Solitons as ground states in supergravity theories

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Outline

1 Introduction

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2 **The model**

- Explicit solutions
- Phase structure
- BPS configurations

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3 Conclusions

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- originally used as “bounce solutions” to discuss the possible instability of the pure Kaluza-Klein vacuum ground state;
- generalizations of these soliton solutions have been also considered in the analysis of the semiclassical stability of non-susy AdS gravity;
- soliton configurations can turn out to be the lowest energy solution with chosen boundary conditions, leading to a new kind of positive energy conjecture;

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- BPS configurations preserving some of the supercharges can be obtained analysing the explicit form of the Killing spinors equations.

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4. We consider an explicit solutions in the T^3 model, the latter resulting in a single dilaton truncations of the maximal $SO(8)$ gauged supergravity in $D = 4$.

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We restrict to purely magnetic solutions. The action has the explicit form:

$$\mathscr{S} = \frac{1}{8\pi G} \int d^4x \, \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} (\partial\phi)^2 + \frac{3}{L^2} \cosh\left(\sqrt{\frac{2}{3}}\phi\right) - \frac{1}{4} e^{3\sqrt{\frac{2}{3}}\phi} \left(F^1\right)^2 - \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\phi} \left(F^2\right)^2 \right).$$

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- in the model we consider there are two Wilson lines,

$$\Phi^1_M = \int F^1, \qquad \Phi^2_M = \int F^2,$$

and there is a one-parameter family of values of the Wilson lines which give supersymmetric solitons;

- the explicit solution has the schematic form

$$\phi = \pm \ell^{-1} \ln(x)\,, \qquad F^{\Lambda}_{\mu\nu}(x, \Gamma^{\Lambda})\,,$$

$$ds^2 = \Upsilon(x) \left(L^2\, dt^2 - \frac{\eta^2}{f(x)}\, dx^2 - f(x)\, d\varphi^2 - L^2\, dz^2 \right)\,;$$

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- Regularity of the metric at $x = x_0$ requires $\varphi \in [0,\Delta]$ where

$$\Delta^{-1} = \left|\frac{1}{4\pi\eta}\frac{df}{dx}\right|_{x=x_0}\,.$$

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- After a suitable change of coordinate $x = x(r)$, the soliton energy parameter μ can be then read-off from the asymptotic expansion of the metric:

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  - fixed fluxes, holding fixed ψ_1, ψ_2 $\Rightarrow$ 0 to 2 sols;
  - fixed charges, holding fixed Q_1, Q_2 $\Rightarrow$ 0 to 4 sols.

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- the above formulae are found imposing the vanishing of SUSY variations (Killing spinor equations);
- for fixed charge boundary conditions there are 2 distinct susy soliton configurations (degeneracy of supersymmetric solutions);
- for the same fixed charge boundary conditions, surprisingly a family of non-susy solutions of lower energy and free energy than the supersymmetric ones can be found.

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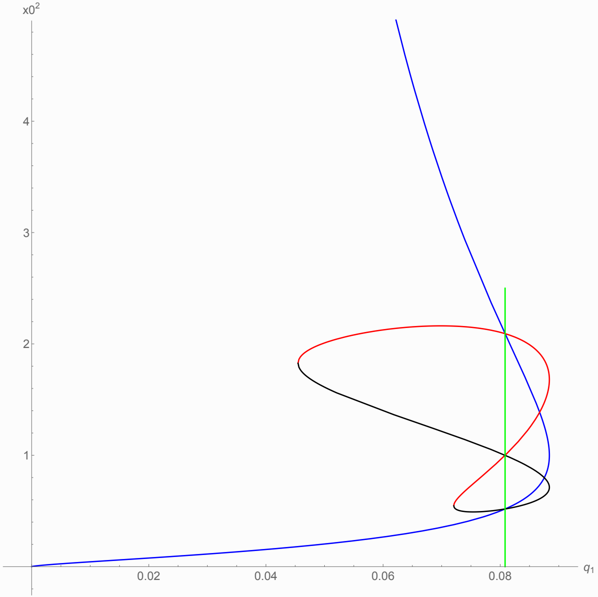

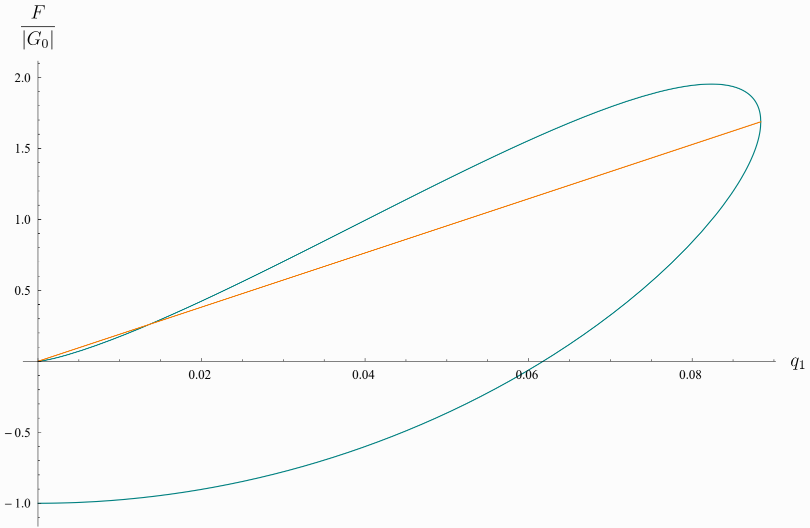
$\frac{F}{|G_0|}$
2.0
1.5
1.0
0.5
−0.5
−1.0
0.02
0.04
0.06
0.08
q_1

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Conclusions Distinctive features

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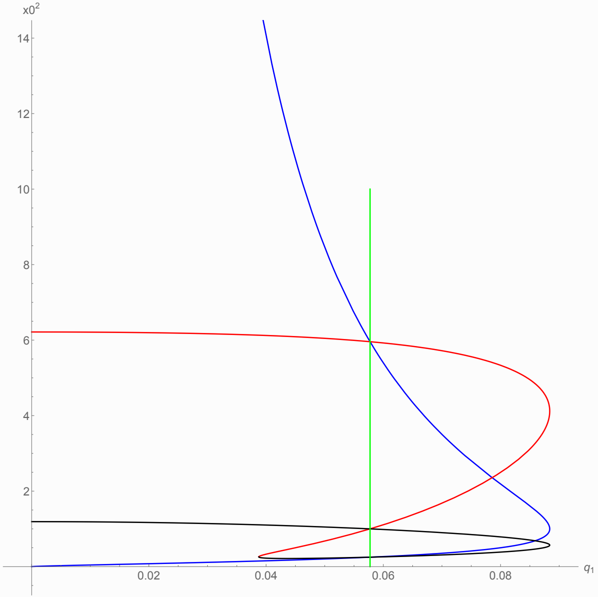
5. One branch of susy solutions has higher energy than a non-susy one with the same boundary conditions.

Thank you for listening!


x0²
14
12
10
8
6
4
2
0.02
0.04
0.06
0.08
q₁