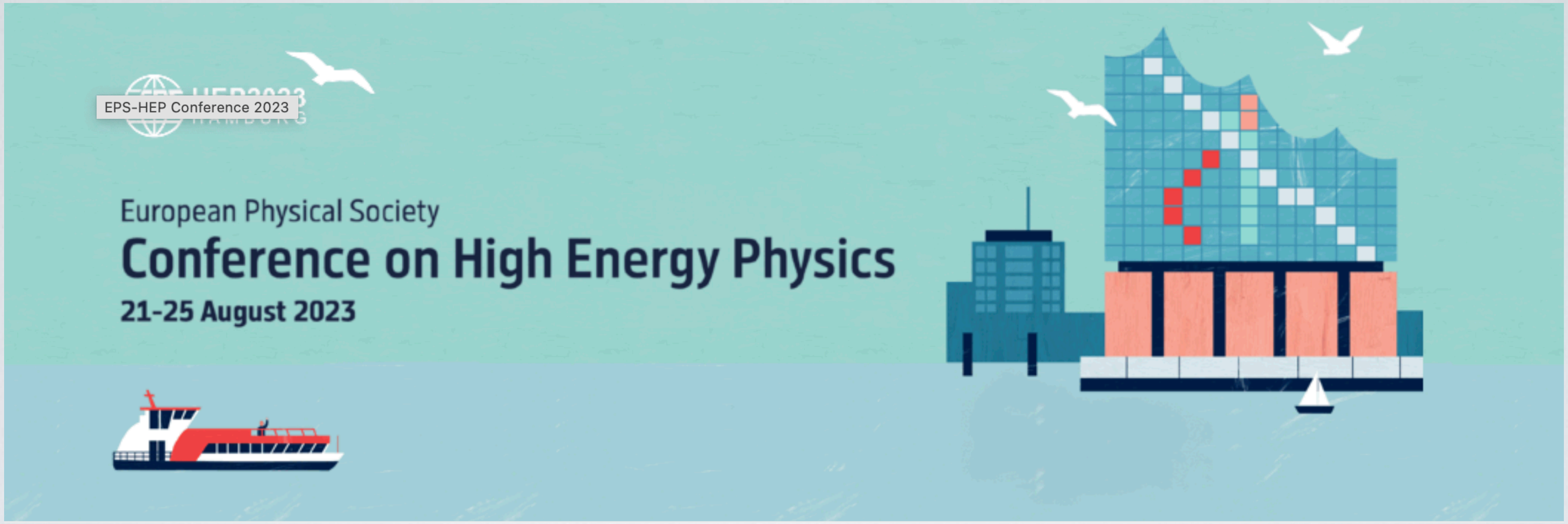

QUANTUM CORRECTIONS TO EFFECTIVE POTENTIAL IN ARBITRARY SCALAR THEORIES

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Effective Potential in Scalar Theory in D=4

Generating functional for Green functions

$$Z(J) = \int \mathcal{D}\phi \ \exp\left(i \int d^4x \ \mathcal{L}(\phi, d\phi) + J\phi \right)$$

$$W(J) = -i \log Z(J)$$

IPI generating functional

Effective action

$$\Gamma(\phi) = W(J) - \int d^4x J(x)\phi(x)$$

Legendre transformation

$$e^{i\Gamma(\hat{\phi})} = \int \mathcal{D}\phi \ e^{i(S[\hat{\phi}+\phi]-\phi S'[\hat{\phi}])}$$

Shifted Classical action

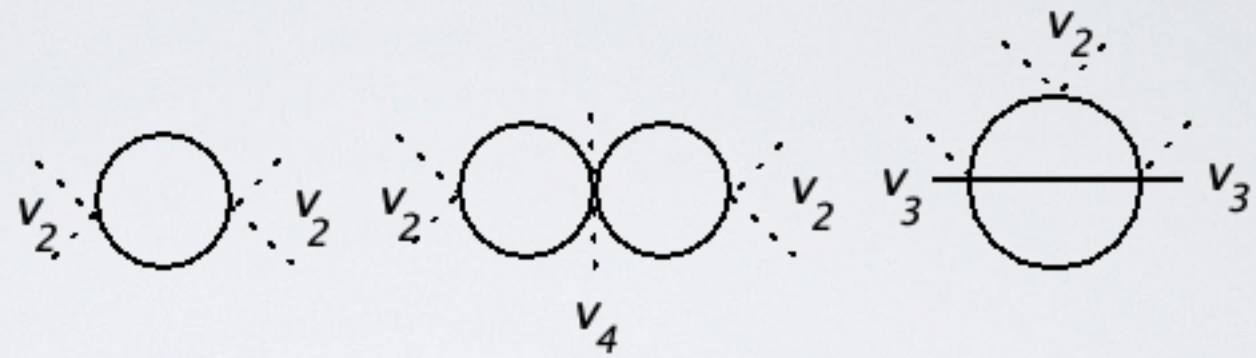
$$S[\hat{\phi} + \phi] = S[\hat{\phi}] + \phi S'[\hat{\phi}] + \frac{1}{2}\phi^2 S''[\hat{\phi}] + \frac{1}{3!}\phi^3 S'''[\hat{\phi}] + \ldots$$

Classical external field ($\hat{\phi}$) — Field dependent mass ($S''[\hat{\phi}]$) — Interaction vertex ($S'''[\hat{\phi}]$)

Effective Potential in Scalar Theory in D=4

General scalar field theory in D=4 $$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - gV_0(\phi)$$

Vacuum diagrams



$$v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2} \qquad v_n \equiv d^n V_0 / d\phi^n$$

One loop $$v_2^2 \frac{1}{\epsilon} \qquad \phi^4 \qquad v_2 \sim \phi^2 \qquad \phi^4 \frac{1}{\epsilon}$$

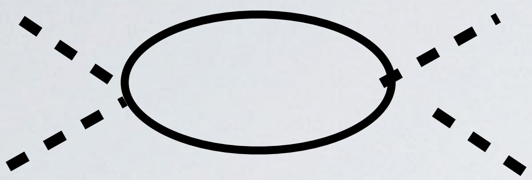
$$\phi^6 \qquad v_2 \sim \phi^4 \qquad \phi^8 \frac{1}{\epsilon}$$

Two loops

$$(v_2^2 v_4 + v_3^2 v_2)/\epsilon^2 \qquad \phi^4 \qquad v_4 \sim 1,\ v_3 \sim \phi \qquad \phi^4/\epsilon^2$$

$$\phi^6 \qquad v_4 \sim \phi^2,\ v_3 \sim \phi^3 \qquad \phi^{10}/\epsilon^2$$

Divergences and Log ϕ behaviour



$$Diag \sim \frac{1}{\epsilon} v_2^2 (\frac{\mu^2}{m^2})^\epsilon \to v_2^2 (\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2}), \quad m^2 = g v_2^2$$

$$\Delta V_1 = \frac{g^2}{16\pi^2} v_2^2 \log \frac{g v_2}{\mu^2}$$

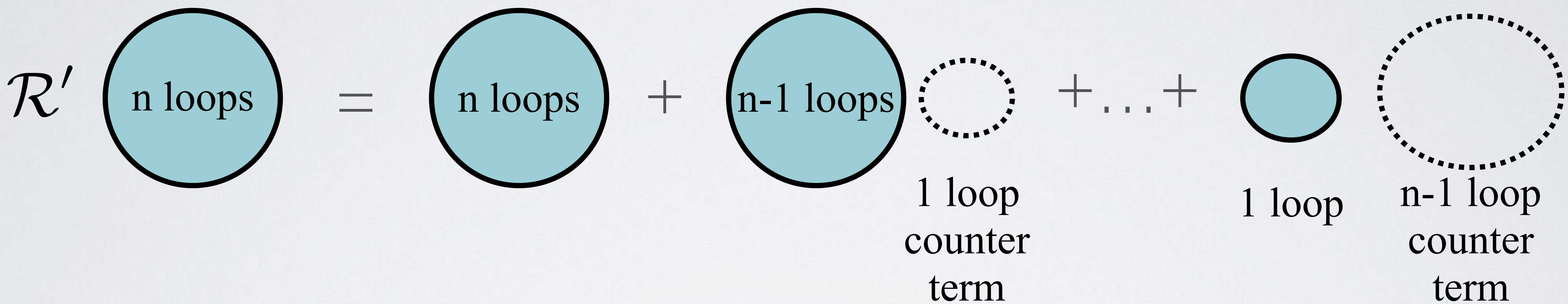
ϕ^4
$$\Delta V_1 = \frac{g^2}{16\pi^2} \phi^4 \log \frac{g\phi^2}{\mu^2}$$

ϕ^6
$$\Delta V_1 = \frac{g^2}{16\pi^2} \phi^8 \log \frac{g\phi^4}{\mu^2}$$

BPHZ R-operation

$$\mathcal{R}'G_n = \frac{A_n^{(n)}(\mu^2)^{n\epsilon}}{\epsilon^n} + \frac{A_{n-1}^{(n)}(\mu^2)^{(n-1)\epsilon}}{\epsilon^n} + \ldots + \frac{A_1^{(n)}(\mu^2)^{\epsilon}}{\epsilon^n}$$

$+$ lower pole terms



$A_k^{(n)}(\mu^2)^{k\epsilon}$ terms appear after subtraction of (n-k) loop counter terms

<u>Statement:</u> $R'G_n$ is local, i.e. terms like $\log^k \mu^2/\epsilon^m$ should cancel for any k and m

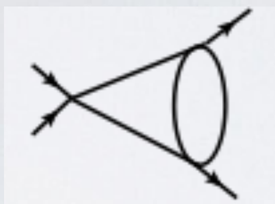
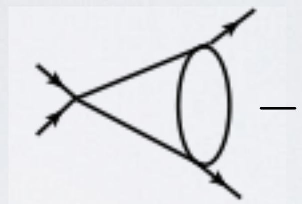
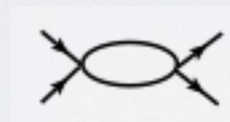
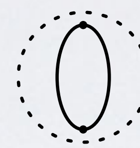
<u>Consequence:</u> $A_n^{(n)} = (-1)^{n+1}\frac{A_1^{(n)}}{n}$

The leading divergences are governed by 1 loop diagrams!

Two loop example

ϕ^4_4

 $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon}$

$\mathcal{R}'$  $=$  $-$   $= \left(\frac{A_2^{(2)}}{\epsilon^2} + \frac{A_1^{(2)}}{\epsilon} \right) (\frac{\mu^2}{s})^{2\epsilon} - \frac{A_1^{(1)}}{\epsilon} (\frac{\mu^2}{s})^{\epsilon} \frac{A_1^{(1)}}{\epsilon}$

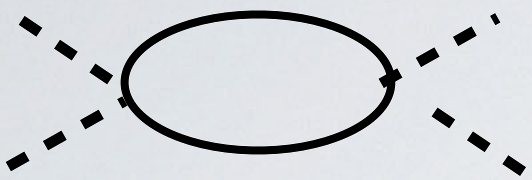
$$= \frac{A_2^{(2)}}{\epsilon^2} - \frac{(A_1^{(1)})^2}{\epsilon^2} + 2\frac{A_2^{(2)}}{\epsilon} \log(\mu^2/s) - \frac{(A_1^{(1)})^2}{\epsilon} \log(\mu^2/s) \quad = -\frac{1}{2}\frac{(A_1^{(1)})^2}{\epsilon^2} + \ldots$$

non-local terms to be cancelled

Leading divergence is given by the one-loop term $A_2^{(2)} = \frac{1}{2}(A_1^{(1)})^2$

- These statements are universal and are valid in non-renormalizable theories as well.
- The only difference is that the counter term $A_1^{(1)}$ depends on kinematics and has to be integrated through the remaining one-loop graph.
- As a result $A_2^{(2)}$ is not the square of $A_1^{(1)}$ anymore but is the integrated square .
- This last statement is the general feature of any QFT irrespective of renormalizability

Divergences and Log ϕ behaviour



$$Diag \sim \frac{1}{\epsilon} v_2^2 \left(\frac{\mu^2}{m^2}\right)^\epsilon \to v_2^2 \left(\frac{1}{\epsilon} - \log\frac{m^2}{\mu^2}\right), \quad m^2 = g v_2^2$$

The leading divergences ⟷ The leading logs

- In non-renormalizable theories divergences cannot be absorbed into the renormalization of the couplings and fields.

- If they are subtracted some way one is left with infinite arbitrariness.

- Coefficients of the leading divergences (logs) do not depend on this arbitrariness !

The aim is to calculate the leading divergences ~ $\frac{1}{\epsilon^n}$ in n-th order of PT

Recurrence relations for the leading poles

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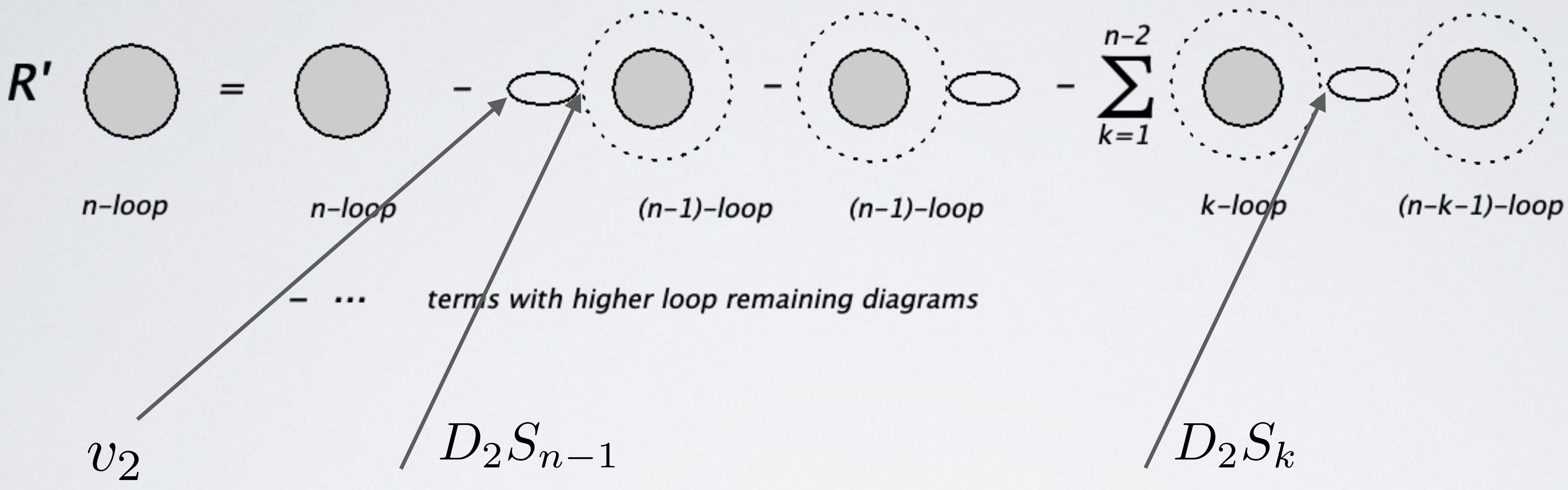
Action of R'-operation on divergent diagram



$$nS_n = \frac{1}{2}v_2 D_2 S_{n-1} + \frac{1}{4}\sum_{k=1}^{n-2} D_2 S_k D_2 S_{n-1-k}, \quad n \geq 2 \qquad S_1 = \frac{1}{4}v_2^2$$

$$nS_n = \frac{1}{4}\sum_{k=0}^{n-1} D_2 S_k D_2 S_{n-1-k}, \quad n \geq 1, \quad S_0 = V_0$$

RG pole equation for arbitrary potential

$$\Sigma(z,\phi) = \sum_{n=0}^{\infty}(-z)^n S_n(\phi)$$

$$z = \frac{g}{\epsilon}$$

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RG pole equation

$$\frac{d\Sigma}{dz} = -\frac{1}{4}(D_2\Sigma)^2 \qquad \Sigma(0,\phi) = V_0(\phi)$$

This a non-linear partial differential equation!

Effective potential

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2}. \qquad v_2(\phi) \equiv \frac{d^2 V_0(\phi)}{d\phi^2}$$

Example I: Power like Potential

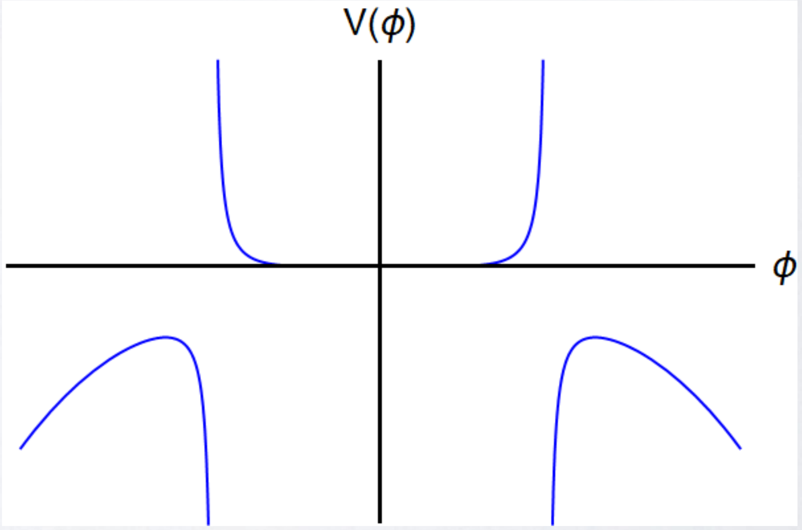
$$gV_0(\phi) = g\frac{\phi^p}{p!} \qquad y = g\phi^{p-4} \qquad \Sigma(z,\phi) = \frac{\phi^p}{p!} f(z\phi^{p-4})$$

$$f'(y) = -\frac{1}{4p!}\left[p(p-1)f(y) + (p-4)(3p-5)yf'(y) + (p-4)^2 y^2 f''(y)\right]^2$$

$$f(0) = 1, f'(0) = -\frac{1}{4}\frac{p(p-1)}{(p-2)!}$$

p=4

$$f'(y) = -\frac{3}{2}f(y)^2 \qquad f(y) = \frac{1}{1+\frac{3}{2}y}$$



$$V_{eff}(\phi) = \frac{g\phi^4/4!}{1 - \frac{3}{2}\frac{g}{16\pi^2}\log\left(\frac{g\phi^2}{2\mu^2}\right)}.$$

Example I: Power like Potential

p>4

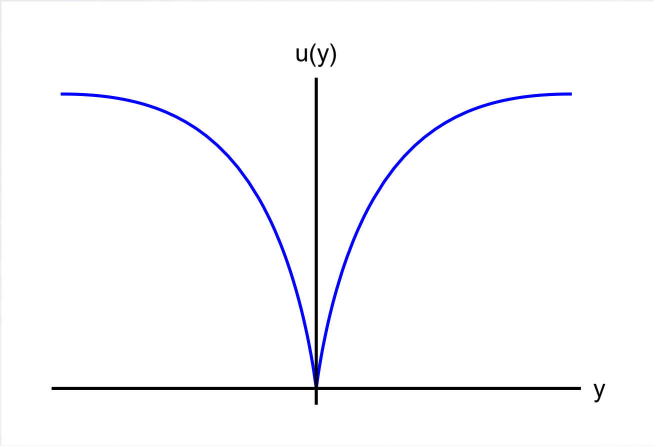
$$gV_0(\phi) = g\frac{\phi^p}{p!}$$

$$f(y) = u(y)/y$$

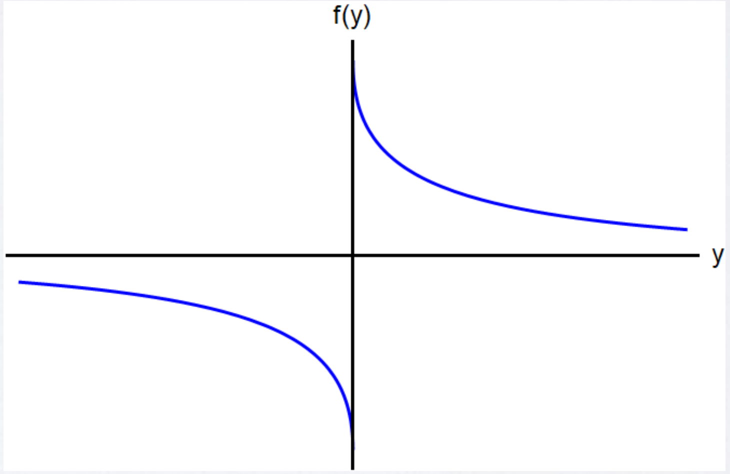
$$yu'(y) - u(y) = -\frac{1}{4p!}[12u(y) + (p-4)(p+3)yu'(y) + (p-4)^2 y^2 u''(y)]^2$$

$$u(\pm 0) = 0, u'(\pm 0) = \pm 1$$

Discontinuity at y=0



$$y = 0$$

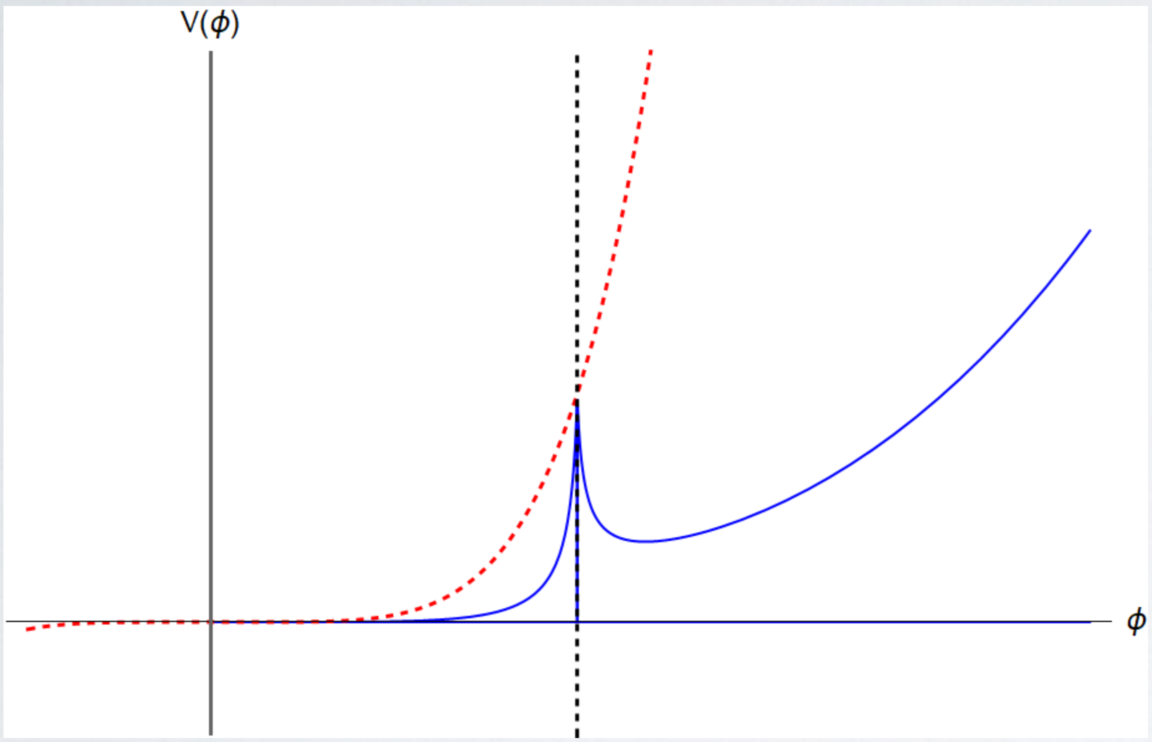


$$y = 0$$

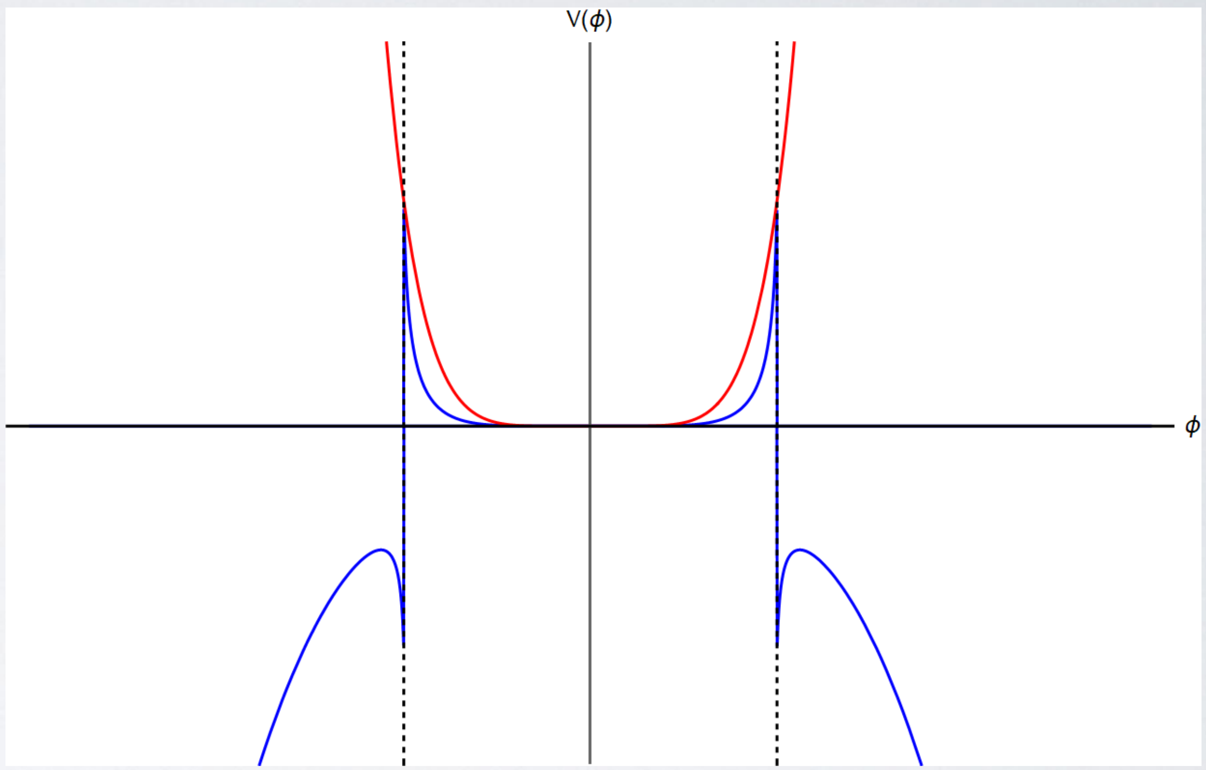
$$y \to -\frac{g}{16\pi^2}\phi^{p-4}\log\frac{g\phi^{p-2}}{\mu^2/(p-2)!}$$

Example I: Power like Potential

p=5



p=6



- Finite gap instead of an infinite barrier as for p=4
- Metastability of the quantum state
- No new minima

Example III: Inflation Potential

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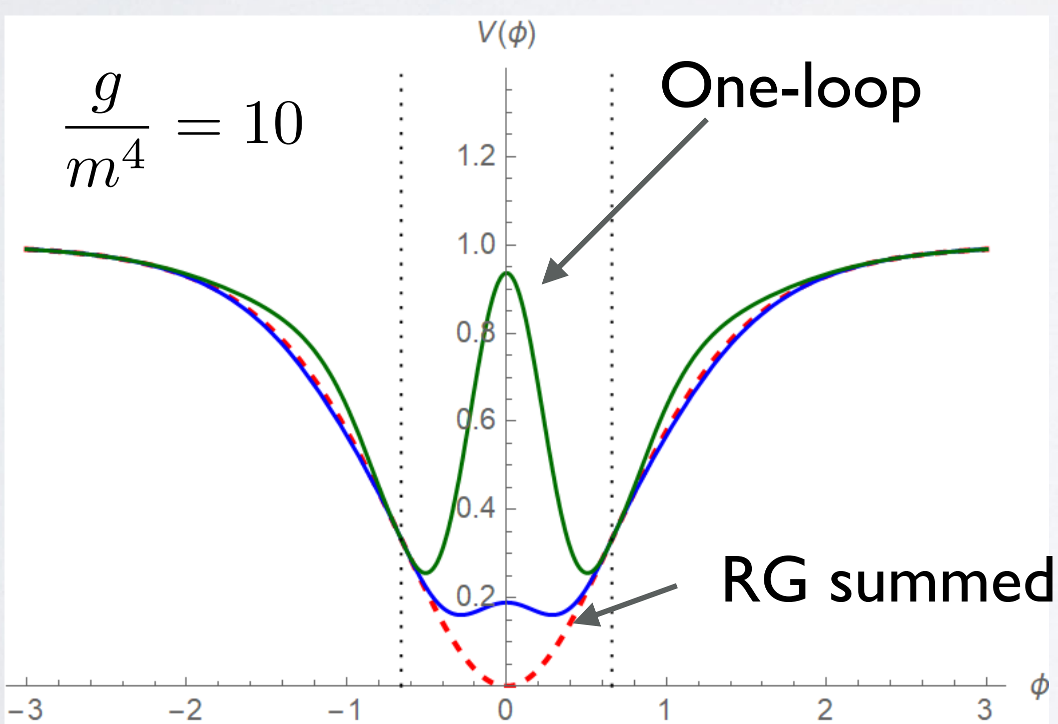
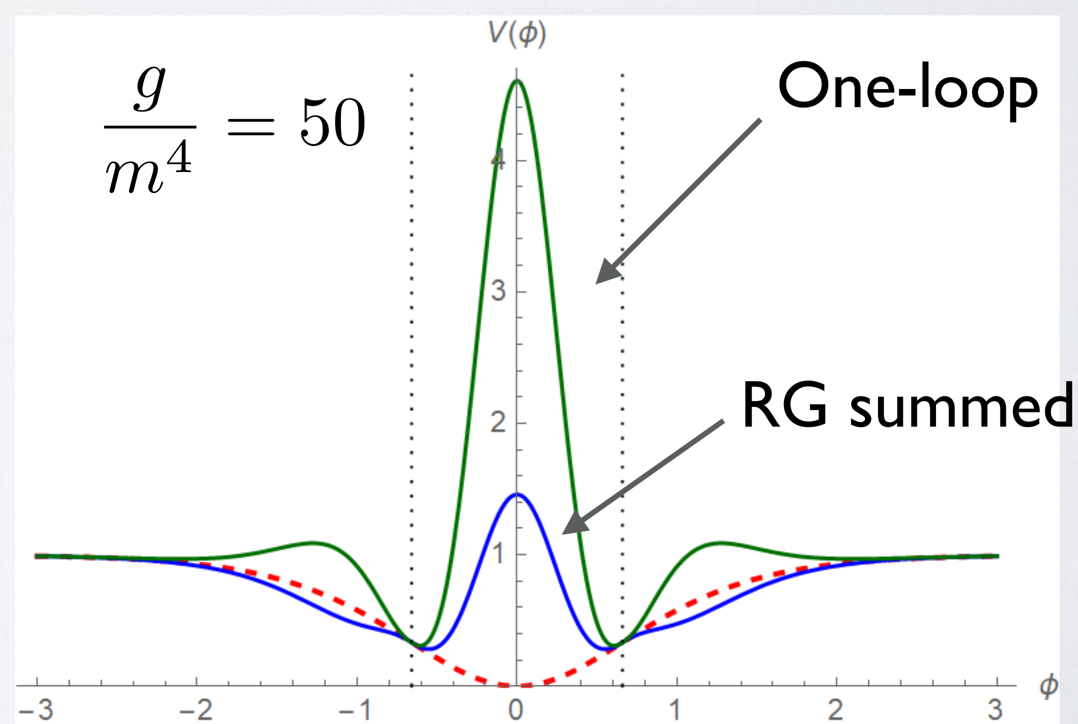
ArXiv: 2308.03872

$$gV_0 = g\tanh^2(\phi/m)$$

$$\frac{d\Sigma}{dz} = -\frac{1}{4}\left(\frac{d^2\Sigma}{d\phi^2}\right)^2 \qquad \Sigma\left(\frac{z}{m^4}, \frac{\phi}{m}\right) \qquad \Sigma|_{\phi\to\infty} \to 1 \quad \Sigma'|_{\phi\to\infty} \to 0 \qquad z = \frac{g}{\epsilon}$$

$$V_{eff}(g,\phi) = g\Sigma(z,\phi)|_{z\to -\frac{g}{16\pi^2}\log g v_2/\mu^2}. \qquad v_2(\phi) \equiv \frac{d^2V_0(\phi)}{d\phi^2}$$





- Peak at the origin
- Additional minima

Conclusion on Effective potential

- The effective potential in the LL approximation obeys the RG master equation which is a partial non-linear differential equation
- In some cases this equation is simplified to the ordinary differential one and can be solved at least numerically. I
- In all the cases that we studied the obtained ordinary differential equations obey the solution with a discontinuity.
- The effective potential has a metastable minima at the origin and no other minima exists.

- The main message is that under certain assumptions while studying the CW mechanism one may not be restricted by the renormalizable potentials but consider much wider possibilities. We provided the method of such analysis.

- This might be useful for cosmological applications where they are usually not limited by renormalizability since gravity makes it non-renormalizable anyway.