac{\boldsymbol{P}^2_{\text{c.m.}}}{4M}}_{\sim T} - \underbrace{\frac{\boldsymbol{p}^4_{\text{rel}}}{4M^3} + \dots}_{\sim M\alpha^4 \text{ suppressed}}$$

$$\sim T\frac{T}{M} \text{ suppressed}$$

$$E_n = 2M - M\frac{\alpha^2}{4n^2} + \frac{\boldsymbol{P}^2}{4M}$$

$$E_p = 2M + \frac{\boldsymbol{p}^2_{\text{rel}}}{M} + \frac{\boldsymbol{P}^2}{4M}$$

- $\phi(t,\boldsymbol{r},\boldsymbol{R})$ bi-local field of the DM pairs (“darkonium”)
- $\boldsymbol{E}(t,\boldsymbol{R}), \boldsymbol{B}(t,\boldsymbol{R})$ ultrasoft dark electric & magnetic fields

Annihilations in pNREFT

Fourier decomposition of darkonium field:

$$\phi_{ij}(t,\boldsymbol{r},\boldsymbol{R}) = \int \frac{d^3P}{(2\pi)^3}\left[\sum_n e^{-iE_nt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_n(\boldsymbol{r})\,S_{ij}\,\phi_n(\boldsymbol{P}) + \sum_{\text{spin}}\int\frac{d^3p}{(2\pi)^3}e^{-iE_pt+i\boldsymbol{P}\cdot\boldsymbol{R}}\,\Psi_{\boldsymbol{p}}(\boldsymbol{r})\,S_{ij}\,\phi_{\boldsymbol{p}}(\boldsymbol{P})\right]$$

- Annihilations are in the imaginary parts of the 4-fermion matching coeff.

- Local interaction terms from NREFT translate directly into local potentials in pNREFT, here for S-waves up to order $1/M^4$:

$$\delta V^{\text{ann}}(\boldsymbol{r}) = -\frac{i}{M^2}\,\delta^3(\boldsymbol{r})\,\left[2\text{Im}(d_s) - \boldsymbol{S}^2\left(\text{Im}(d_s)-\text{Im}(d_v)\right)\right] - \frac{i}{M^4}\,\delta^3(\boldsymbol{r})\,\boldsymbol{\nabla}^2_{\boldsymbol{R}}\left[2\text{Im}(\tilde{d}_s) - \boldsymbol{S}^2\left(\text{Im}(\tilde{d}_s)-\text{Im}(\tilde{d}_v)-\text{Im}(\tilde{d}'_v)\right)\right]$$

- Annihilations with higher angular momenta arise from higher dimensional local interaction terms, e.g. P-waves from dim.-8 operators

- From optical theorem, compute observables for **S-wave annihilation**, in the **lab frame**:

  - $$(\sigma_{\text{ann}}v_{\text{rel}})(\boldsymbol{p}) = \underbrace{\frac{\text{Im}(d_s)+3\text{Im}(d_v)}{M^2}}_{\text{hard corrections}}\underbrace{|\Psi_{\boldsymbol{p}0}(\boldsymbol{0})|^2}_{\text{soft corrections}}\overbrace{\left(1-\frac{\boldsymbol{P}^2}{8M^2}\right)}^{\text{Lorentz contraction}}$$

    Sommerfeld enhancement

  - Paradarkonium $X\bar{X}\to\gamma\gamma$: $\Gamma^{n,\text{para}}_{\text{ann}} = \frac{4\text{Im}(d_s)}{M^2}|\Psi_n(\boldsymbol{0})|^2\left(1-\frac{\boldsymbol{P}^2}{8M^2}\right)$

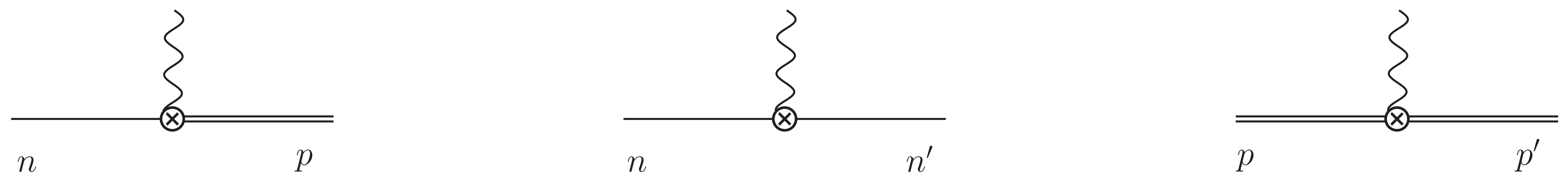
  - Orthodarkonium $X\bar{X}\to\gamma\gamma\gamma$: $\Gamma^{n,\text{ortho}}_{\text{ann}} = \frac{4\text{Im}(d_v)}{M^2}|\Psi_n(\boldsymbol{0})|^2\left(1-\frac{\boldsymbol{P}^2}{8M^2}\right)$

→ Factorization of the hard and soft scales! Lorentz contraction due to c.m. motion, but suppressed since $\frac{\boldsymbol{P}^2}{M^2}\sim\frac{T}{M}\ll 1$

Near-threshold processes

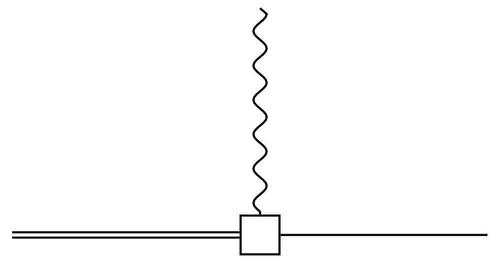
- Feature of EFTs: For near-threshold processes the interaction rates will factorize in terms coming from the soft and ultrasoft scales

- At LO in r, close-to-threshold processes are induced by the **electric dipole vertex** and include:
  - Formation of bound states (**bsf**)
  - Dissociation into scattering states (**bsd**)
  - Bound-to-bound transitions: (De-)excitations
  - Scattering-to-scattering processes: Bremsstrahlung and thermal absorption



- Distinguish between two frames:
  - Thermal medium at rest, DM pair initially moves with c.m. momentum $\boldsymbol{P} = 2M\boldsymbol{V} \sim \sqrt{MT}$
  - DM pair initially at rest, thermal medium moves with velocity $\boldsymbol{v}$

⟶ Additional vertex due to **Lorentz force** (velocity suppressed):



- Dynamical d.o.f. in the process:
  - Thermal DM pairs with momenta of order $M\alpha, \sqrt{MT}$ and energies $M\alpha^2, T$
  - Ultrasoft, thermal dark gauge fields with momenta and energies of order $M\alpha^2, T$

Electric dipole transitions

- Contrarily to annihilations, near-threshold processes happen in the thermal medium ⟶ pNREFTs at finite T !

- Use **Schwinger-Keldysh formalism**, advantage: Though d.o.f. double, they decouple for non-relativistic DM pairs

At LO:
$$G^{\phi}(p_0) = \begin{pmatrix} \frac{i}{p_0 - H + i\epsilon} & 0 \\ 2\pi\delta(p_0 - H) & \frac{-i}{p_0 - H - i\epsilon} \end{pmatrix} + 2\pi\delta(p_0 - H)\, n_{\mathrm{B}}(p_0) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \approx \begin{pmatrix} \frac{i}{p_0 - H + i\epsilon} & 0 \\ 2\pi\delta(p_0 - H) & \frac{-i}{p_0 - H - i\epsilon} \end{pmatrix}$$

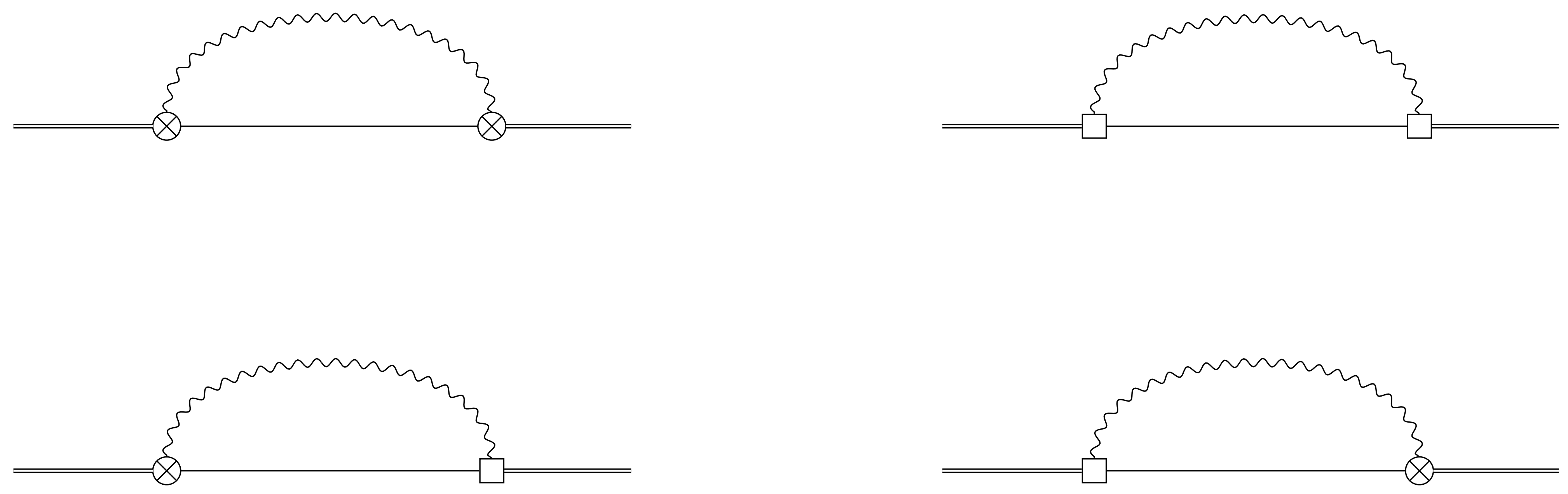
- Use Coulomb gauge, advantage: Longitudinal propagator independent of T

$$D_{00}(|\boldsymbol{k}|) = \begin{pmatrix} \frac{i}{|\boldsymbol{k}|^2 + i\epsilon} & 0 \\ 0 & \frac{-i}{|\boldsymbol{k}|^2 - i\epsilon} \end{pmatrix}$$
$$D_{ij}(k) = \left(\delta_{ij} - \frac{k_i k_j}{|\boldsymbol{k}|^2}\right) \left[\begin{pmatrix} \frac{i}{k^2 + i\epsilon} & \theta(-k_0)2\pi\delta(k^2) \\ \theta(k_0)2\pi\delta(k^2) & \frac{-i}{k^2 - i\epsilon} \end{pmatrix} + 2\pi\delta(k^2)\, n_{\mathrm{B}}(|k_0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right]$$

- Medium breaks Lorentz invariance ⟶ choose frame where medium at rest

Lab frame: Thermal medium at rest, DM pair with non-vanishing c.m. momentum

- Using the optical theorem, obtain **bsf cross section in the dipole limit** by cutting self-energy diagrams (at finite T):



$$\frac{i}{\Delta + \frac{2\boldsymbol{P}\cdot\boldsymbol{k} - \boldsymbol{k}^2}{4M} - k_0 + i\epsilon}$$

- In the **bsf process** $(X\bar{X})_p \to \gamma + (X\bar{X})_n$ the DM pair recoils when emitting a thermal or ultrasoft photon! Recoil term of order $T\sqrt{\frac{T}{M}} \ll T$

  ⟶ Expand the DM pair propagator!

- Electric-electric correlator:

$$\langle E_i(t,\boldsymbol{R})E_j(0,\boldsymbol{R}')\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik^0t + i\boldsymbol{k}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \left[k_0^2 D_{ij}(k) + k_i k_j D_{00}(k)\right]$$

- Magnetic-magnetic correlator:

$$\langle B_i(t,\boldsymbol{R})B_j(0,\boldsymbol{R}')\rangle = \epsilon_{iab}\epsilon_{jcd} \int \frac{d^4k}{(2\pi)^4} e^{-ik^0t + i\boldsymbol{k}\cdot(\boldsymbol{R}-\boldsymbol{R}')} k_a k_c D_{bd}(k)$$

- Electric-magnetic correlator:

$$\langle B_i(t,\boldsymbol{R})E_j(0,\boldsymbol{R}')\rangle = -\epsilon_{iab} \int \frac{d^4k}{(2\pi)^4} e^{-ik^0t + i\boldsymbol{k}\cdot(\boldsymbol{R}-\boldsymbol{R}')} k_a k_0 D_{bj}(k)$$

- Magnetic-electric correlator:

$$\langle E_i(t,\boldsymbol{R})B_j(0,\boldsymbol{R}')\rangle = -\epsilon_{jab} \int \frac{d^4k}{(2\pi)^4} e^{-ik^0t + i\boldsymbol{k}\cdot(\boldsymbol{R}-\boldsymbol{R}')} k_a k_0 D_{ib}(k)$$

BSF cross section via thermal photo-emission

- **Final result**: BSF cross section up to $\mathcal{O}\left(\frac{\Delta}{M}, \frac{\boldsymbol{P}^2}{M^2}\right) \sim \alpha^2, \frac{T}{M}$

$$(\sigma_{\rm bsf}\, v_{\rm rel})(\boldsymbol{p},\boldsymbol{P}) = \frac{4}{3}\alpha \sum_n \Delta^3[1+n_{\rm B}(\Delta)]\left\{|\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle|^2 X_1^n(\boldsymbol{p},\boldsymbol{P}) + |\langle n|\boldsymbol{r}\cdot\frac{\boldsymbol{P}}{2M}|\boldsymbol{p}\rangle|^2 X_2^n(\boldsymbol{p},\boldsymbol{P})\right\}$$

$$\text{with} \quad \Delta = \Delta E_n^p$$

$$\text{with} \quad X_1^n(\boldsymbol{p},\boldsymbol{P}) = 1 - \frac{3\Delta}{4M} + \frac{\boldsymbol{P}^2}{4M^2} + \frac{\Delta}{T}n_{\rm B}(\Delta)\left[\frac{\Delta}{4M} - \frac{\boldsymbol{P}^2}{4M^2}\left(1+\frac{\Delta}{5T}\right)\right] + \left(\frac{\Delta}{T}\right)^2 n_{\rm B}(\Delta)[1+n_{\rm B}(\Delta)]\frac{\boldsymbol{P}^2}{10M^2}$$

$$X_2^n(\boldsymbol{p},\boldsymbol{P}) = 1 + \frac{1}{10}\left(\frac{\Delta}{T}\right)^2 n_{\rm B}(\Delta) - \frac{1}{5}\left(\frac{\Delta}{T}\right)^2 n_{\rm B}(\Delta)[1+n_{\rm B}(\Delta)]$$

What is the impact of these corrections?

- Compare with LO result known in literature:

$$(\sigma_{\rm bsf}\, v_{\rm rel})(\boldsymbol{p}) = \frac{4}{3}\alpha \sum_n [1+n_{\rm B}(\Delta E_n^p)]\,|\langle n|\boldsymbol{r}|\boldsymbol{p}\,\rangle|^2 (\Delta E_n^p)^3$$

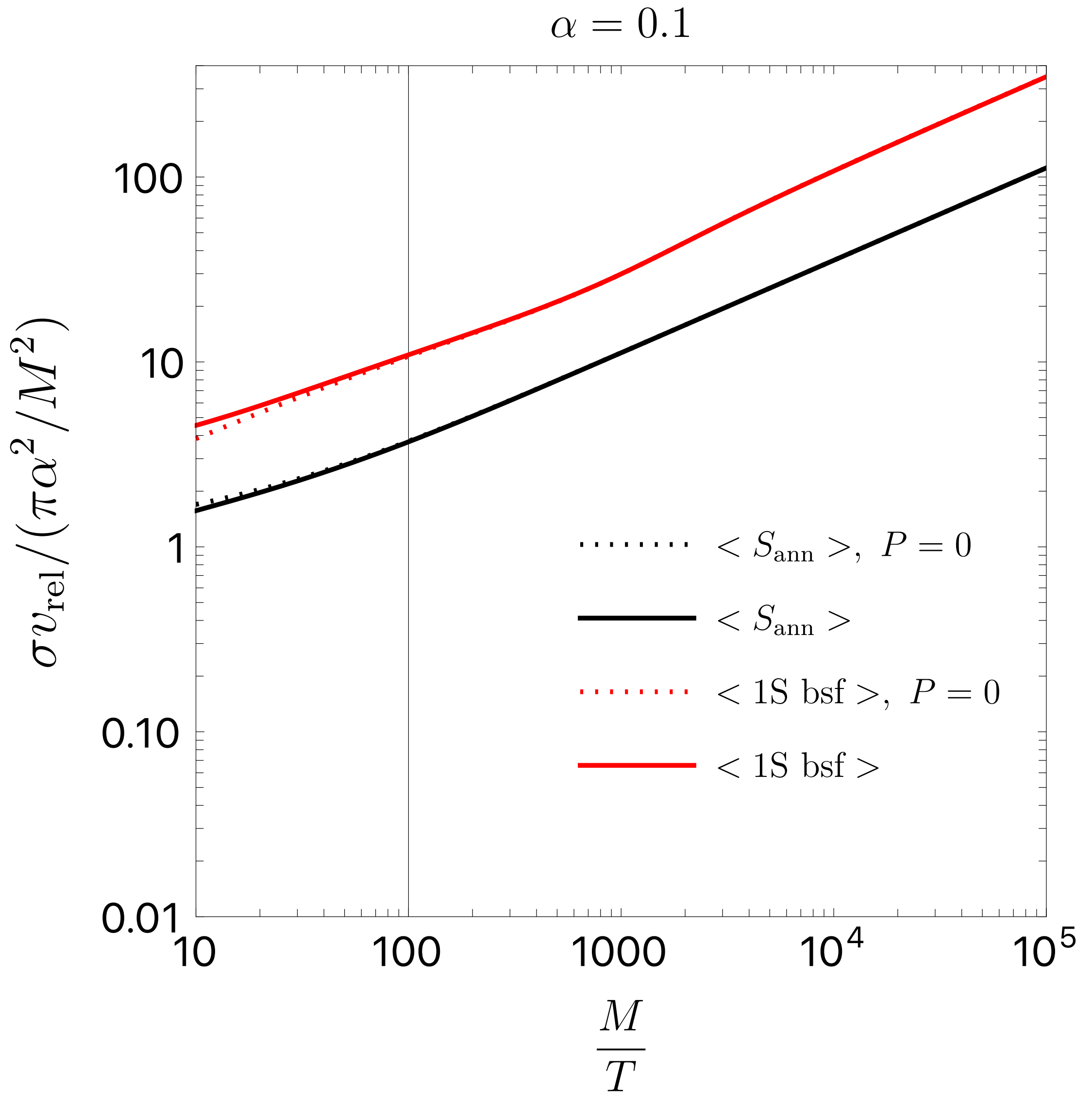
See also 1407.7874, 1109.5826, 2002.07145, 1805.01200, 2112.01499, ...

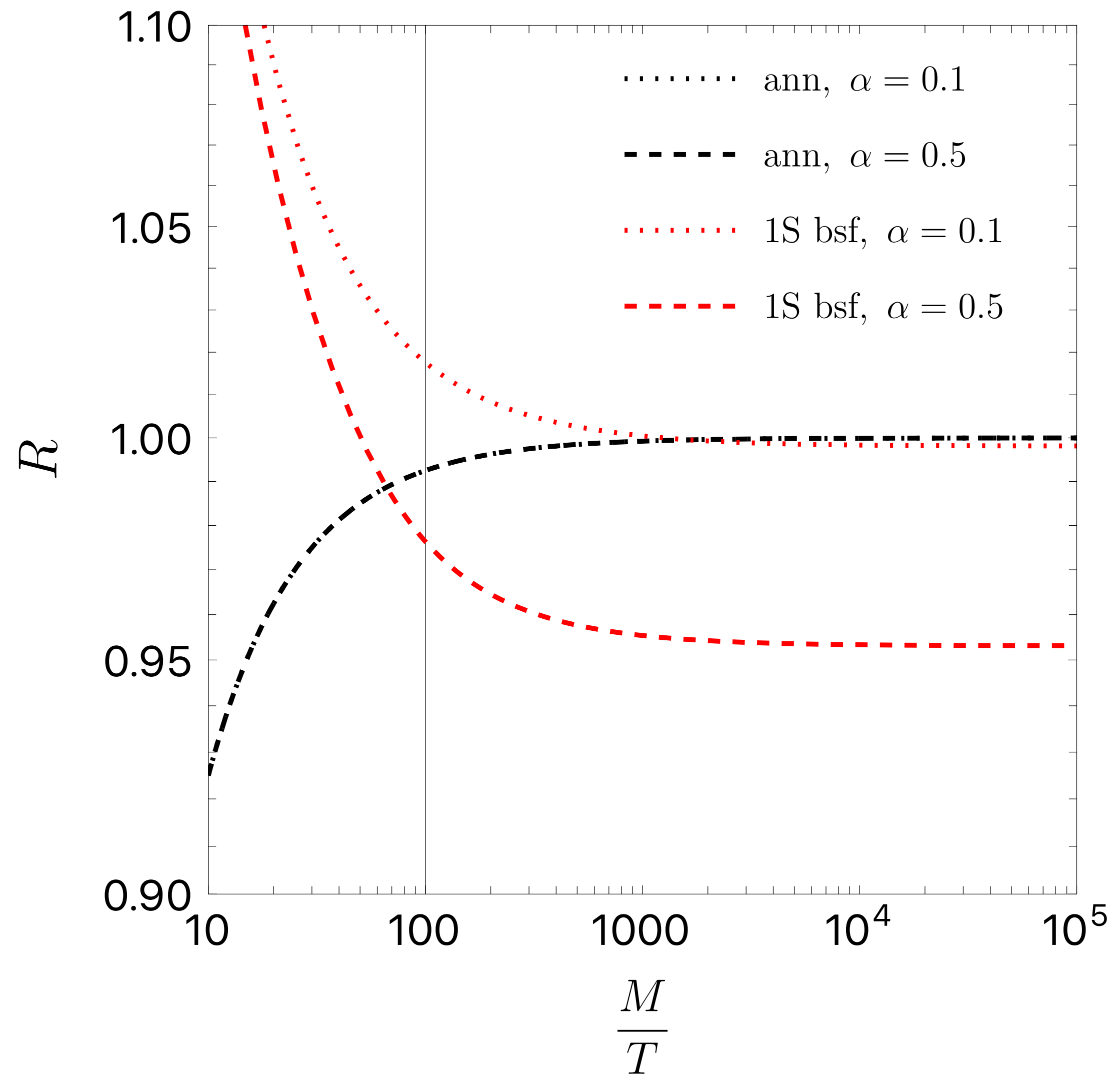
BSF: Impact of the center-of-mass motion and recoil correction

- Thermally average over the incoming relative & c.m. momentum of the scattering state, as $\langle(\sigma_{\rm bsf} v_{\rm rel})(\boldsymbol{p})\rangle$ enters the Boltzmann eqs.!

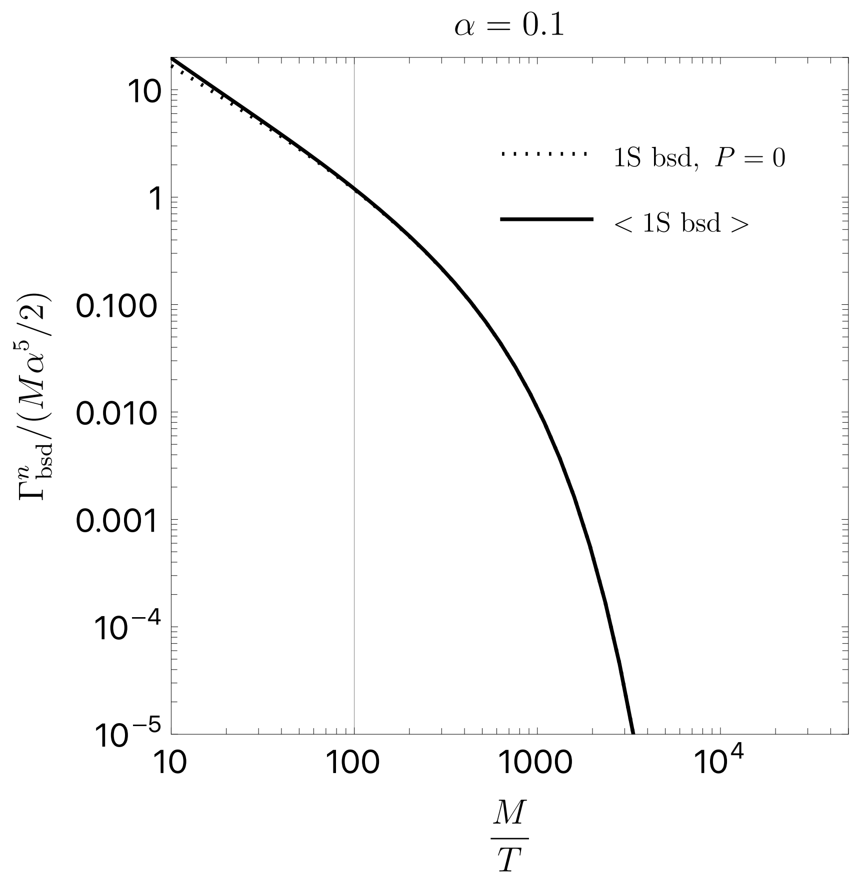
- Result: **Impact of c.m. motion & recoil quite low** (few percent) for most of the time after thermal freeze-out

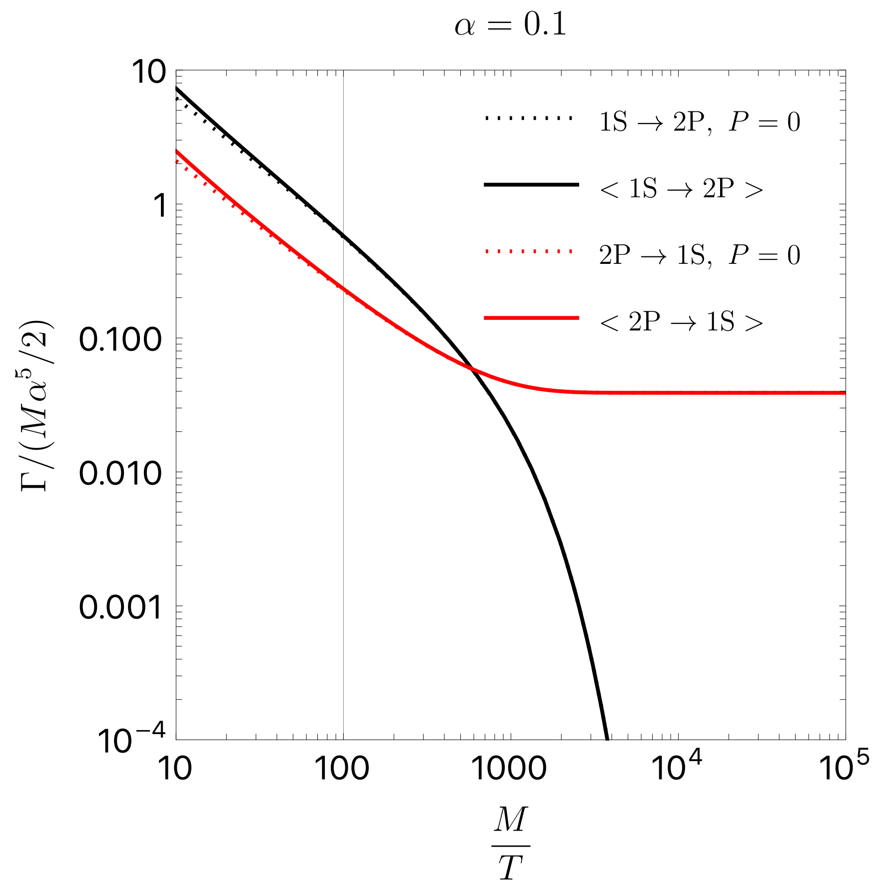
- Example: Formation of 1S ground state, plot annihilation & bsf cross sections





Similarly for bsd & (de-)excitation widths:



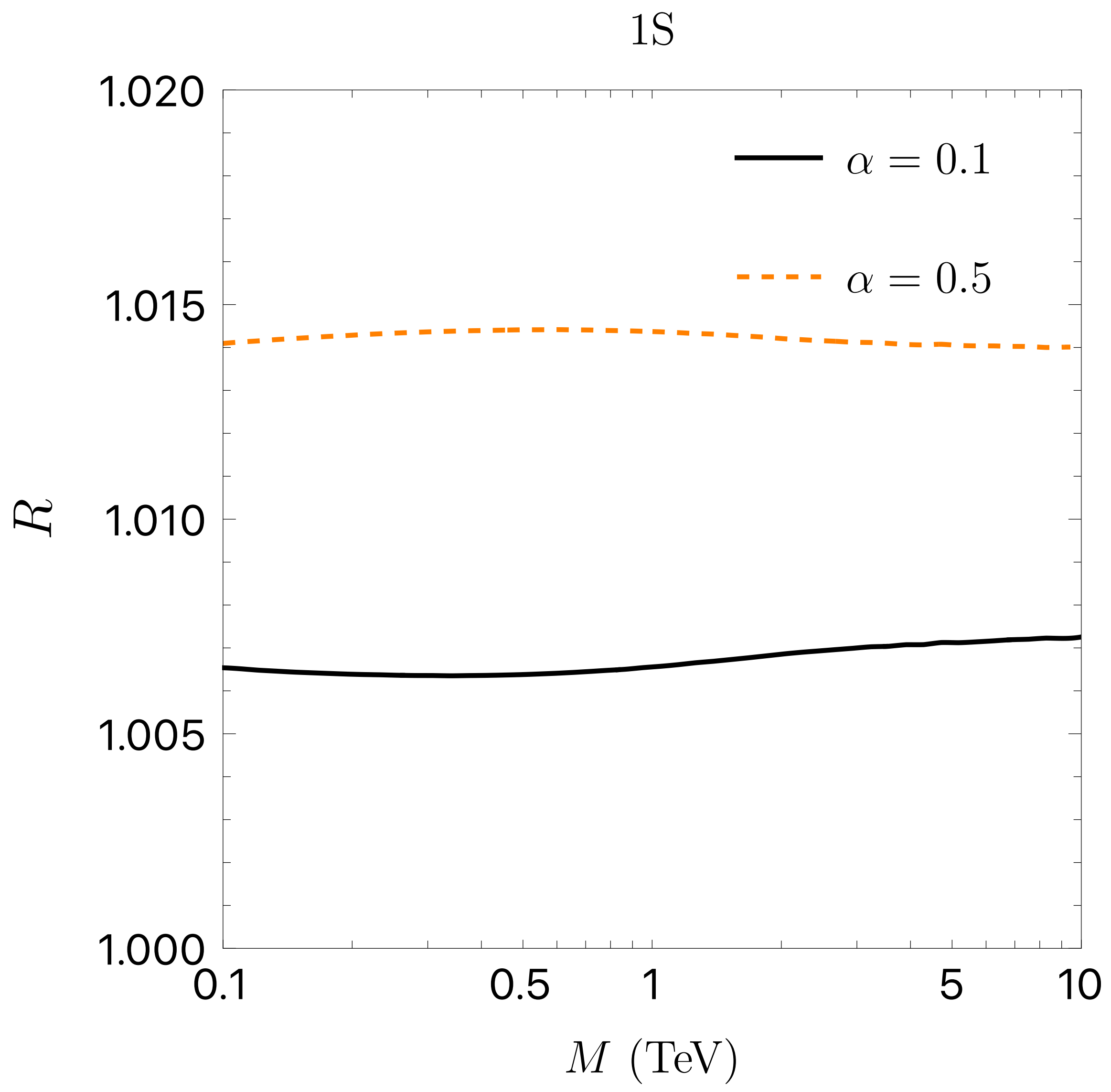


- Known result gives already good estimate for the BSF cross section, for $T \lesssim M\alpha^2$:

$$(\sigma_{\rm bsf}\, v_{\rm rel})(\boldsymbol{p}) = \frac{g^2}{3\pi}\sum_n \left[1 + n_{\rm B}(\Delta E_n^p)\right] |\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle|^2 (\Delta E_n^p)^3$$

Conclusions

- For **thermal heavy DM**, chemical freeze-out of the energy density is set by the non-relativistic regime
- Accurate estimation of interaction rates necessary in order to predict the relic abundance of DM
- In the framework of NREFTs, we compute the relevant thermal rates and highlight their range of validity
- We observe that the **center-of-mass motion & recoil corrections are small** & can eventually be neglected
- The correction on the DM relic energy density is only about 1% !



Thank you!