The Euclidean Adler Function at the edge of the perturbative breakdown

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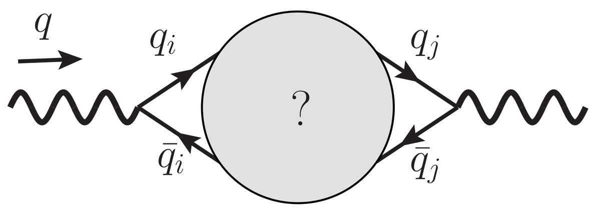
The Euclidean Adler function

$\Pi_{ij}(q) \sim$ 

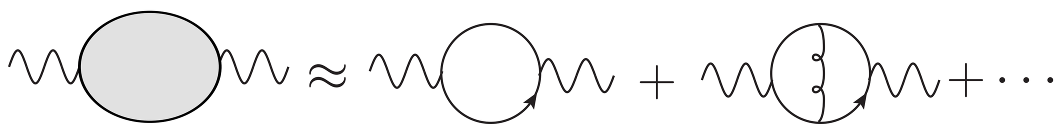
EM $\Pi = \sum Q_i Q_j \Pi_{ij}$

$$\boxed{D_{ij}(Q^2) \sim Q^2 \frac{d\Pi_{ij}(Q^2)}{dQ^2}}$$

$Q^2 \equiv -q^2$, $D(Q^2) = \sum Q_i Q_j D_{ij}(Q^2)$

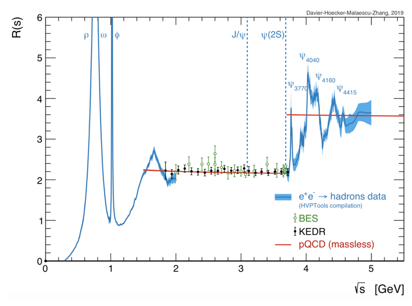
$$\Delta\alpha_{\rm had}(Q^2) - \Delta\alpha_{\rm had}(Q_0^2) = \frac{\alpha}{3\pi}\int_{Q_0^2}^{Q^2} \frac{dQ'^2}{Q'^2} D(Q'^2)\,.$$

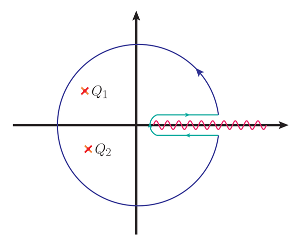
From perturbative QCD



Leading terms of the OPE. Valid at $Q^2 \gg \Lambda^2_{\rm QCD}$

From e^+e^- data





$$D(Q^2) = Q^2 \int_{s_{th}}^{\infty} ds \frac{R(s)}{(s+Q^2)^2}$$

DHMZ compilation

From the lattice

$$\bar{\Pi}_{ij}(Q^2) = \int_0^\infty dt\, G_{ij}(t)\, K(t, Q^2)$$

$$\bar{\Pi}(Q^2) \approx \frac{\sum_{n=1}^3 a_n x^n}{1+\sum_{n=1}^3 b_n x^n} = \frac{0.1094\,(23)\,x + 0.093\,(15)\,x^2 + 0.0039\,(6)\,x^3}{1 + 2.85\,(22)\,x + 1.03\,(19)\,x^2 + 0.0166\,(12)\,x^3}$$

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The perturbative Adler function within EFT approach

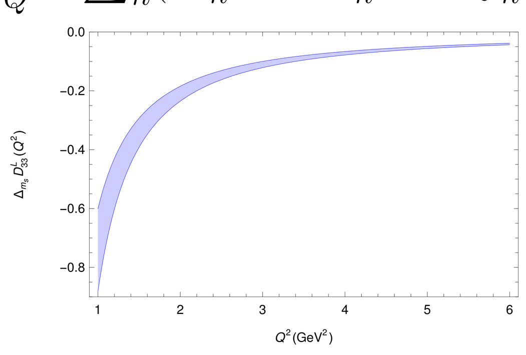
- Light-quark correlators, massless

$$D_{ii}^{L,(0)}(Q^2) \sim \sum K_n \left(\frac{\alpha_s(Q^2)}{\pi}\right)^n$$

			$D_{ii}^{L,(0)}(Q^2)$					
$\alpha_s^{(n_f=5)}(M_Z^2)$	Q	$\alpha_s(Q^2)$	0	1	2	3	4	5
0.115	1.0	0.4227	3	3.4036	3.4927	3.5392	3.5874	3.6238
	1.5	0.3197	3	3.3053	3.3562	3.3764	3.3921	3.4011
	2.0	0.2751	3	3.2627	3.3005	3.3133	3.3219	3.3262
0.120	1.0	0.5277	3	3.5039	3.6427	3.7332	3.8504	3.9606
	1.5	0.3681	3	3.3515	3.4191	3.4498	3.4776	3.4958
	2.0	0.3085	3	3.2946	3.3420	3.3601	3.3738	3.3813

- Light-quark correlators, strange mass

$$\Delta_{m_s} D_{33}^L(Q^2) = -3N_C \frac{m_s^2(Q^2)}{Q^2} \sum_n (2c_n^{L+T} + e_n^{L+T} + f_n^{L+T}) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^n + \mathcal{O}\left(\frac{m_s^4}{Q^4}\right)$$

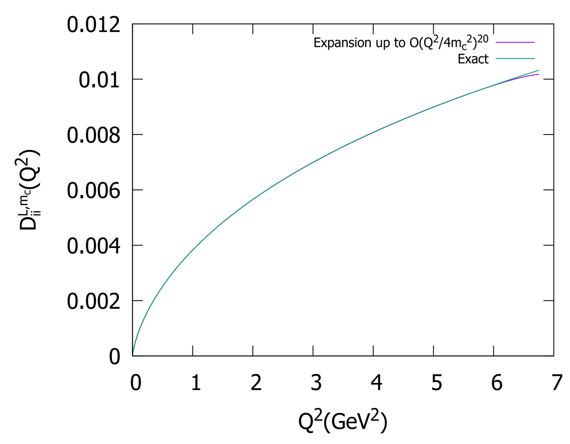


- Light-quark correlators, charm mass

$$D_{ii}^{L,m_c}(Q^2) \sim \int_0^{4m_c^2} ds \frac{\rho_V(s)}{(s+Q^2)^2} + \int_{4m_c^2}^{\infty} ds \frac{\rho_R(s)+\rho_V(s)}{(s+Q^2)^2}$$

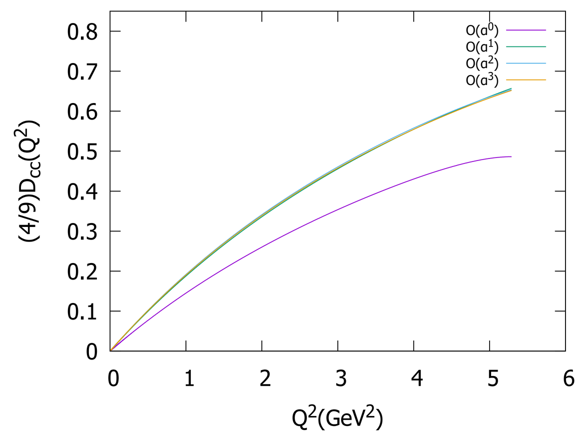
NB, at $Q^2 \gg 4m_c^2$ massless $n_f = 4$ recovered:

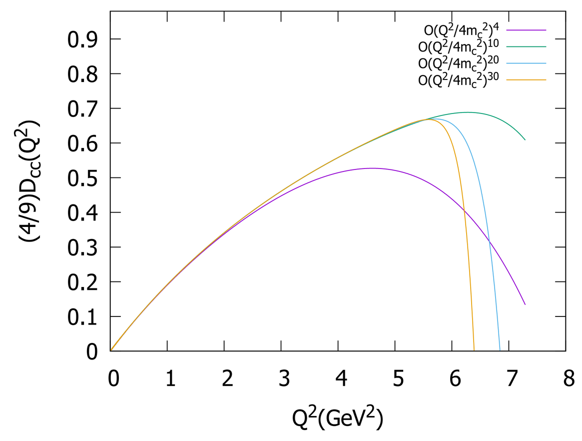
- Divergent logarithm absorbed by $\alpha_s^{n_f=4} - \alpha_s^{n_f=3}$
- constant piece by $K_2^{n_f=4} - K_2^{n_f=3}$



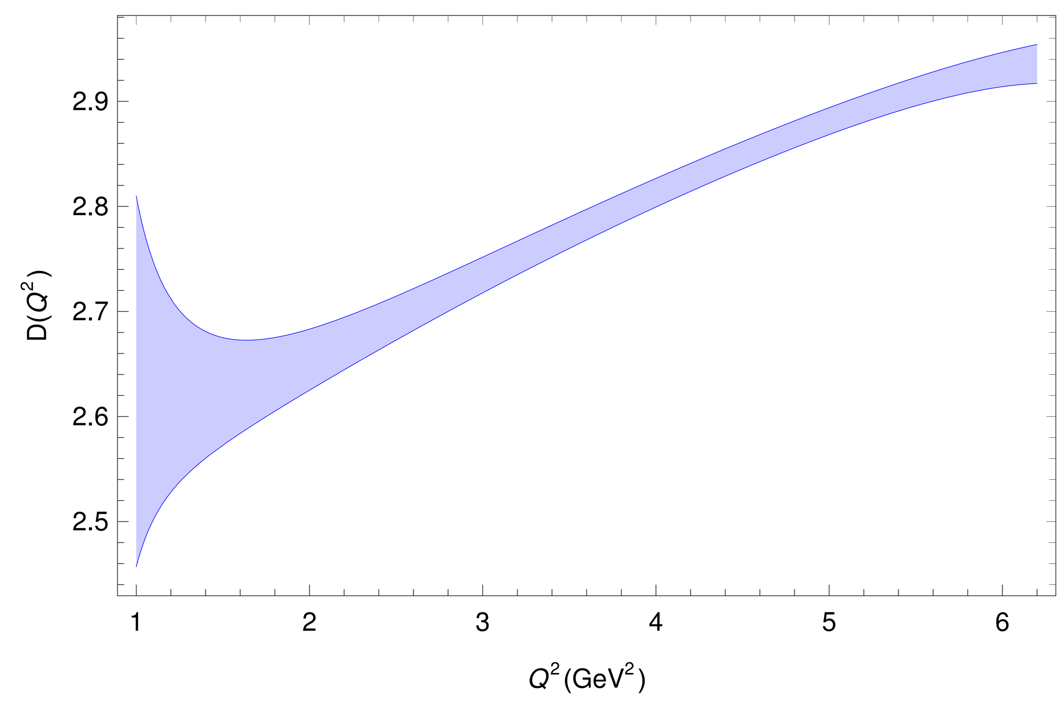
- Heavy correlators

$$D_{ii}(Q^2) = -\frac{9}{4}\sum_j (-1)^j\, j\, \overline{C}_j(\mu) \left(\frac{Q^2}{4\hat{m}_i^2(\mu^2)}\right)^j$$



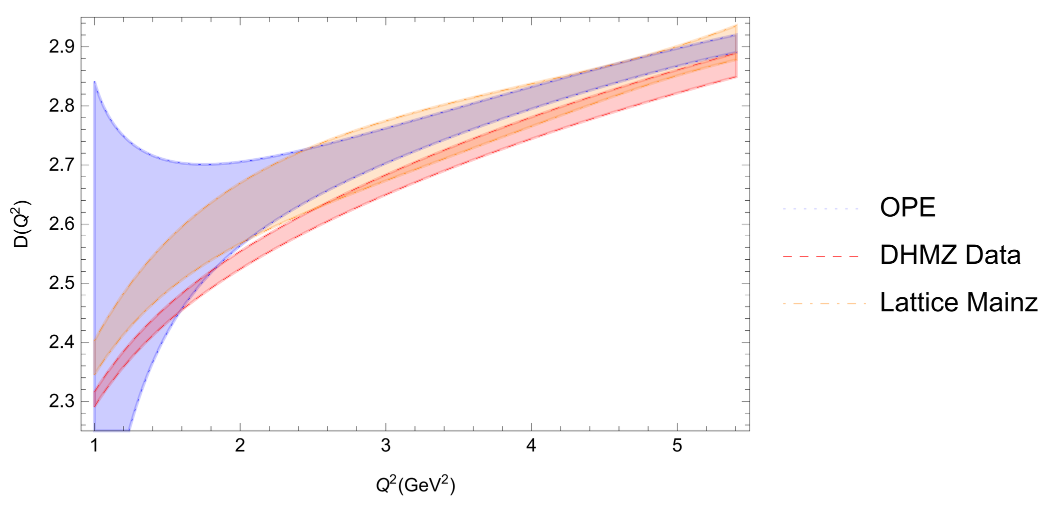


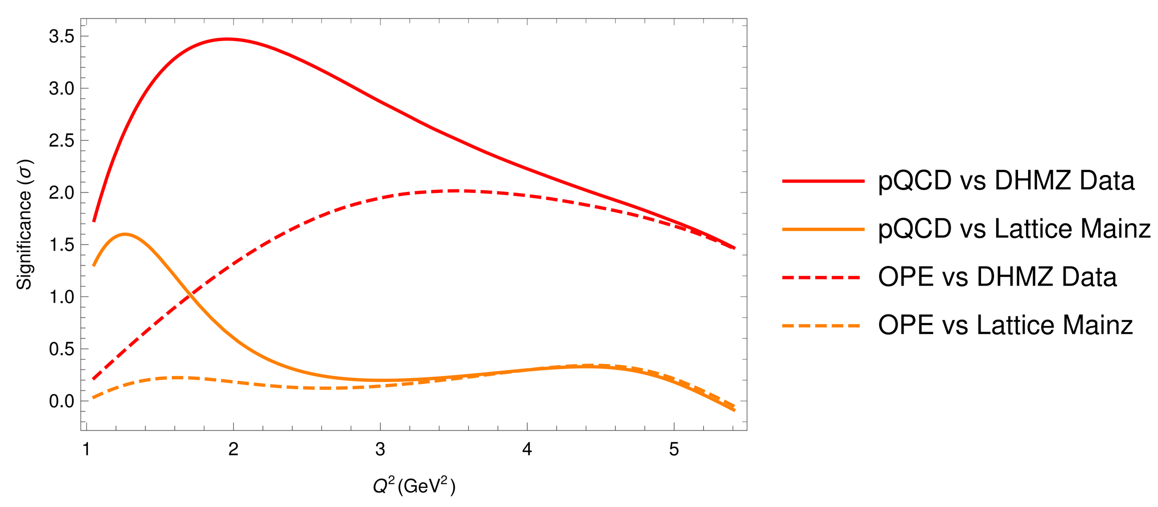
$\alpha_s^{(n_f=5)}(M_Z^2)$	Q^2	$\frac{2}{3}D_{ii}^{L,(0)}$	$\frac{1}{9}\Delta_{m_s}D_{33}^L$	$\frac{2}{3}D_{ii}^{L,m_c}$	$\frac{4}{9}D_{cc}$	$\frac{4}{9}D_{cc}^s$	$\frac{1}{9}D_{bb}$	$\Delta D_{\rm QED}$	D
0.115	3	2.2395(77)	−0.0123(12)	0.0039(10)	0.4484(21)(24)	0.0000(00)	0.0130(01)	0.0012(04)	2.694(09)
	4	2.2175(52)	−0.0080(07)	0.0045(12)	0.5435(24)(26)	0.0000(01)	0.0171(02)	0.0012(04)	2.776(07)
	5	2.2033(39)	−0.0058(04)	0.0050(13)	0.6197(31)(26)	0.0001(03)	0.0212(02)	0.0012(04)	2.845(06)
0.120	3	2.2866(156)	−0.0141(17)	0.0053(21)	0.4649(47)(24)	0.0000(01)	0.0132(01)	0.0012(04)	2.757(17)
	4	2.2542(98)	−0.0089(09)	0.0062(24)	0.5629(53)(26)	0.0001(02)	0.0174(02)	0.0012(04)	2.833(12)
	5	2.2343(70)	−0.0063(06)	0.0069(26)	0.6429(65)(27)	0.0002(05)	0.0215(02)	0.0012(04)	2.900(11)
0.1184(8)	3	2.2699(124)	−0.0134(15)	0.0048(17)	0.4591(36)(24)	0.0000(01)	0.0131(01)	0.0012(04)	2.735(17)
	4	2.2414(79)	−0.0086(08)	0.0055(19)	0.5561(41)(26)	0.0001(02)	0.0173(02)	0.0012(04)	2.813(14)
	5	2.2236(58)	−0.0061(05)	0.0062(20)	0.6348(51)(27)	0.0002(04)	0.0214(02)	0.0012(04)	2.881(13)

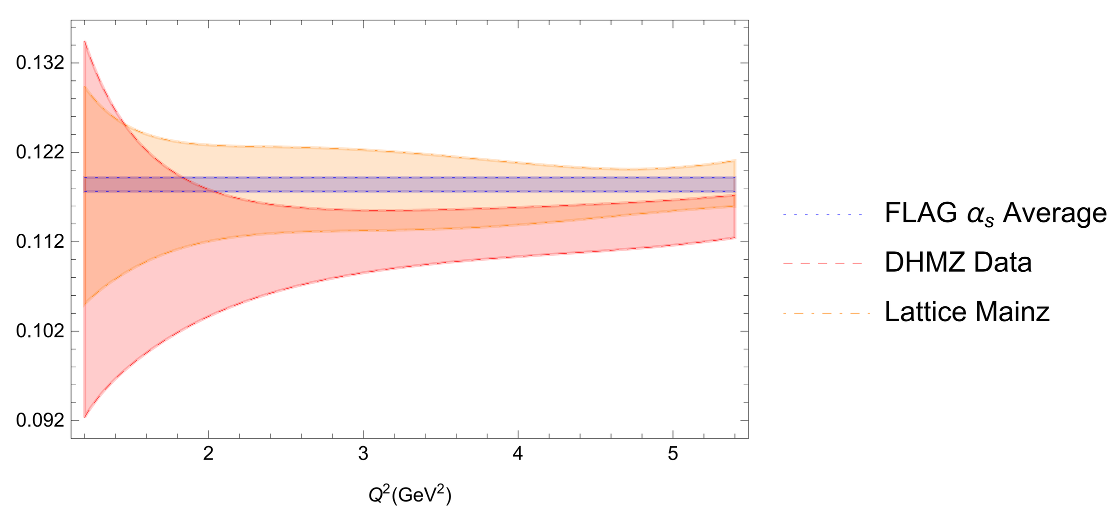


Power corrections (from OPE): $\delta D^{\rm NP}_{\rm em} \approx \frac{4\pi^2}{3Q^4}\left\{\left(1+\frac{7}{6}a_s\right)\langle a_s GG\rangle + 4\left[1+\frac{a_s}{3}+\left(\frac{27}{8}+4\,\zeta_3\right)a_s^2\right] m_s\,\langle\bar{s}s\rangle\right]\right\} + 24\pi^2\frac{\mathcal{O}_{6,V}}{Q^6}$

Results







Discussion and Conclusions

- Three precise descriptions of $D(Q^2)$ at $Q^2 \approx 4\,{\rm GeV}^2$.
- In line with other observables (but with less significance), e^+e^- data below lattice.
- e^+e^- data-based Adler function is $1.5-2.0\sigma$ below OPE prediction.
- Different strategies to extract α_s are studied. The possibility of testing the RGE is explored.

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