New constraints on extended Higgs sectors from the trilinear Higgs coupling

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY.

Why study the trilinear Higgs coupling?

- **Probing the Higgs potential:**
  Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  → the location of the EW minimum:

  v = 246 GeV

  → the curvature of the potential around the EW minimum:

  m_h = 125 GeV

  However we still don't know the **shape** of the potential, away from EW minimum → **depends on λ_{hhh}**

- **λ_{hhh} determines the nature of the EWPT**!
  ⇒ deviation of λ_{hhh} from its SM prediction typically needed to have a strongly first-order EWPT → necessary for EWBG [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

- *New in this talk*: **studying λ_{hhh} can also serve to constrain the parameter space of BSM models!**
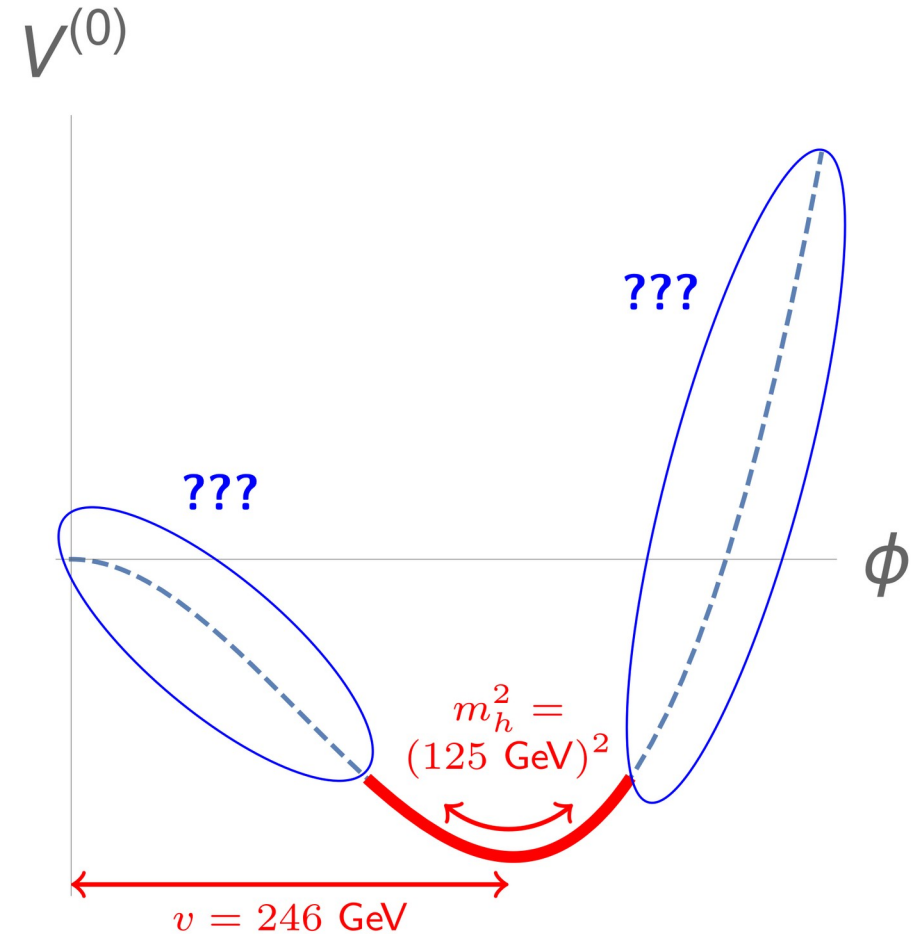
# BSM contributions to λ_{hhh}

The Two-Higgs-Doublet Model

- 2 SU(2)$_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z$_2$ symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{2\mathrm{HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2)$$

$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\left((\Phi_2^\dagger\Phi_1)^2 + \mathrm{h.c.}\right)$$

$$v_1^2 + v_2^2 = v^2 = (246\ \mathrm{GeV})^2$$

- **Mass eigenstates**:
    h, H: CP-even Higgs bosons *(h → 125-GeV SM-like state);* A: CP-odd Higgs boson;
    H$^\pm$: charged Higgs boson

- **BSM parameters**: 3 BSM masses m_H, m_A, $m_{H\pm}$, BSM mass scale M (defined by $M^2 \equiv 2m_3^2/s_{2\beta}$), angles α (CP-even Higgs mixing angle) and β (defined by $\tan\beta = v_2/v_1$)

- **BSM-scalar masses** take form $\quad m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2\,, \quad \Phi \in \{H, A, H^\pm\}$

- We take the **alignment limit α=β-π/2** → all Higgs couplings are SM-like at tree level
    → compatible with current experimental data!

Mass splitting effects in λ_{hhh}

- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM:
  [Kanemura, (Kiyoura), Okada, Senaha, Yuan '02, '04]

$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$$

$$(\Phi \in \{H, A, H^\pm\})$$
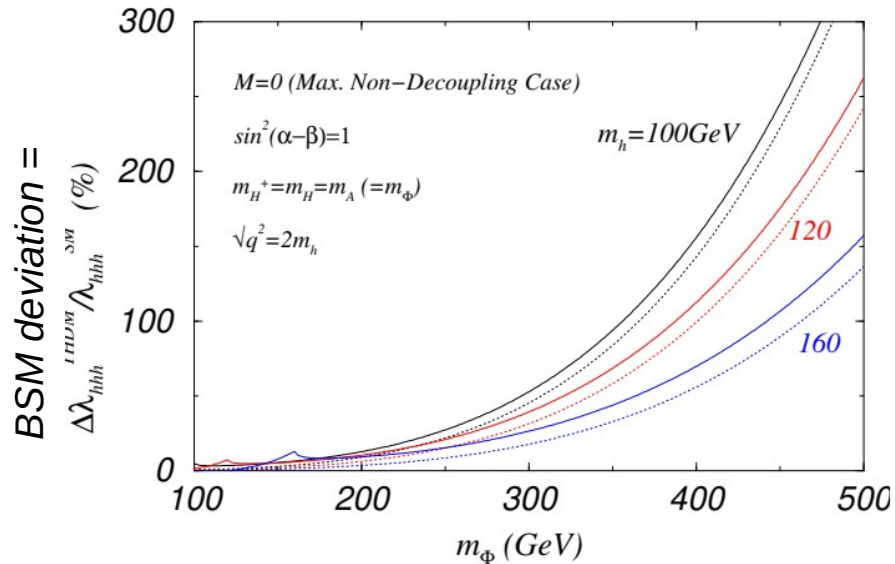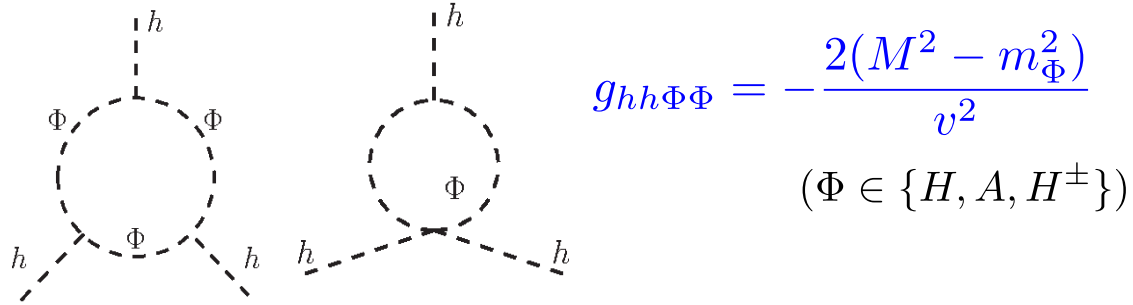
- **Deviations of tens/hundreds of % from SM possible,** for
  large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings
- **Mass splitting effects**, now found in various models (2HDM,
  inert doublet model, singlet extensions, etc.)

Mass splitting effects in λ_{hhh}

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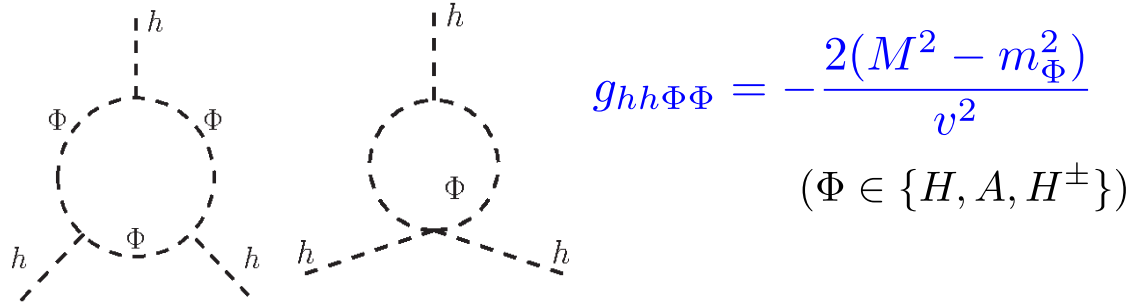
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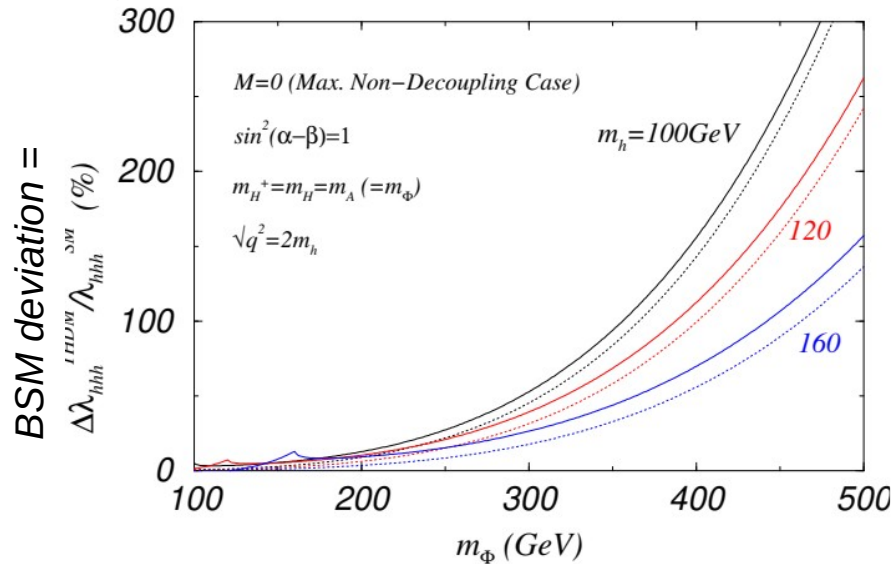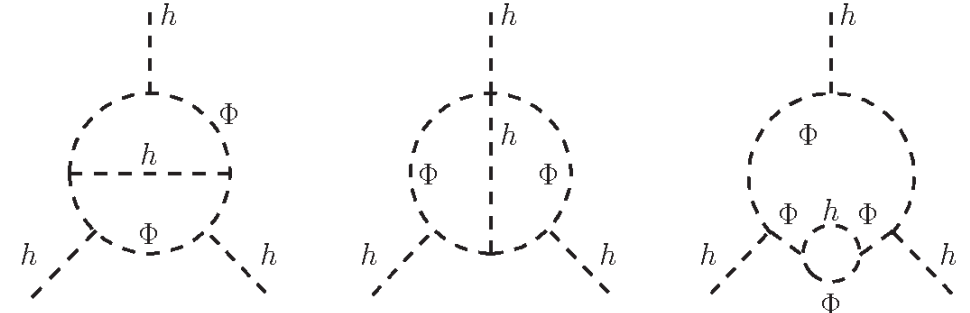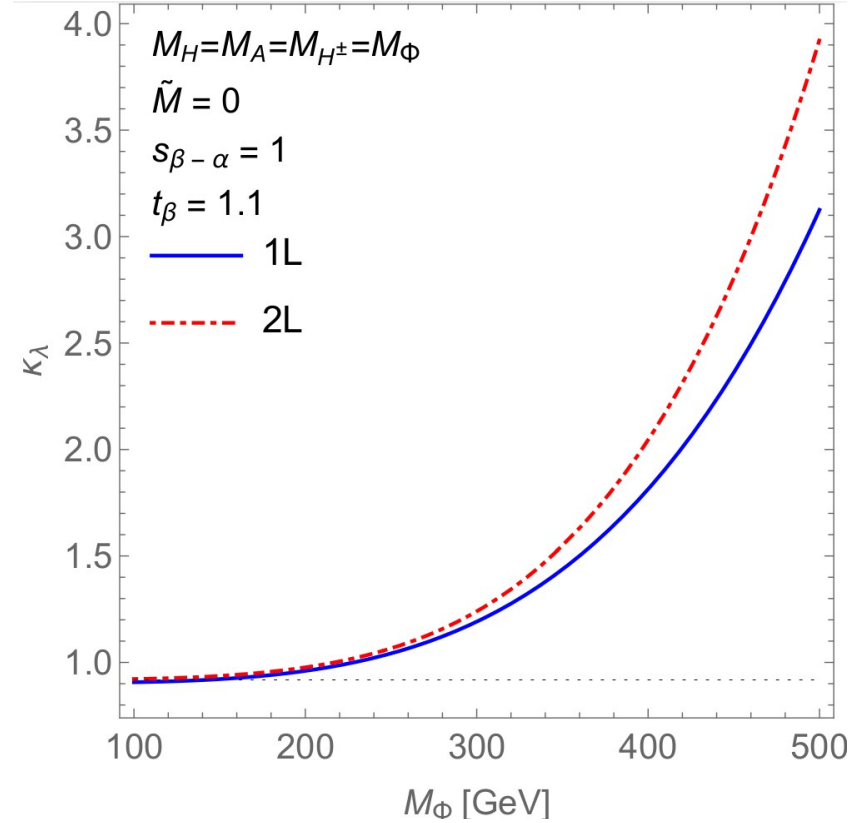
- Large effects **confirmed at 2L** in [JB, Kanemura '19]
→ **leading 2L corrections involving BSM scalars (H,A,H±) and top quark**, computed in effective potential approximation

Constraining the 2HDM with λ$_{hhh}$

i. Can we apply the limits on κ_λ, extracted from experimental searches for double-Higgs production, for BSM models?

ii. Can large BSM deviations occur for points still allowed in light of theoretical and experimental constraints? If so, how large can they become?

Can we apply di-Higgs results for the aligned 2HDM?

➢ Current strongest limit on κ_λ are from ATLAS double- (+ single-) Higgs searches

<div align="center">

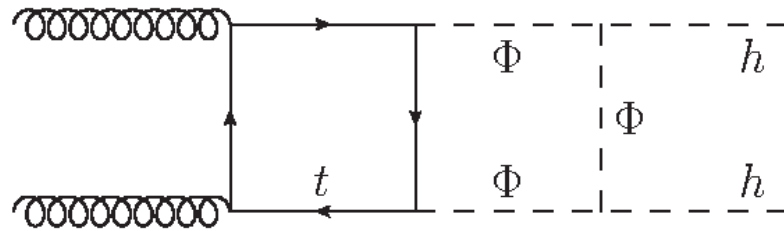**-0.4 < κ_λ < 6.3** [ATLAS-CONF-2022-050]   [recall $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$]
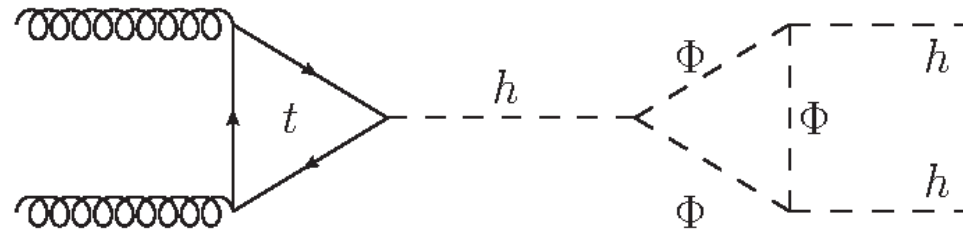
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➢ What are the *assumptions* for the ATLAS limits?

- All other Higgs couplings (to fermions, gauge bosons) are SM-like

  → this is **ensured by the alignment** ✓

- The modification of λ_{hhh} is the only source of deviation of the *non-resonant Higgs-pair production cross section* from the SM

$$\propto \mathcal{O}(y_t^2 g_{hh\Phi\Phi}^2) \quad \textit{not included} \qquad\qquad \propto \mathcal{O}(y_t g_{hh\Phi\Phi}^3) \quad \textit{included}$$

→ We **correctly include all leading BSM effects to double-Higgs production, in powers of $g_{hh\Phi\Phi}$, up to NNLO!** ✓

➢ **We can apply the ATLAS limits to our setting!**

(Note: BSM resonant Higgs-pair production cross section also suppressed at LO, thanks to <u>alignment</u>)

A parameter scan in the aligned 2HDM

[Bahl, JB, Weiglein PRL '22]

- Our strategy:

  1. **Scan BSM parameter space**, keeping only points passing various theoretical and experimental constraints (*see below*)

  2. Identify regions with **large BSM deviations in λ_{hhh}**

  3. Devise a **benchmark scenario** allowing large deviations and investigate impact of experimental limit on λ_{hhh}

- *Here*: we consider an **aligned 2HDM of type-I**, but similar results expected for other 2HDM types, or other BSM models with extended Higgs sectors

- Constraints in our parameter scan:

  experimental
  - SM-like Higgs measurements with `HiggsSignals`
  - Direct searches for BSM scalars with `HiggsBounds`
  - b-physics constraints, using results from [Gfitter group 1803.01853]

  Checked with `ScannerS`
  [Mühlleitner et al. 2007.02985]

  - EW precision observables, computed at two loops with `THDM_EWPOS` [Hessenberger, Hollik '16]

  theoretical
  - Vacuum stability
  - Boundedness-from-below of the potential

  Checked with `ScannerS`

  - NLO perturbative unitarity, using results from [Grinstein et al. 1512.04567], [Cacchio et al. 1609.01290]

- For points passing these constraints, we **compute κ_λ at 1L and 2L**, using results from [JB, Kanemura '19]

Parameter scan results

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in (m_H-$m_{H\pm}$, m_A-$m_{H\pm}$) plane
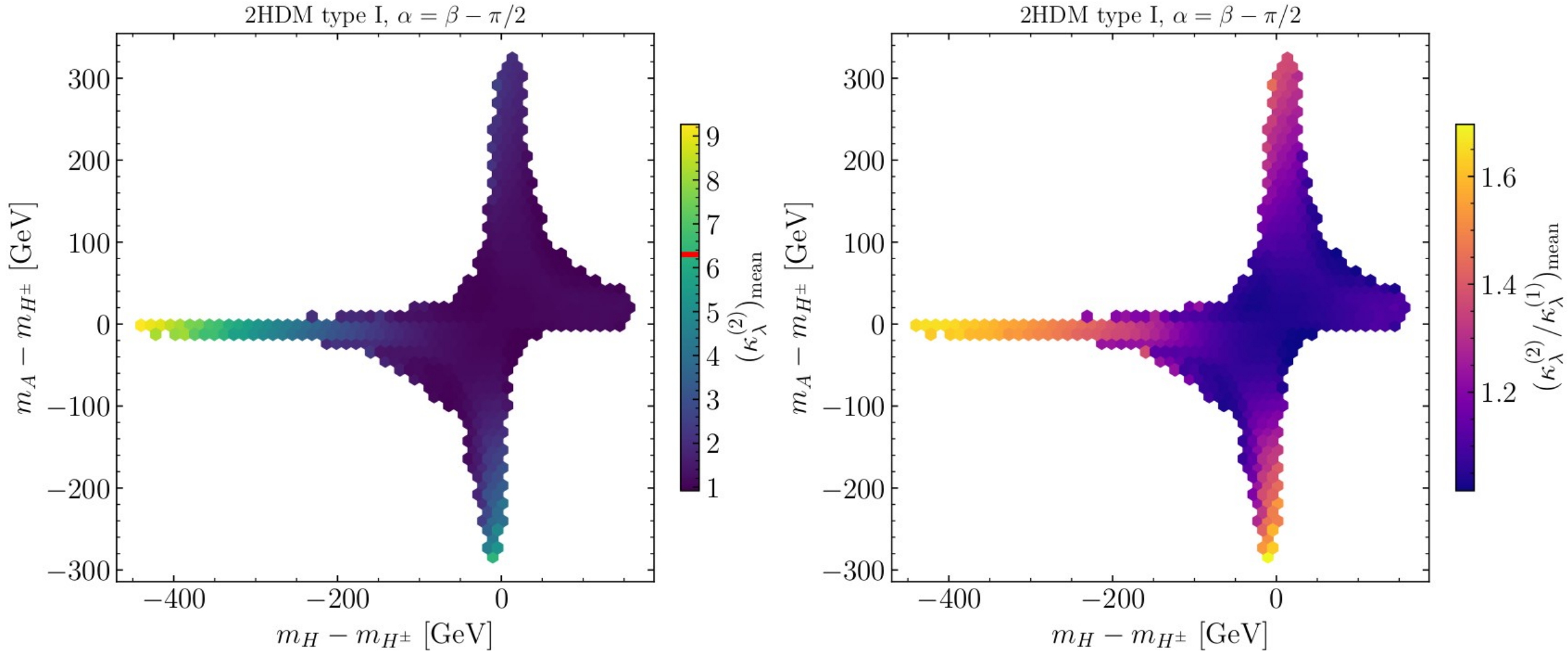
NB: all previously mentioned constraints are fulfilled by the points shown here

Parameter scan results

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ [left] and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ [right] in $(m_H - m_{H\pm}, m_A - m_{H\pm})$ plane
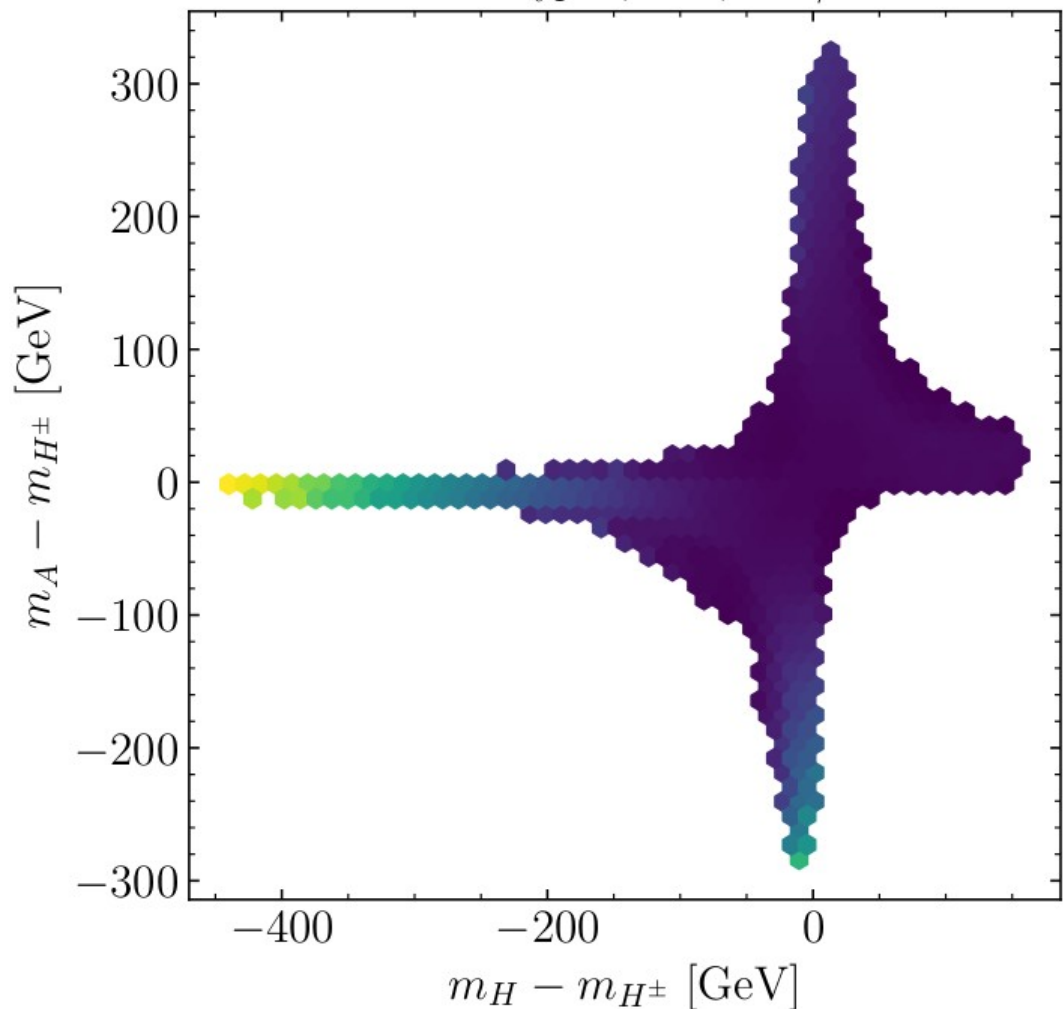

> ➢ 2L corrections can become **significant** (up to ~70% of 1L)

Parameter scan results

Mean value for $\kappa_\lambda^{(2)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(0)})^{SM}$ **[left]** and $\kappa_\lambda^{(2)}/\kappa_\lambda^{(1)} = (\lambda_{hhh}^{(2)})^{2HDM}/(\lambda_{hhh}^{(1)})^{2HDM}$ **[right]** in (m_H-$m_{H\pm}$, m_A-$m_{H\pm}$) plane

2HDM type I, $\alpha = \beta - \pi/2$

Huge deviations, up to ~ x10 wrt SM, possible !

Upper limit from ATLAS

2HDM type I, $\alpha = \beta - \pi/2$

2L corrections can reach 70% of 1L ones!
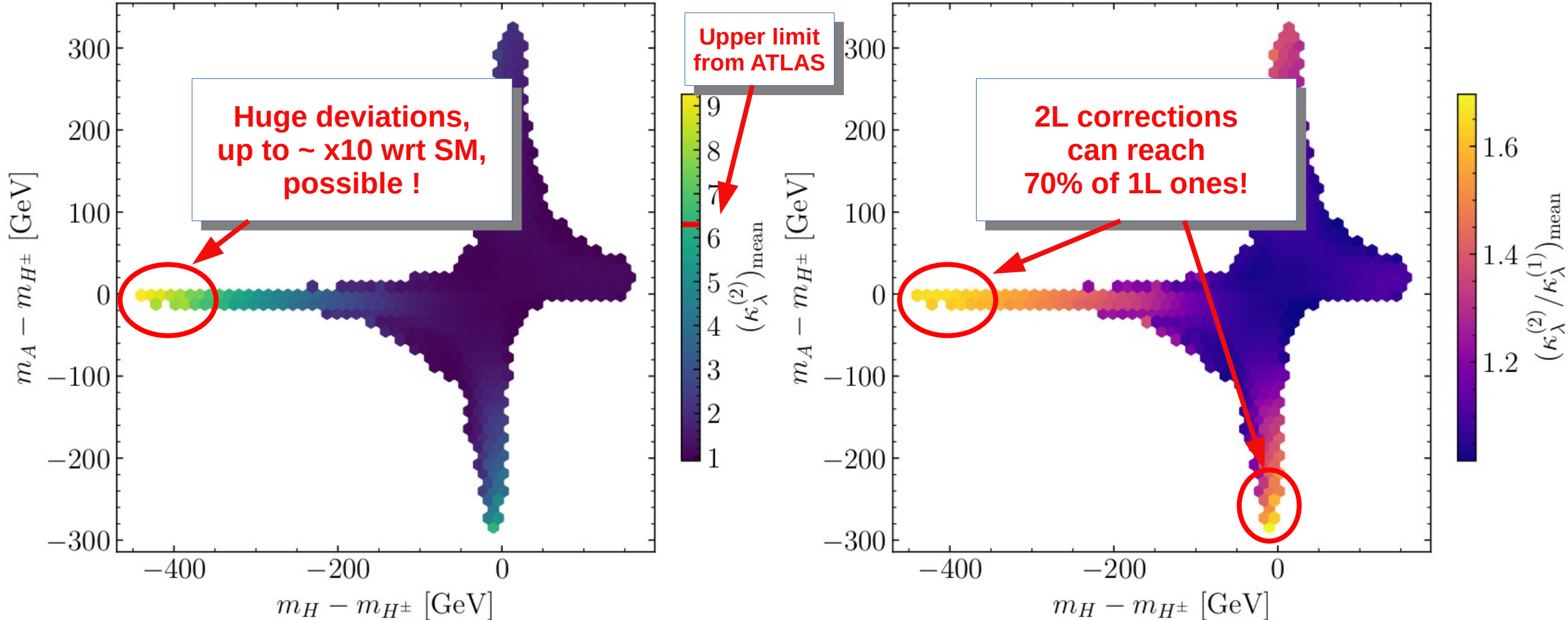
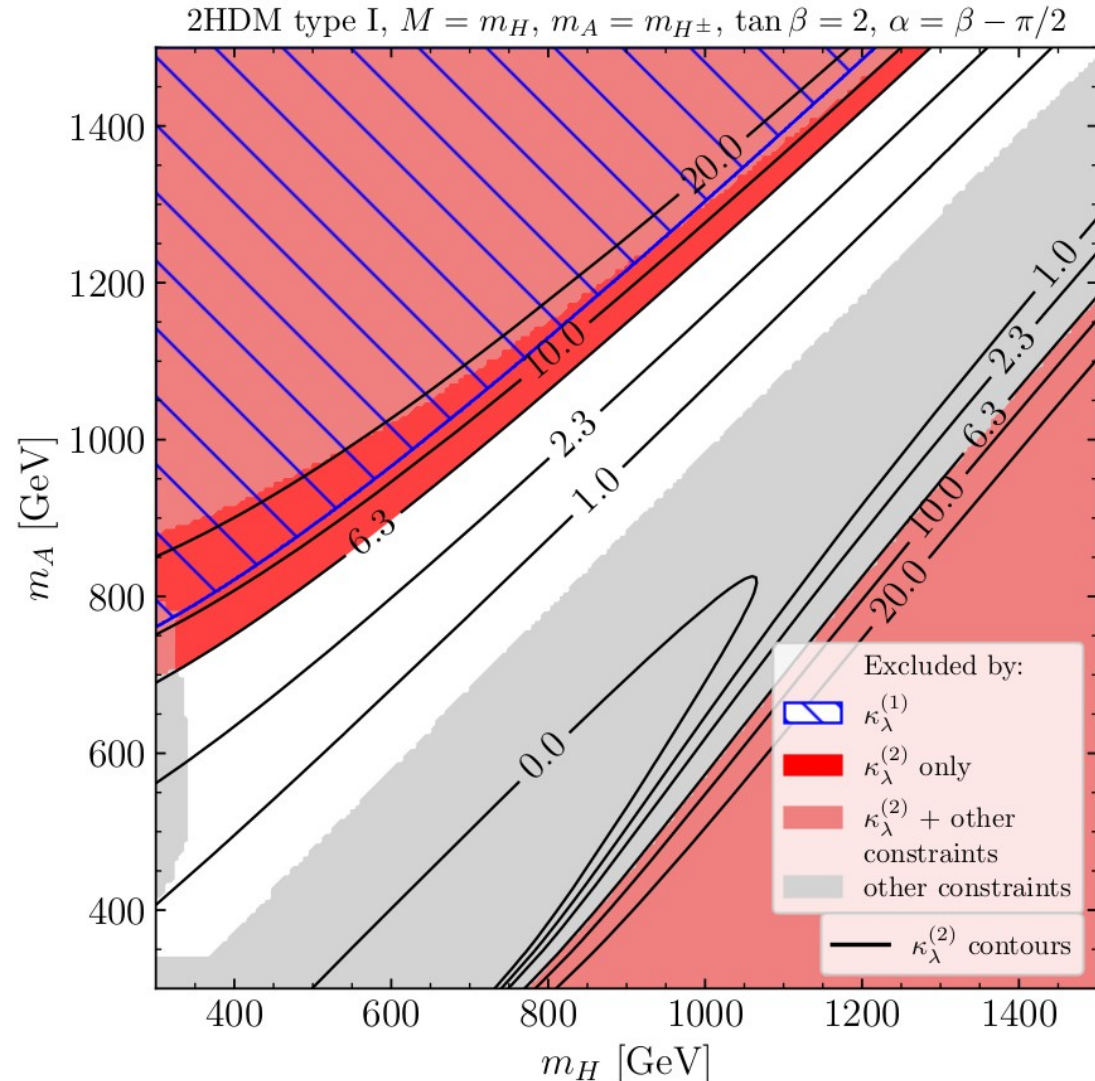- ➤ 2L corrections can become **significant** (up to ~70% of 1L)
- ➤ **Huge enhancements** (by a factor ~10) of λ_{hhh} possible for $m_A \sim m_{H\pm}$ and $m_H \sim M$

A benchmark scenario in the aligned 2HDM

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)
We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$

- ➢ ***Grey area:*** area excluded by other constraints, in particular Higgs physics, boundedness-from-below (BFB), perturbative unitarity

- ➢ ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]

- ➢ ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!

- ➢ ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!
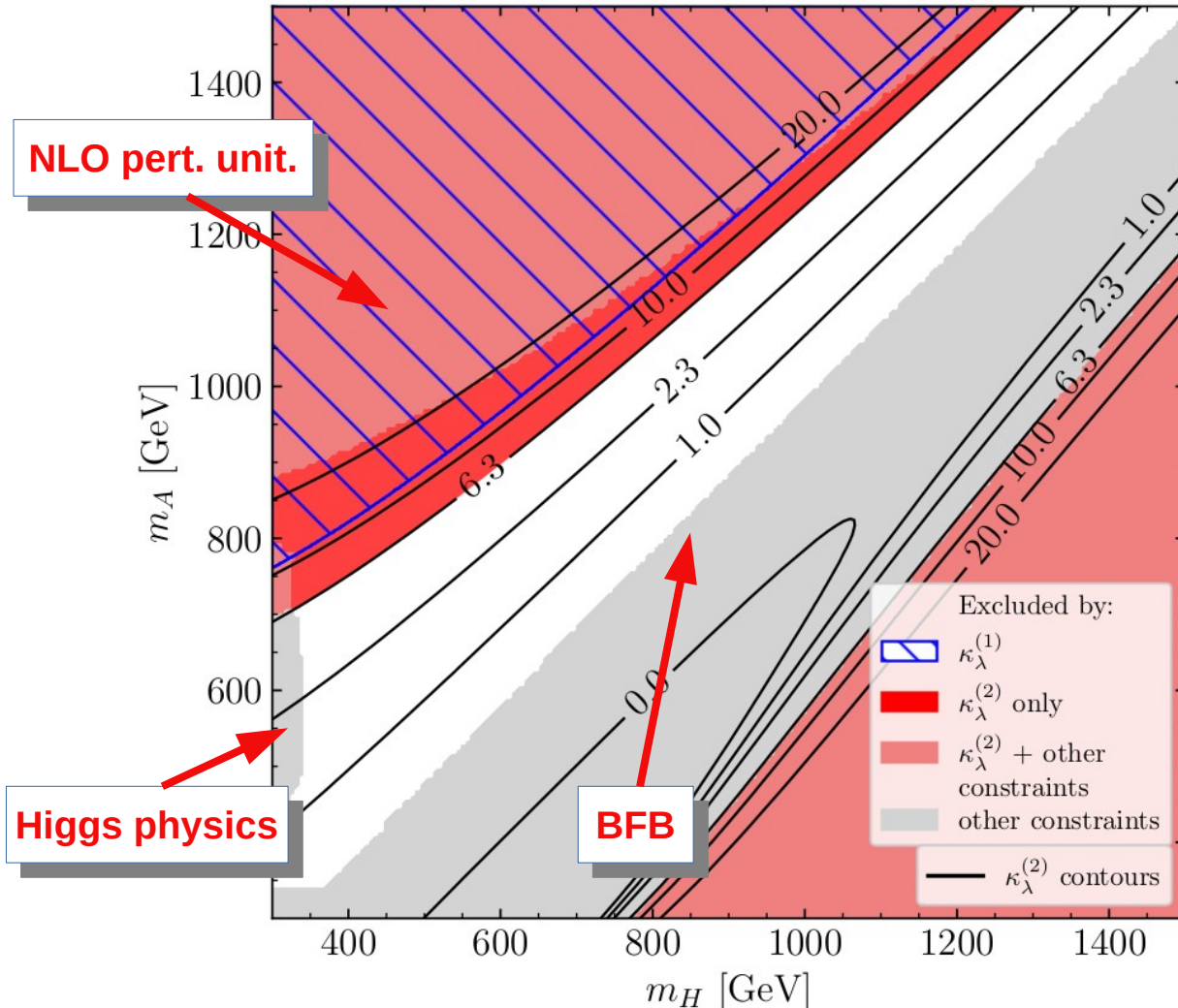
A benchmark scenario in the aligned 2HDM

Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A=m_{H\pm}$, M=m_H, tanβ=2

2HDM type I, $M = m_H$, $m_A = m_{H\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

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- ➤ ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3 \rightarrow$ impact of including 2L corrections is significant!

A benchmark scenario in the aligned 2HDM – future prospects

Suppose for instance the upper bound on κ_λ becomes $\kappa_\lambda < 2.3$

2HDM type I, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$
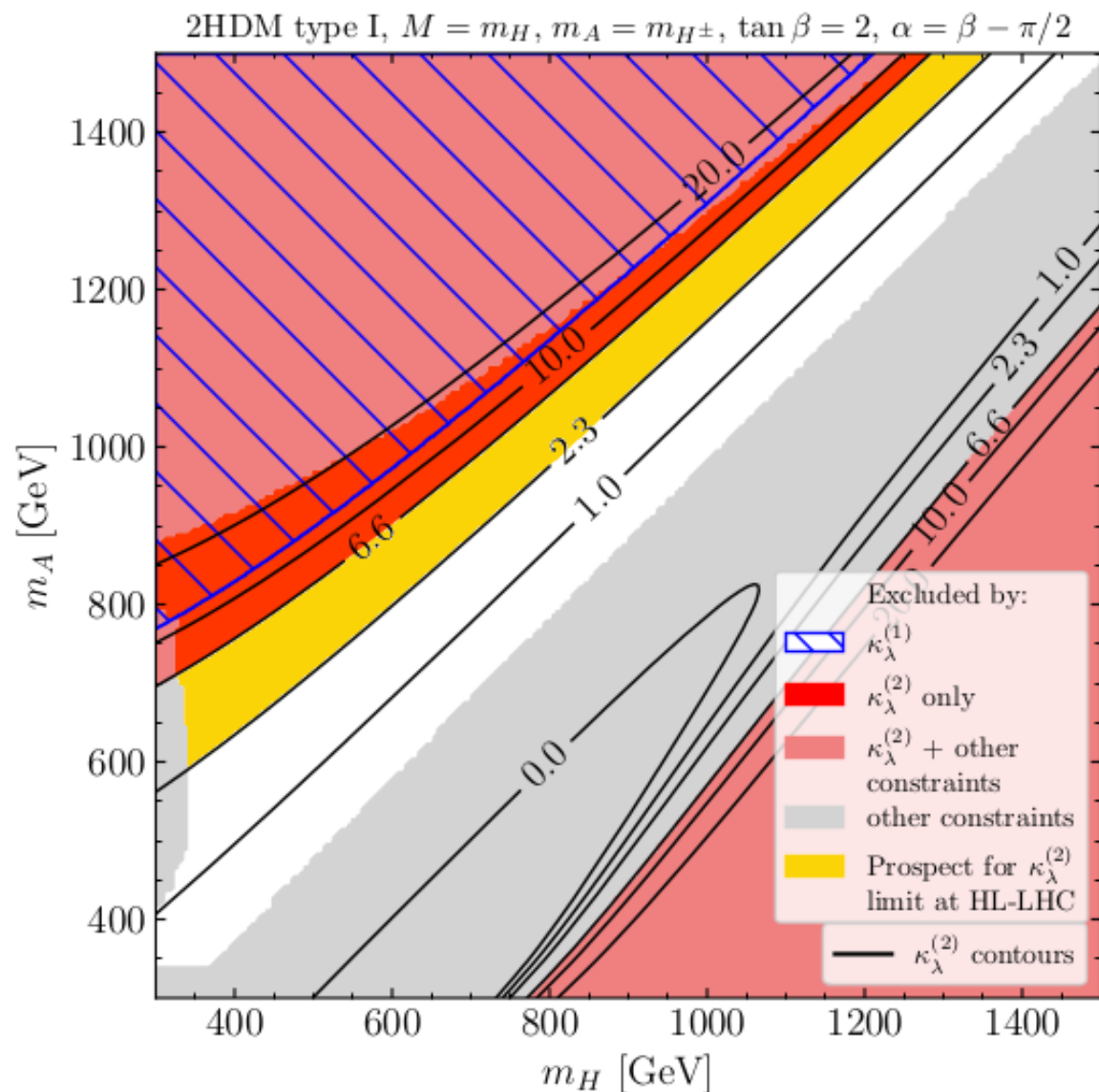
- ➢ ***Golden area:*** additional exclusion if the limit on κ_λ becomes $\kappa_\lambda^{(2)} < 2.3$ (achievable at HL-LHC)

- ➢ Of course, **prospects even better with an e⁺e⁻ collider!!**

- ➢ Experimental constraints, such as Higgs physics, may also become more stringent, however **not** theoretical constraints (like BFB or perturbative unitarity)

A benchmark scenario in the aligned 2HDM – a 2023 update
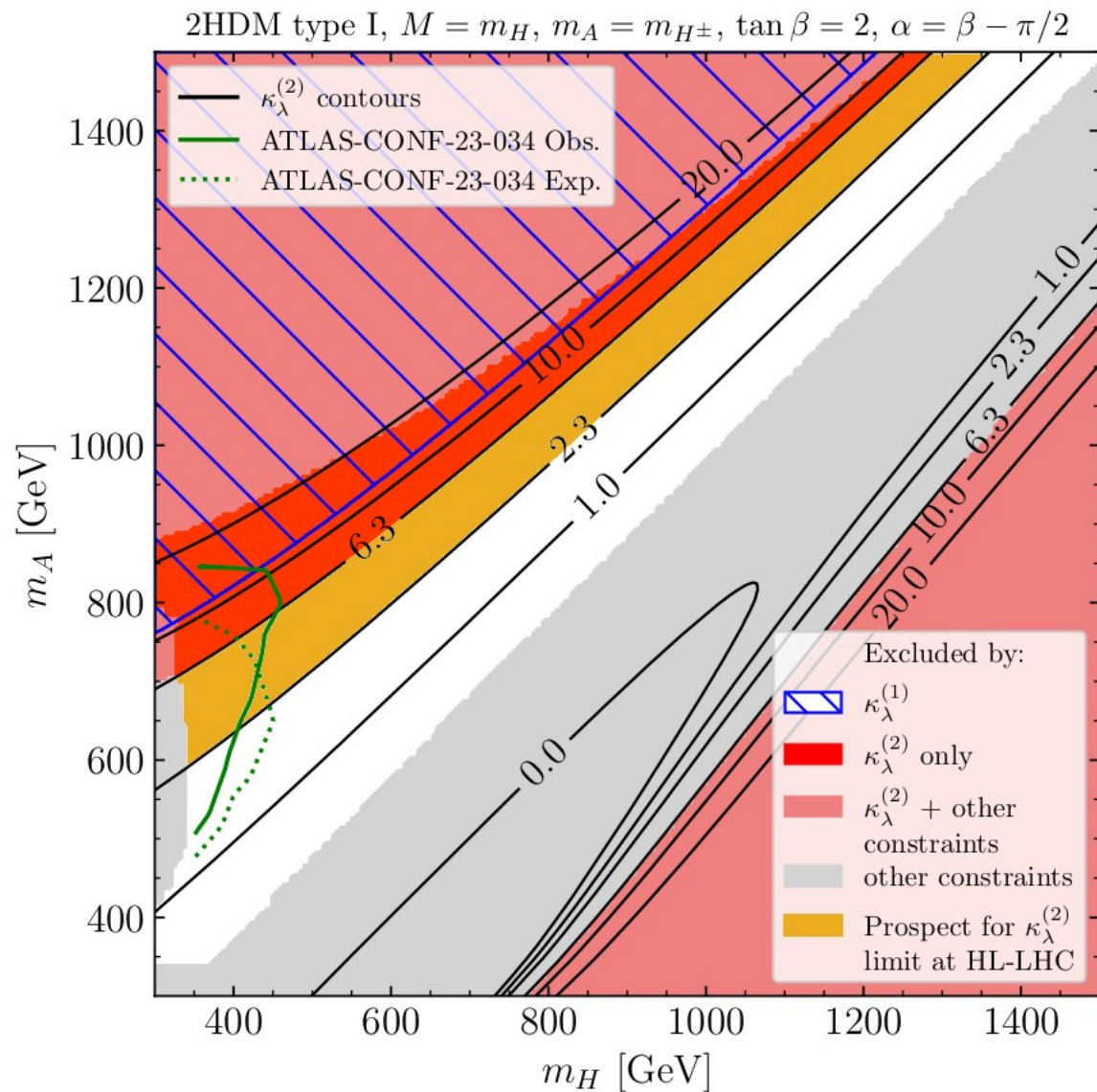
In view of [ATLAS-CONF-23-034]

2HDM type I, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

- ➤ **Green lines**: New results (exp./obs.) in direct searches for heavy Higgs bosons, via

  A → Z H

  with full LHC-Run2 data
  [ATLAS-CONF-23-034]

A benchmark scenario in the aligned 2HDM – a 2023 update

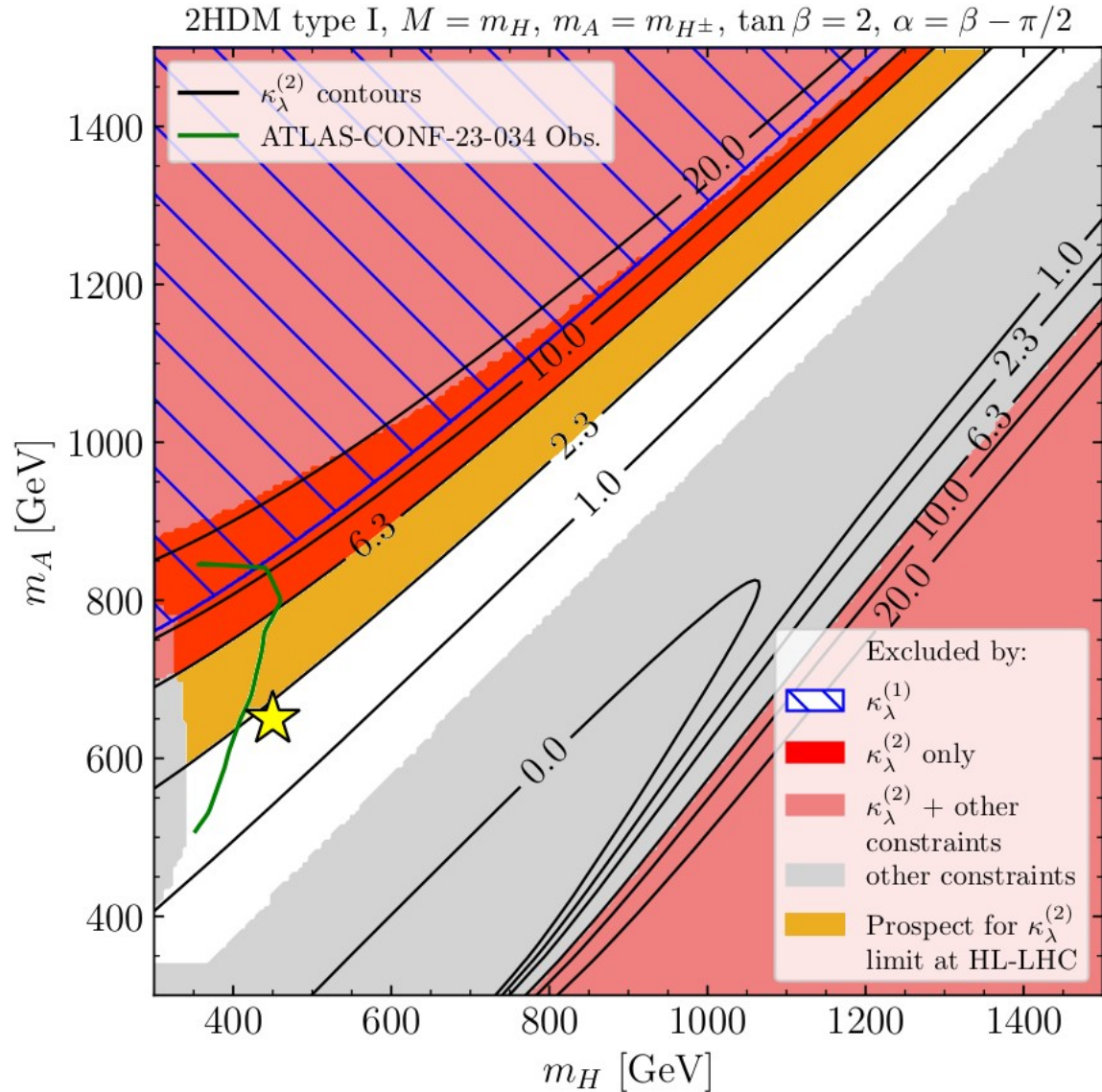**In view of [ATLAS-CONF-23-034]**

- ➢ **Green lines**: New results (exp./obs.) in direct searches for heavy Higgs bosons, via

  A→Z H

  with full LHC-Run2 data
  [ATLAS-CONF-23-034]

- ➢ **Small excess (2.9 σ) for m_H ~ 450 GeV and m_A ~ 650 GeV**
  - → near region probed by κ_λ at HL-LHC
  - → complementarity between direct and indirect searches!

A benchmark scenario in the aligned 2HDM – 1D scan

Within the previously shown plane, we fix M=m$_H$=600 GeV, and vary m$_A$=m$_{H^\pm}$ [Bahl, JB, Weiglein PRL '22]

2HDM type I, $\alpha = \beta - \pi/2$, $m_A = m_{H^\pm}$, $M = m_H = 600$ GeV, $\tan\beta = 2$
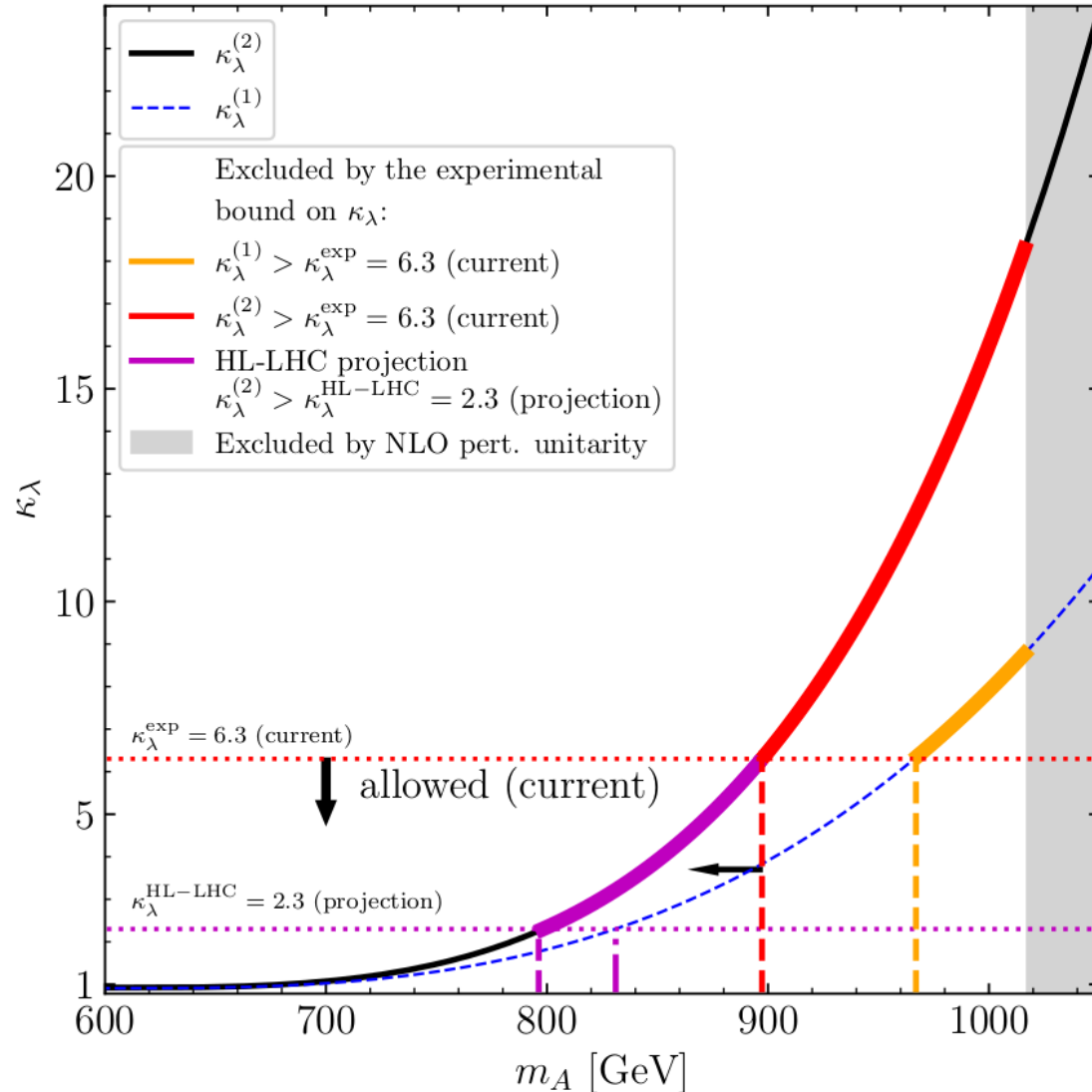
- ➤ Illustrates the significantly improved reach of the experimental limit when including **2L corrections** in calculation of κ$_\lambda$

- ➤ A stricter choice for the perturbativeunitarity constraint (grey) does not significantly change the region excluded by κ$_\lambda^{(2)}$

Mass splitting effects in various BSM models
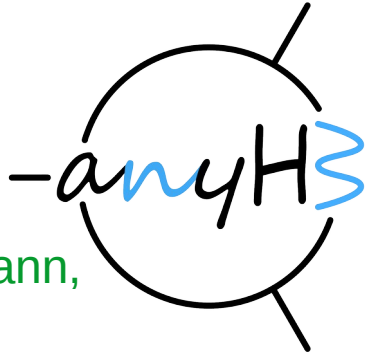
[Bahl, JB, Gabelmann, Weiglein '23]

- ➤ Consider the non-decoupling limit in several BSM models

$$M_{\text{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

- ➤ Increase M_{BSM}, keeping $\mathcal{M}$ fixed
  - → large mass splittings
  - → **large BSM effects!**

- ➤ Perturbative unitarity checked with `anyPerturbativeUnitarity`

Mass-splitting effects on κ_λ for various aligned $SU(2)_L$ multiplets

ATLAS-CONF-2022-050

- SM
- IDM   SM + doublet
- THDM-II
- TSM$_{Y=1}$ SM + triplet
- GeorgiMachacek   SM + 2 triplets

HL-LHC

κ_λ

M_{BSM} [GeV]
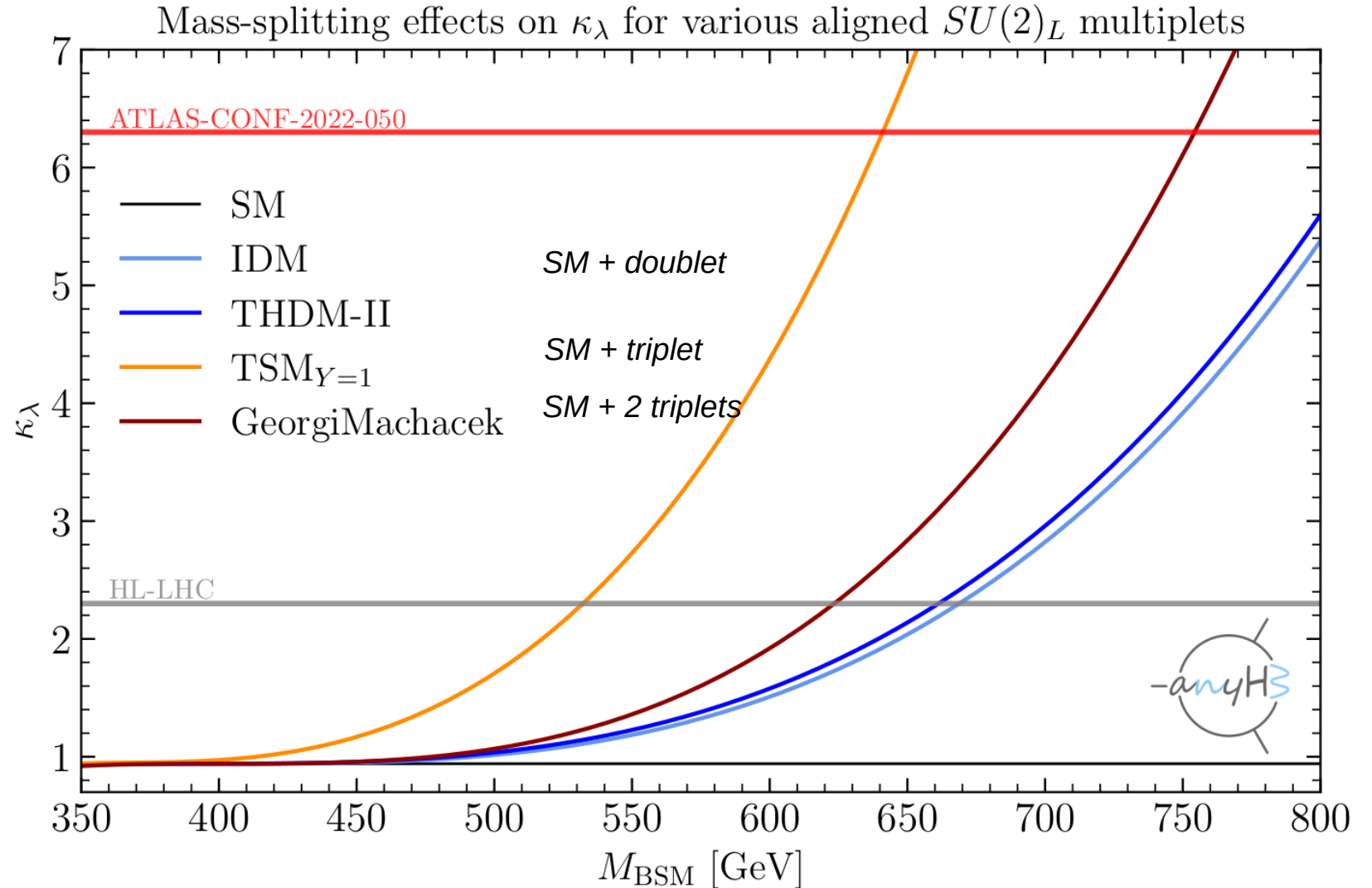
Here: scenarios with lightest BSM scalar mass & BSM mass param. $\mathcal{M}$ at 400 GeV; other BSM scalar masses = M_{BSM}

Summary

➢ λ_{hhh} plays a crucial role to understand the shape of the Higgs potential, and probe indirectly signs of New Physics

➢ λ_{hhh} can **deviate significantly from SM** prediction (by up to a **factor ~10**), for otherwise theoretically and experimentally **allowed points**, due to mass splitting effects in radiative corrections involving BSM scalars

➢ Current experimental bounds on λ_{hhh} can **already exclude significant parts of otherwise unconstrained BSM parameter space**, and future prospects even better! Inclusion of 2L corrections [JB, Kanemura '19] has significant impact.

➢ In this talk, 2HDM taken as an *example*, but similar results are expected for a wider range of BSM models with extended scalar sectors
→ further motivates **automating calculations of λ_{hhh}** → see **Martin Gabelmann's talk tomorrow (Hörsaal B, 8:30)!!**

Thank you for your attention!

Contact

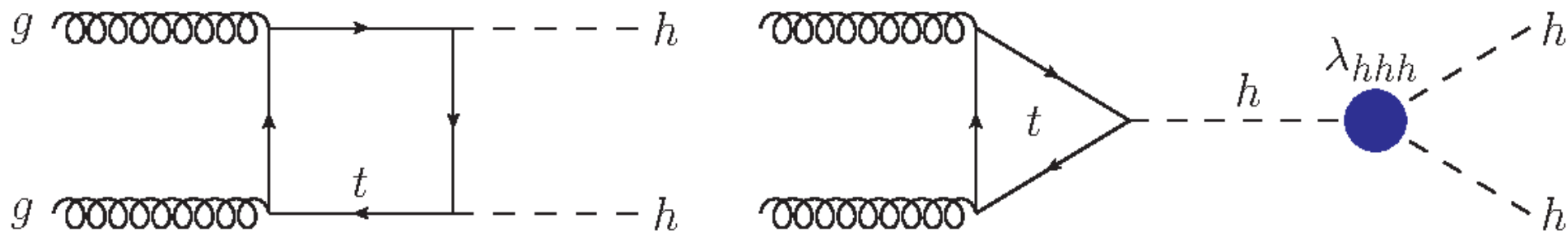**DESY.** Deutsches
Elektronen-Synchrotron

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DESY.

Backup

Accessing λ_{hhh} via double-Higgs production

- **Double-Higgs production** $\to \lambda_{hhh}$ enters at LO $\to$ **most direct probe of λ_{hhh}**

[Note: Single-Higgs production (EW precision observables) $\to \lambda_{hhh}$ enters at NLO (NNLO)]

- Box and triangle diagrams **interfere destructively**
  $\to$ small prediction in SM

  $\to$ BSM deviation in λ_{hhh} can **significantly enhance hh-production!**

- Upper limit on hh-production cross-section $\to$ **limits on**
  $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$
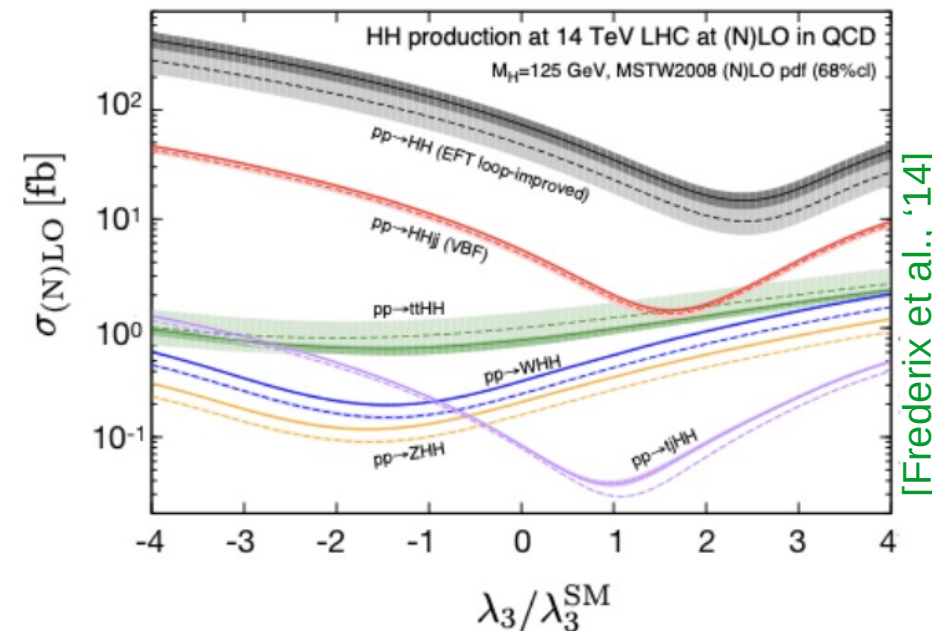
[Frederix et al., '14]

Accessing λ_{hhh} via double-Higgs production

> **Dou**

> Box
> → s
> → B

hh-p

> Upper limit on hh-production cross-section → **limits on**
> $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$

[Not

Recent results from ATLAS hh-searches [ATLAS-CONF-2021-052]
yield the limits:

-1.0 < κ_λ < 6.6 at 95% C.L.

→ factor ~2 improvement compared to
previously best ATLAS limits (from single h prod.)
[ATLAS-PHYS-PUB-2019-009]

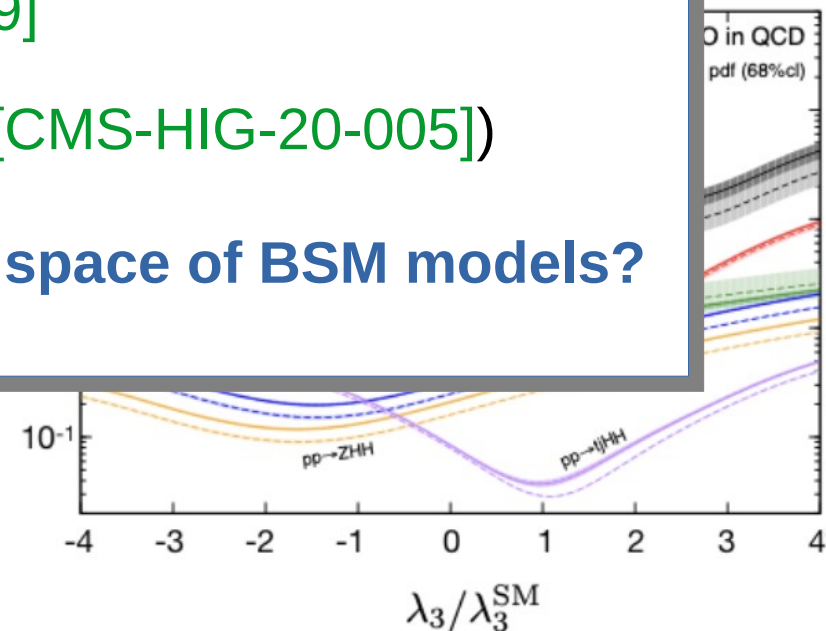(CMS recently gave **-2.3 < κ_λ < 9.4** at 95% C.L. [CMS-HIG-20-005])

→ **Can κ_λ now be used to constrain the parameter space of BSM models?**

[Frederix et al., '14]

Future determination of λhhh

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

Plot taken from
[de Blas et al., 1905.03764]

di-Higgs exclusive result

single-Higgs exclusive

single-Higgs global

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}
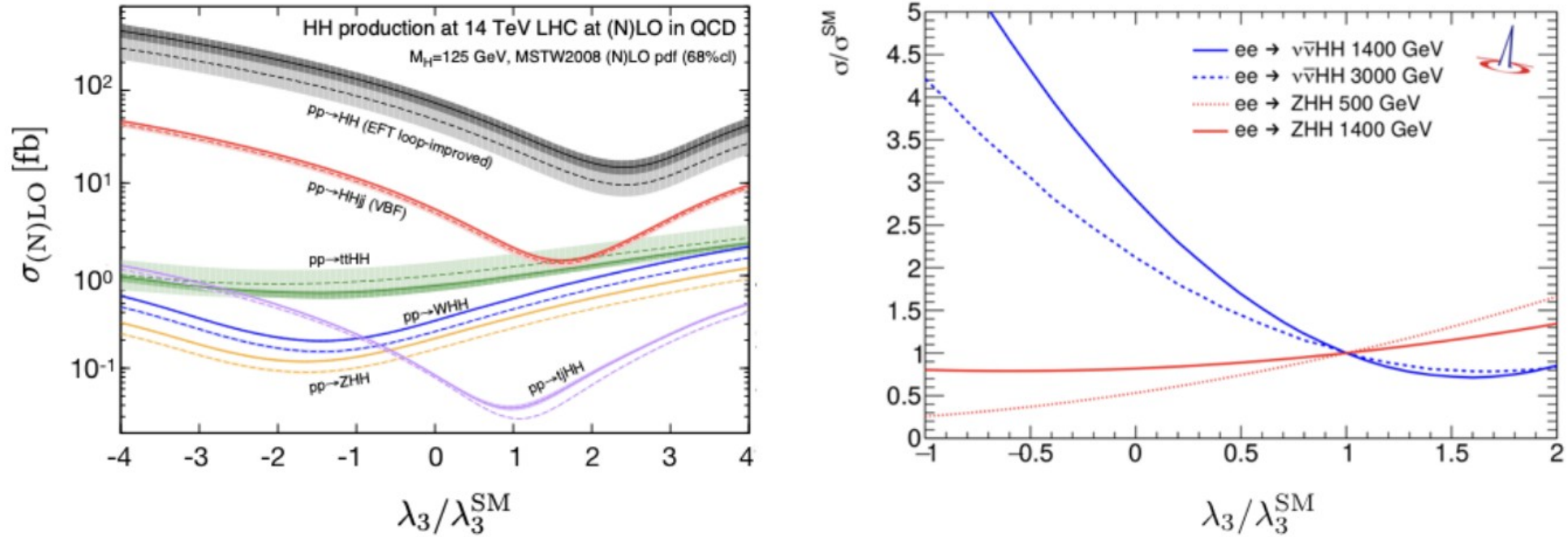
Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

Plots taken from
[de Blas et al., 1905.03764]

[Frederix et al., 1401.7340]

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}
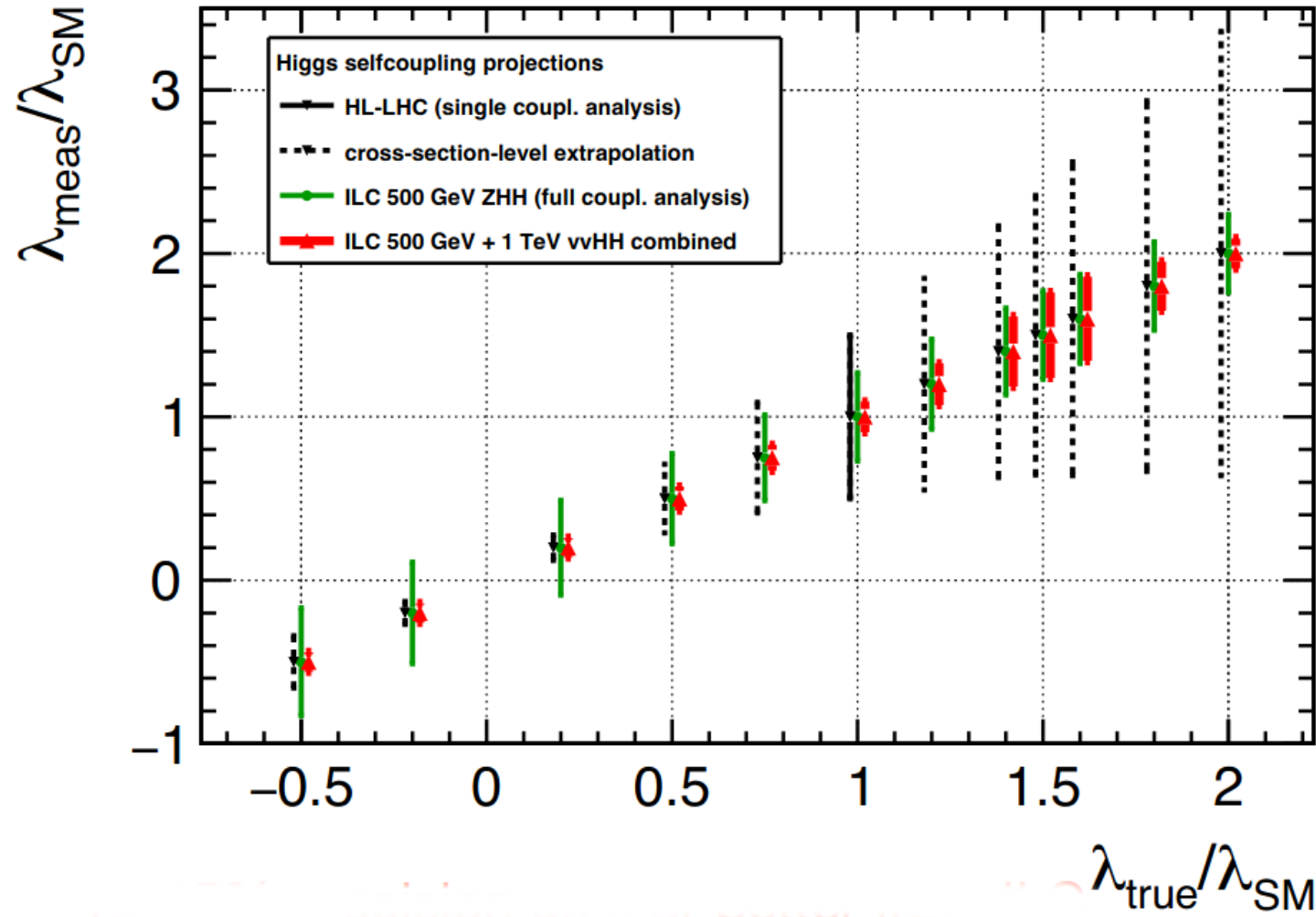
[J. List et al. '21]

See also [Dürig, DESY-THESIS-2016-027]

The Two-Higgs-Doublet Model

➢ 2 SU(2)$_L$ doublets $\Phi_{1,2}$ of hypercharge ½

➢ CP-conserving 2HDM, with softly-broken Z$_2$ symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V_{\mathrm{2HDM}}^{(0)} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2)$$

$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\left((\Phi_2^\dagger\Phi_1)^2 + \mathrm{h.c.}\right)$$

➢ m$_1$,m$_2$ eliminated with tadpole equations, and $v_1^2 + v_2^2 = v^2 = (246~\mathrm{GeV})^2$

➢ 7 free parameters in scalar sector: m$_3$, λ_i (i=1,..,5), tanβ≡v$_2$/v$_1$

➢ Mass eigenstates: h, H: CP-even Higgses, A: CP-odd Higgs, H$^\pm$: charged Higgs, α: CP-even Higgs mixing angle

➢ λ_i (i=1,..,5) traded for mass eigenvalues m$_h$, m$_H$, m$_A$, m$_{H\pm}$ and angle α

➢ m$_3$ replaced by a Z$_2$ soft-breaking mass scale

$$M^2 = \frac{2m_3^2}{s_{2\beta}}$$

One-loop non-decoupling effects

- **Leading one-loop** corrections to λ_{hhh} in models with extended sectors (like 2HDM):
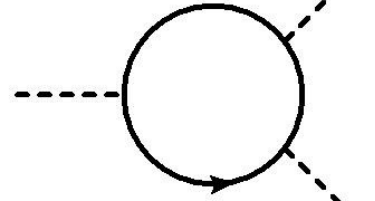
SM top quark loop   *BSM scalar loops*
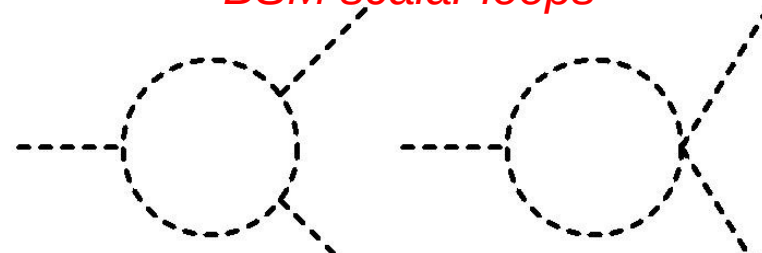
$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} + \sum_\Phi \frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

First found in 2HDM:
[Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases} \longrightarrow$$ **Huge BSM effects possible!**

One-loop non-decoupling effects
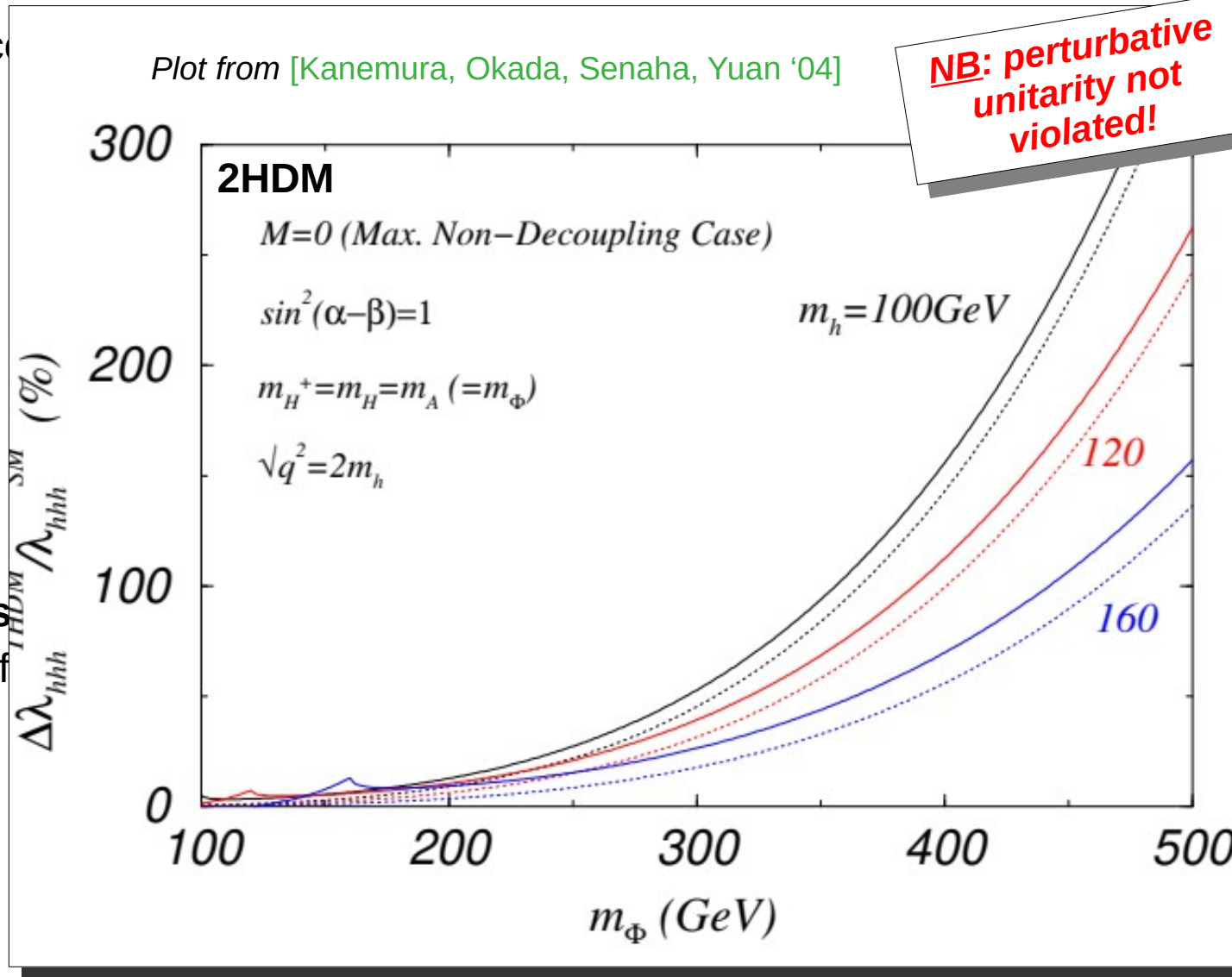
> Leading one-loop c[...]

$$\delta^{(1)}\lambda_{hhh} \supset$$

$\mathcal{M}$: **BSM mass**
n_Φ : # of d.o.f of

> Size of new effects

Plot from [Kanemura, Okada, Senaha, Yuan '04]

2HDM

$M=0$ (Max. Non−Decoupling Case)

$\sin^2(\alpha-\beta)=1$

$m_{H^+}=m_H=m_A\ (=m_\Phi)$

$\sqrt{q^2}=2m_h$

$m_h=100GeV$

NB: perturbative unitarity not violated!

First found in 2HDM: [Kanemura, Kiyoura, Okada, Senaha, Yuan '02]

$$\Lambda^2 + \tilde{\lambda}v^2$$

Huge BSM effects possible!

Our calculation

Goal: How large can the two-loop corrections to λ_{hhh} become?

An effective Higgs trilinear coupling

➢ In principle: consider 3-point function Γ_{hhh}

but this is momentum dependent → very difficult beyond one loop

$$\equiv \Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$$
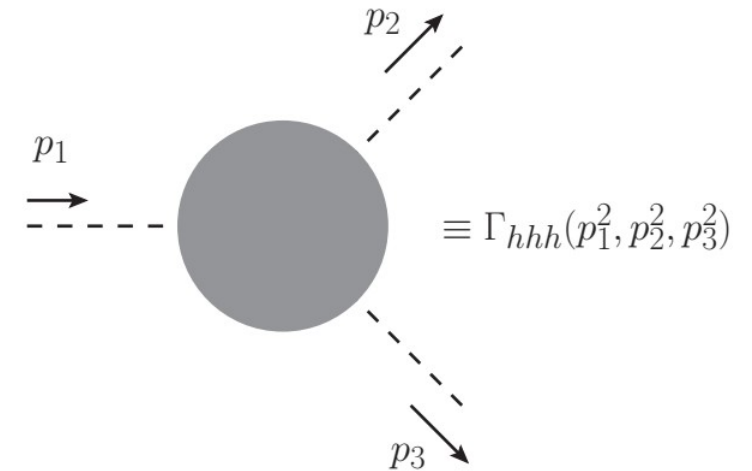
➢ Instead, consider an **effective trilinear coupling**

$$\lambda_{hhh} \equiv \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min}}$$

➢ Momentum effects are neglected, but are expected to be *sub-leading* anyway

  – At one loop [Kanemura, Okada, Senaha, Yuan '04]: effects of a few % (away from thresholds)

  – At two loops, no study for 3-pt. functions but experience from Higgs mass calculations

Our effective-potential calculation

➢ **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \dfrac{1}{16\pi^2} V^{(1)} + \dfrac{1}{(16\pi^2)^2} V^{(2)}$ ($\overline{\text{MS}}$ result)

➔ $V^{(2)}$: 1PI vacuum bubbles

➔ *Dominant BSM contributions to $V^{(2)}$* = diagrams involving **heavy BSM scalars and top quark**

$$V^{(2)}_{SSS} \qquad V^{(2)}_{SS} \qquad V^{(2)}_{FFS}$$

➔ **Aligned scenarios** → no mixing + compatible with experimental results
➔ **Neglect masses of light states** (SM-like Higgs, light fermions, ...)

Our effective-potential calculation

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \dfrac{1}{16\pi^2} V^{(1)} + \dfrac{1}{(16\pi^2)^2} V^{(2)}$ ($\overline{\text{MS}}$ result)

  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to $V^{(2)}$* = diagrams involving **heavy BSM scalars and top quark**
  - *Aligned scenarios + neglect light masses*

- **Step 2**:
$$\lambda_{hhh} \equiv \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left[\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left(\frac{\partial^2}{\partial h^2} - \frac{1}{v}\frac{\partial}{\partial h}\right)\right]\Delta V\bigg|_{\text{min.}}$$
($\overline{\text{MS}}$ result too)

Express tree-level result in terms of effective-potential Higgs mass

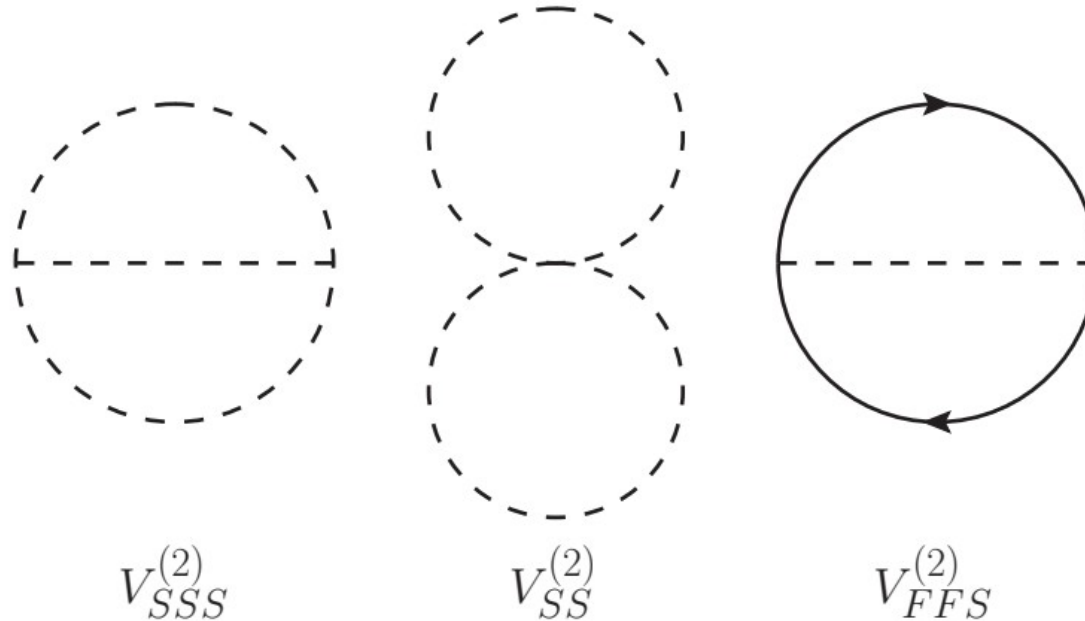# Our effective-potential calculation

- **Step 1**: compute $V_{\text{eff}} = V^{(0)} + \dfrac{1}{16\pi^2} V^{(1)} + \dfrac{1}{(16\pi^2)^2} V^{(2)}$ ($\overline{\text{MS}}$ result)

  - $V^{(2)}$: 1PI vacuum bubbles
  - *Dominant BSM contributions to $V^{(2)}$* = diagrams involving **heavy BSM scalars and top quark**
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  ($\overline{\text{MS}}$ result too)

- **Step 3**: conversion from $\overline{\text{MS}}$ to OS scheme (*details in the following*)
  - Express result in terms of **pole masses**: M_t, M_h, M_Φ (Φ=H,A,H$^\pm$); OS Higgs VEV $v_{\text{phys}} = \dfrac{1}{\sqrt{\sqrt{2}G_F}}$
  - Include **finite WFR**: $\hat{\lambda}_{hhh} = (Z_h^{\text{OS}}/Z_h^{\overline{\text{MS}}})^{3/2}\lambda_{hhh}$
  - Prescription for M to ensure **proper decoupling** with $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

Our effective-potential calculation – scheme conversion

▶ OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\mathsf{OS}}}{Z_h^{\overline{\mathsf{MS}}}} \right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\mathsf{MS}} \text{ parameters} \\ \text{translated to OS ones}}}$$

▶ Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\mathsf{MS}}}) + \kappa f^{(1)}(x^{\overline{\mathsf{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\mathsf{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2} \right)$$

and

$$x^{\overline{\mathsf{MS}}} = X^{\mathsf{OS}} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\mathsf{OS}}) + \kappa \left[f^{(1)}(X^{\mathsf{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\mathsf{OS})}\delta^{(1)} x \right]$$

$$+ \kappa^2 \left[f^{(2)}(X^{\mathsf{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\mathsf{OS}})\delta^{(1)} x + \frac{\partial f^{(0)}}{\partial x}(X^{\mathsf{OS}})\delta^{(2)} x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\mathsf{OS}})(\delta^{(1)} x)^2 \right]$$

Our effective-potential calculation – scheme conversion

▶ OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}} \text{ parameters} \\ \text{translated to OS ones}}}$$

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and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa \left[f^{(1)}(X^{\text{OS}}) + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS})\delta^{(1)}x}} \right]$$

$$+ \kappa^2 \left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS})\delta^{(2)}x}} + \cancel{\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS})(\delta^{(1)}x)^2}} \right]$$

because we neglect m_h in the loop corrections and $\lambda_{hhh}^{(0)} = 3m_h^2/v$ (in absence of mixing)

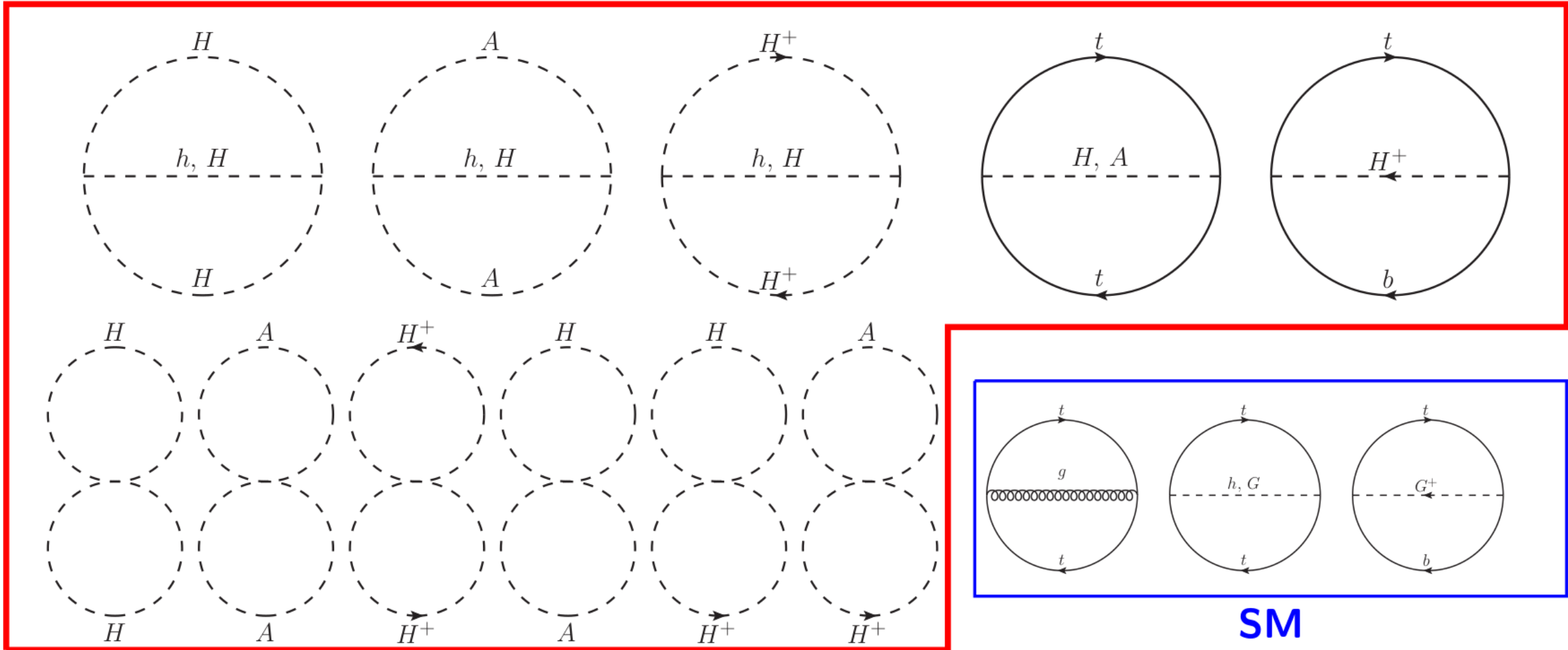# Effective potential in the 2HDM

$$\lambda_{hhh} \equiv \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min.}}$$
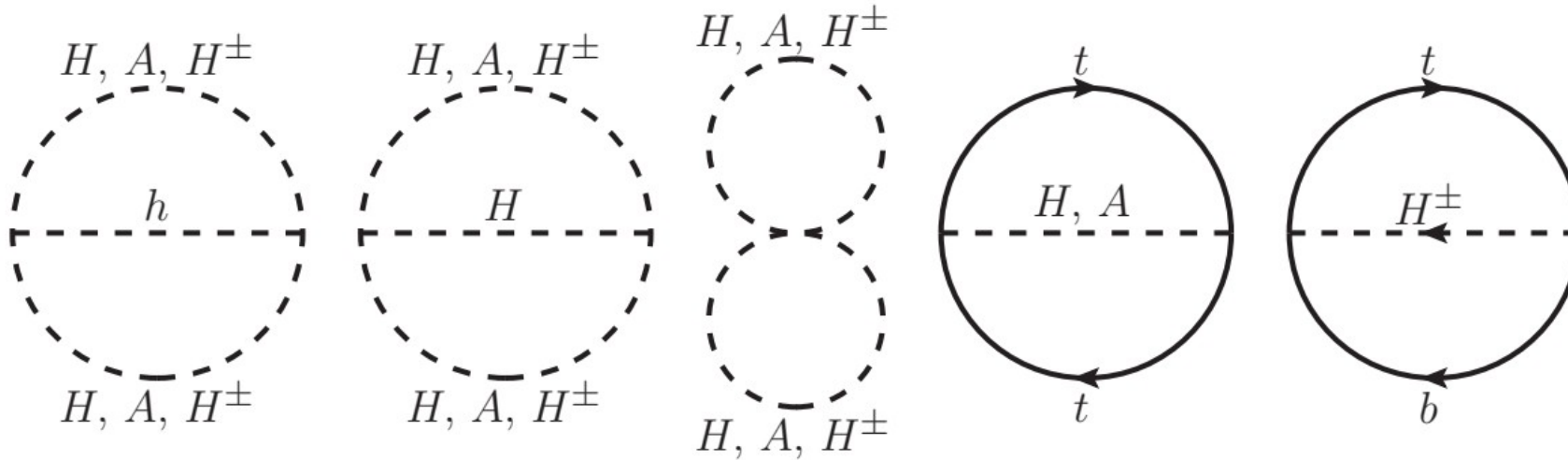
➤ 2HDM → **15 new BSM diagrams** appearing in $V^{(2)}$ w.r.t. the SM case

[JB, Kanemura '19]

2HDM

SM

$\overline{\text{MS}}$ result

> Taking BSM scalars to be degenerate $M_\Phi = M_H = M_A = M_{H^\pm}$ we obtain in the $\overline{\text{MS}}$ scheme:

(expressions for non-degenerate masses → see [JB, Kanemura `1911.11507`])

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\,\overline{\log}\,m_\Phi^2\right]$$

$$+ \frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[1 + 2\,\overline{\log}\,m_\Phi^2\right]$$

$$+ \frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3\left[-1 + 2\,\overline{\log}\,m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

Decoupling property in $\overline{\text{MS}}$ scheme

▶ Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\overline{\log}\, m_\Phi^2\right]$$

$$\delta^{(1)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^3}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3 + \frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4\left[1 + 2\overline{\log}\, m_\Phi^2\right]$$

$$+ \frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3\left[-1 + 2\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

where $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

▶ To have $m_\Phi \to \infty$, then we must take $M \to \infty$, otherwise the quartic couplings grow out of control

▶ Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1 - \frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2 + \tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M \to \infty} 0$$

$\overline{\text{MS}} \to$ OS scheme conversion

- To express $\delta^{(2)}\lambda_{hhh}$ in terms of physical parameters $(v_{\text{phys}}, M_t, M_A = M_H = M_{H\pm} = M_\Phi)$, we replace

$$m_A^2 \to M_A^2 - \Pi_{AA}(M_A^2), \quad m_H^2 \to M_H^2 - \Pi_{HH}(M_H^2), \quad m_{H\pm}^2 \to M_{H\pm}^2 - \Pi_{H^+H^-}(M_{H\pm}^2),$$

$$v \to v_{\text{phys}} - \delta v, \quad m_t^2 \to M_t^2 - \Pi_{tt}(M_t^2)$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable

  ... but then, expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!

- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly

- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that
  1. the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$
  2. all the log terms in $\delta^{(2)}\hat{\lambda}_{hhh}$ are absorbed in δM^2

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48 M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4 \left\{4 + 3\cot^2 2\beta \left[3 - \frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2} + 2\right)\right]\right\} + \frac{576 M_\Phi^6 \cot^2 2\beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$

$$+ \frac{288 M_\Phi^4 M_t^2 \cot^2 \beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 + \frac{168 M_\Phi^4 M_t^2}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 - \frac{48 M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^5 + \mathcal{O}\left(\frac{M_\Phi^2 M_t^4}{v_{\text{phys}}^5}\right)$$
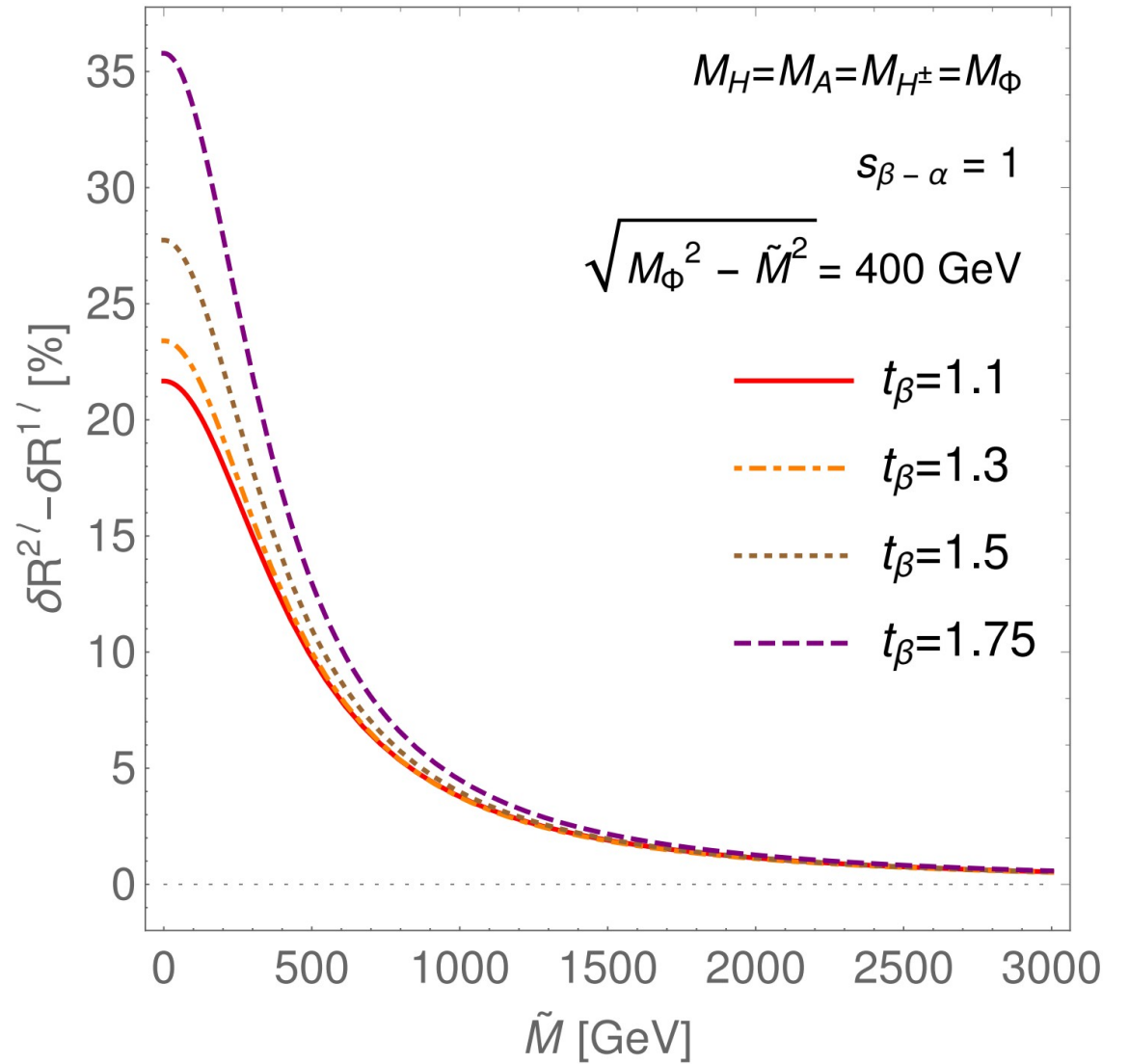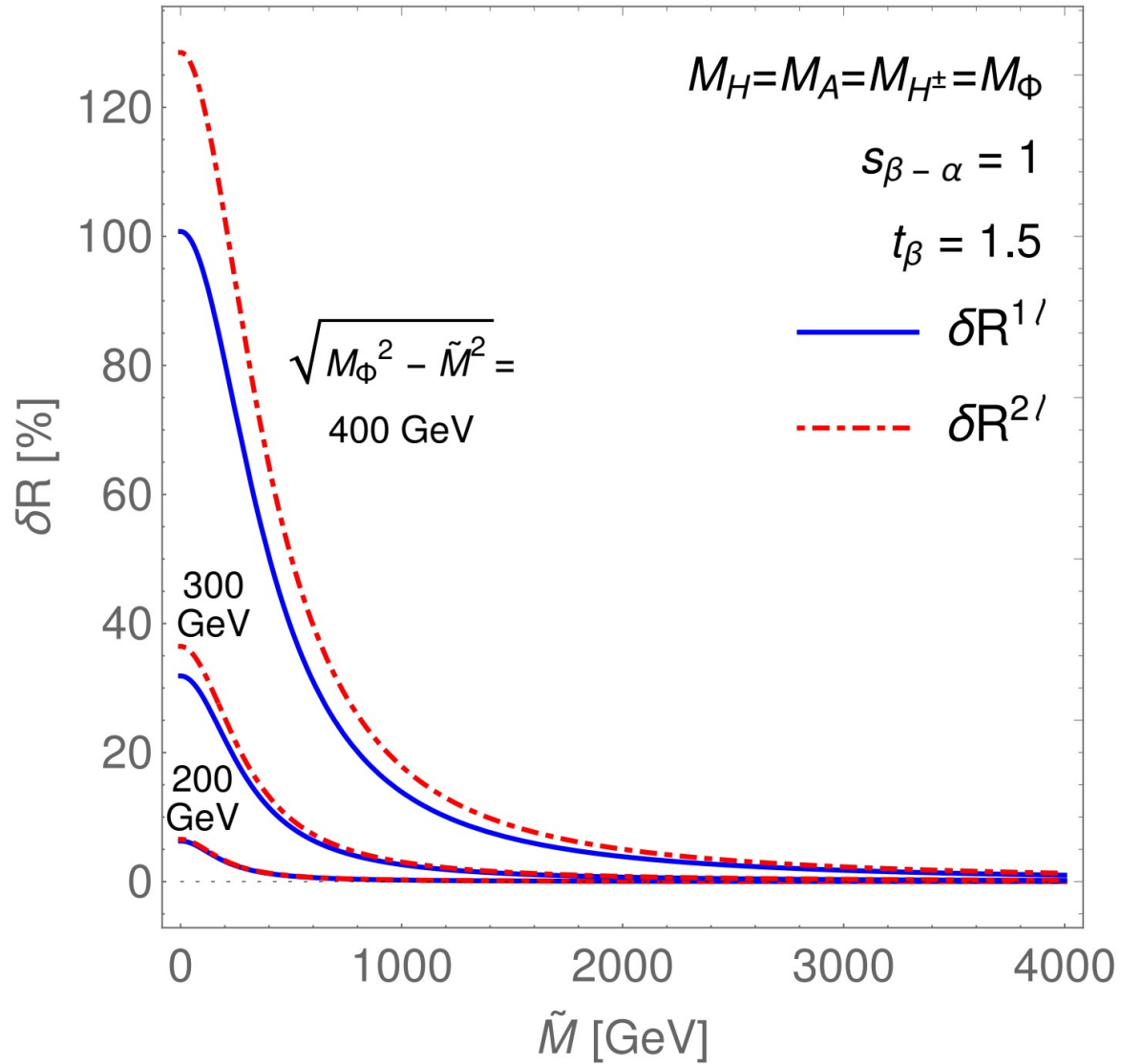
Numerical results in an aligned 2HDM

$$\delta R \equiv \frac{\hat{\lambda}_{hhh}^{\text{2HDM}} - \hat{\lambda}_{hhh}^{\text{SM}}}{\hat{\lambda}_{hhh}^{\text{SM}}}$$

Decoupling of BSM effects

$\tilde{M}$: modified "OS" version of Z_2 breaking scale   [JB, Kanemura '19]

Decoupling of BSM effects

$\tilde{M}$: modified "OS" version of Z_2 breaking scale

$M_H = M_A = M_{H^\pm} = M_\Phi$

$s_{\beta - \alpha} = 1$

$\sqrt{M_\Phi^2 - \tilde{M}^2} = 400$ GeV

$\sqrt{M_\Phi^2 - \tilde{M}^2} = 400$ GeV

300 GeV

200 GeV

$t_\beta = 1.1$

$t_\beta = 1.3$

$t_\beta = 1.5$

$t_\beta = 1.75$

➤ Radiative corrections from additional scalars + top quark indeed decouple properly when taking $\tilde{M} \rightarrow \infty$

➤ $\tilde{M}$: controls the decoupling behaviour in OS result
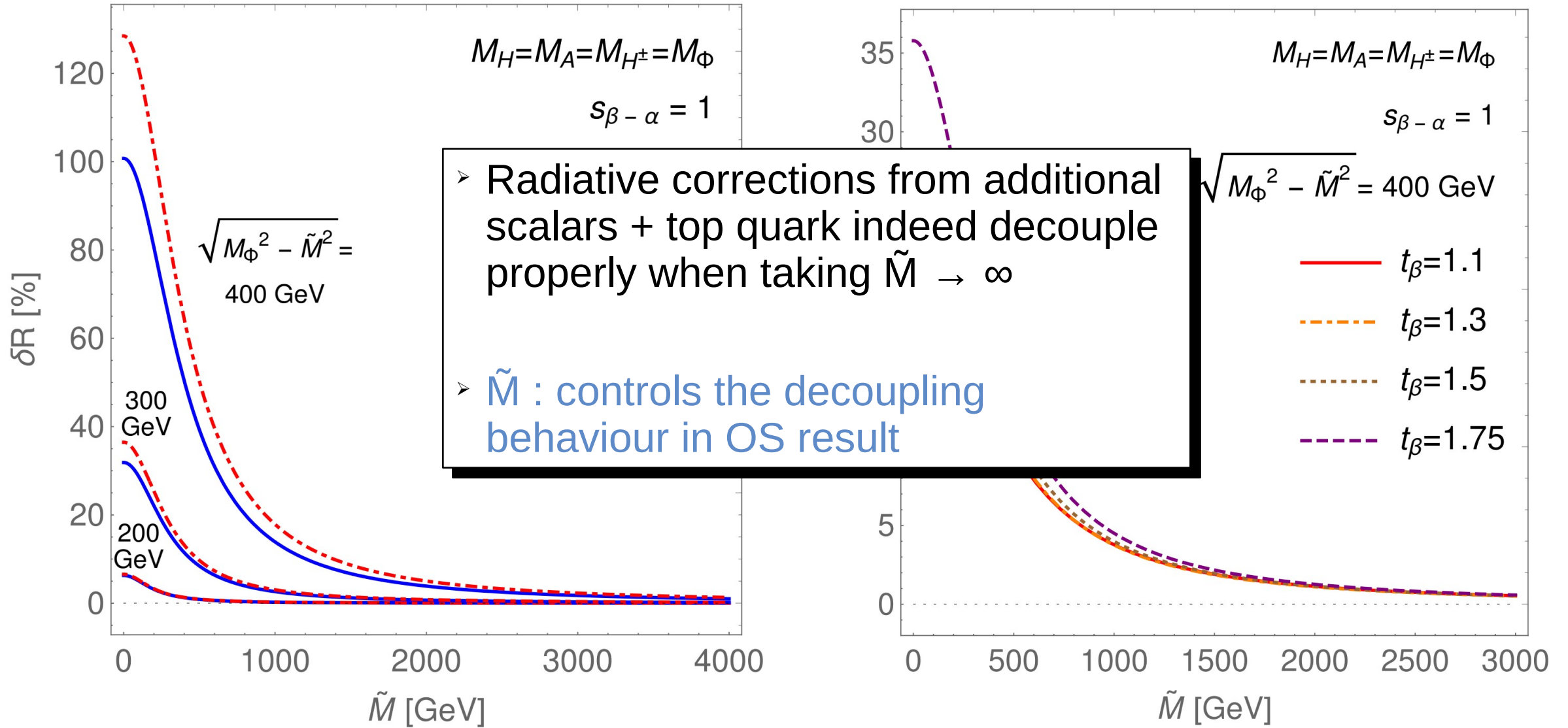
BSM deviation of λ_{hhh} in non-decoupling limit

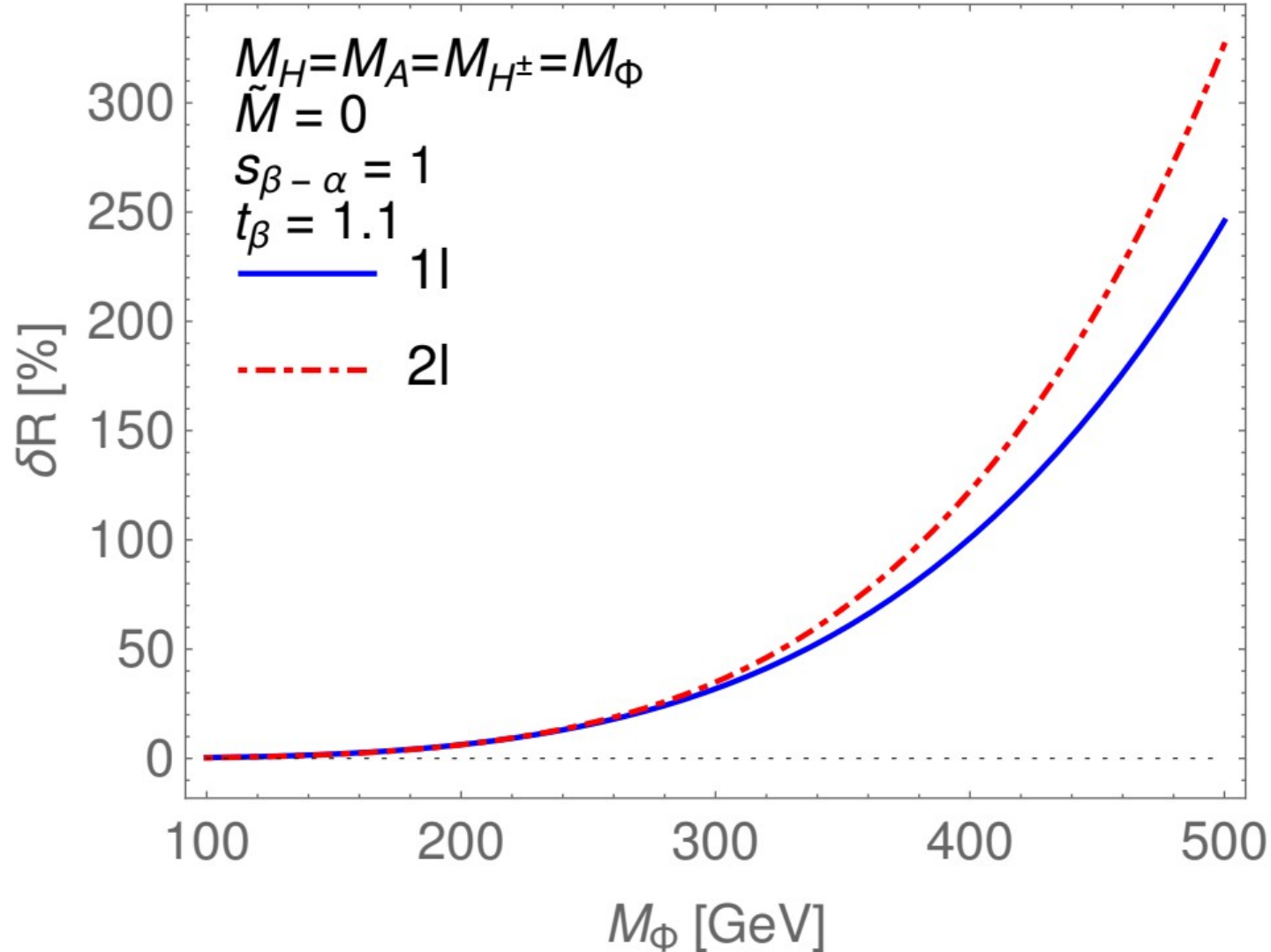**Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_H{}^{\pm}$**

[JB, Kanemura 1903.05417]

$M_H = M_A = M_{H^{\pm}} = M_\Phi$
$\tilde{M} = 0$
$s_{\beta-\alpha} = 1$
$t_\beta = 1.1$
— 1l
--- 2l
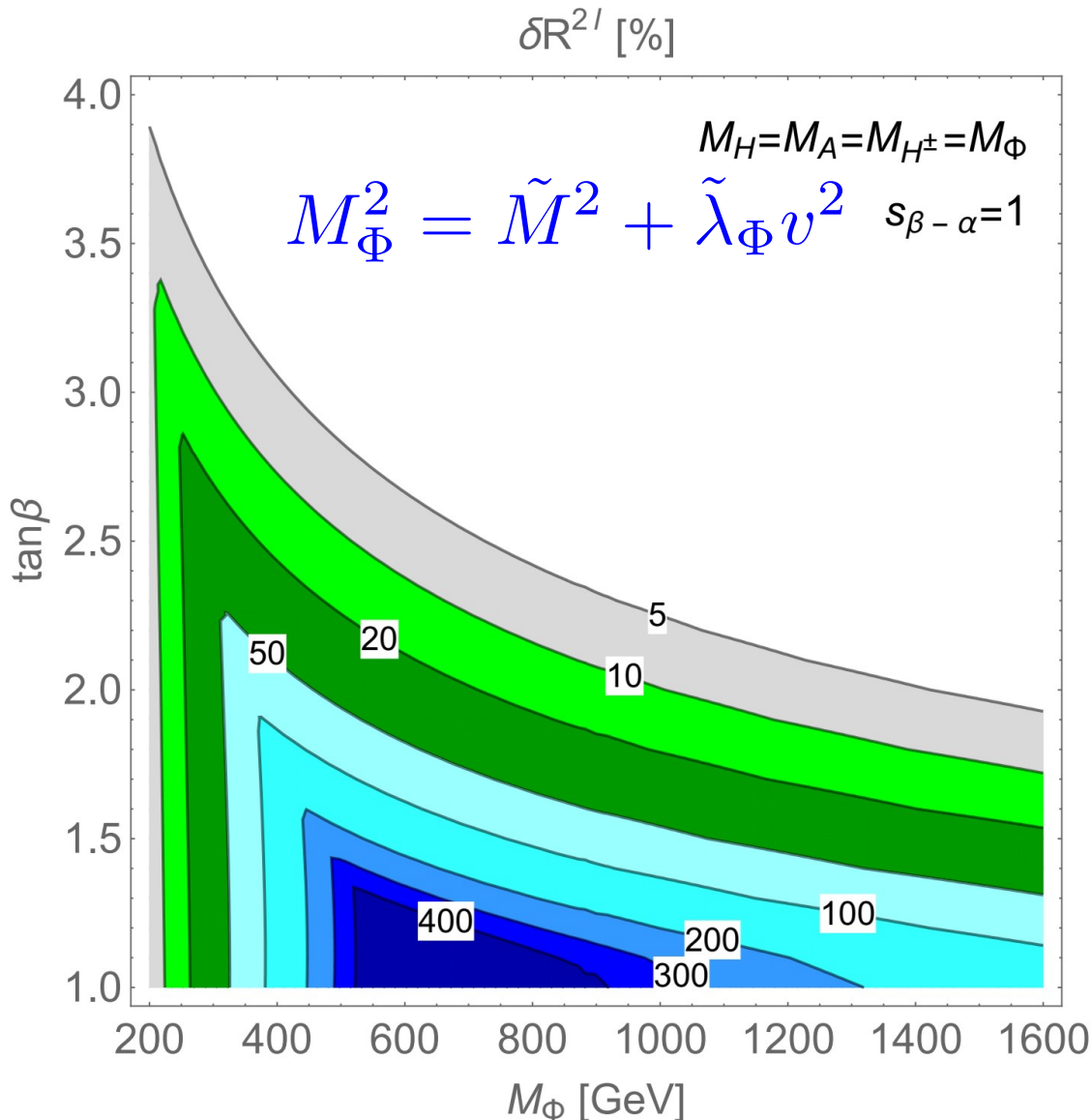
- $\tilde{M} = 0 \to$ maximal non-decoupling effects

- 1 loop: $\propto M_\Phi^4$
- 2 loops: $\propto M_\Phi^6$

- $\delta^{(2)}\lambda_{hhh}$ typically 10-20% of $\delta^{(1)}\lambda_{hhh}$ for most of mass range, at most 30%

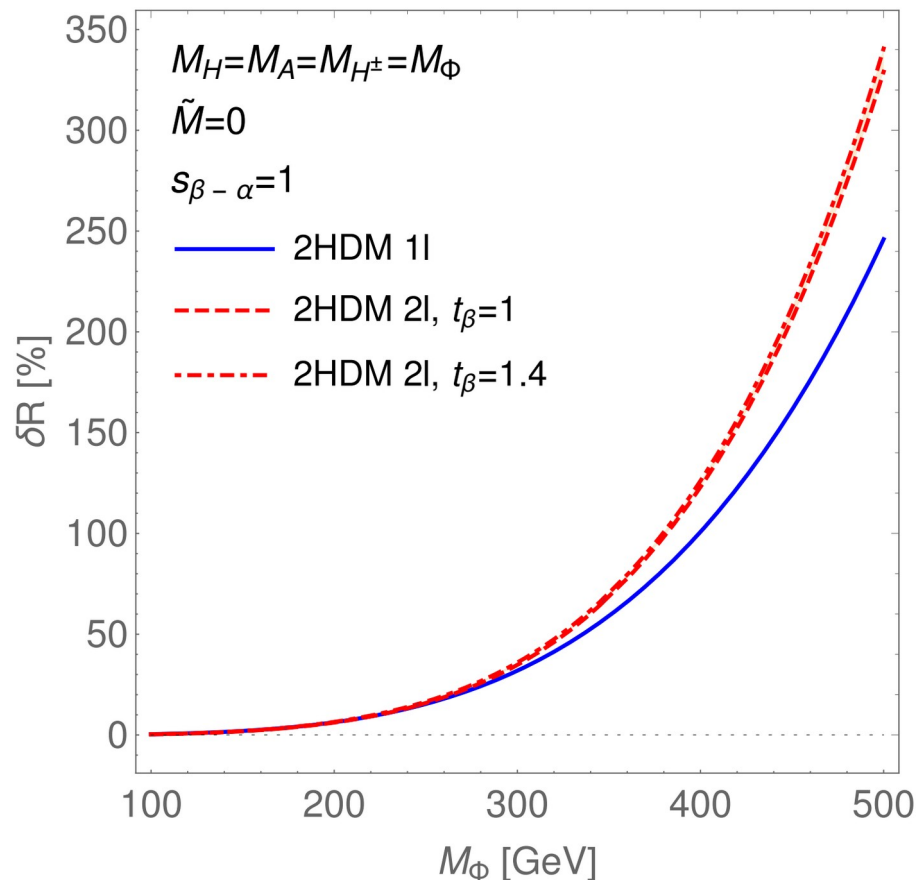# Maximal BSM deviation in an aligned 2HDM scenario

[JB, Kanemura 1911.11507]

δR^{2l} [%]

$M_H = M_A = M_{H^\pm} = M_\Phi$

$$M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$$

$s_{\beta-\alpha} = 1$

- Maximal δR (1l+2l) allowed while fulfilling perturbative unitarity [Kanemura, Kubota, Takasugi '93]

- Max. deviations for low tanβ and M_Φ~600-800 GeV
  → heavy BSM scalars acquiring their mass from Higgs VEV **only**
  - 1 loop: up to ~300% deviation at most
  - 2 loops: additional 100% (for same points)
- For increasing tanβ, unitarity constraints become more stringent → smaller δR

- **Blue region:** probed at **HL-LHC** (50% accuracy on λ_{hhh})
- **Green region:** probed at lepton colliders, e.g. **ILC** (50% accuracy at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)

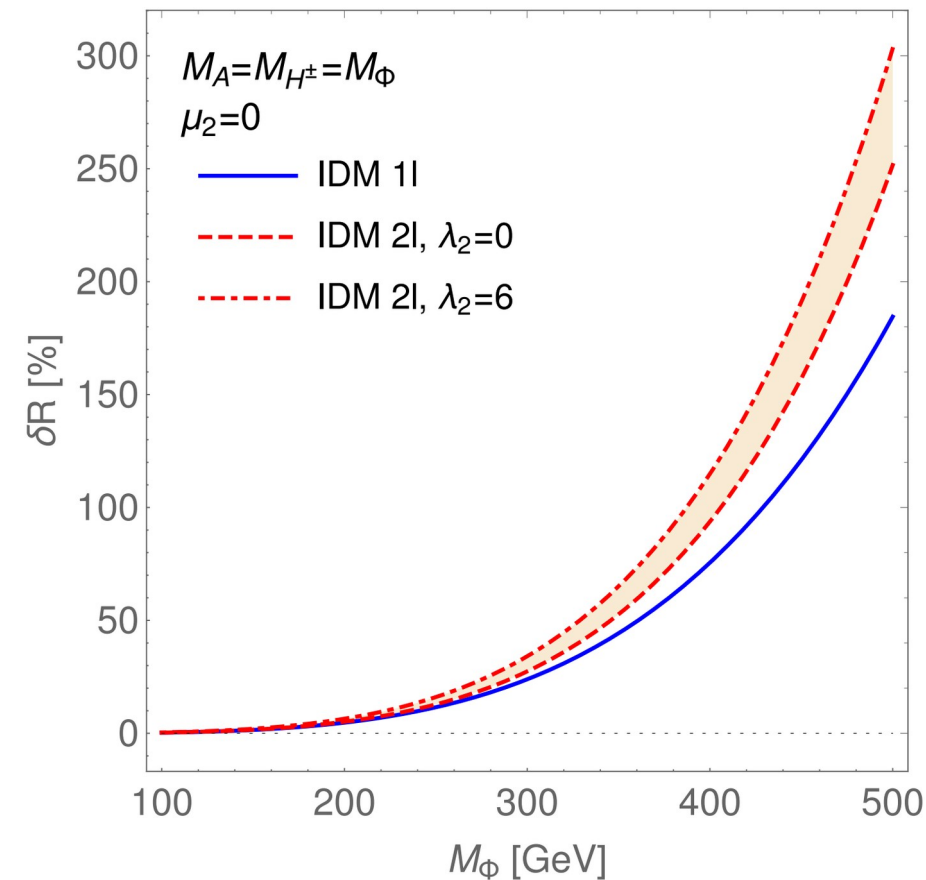# λ_{hhh} at two loops in more models

- ➤ Calculations in several other models: *IDM, singlet extension of SM*
- ➤ Each model contains a **new parameter appearing from two loops**:

Aligned 2HDM → tanβ

$M_H = M_A = M_{H^\pm} = M_\Phi$
$\tilde{M} = 0$
$s_{\beta-\alpha} = 1$

— 2HDM 1l
- - - 2HDM 2l, $t_\beta = 1$
- · - 2HDM 2l, $t_\beta = 1.4$

tanβ constrained by perturbative unitarity
→ only small effects

IDM → λ_2 (quartic coupling of inert doublet)

$M_A = M_{H^\pm} = M_\Phi$
$\mu_2 = 0$

— IDM 1l
- - - IDM 2l, $\lambda_2 = 0$
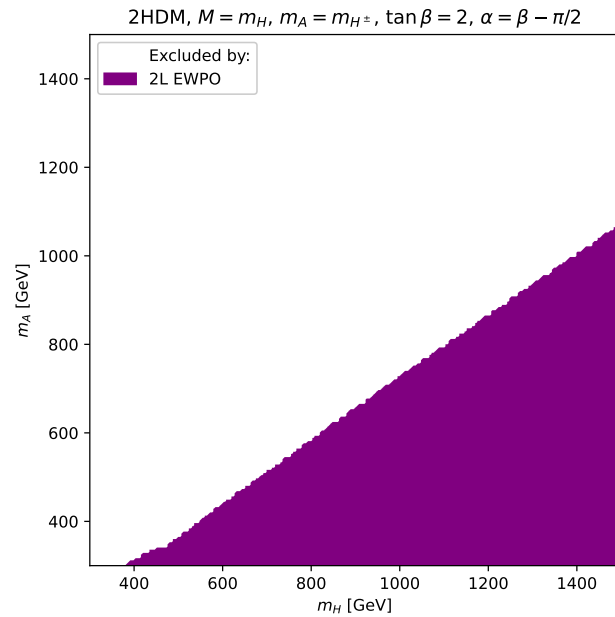- · - IDM 2l, $\lambda_2 = 6$

λ_2 is less contrained → **enhancement is possible**
(but 2l effects remains <u>well smaller</u> than 1l ones)

2HDM benchmark plane – individual theoretical constraints

Constraints shown below are independent of 2HDM type

2HDM, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$
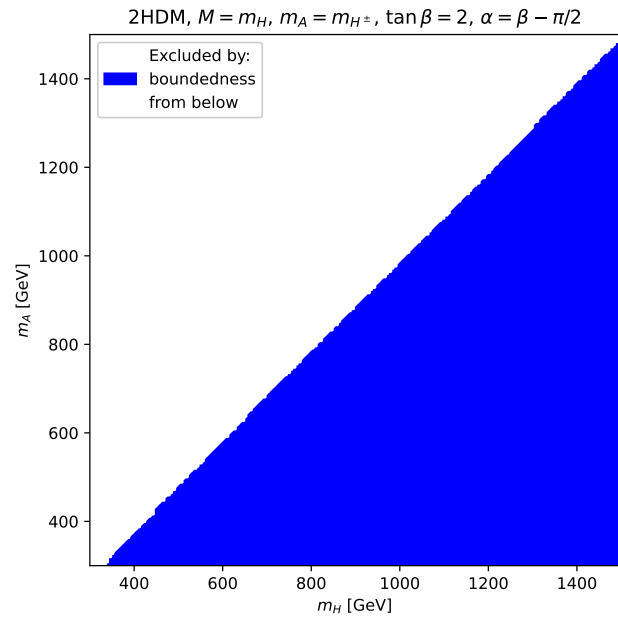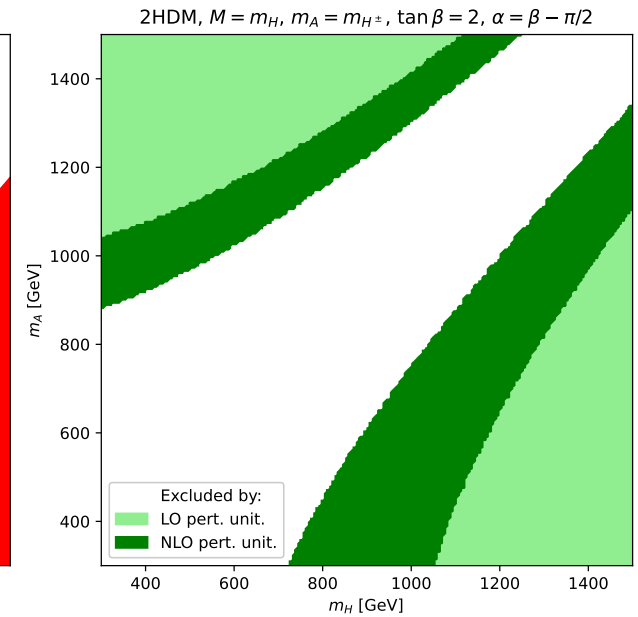
Boundedness from below

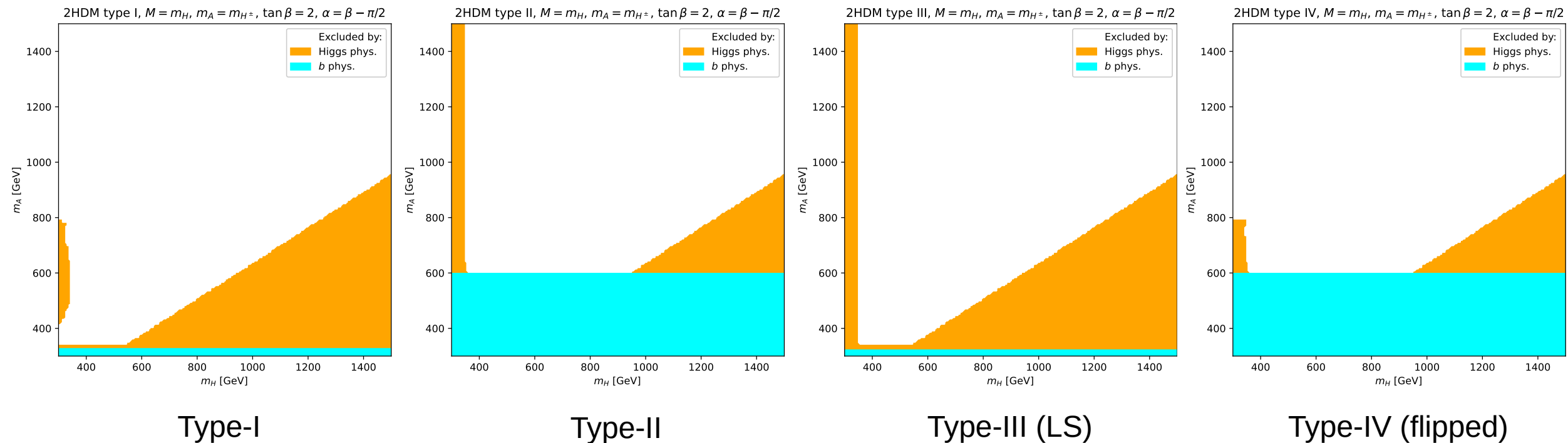EW precision observables computed at 2L
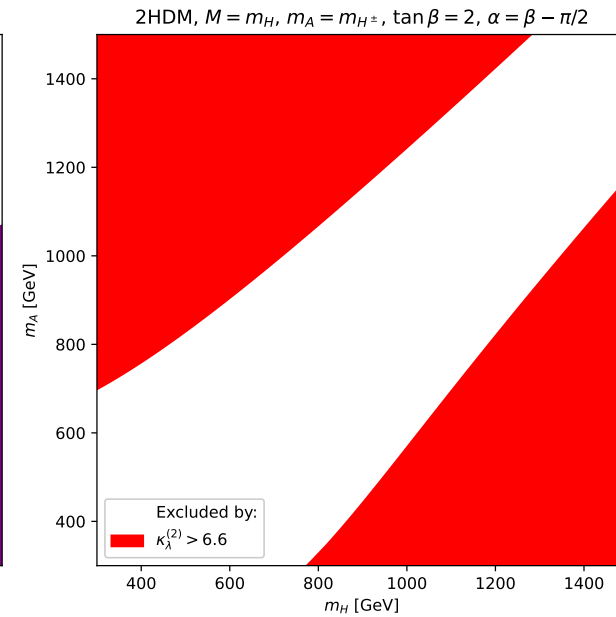
$\kappa_\lambda^{(2)} > 6.6$

Perturbative unitarity at (N)LO

2HDM benchmark plane – experimental constraints

i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])

Type-I

Type-II
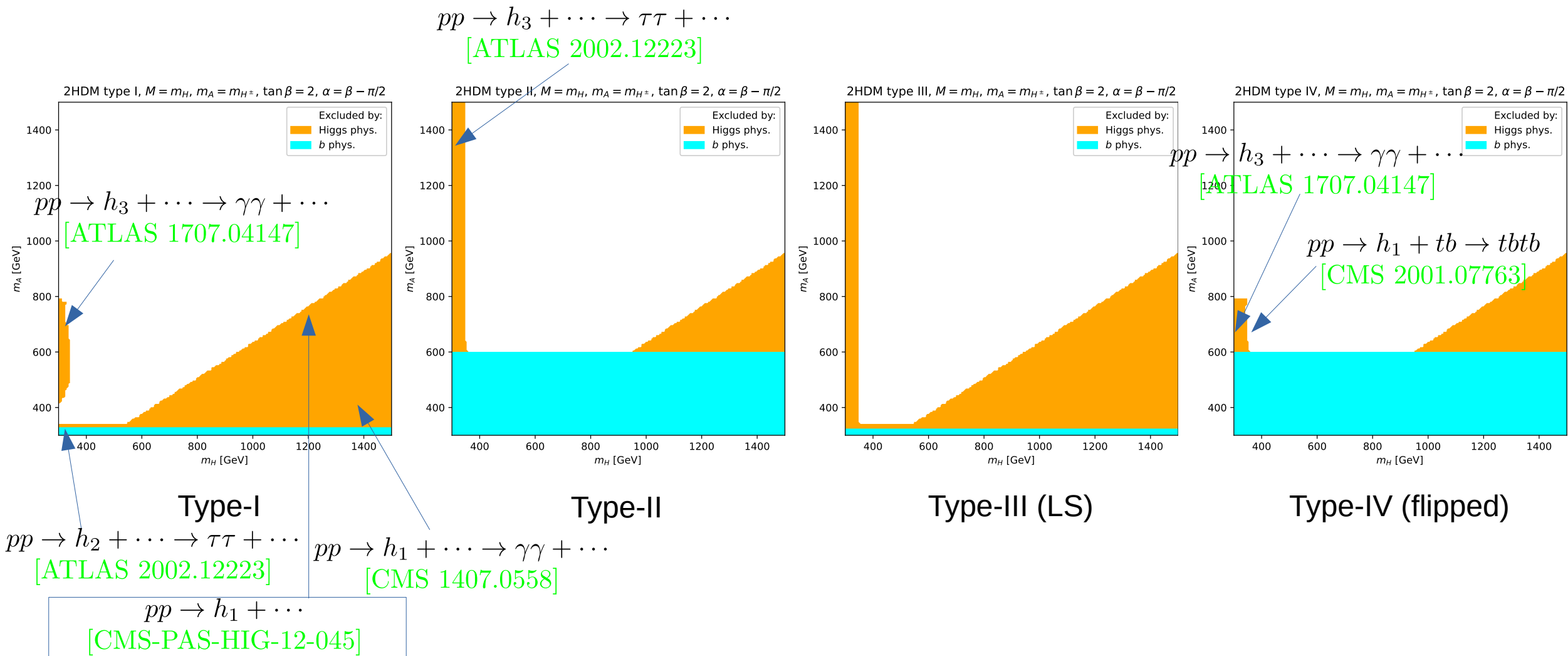
Type-III (LS)

Type-IV (flipped)

2HDM benchmark plane – experimental constraints

i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])

$pp \to h_3 + \cdots \to \tau\tau + \cdots$
[ATLAS 2002.12223]

$pp \to h_3 + \cdots \to \gamma\gamma + \cdots$
[ATLAS 1707.04147]

$pp \to h_3 + \cdots \to \gamma\gamma + \cdots$
[ATLAS 1707.04147]

$pp \to h_1 + tb \to tbtb$
[CMS 2001.07763]

2HDM type I, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

2HDM type II, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

2HDM type III, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

2HDM type IV, $M = m_H$, $m_A = m_{H^\pm}$, $\tan\beta = 2$, $\alpha = \beta - \pi/2$

Excluded by:
Higgs phys.
b phys.

Type-I

Type-II

Type-III (LS)

Type-IV (flipped)

$pp \to h_2 + \cdots \to \tau\tau + \cdots$
[ATLAS 2002.12223]

$pp \to h_1 + \cdots \to \gamma\gamma + \cdots$
[CMS 1407.0558]

$pp \to h_1 + \cdots$
[CMS-PAS-HIG-12-045]

2HDM benchmark plane – results for all types

Type-I   Type-II   Type-III (LS)   Type-IV (flipped)