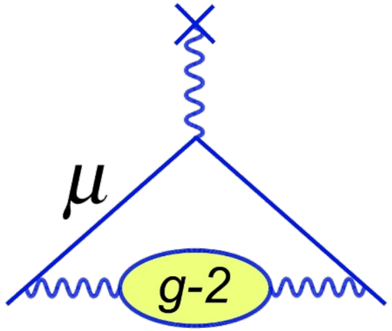
Beam Dynamics Correction to the Anomalous Spin Precession Frequency in the Muon $g - 2$ experiment at Fermilab

On Kim
On behalf of the Muon $g - 2$ Collaboration

EPS-HEP 2023
2023 Aug. 21st



Introduction

- Measurement of ω_a was covered in detail in Sean's talk.

Measurement of the muon anomalous precession frequency ω_a in the Fermilab $g-2$ experiment *Sean Foster*
Hörsaal C, Historic main building 16:45 - 17:02

- Magnetic field measurement was covered in detail in Saskia's talk.

Measurement of the precision magnetic field in the Fermilab Muon g–2 experiment *Saskia Charity*
Hörsaal C, Historic main building 17:02 - 17:19

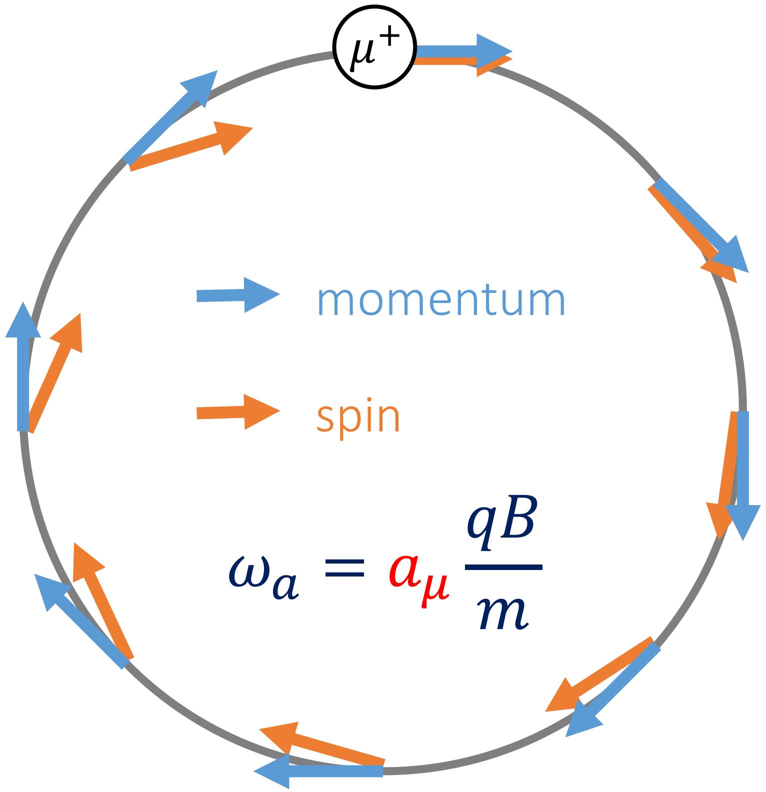
- Summary of the new result will be given by Graziano tomorrow (plenary).

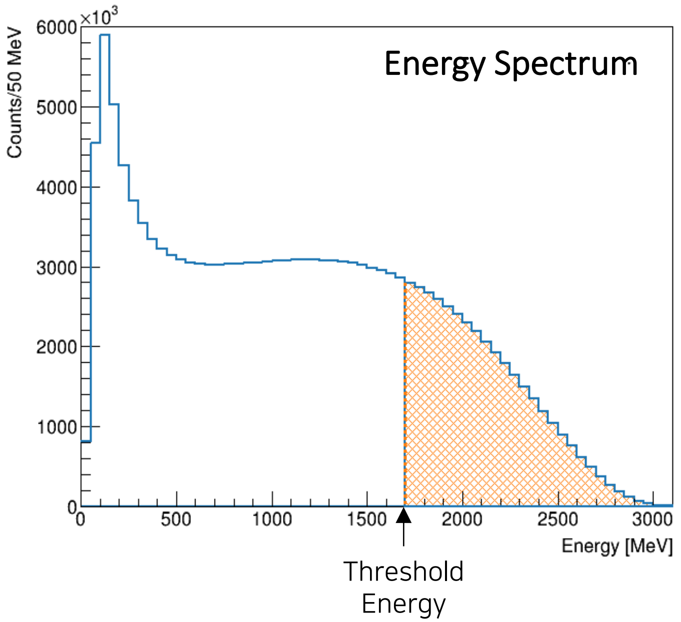
News on muon g-2 *Graziano Venanzoni*
Audimax, Universität Hamburg 15:15 - 15:30

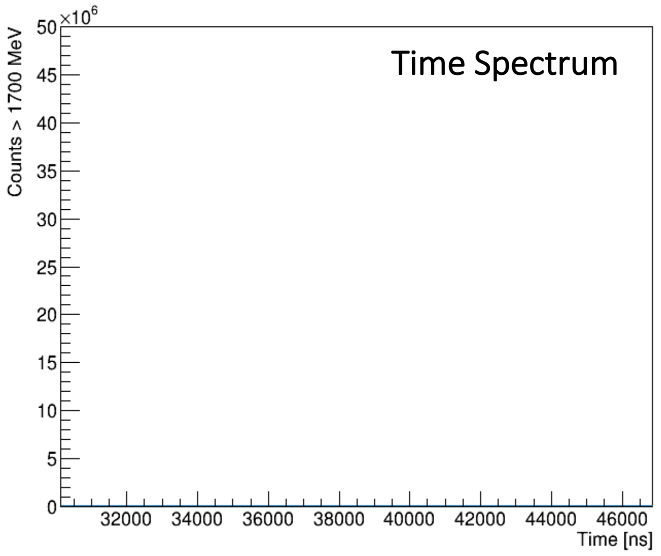
- I am going to talk about the beam dynamics correction to ω_a.

Quick Recap: ω_a measurement from Wiggle Plot

- Self-analyzing muon decay: Parity-violating weak decay causes the high energy decay positrons are preferred to be emitted along the muon spin direction.
- Detected e^+ time spectrum wiggles at the spin precession frequency ω_a.



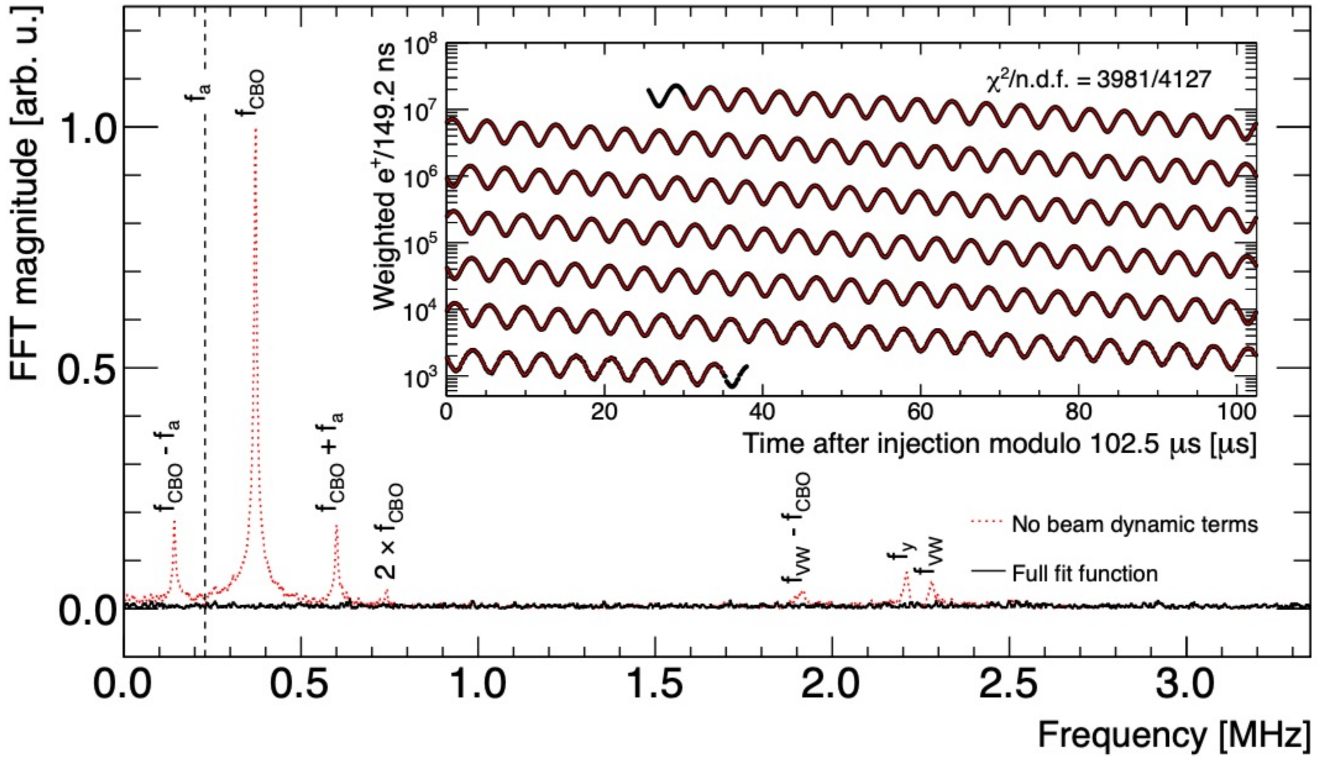




Quick Recap: ω_a measurement from Wiggle Plot

- Fitting the wiggle plot to extract ω_a^m.

$$N(t) = N_0 \eta_N(t) e^{-t/\gamma\tau_\mu} \times \{1 + A\eta_A(t)\cos[\omega_a^m t + \varphi_0 + \eta_\phi(t)]\}$$



Beam Dynamics Correction to ω_a^m

- Is measured ω_a^m precisely the spin precession frequency of the muons? In practice, NO!

$$\omega_a = \omega_a^m \left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)$$

Correction to ω_a

E-field effect to off-design-momentum particles.
Coupled precession effect from vertical motions.

Correction to $\phi = \phi(t)$

Spin precession phase changes over the muon storage due to acceptance effect, differential decay and momentum-dependent losses.

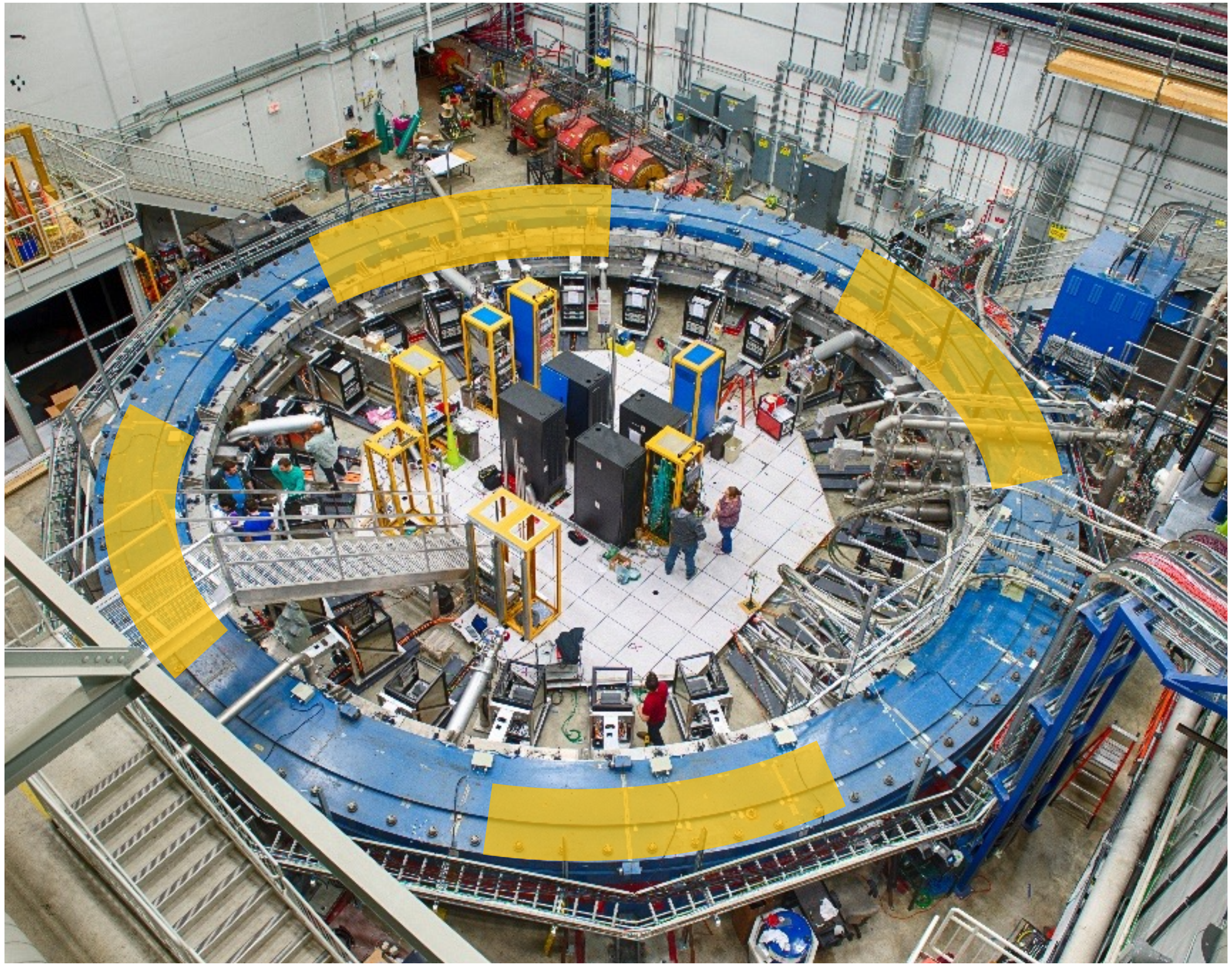
$$\cos(\omega_a^m t + \phi(t)) = \cos(\omega_a^m t + \phi_0 + \phi_1 t + \cdots)$$
$$\approx \cos((\omega_a^m + \phi_1)t + \phi_0)$$

- How significant those corrections are?

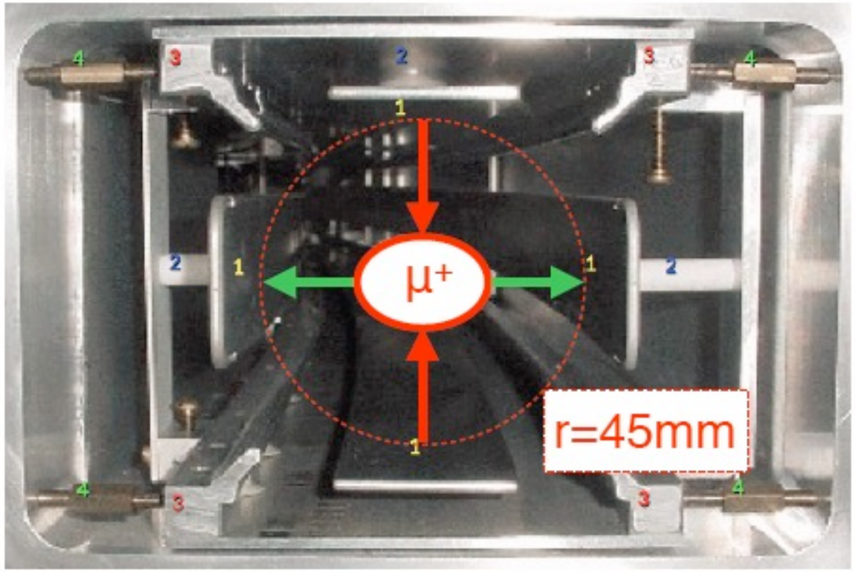
BD Corrections to ω_a^m	Net Correction [ppb]	Net Uncertainty [ppb]	Total Systematic Uncertainty on a_μ [ppb]
Run-1	500	93	159
Run-2/3	580	40	74

> factor of 2 improvement

Electrostatic Quadrupole (ESQ)

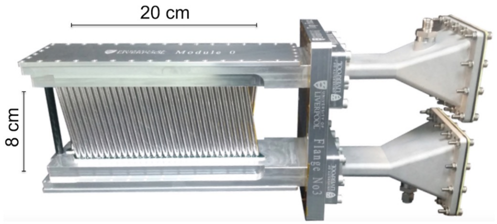


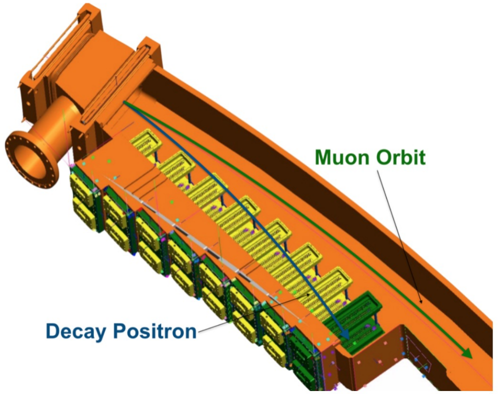
- The electrostatic quadrupoles (ESQ) are used to focus the beam vertically.
- Four sections cover 43% of the circumference.
- Each plate is charged to ± 18 kV.

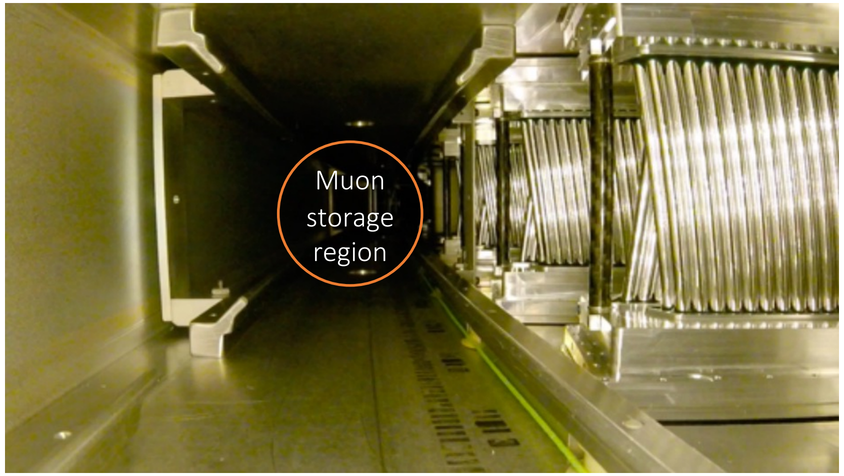


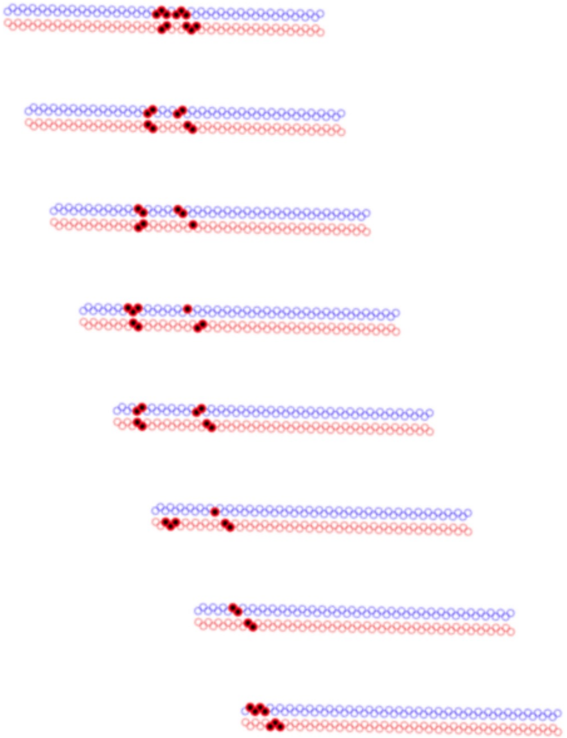
Straw Trackers

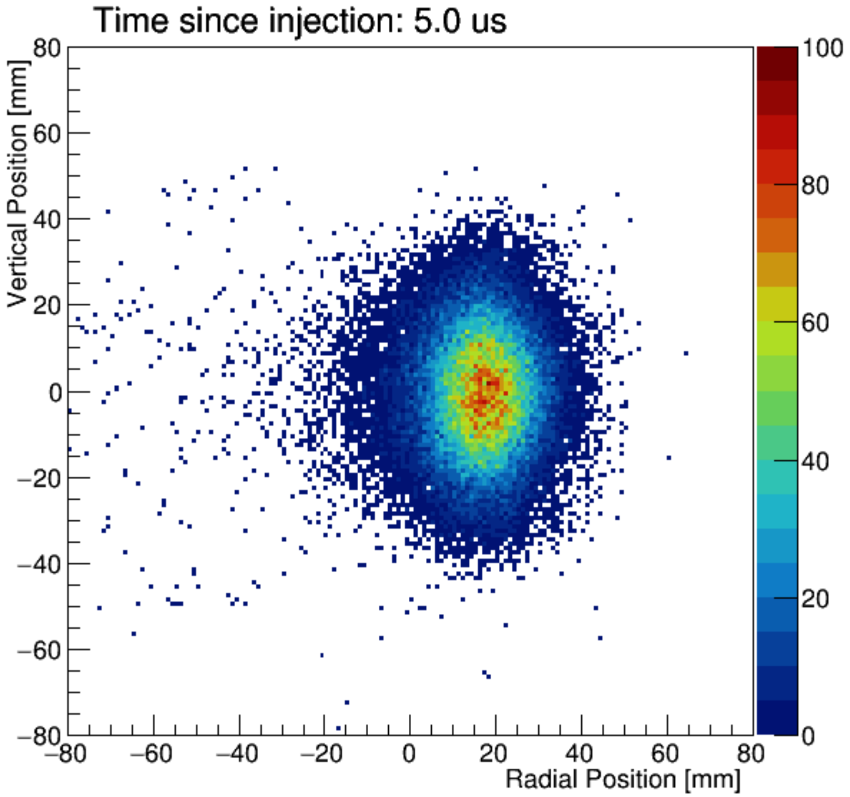
- The muon distribution is measured by the straw tracker.
  - A straw is an ionization chamber filled with an Ar-Ethane gas and a central wire at 1.6 kV.
  - It finds the trajectories of decay positrons, which are extrapolated to reconstruct the muon distribution.











E-field Correction C_e

$$\omega_a^m\,(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- The original spin precession frequency is much more complicated. Imposing the accelerator conditions ($\boldsymbol{\beta}\cdot\mathbf{B} = \boldsymbol{\beta}\cdot\mathbf{E} = 0$) and neglecting the EDM terms, one gets

Cancels out for the magic momentum (p_m) muons.

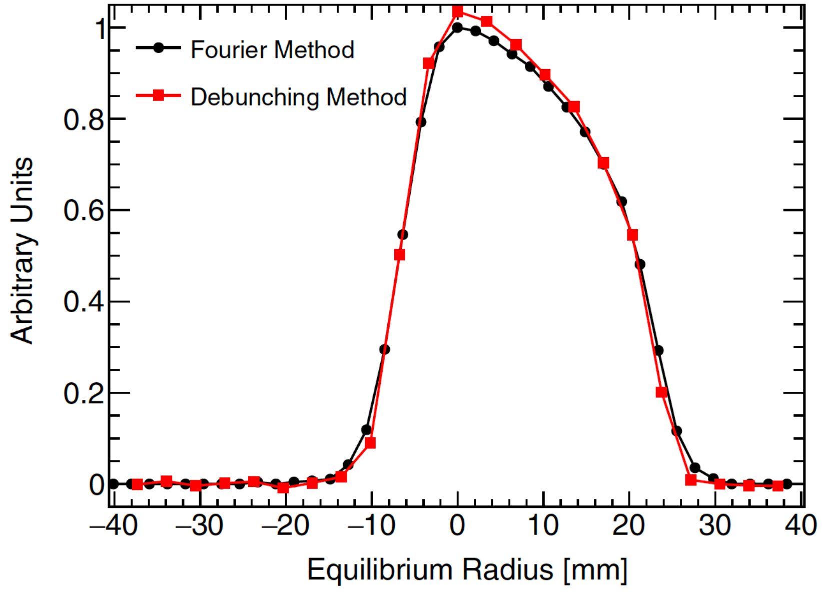
$$\boldsymbol{\omega}_a \approx -\frac{q}{m}\left[a\mathbf{B} - \left(a - \frac{1}{\gamma^2-1}\right)\frac{\boldsymbol{\beta}\times\mathbf{E}}{c}\right]$$

- The magic momentum ($p_m = mc/\sqrt{a}$) cancels the second term, leading to $\omega_a = a_\mu\, qB/m$.

- In practice, muon momenta are spread around the magic momentum.
  - $\Delta\omega_a/\omega_a$ coming from this term is called the E-field correction C_e.

$$C_e \approx 2n(1-n)\beta_0^2\frac{\langle x_e^2\rangle}{R_0^2}$$

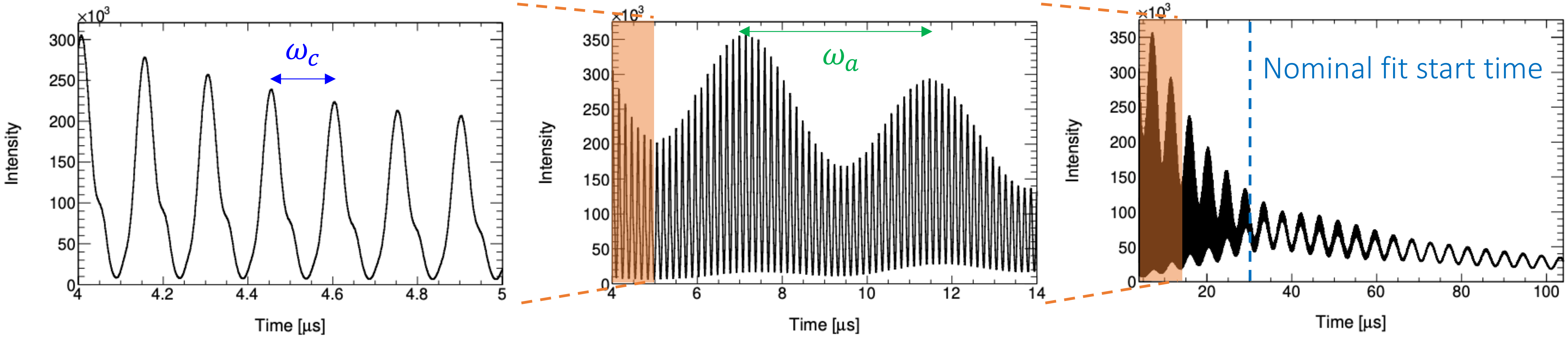
  - n is the weak focusing field index ($n = \frac{\partial E}{\partial x}\frac{r_0}{vB_0}$).
  - The radial equilibrium position (x_e) is proportional to the momentum offset.



E-field Correction C_e

$$\omega_a^m (1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- The radial equilibrium distribution can be measured by Fast Rotation Analysis.



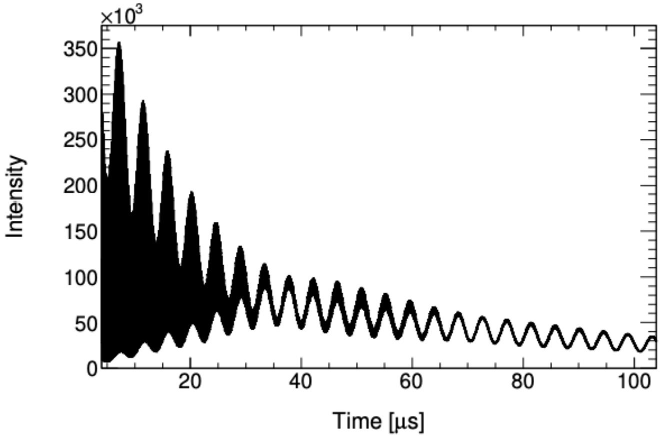
- Fast Rotation: Signal from the bunched beam's cyclotron motion.
  - The beam debunches due to the mixed cyclotron frequencies.

$$\omega_c \approx \omega_{c0}\left(1 - \frac{1}{1-n}\frac{\Delta p}{p_0}\right)$$

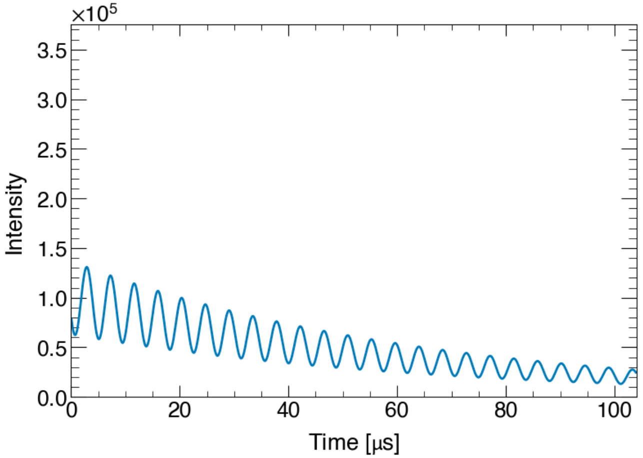
  - Debunching characteristics depend on the momentum distribution.

E-field Correction C_e: Fast Rotation Analysis

Finely-binned decay e^+ spectrum



Simple fit (FR was removed)



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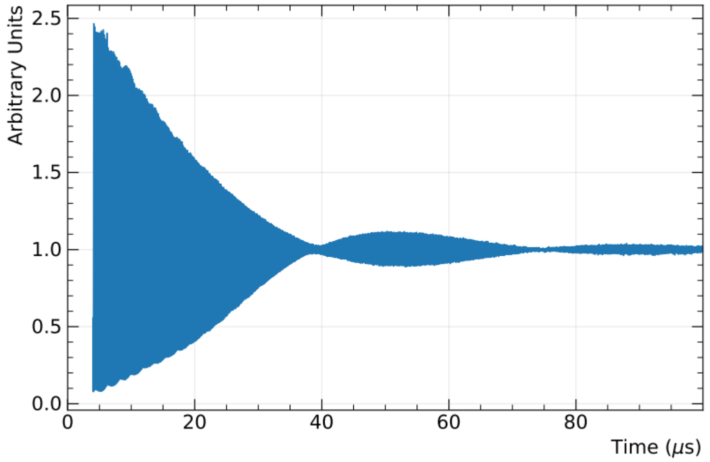
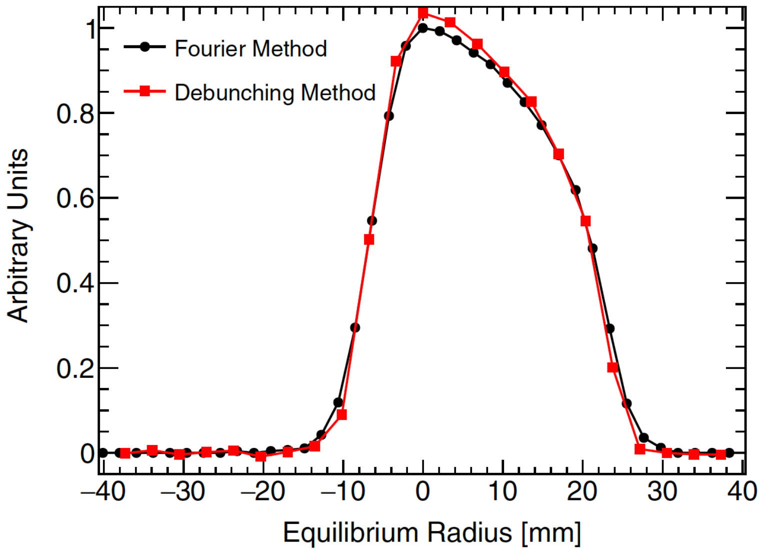
Isolated FR signal $S(t)$



Reconstructed x_e distribution



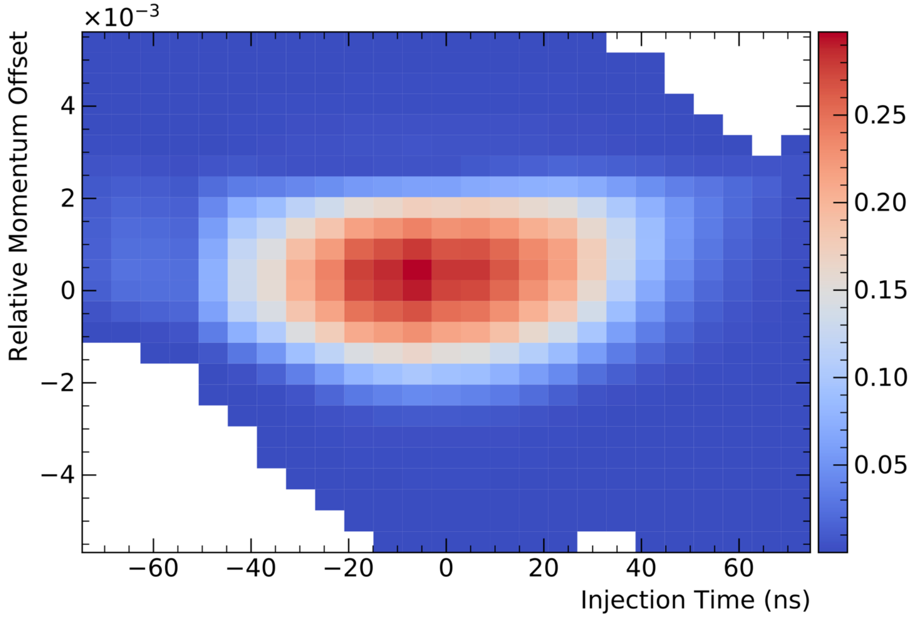
1. Fourier method (freq. domain)
2. Debunching method (time domain)

E-field Correction C_e

$$\omega_a^m \left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)$$

- Dominant systematics: correlation between the injection time and momentum distribution of stored muons.
  - Reconstruction of x_e becomes more complicated if there's a p-t correlation.
  - A time-dependent kick induces the correlation (under-kick prefers high p, and over-kick prefers low p).

- Improvement after Run-1
  - Bunch-level analysis to sort out the effect of the p-t correlation.
  - Complementary Tracker-based analysis.

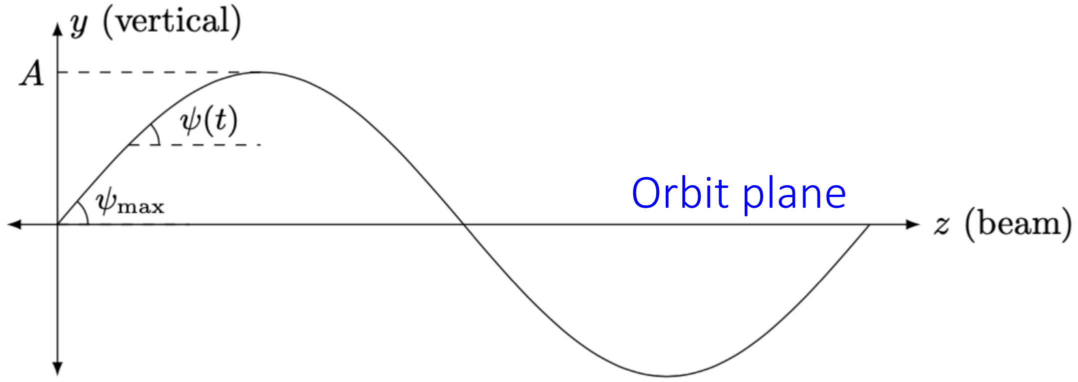


C_e	Correction [ppb]	Uncertainty [ppb]
Run-1	489	53
Run-2/3	451	32

Pitch Correction C_p

$$\omega_a^m (1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- The vertical motion (pitch motion) of the muon causes the vertical spin precession.



Pitch-driven radial component of $\boldsymbol{\omega}_s$

$$\omega_{sx} \approx \psi_0 \omega_y \sin(\omega_y t + \phi_y)$$

- The horizontal precession (ω_a) is affected by coupled in-plane and out-of-plane precessions due to the vertical motion. In such case, $\omega_a = \omega_{cy} - \omega_{sy}$ no longer holds.

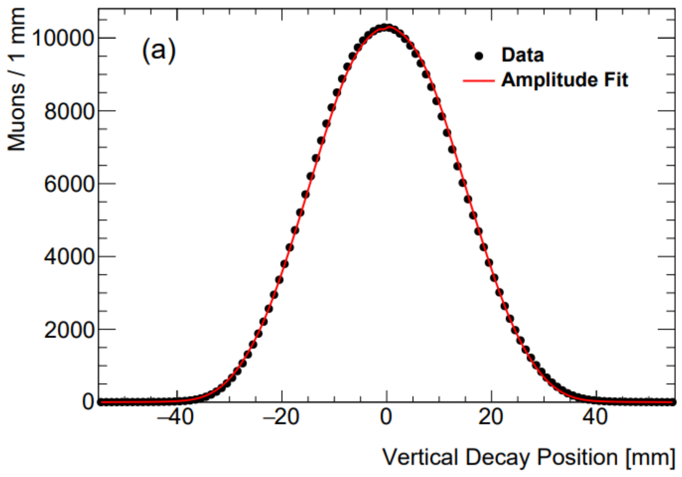
$$C_p = \frac{n}{2}\frac{\langle y^2 \rangle}{R_0^2} = \frac{n}{4}\frac{\langle A^2 \rangle}{R_0^2}$$

Pitch Correction C_p

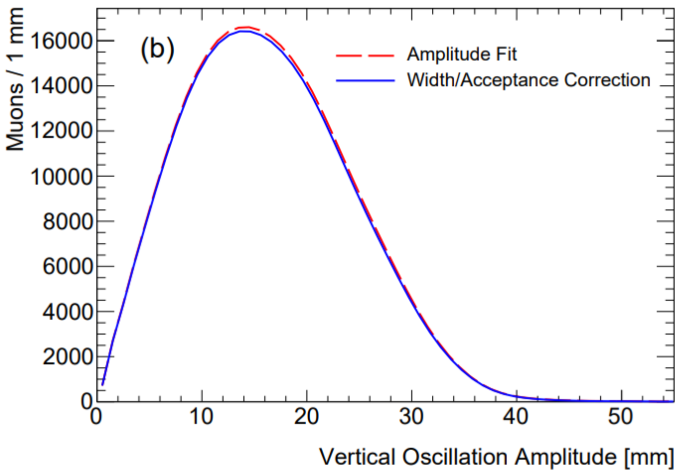
$$\omega_a^m (1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- Estimate C_p from the amplitude distribution rather than the vertical distribution.
  - Due to the calorimeter acceptance, the vertical positions are not evenly weighted.
  - This makes the measured $\langle y^2 \rangle$ systematically biased from the actual $\langle y^2 \rangle$.
  - The amplitude is reconstructed from the position distribution, including the acceptance correction.
- Dominant systematic error source: tracker alignment and reconstruction.
- Improvement after Run-1: Independent & different method analysis cross-check.

Vertical position



Vertical oscillation amplitude



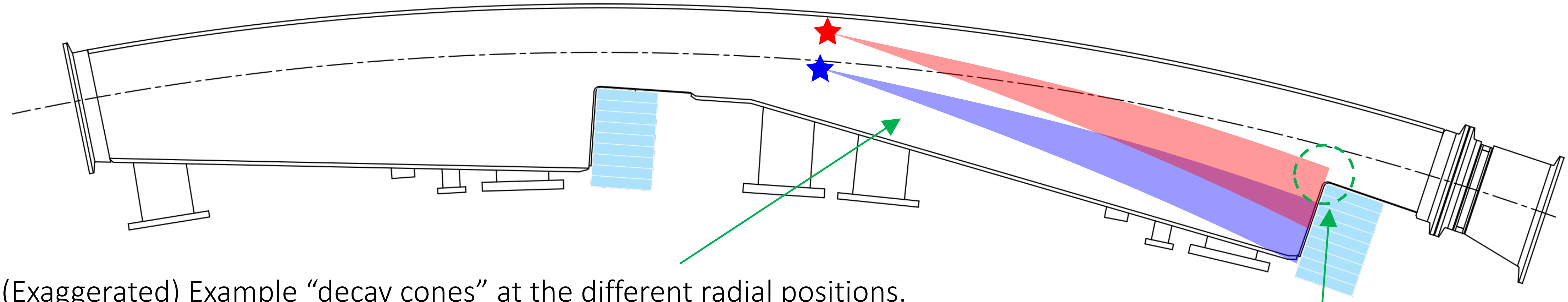
$$C_p = \frac{n}{2}\frac{\langle y^2 \rangle}{R_0^2} = \frac{n}{4}\frac{\langle A^2 \rangle}{R_0^2}$$

C_p	Correction [ppb]	Uncertainty [ppb]
Run-1	180	13
Run-2/3	170	10

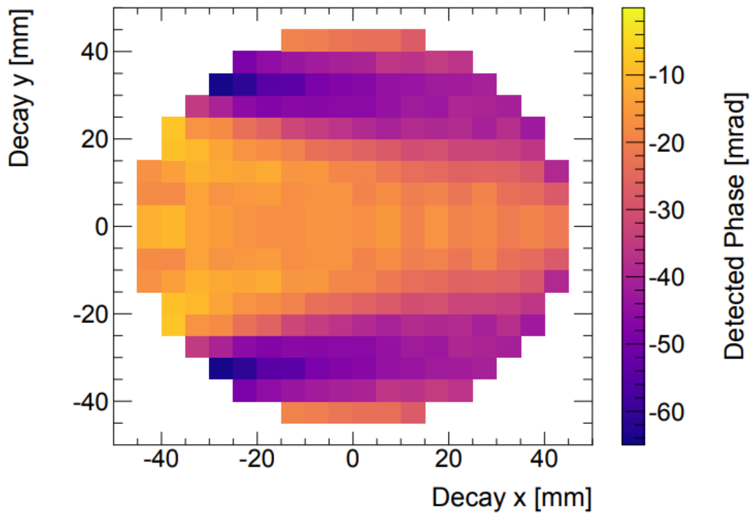
Phase-Acceptance Correction C_{pa}

$$\omega_a^m(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- The $g-2$ phase of the accepted positrons depends on the muon decay position (x, y, ϕ) and energy.



(Exaggerated) Example "decay cones" at the different radial positions.



Unlike the blue cone, some e^+ in the red cone are not accepted because of the finite geometry of the calorimeter.
Accepting more inward decay e^+ than outward e^+, the muon spin maximizing the number of accepted e^+ (≈ phase) is altered accordingly.

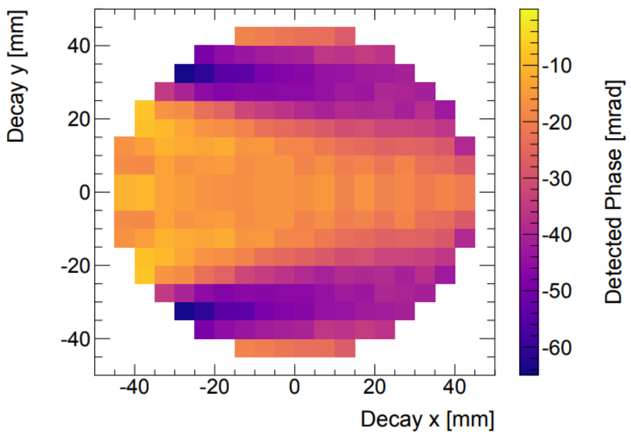
The phase of the accepted e^+ wiggle depends on the decay positions.
This itself does not bias ω_a unless the beam profile changes over time.

Phase-Acceptance Correction C_{pa}

$$\omega_a^m \left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)$$

- The (azimuthally-averaged) muon position distribution does change over time.

Phase (x, y)



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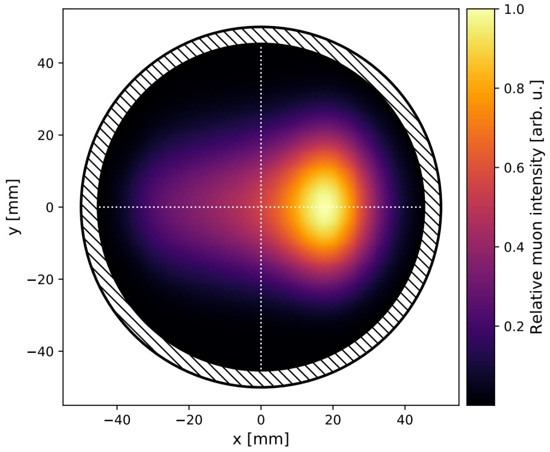
Muon distribution (x, y, t)



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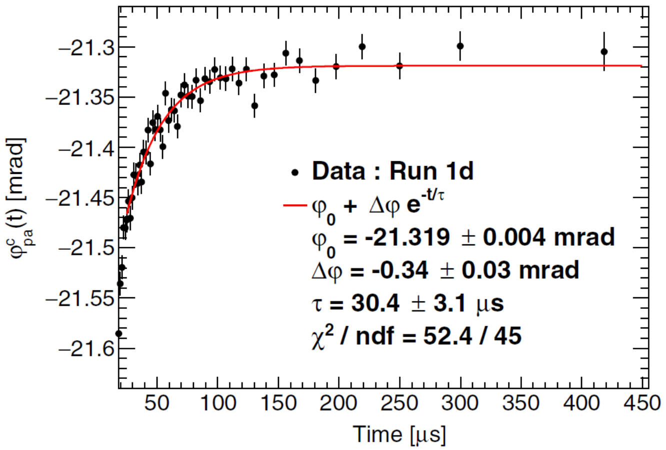
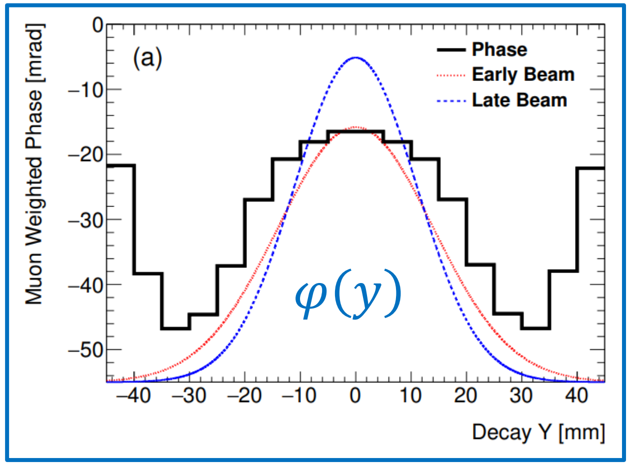
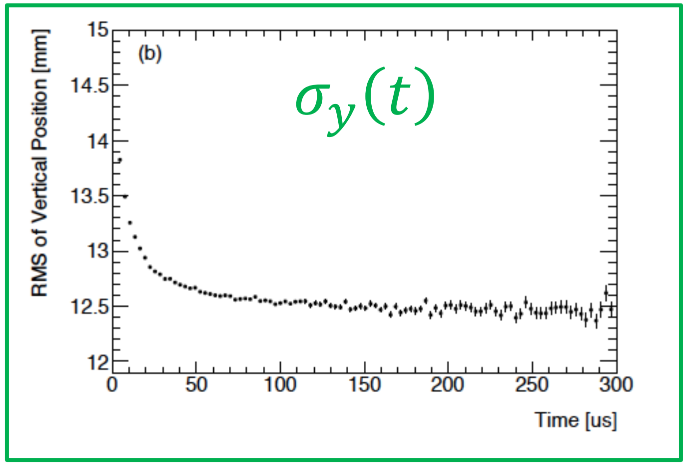
Phase (t)



- The dominant effect from Run-1 came from the early-to-late vertical distribution change.

$$\frac{d\varphi}{dt} = \frac{d\varphi}{d\langle\sigma_y^2\rangle}\frac{d\langle\sigma_y^2\rangle}{dt}$$





Phase-Acceptance Correction C_{pa}

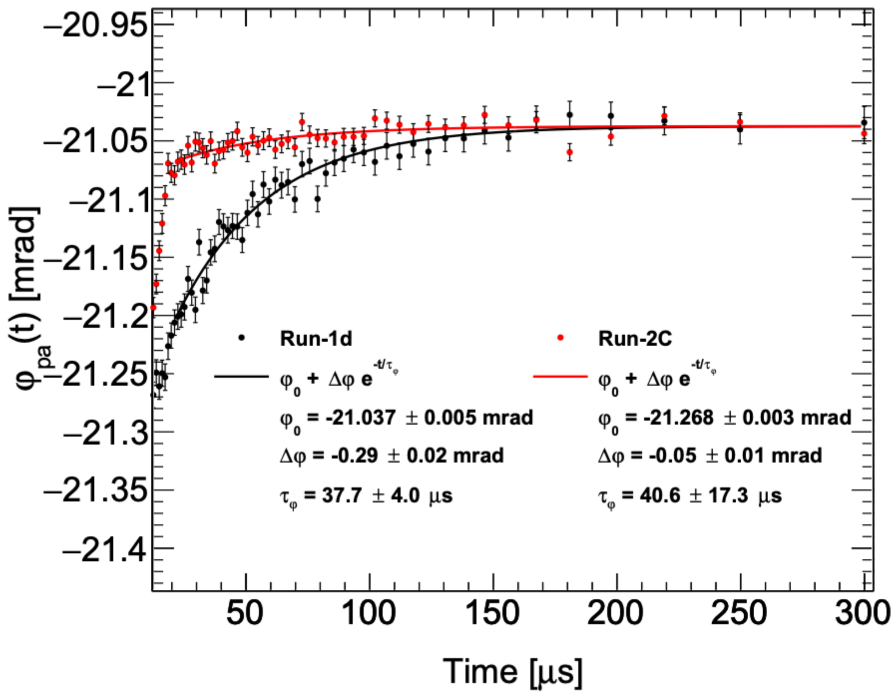
$$\omega_a^m (1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- All acceptance information is incorporated to compute the time-varying average phase.

$$\varphi^c_{pa}(t) = \arctan \frac{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j) \cdot A^c(x_i, y_j) \cdot \sin(\varphi^c_{pa}(x_i, y_j))}{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j) \cdot A^c(x_i, y_j) \cdot \cos(\varphi^c_{pa}(x_i, y_j))}$$

Decay distribution (t) — $M^c(x_i, y_j, t)$; Asymmetry map — $A^c(x_i, y_j)$; Acceptance map — $\varepsilon^c(x_i, y_j)$; Phase map — $\varphi^c_{pa}(x_i, y_j)$

- In Run-1, the phase-acceptance effect was amplified by the damaged ESQ resistors.
  - The damaged resistors were replaced before Run-2.
  - It significantly improved the beam early-to-late stability, and so are C_{pa} and ΔC_{pa} accordingly.



C_{pa}	Correction [ppb]	Uncertainty [ppb]
Run-1	−158	75
Run-2/3	−27	13

Spin-Momentum & Momentum-Time Correlations

- Phase changes due to the coupled effects from ϕ-$\langle p\rangle$ correlation & $\langle p\rangle$-t correlation.

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

- Each correlation can be decomposed as follows:

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{bml}$$

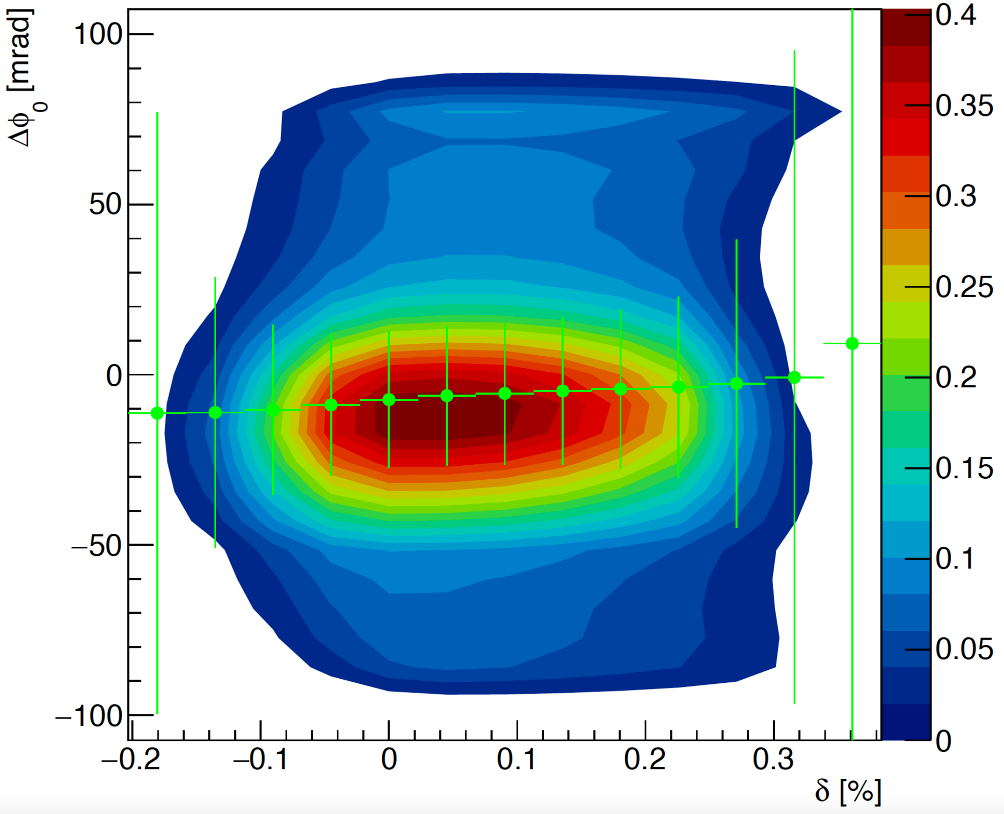
Beamline
Upstream dipole bending magnet.

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{p\text{-}x}$$

p-x
Inflector geometry (especially radial coordinates).

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{p\text{-}t_0}$$

p-t_0
Head-to-tail phase difference &
Head-to-tail stored momentum distribution.



Spin-Momentum & Momentum-Time Correlations

- Phase changes due to the coupled effects from ϕ-$\langle p\rangle$ correlation & $\langle p\rangle$-t correlation.

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

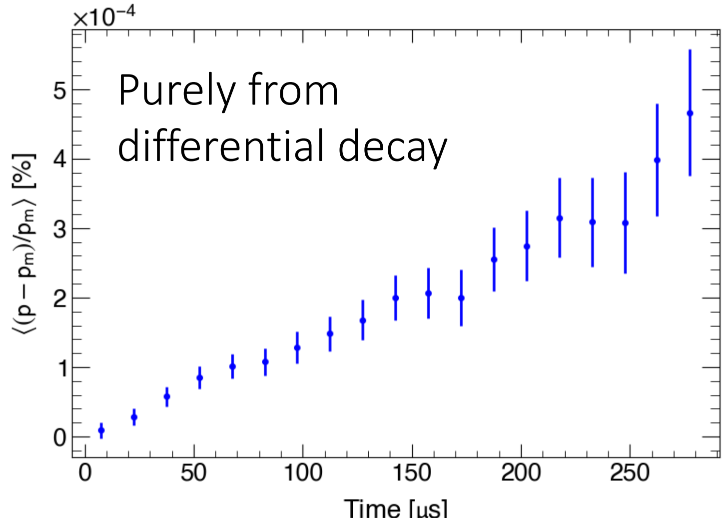
- Each correlation can be decomposed as follows:

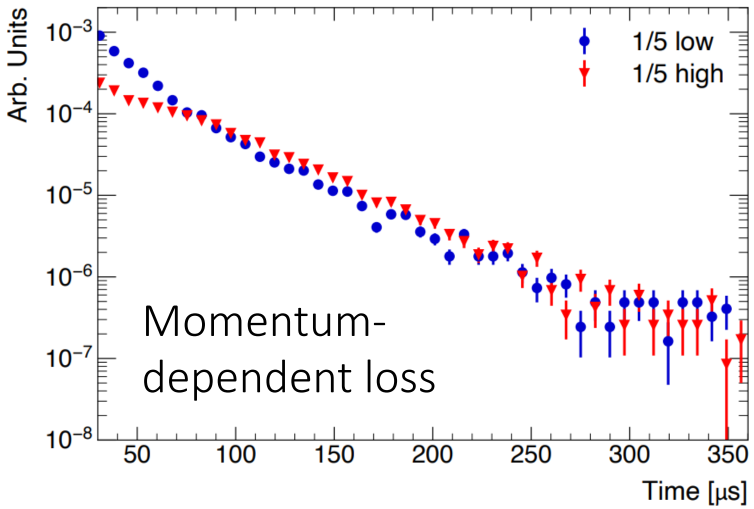
$$\left(\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}\right)_{dd}$$

Differential decay
Muons have different lifetimes depending on their energies.

$$\left(\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}\right)_{ml}$$

Momentum-dependent loss
Muon loss spectrum depends on the momentum.





Differential Decay Correction C_{dd} Muon Loss Correction C_{ml}

$$\omega_a^m \left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)$$

- In Run-1, we neglected C_{dd}. We were at the early stage of understanding the p-x and p-t_0 effects and the beamline C_{dd} was negligible compared to C_{ml} which was enhanced due to the damaged resistors.

- Dominant systematics comes from the bunch-by-bunch deviations in $(\mathrm{d}\phi/\mathrm{d}\langle p\rangle)_{p\text{-}t_0}$.

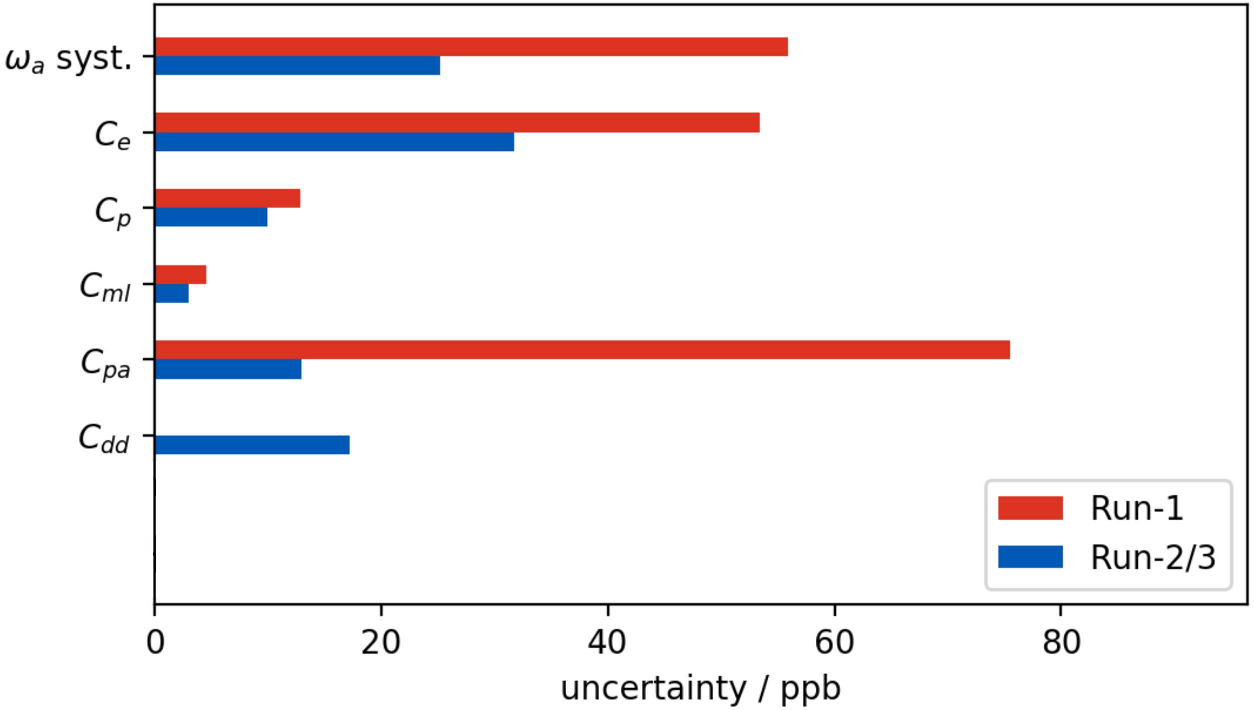
C_{dd}	Correction [ppb]	Uncertainty [ppb]
Run-1		
Run-2/3	-15	17

C_{ml}	Correction [ppb]	Uncertainty [ppb]
Run-1	-11	5
Run-2/3	0	3

Summary

- Beam dynamics corrections to anomalous spin precession frequency ω_a^m.
- The net uncertainty of the BD corrections was reduced by more than a factor of 2 in Run-2/3.

BD Corrections [ppb]	Run-1	Run-2/3
C_e	489 ± 53	451 ± 32
C_p	180 ± 13	170 ± 10
C_{pa}	-158 ± 75	-27 ± 13
C_{dd}	-	-15 ± 17
C_{ml}	-11 ± 5	0 ± 3
Sum	**500 ± 93**	**580 ± 40**



Thanks for your attention!

Acknowledgement

- Department of Energy (USA)
- National Science Foundation (USA)
- Istituto Nazionale di Fisica Nucleare (Italy)
- Science and Technology Facilities Council (UK)
- Royal Society (UK)
- Leverhulme Trust (UK)
- European Union's Horizon 2020
- Strong 2020 (EU)
- German Research Foundation (DFG)
- National Natural Science Foundation of China
- MSIP, NRF and IBS-R017-D1 (Republic of Korea)

Backups

17:00

Measurement of the muon anomalous precession frequency ω_a in the Fermilab $g-2$ experiment	*Sean Foster*
Hörsaal C, Historic main building	16:45 - 17:02
Measurement of the precision magnetic field in the Fermilab Muon g–2 experiment	*Saskia Charity*
Hörsaal C, Historic main building	17:02 - 17:19
Beam dynamics corrections to measurements of the muon anomalous magnetic moment	*On Kim*
Hörsaal C, Historic main building	17:19 - 17:36

News on muon g-2	*Graziano Venanzoni*
Audimax, Universität Hamburg	15:15 - 15:30

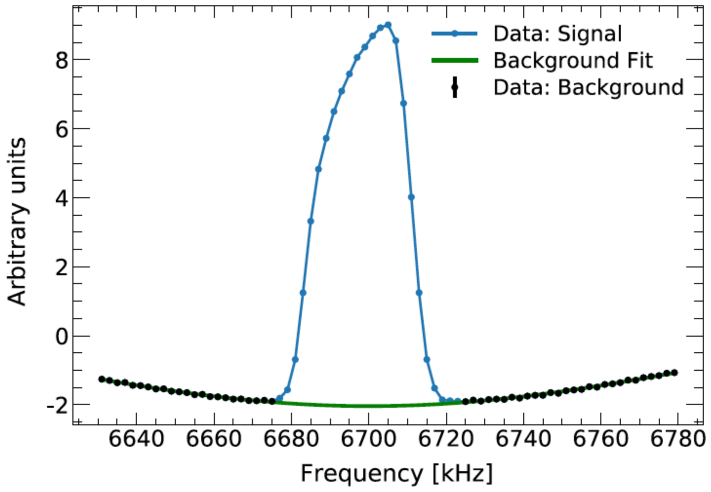
E-field Correction C_e: Fast Rotation Analysis

Fourier Method (Frequency Domain)

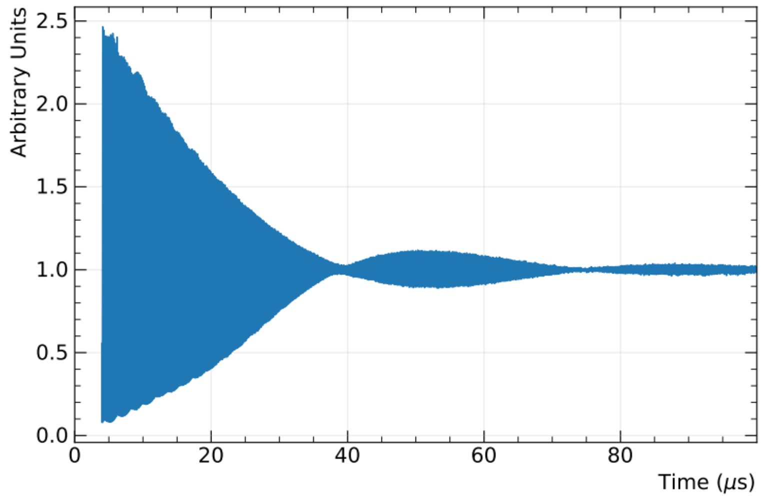
$$\hat{S}(\omega) = 2\int_0^\infty S(t+t_0)\cos(\omega t)dt$$

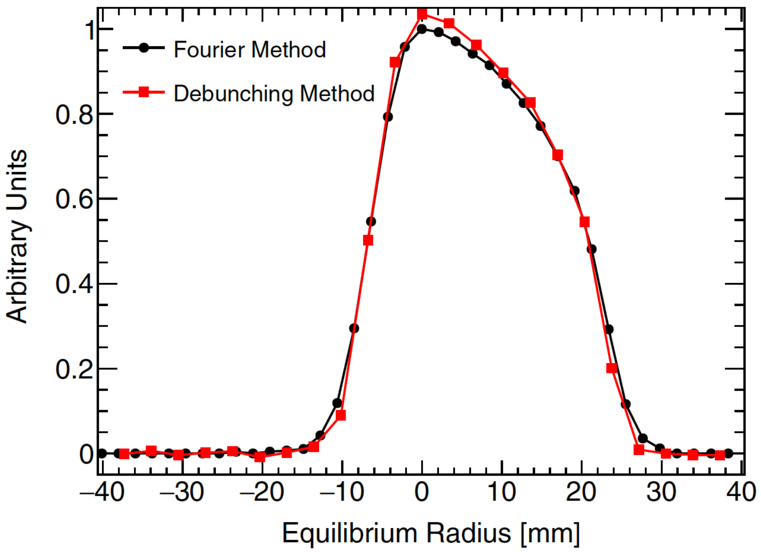
Tunable time-shift

1. Produce f_c distribution.
2. Identify background & optimize t_0.
3. Reconstruct radial distribution.
4. Estimate C_e.



Fast Rotation Signal $S(t)$





Debunching Method (Time Domain)

$$S_j = \sum_{ik} \beta_{ijk} f_i I_k$$

i: Radial bin.
j: (Measurement) time bin.
k: (Injection) time bin.

f_i: Radial distribution.
I_k: Injection time distribution.
β_{ijk}: Model coefficient between $f_i I_k$ and N_j.

1. Obtain β_{ijk} geometric factors.
2. Optimize f_i & I_k with χ^2-minimization fit.
3. Estimate C_e.

Model prediction

Measurements (~10^5 for 1 ns bins)

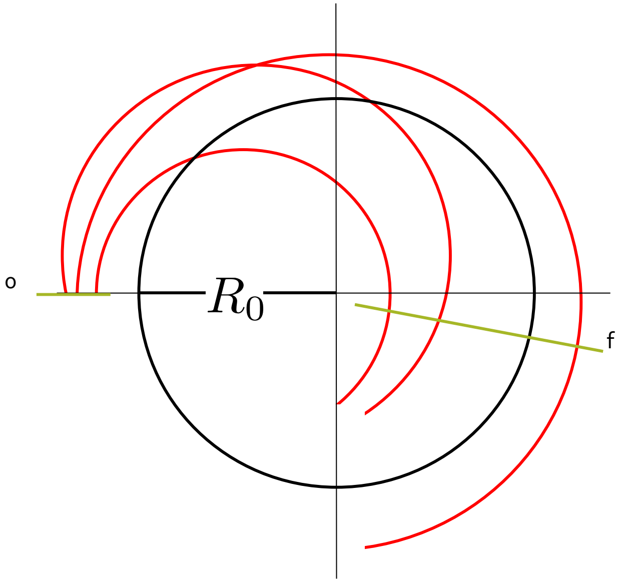
$$\chi^2 = \sum_j \frac{\left(\sum_{ik} \beta_{ijk} f_{ik} - N_j\right)^2}{\sigma_j^2}$$

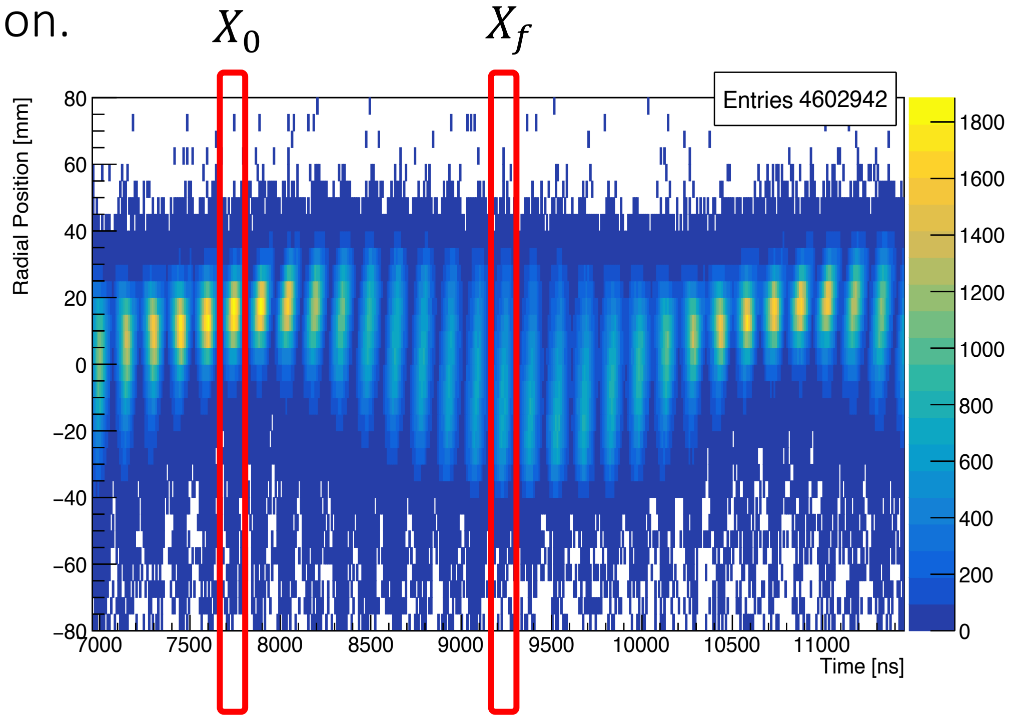
E-field Correction C_e

$$\omega_a^m (1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- Improvement after Run-1
  - Bunch-level analysis to sort out the effect of the p-t correlation.
  - Complementary Tracker-based analysis.

- Momentum spread is imprinted in the radial spread "breathing," which can be seen in the tracker.



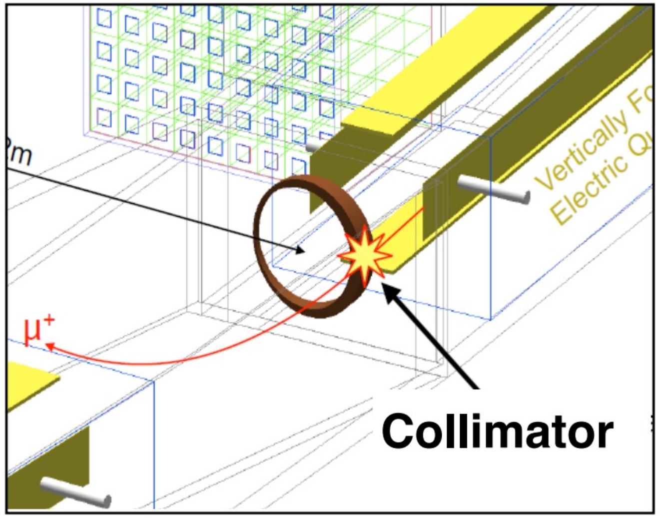


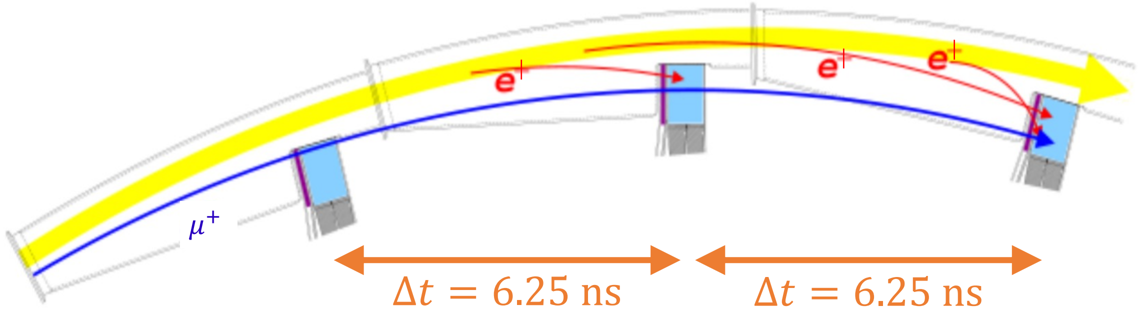
C_e	Correction [ppb]	Uncertainty [ppb]
Run-1	489	53
Run-2/3	451	32

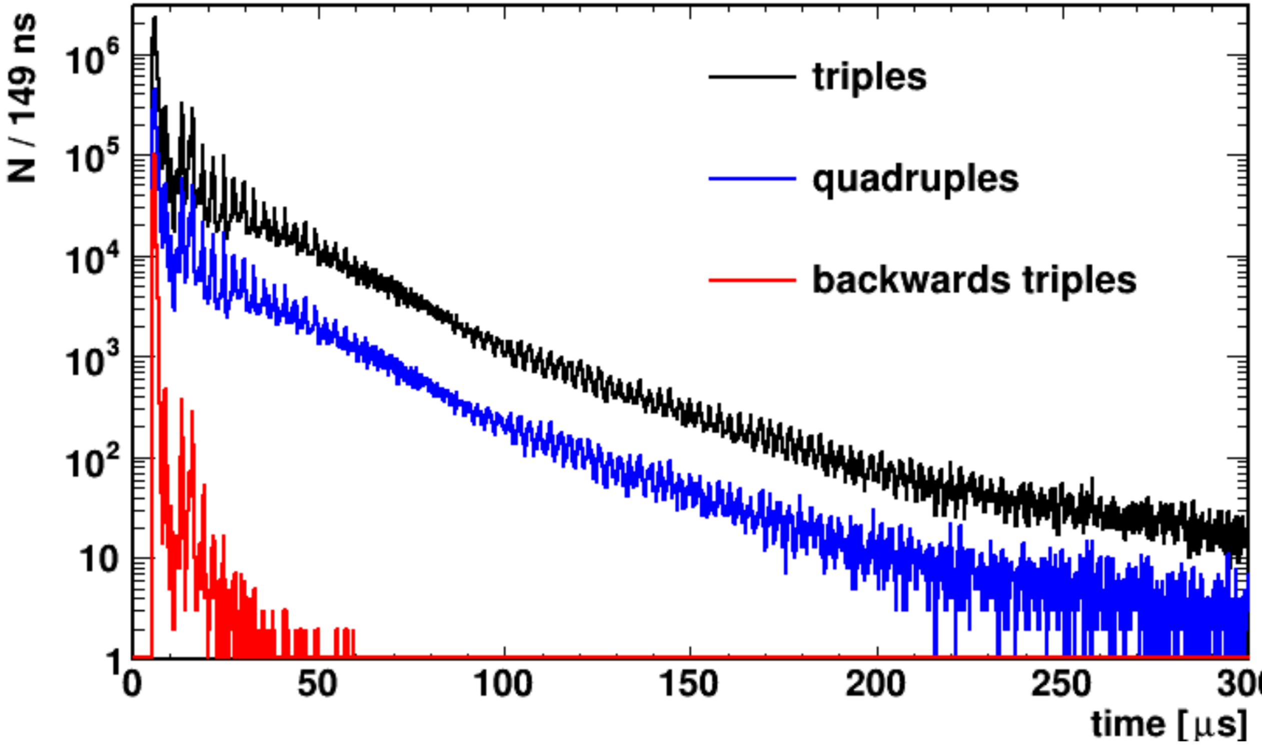
Muon Loss Correction C_{ml}

$$\omega_a^m \left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)$$

- Muon loss: Unwanted muon depletion due to interactions with materials during storage.
- The fraction of the muon losses can be measured by counting the triple coincidences.







Muon Loss Correction C_{ml}

$$\omega_a^m(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml})$$

- Momentum-dependent muon losses coupled with the spin-momentum correlation induces bias to ω_a.

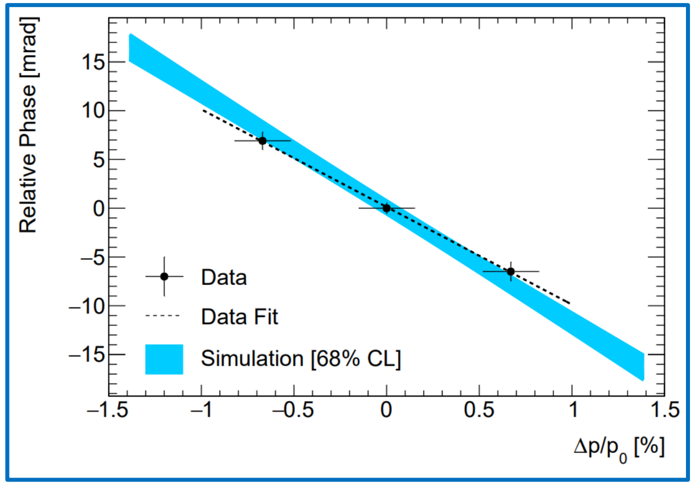
$$\cos(\omega_a^m t + \varphi(t)) = \cos(\omega_a^m t + \varphi_0 + \varphi_1 t + \cdots)$$
$$\approx \cos((\omega_a^m + \varphi_1)t + \varphi_0)$$

Bias on ω_a

$$\frac{d\varphi}{dt} = \frac{d\varphi}{d\langle p\rangle}\frac{d\langle p\rangle}{dt}$$

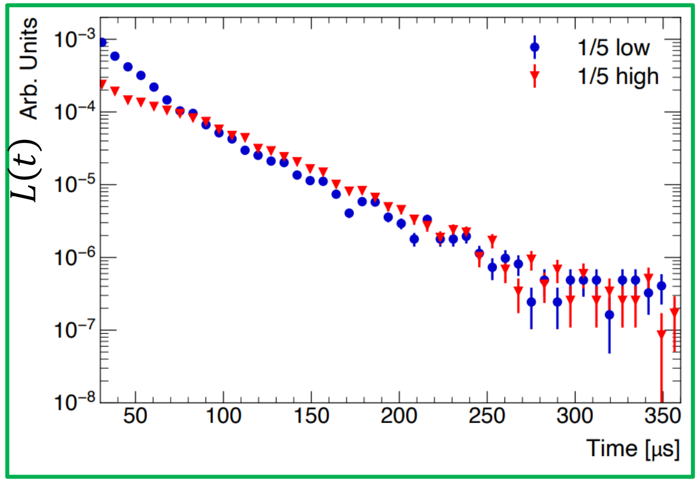
Spin-momentum correlation

Pre-exist before the injection due to the dipole bending magnet.



Momentum-dependent losses

Low momenta muons are lost faster than the high momenta muons at early times.



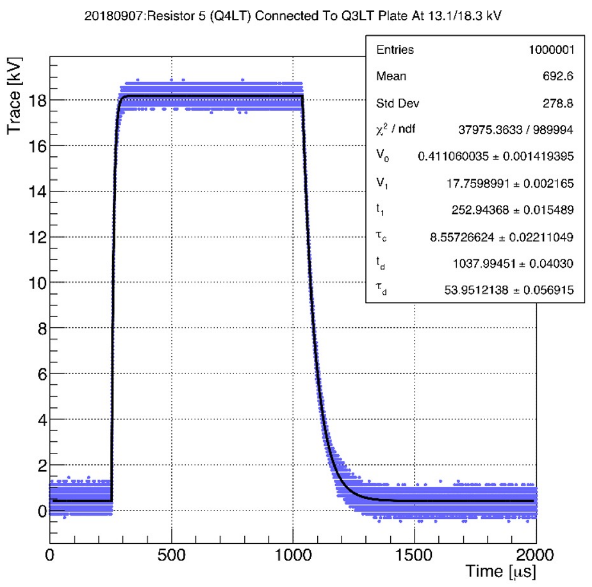
Data Set	Run-1a	Run-1b	Run-1c	Run-1d
C_{ml}	-14	-3	-7	-17
Phase-momentum	2	0	1	3
Form of $l(t)$	2	0	1	1
f_{loss} function	2	1	2	2
Linear sum ($\sigma_{C_{ml}}$)	6	2	4	6

Run-1: $C_{ml} = -11\ (5)$ ppb

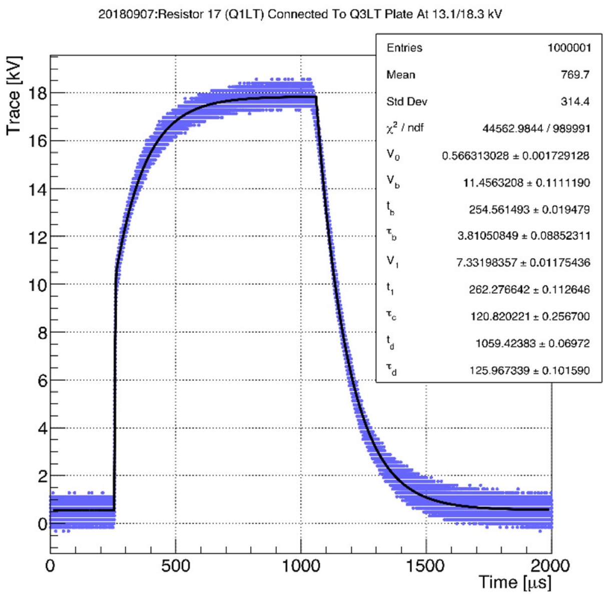
Challenge in Run-1: Damaged ESQ Resistors

- The RC time constant of the ESQ charging is designed to be around 5 us.
  - Two resistors were damaged and their resistance increased significantly. It induced slow changes to beam dynamics.
- Early-to-late BD effect
  - The CBO (horizontal oscillation) frequency drifted in time.
  - Vertical width changed slowly in time.

ESQ HV Trend



Nominal



Damaged resistor