Freeze-In at Finite Temperature

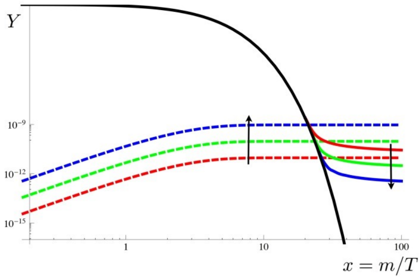
in collaboration with

Emanuele Copello, Julia Harz, and Carlos Tamarit

based on ongoing work

supported by DFG Emmy Noether Grant No. HA 8555/1-1.

Motivation: Relevant Temperatures



Freeze-In

$$T_{f.i} \sim \frac{M}{2} - \frac{M}{5}$$

Freeze-Out

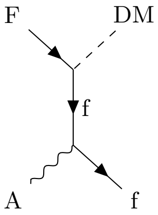
$$T_{f.o} \sim \frac{M}{25} - \frac{M}{30}$$

→ Finite Temperature Corrections relevant for Freeze-In

Motivation: Infrared Divergencies at NLO

Example: η (scalar DM), F (gauge charged Parent)

$$\mathcal{L}_{\text{int}} = y_{\text{DM}}\, \bar{f}_{\text{SM}}\, F\, \eta + \text{h.c.}$$

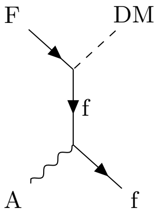


$$\rightarrow \sigma \sim \int dt |\mathcal{M}|^2 \sim \int \frac{dt}{t} \sim \ln\left(\frac{m_f}{T}\right) \Rightarrow \text{divergent for } m_f \ll T$$

Motivation: Infrared Divergencies at NLO

Example: η (scalar DM), F (gauge charged Parent)

$$\mathcal{L}_{\rm int} = y_{\rm DM}\,\bar{f}_{\rm SM}\,F\,\eta + \text{h.c.}$$



$$\to \sigma \sim \int dt |\mathcal{M}|^2 \sim \int \frac{dt}{t} \sim \ln\left(\frac{m_f}{T}\right) \Rightarrow \text{divergent for } m_f \ll T$$

Common Treatment: Thermal masses $m_f \to m_f(T) \sim T$

see for instance [Belanger et. al(2020)],[No et. al(2020)],[Calibbi et. al(2021)]

What do we do?

→ Calculate the DM production rate in the real time formalism of thermal QFT

→ We consider DM feebly coupled to a gauge charged Parent (neglecting potential Yukawa or quartic interactions)

→ Compare our results to:

Thermal QFT calculations in Hard Thermal Loop approximation

Boltzmann approach employing scattering rates regulated with thermal masses

Closed Time Path (Keldyish-Schwinger/real time) formalism

Construct 2 particle irreducible (2PI) effective action Γ

↓ Find equations of motion $\frac{\delta\Gamma}{\delta\Delta} = \frac{\delta\Gamma}{\delta S} = 0$

Kadanoff-Baym equations

↓ Kadanoff-Baym Ansatz

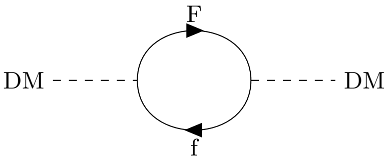
Boltzmann-like evolution equation

for details see [Garbrecht(2018)],[Berges(2004)],[Prokopec et. al(2003)]

DM Time Evolution

$$\dot{n}_{\mathrm{DM}} + 3Hn_{\mathrm{DM}} = \gamma_{\mathrm{DM}} \sim \int d^3p \frac{\Pi^{\mathcal{A}}_{\mathrm{DM}}}{E_{\mathrm{DM}}} f_{\mathrm{DM}}\left(E_{\mathrm{DM}}\right)$$

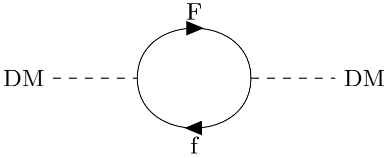
Spectral Self-Energy $\Pi^{\mathcal{A}}_{\mathrm{DM}} = -\mathrm{Im}\Pi^{R}_{\mathrm{DM}} =$



DM Time Evolution

$$\dot{n}_{DM} + 3Hn_{DM} = \gamma_{DM} \sim \int d^3p \frac{\Pi^{\mathcal{A}}_{DM}}{E_{DM}} f_{DM}(E_{DM})$$

Spectral Self-Energy $\Pi^{\mathcal{A}}_{DM} = -\mathrm{Im}\Pi^{R}_{DM} =$



DM Self-Energy

$$\Pi^{\mathcal{A}}_{DM}(P) \sim \int dK\, S^{\mathcal{A}}_{F}(K)\, S^{\mathcal{A}}_{f}(K-P) + \text{higher order contributions}$$

Level of approximation depends on

- Loop order at which $\Pi^{\mathcal{A}}_{\mathrm{DM}}$ is evaluated
  $\rightarrow$ LO (this work), NLO, ..
- Which propagators are used to derive $\Pi^{\mathcal{A}}_{\mathrm{DM}}$
  $\rightarrow$ Tree Level, perturbative 1-Loop, HTL approximated resummed, fully resummed (this work)

Example: Tree-Level Propagators

Spectral Propagator(Tree-Level)

$$\not{S}^{\mathcal{A}}_{\mathrm{F/f}} \sim (\not{k} + \mathrm{m_{F/f}})\delta\left(\mathrm{k}^2 - \mathrm{m}^2_{\mathrm{F/f}}\right)$$

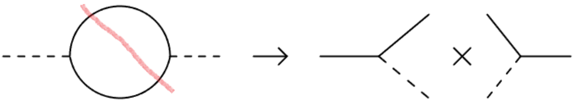
Implies a dispersion relation $\mathrm{k}^0 = \pm\sqrt{|\vec{\mathrm{k}}|^2 + \mathrm{m}^2_{\mathrm{F/f}}}$

Example: Tree-Level Propagators

Spectral Propagator(Tree-Level)

$$\not{S}^{\mathcal{A}}_{\mathrm{F/f}} \sim (\not{k} + m_{\mathrm{F/f}})\delta\left(k^2 - m^2_{\mathrm{F/f}}\right)$$

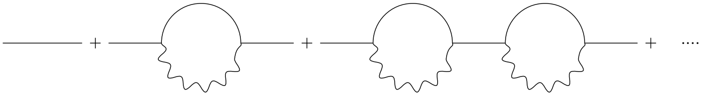
Implies a dispersion relation $k^0 = \pm\sqrt{|\vec{k}|^2 + m^2_{\mathrm{F/f}}}$



$\Rightarrow$ recovers Boltzmann equation for a tree-level decay!

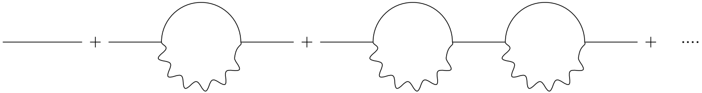
Resummed Propagators

We resum the gauge boson contribution (remember $y_{\mathrm{DM}} \ll g$)

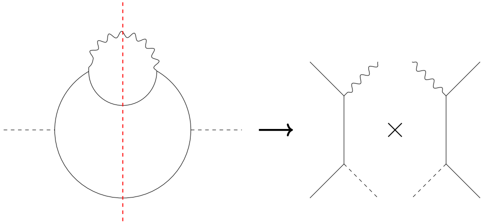


Resummed Propagators

We resum the gauge boson contribution (remember $y_{DM} \ll g$)

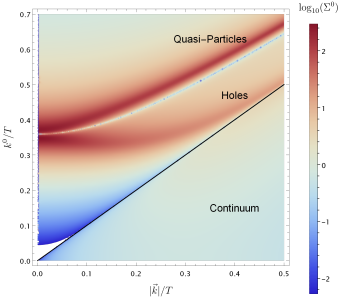


$\rightarrow$ scattering contributions also arise at leading order



Form of the spectral propagator with a narrow width

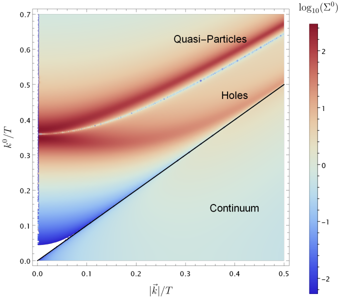
$$\not{S}^{\mathcal{A}} \overset{\Gamma \ll k, \Sigma^{\mathcal{H}}}{\sim} \frac{\Gamma}{\left((k - \Sigma^{\mathcal{H}})^2 - m^2\right)^2 + \Gamma^2} \overset{\Gamma \to 0}{\sim} \delta\left((k - \Sigma^{\mathcal{H}})^2 - m^2\right)$$



- Much simpler analytic form in the Hard Thermal Loop (HTL) approximation. But only reliable for $T \gg m$!
- Previous calculations assume the HTL [Garbrecht et. al(2019)] or interpolate between HTL and non-relativistic results [Biondini et. al(2020)]

Form of the spectral propagator with a narrow width

$$\not{S}^{\mathcal{A}} \overset{\Gamma\ll k,\Sigma^{\mathcal{H}}}{\sim} \frac{\Gamma}{\left((k-\Sigma^{\mathcal{H}})^2-m^2\right)^2+\Gamma^2} \overset{\Gamma\to 0}{\sim} \delta\left((k-\Sigma^{\mathcal{H}})^2-m^2\right)$$



- Much simpler analytic form in the Hard Thermal Loop (HTL) approximation. But only reliable for $T \gg m$!
- Previous calculations assume the HTL [Garbrecht et. al(2019)] or interpolate between HTL and non-relativistic results [Biondini et. al(2020)]

Remember: Freeze-In occurs around $T \sim M \Rightarrow$ Accuracy in intermediate regime required.

Results

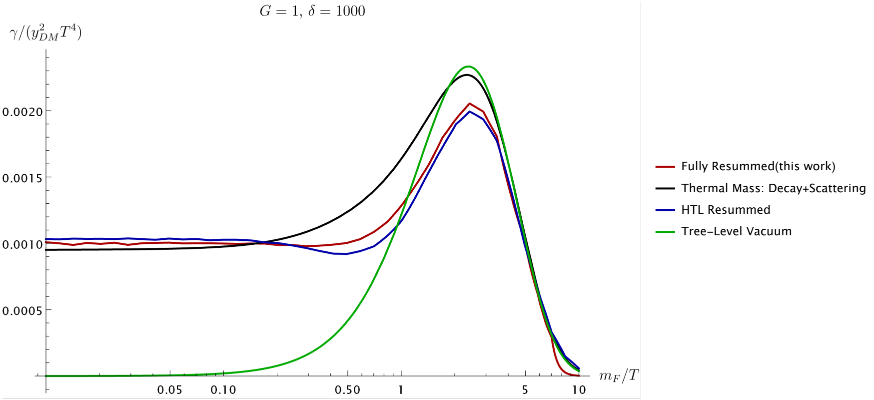
We compare

- Fully resummed results (our work in progress)
- Hard Thermal Loop resummed results
- Boltzmann Equations with decays and scatterings regulated by thermal masses
- Boltzmann Equations with only decays and in vacuum masses (Tree-Level Vacuum)

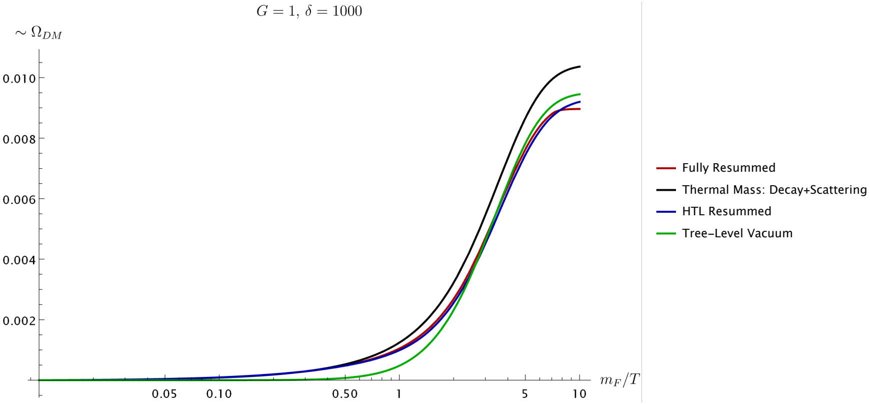
in terms of two relevant parameters

- the effective gauge coupling G
- the mass splitting between the parent F and DM $\delta = 1 - \mathrm{m_{DM}}/\mathrm{m_F}$

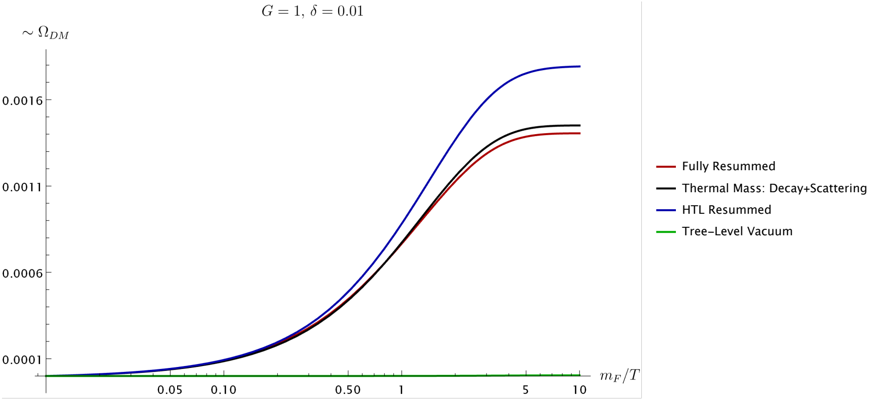
Preliminary Results (Interaction Rate)



Preliminary Results (Relic Density, Large Mass Splitting)



Preliminary Results (Relic Density, Small Mass Splitting)



Conclusions

Finte temperature corrections expected to be relevant for freeze-in as $T_{f.i} \sim M$.

$\rightarrow$ We compare finite temperature QFT results using complete 1PI resummed propagators with

- a Boltzmann approach regularizing IR divergencies with thermal masses
- finite temperature results using the Hard Thermal Loop approximation

Our preliminary results indicate:

- For large mass splittings between Dark Matter and its Parent we find a $\sim 20\,\%$ difference of the thermal mass regulated and our result.
- For small mass splittings the thermal mass approach only differs by $\sim 3\,\%$