On the normalization of UPDFs

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Motivations

1. LO calculations using the KMRW UPDFs (with the angular ordering cutoff) can reproduce Drell-Yan data, *Nefedov et al., 2009.13188*. No place left for NLO...
2. Issues of the KMRW UPDFs discussed in several papers in the last few years, no fully satisfying solution.

The first point is related to the large tail of the KMRW distributions (but is it an issue?).

I realized that some of the issues could be related to the normalization of UPDFs.

I thought that changing the normalization may change the shape of the distribution. Not really the case!

Reminder: The KMRW UPDFs

Normalization: $\widetilde{f}_a(x,\mu^2) = \int_0^{\mu^2} F_a(x,k_t^2;\mu^2)dk_t^2$

Differential definition:

$$F_a(x,k_t^2;\mu^2) = \frac{\partial}{\partial k_t^2}\left[T_a(k_t,\mu)\widetilde{f}_a(x,k_t)\right], \quad k_t \geq \mu_0$$

$$F_a(x,k_t^2;\mu^2) = \frac{1}{\mu_0^2}T_a(\mu_0,\mu)\widetilde{f}_a(x,\mu_0), \quad k_t < \mu_0$$

Sudakov factor (k_t appears only at one place):

$$T_a(k_t,\mu) = \exp\left(-\int_{k_t^2}^{\mu^2}\frac{dq^2}{q^2}\frac{\alpha_s(q^2)}{2\pi}\sum_b\int_{z_{ab}^{\mathrm{min}}(q^2,\mu^2)}^{z_{ab}^{\mathrm{max}}(q^2,\mu^2)} dz\, z\hat{P}_{ba}(z)\right)$$

1st issue: $T_a > 1$ when $k_t > \mu$, which clearly happens for the cross section.

Reminder: The KMRW UPDFs

Usual solution: $T_a^{new} = \Theta(\mu^2 - k_t^2) T_a + \Theta(k_t^2 - \mu^2)$

Taking the derivative (w.r.t. k_t) and using the cutoff dependent DGLAP eq., we obtain the **integral definition**

$$F_a(x, k_t^2; \mu^2) = \frac{\alpha_s(k_t^2, \mu^2)}{2\pi k_t^2} \left(T_a^{new}(k_t, \mu) \sum_b \int_x^{z_{ab}^{\mathsf{max}}(k_t^2, \mu^2)} dz \hat{P}_{ab}(z) \widetilde{f}_a\left(\frac{x}{z}, k_t\right) \right.$$

$$\left. - \Theta(k_t^2 - \mu^2) \widetilde{f}_a(x, k_t) \sum_b \int_{z_{ab}^{\mathsf{min}}(k_t^2, \mu^2)}^{z_{ab}^{\mathsf{max}}(k_t^2, \mu^2)} dz\, z \hat{P}_{ba}(z) \right)$$

2^o issue: this definition does not obey exactly

$$\widetilde{f}_a(x, \mu^2) = \int_0^{\mu^2} F_a(x, k_t^2; \mu^2) dk_t^2$$

$z_{ab} = \frac{\mu}{\mu + k_t}$ too far from 1: The cutoff-dependent DGLAP eq. is not a good approximation of the DGLAP eq.

Proposed solutions (to the second issue)

Cutoff dependent PDFs: *Golec-Biernat and Staśto, Phys. Lett. B 781, 633-638 (2018); Valeshabadi and Modarres, Eur. Phys. J. C 82 (2022) no.1, 66*

$$\widetilde{f}_a(x,\mu^2,\Delta) = \int_0^{\mu^2} F_a(x,k_t^2;\mu^2,\Delta)dk_t^2$$

- Interpretation? $\int_0^1 u_{d/p}(x,\mu^2,\Delta)dx \neq 2$.

x-dependent Sudakov factor: *Nefedov and Saleev, Phys. Rev. D 102, 114018 (2020)*

1. Impose the usual integral definition and exact equivalence betwwen the dif. and int. definitions.
2. Find a new, x dependent, Sudakov factor.

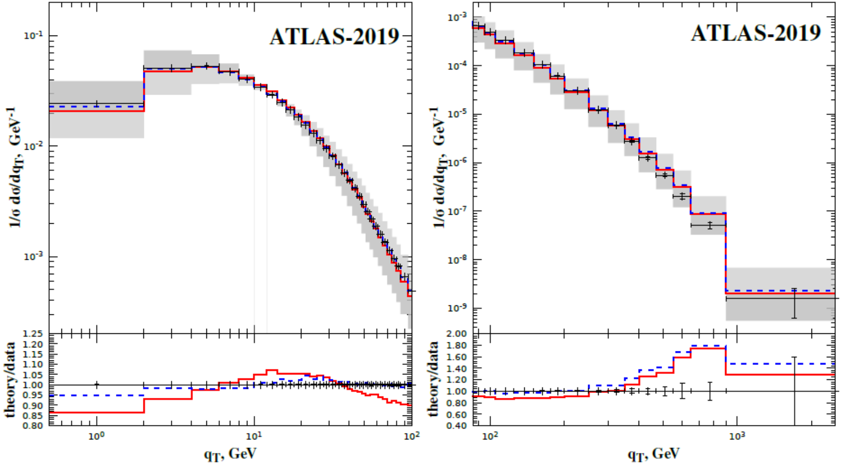
However, $T_a(k_t,\mu,x) > 1$ when $k_t > \mu$.

Solution $T_a \to \Theta(\mu^2 - k_t^2)T_a + \Theta(k_t^2 - \mu^2)$ not applicable.

Additional issues

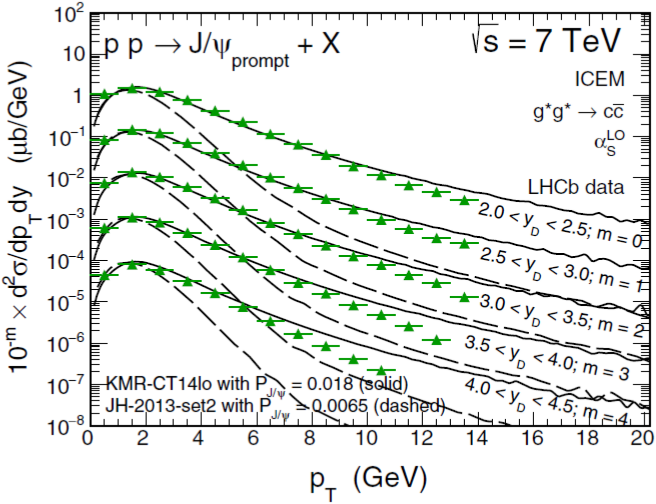
1. Theoretical issues related to the fact that the integral on k_t is cut off.
2. Phenomenological issues:
   - It reproduces Drell-Yan and J/ψ data with LO calculations (space for NLO corrections??). Looks like an advantage...
   - However, the $cg \to cg$ process alone overestimates the D meson cross section.

Drell-Yan



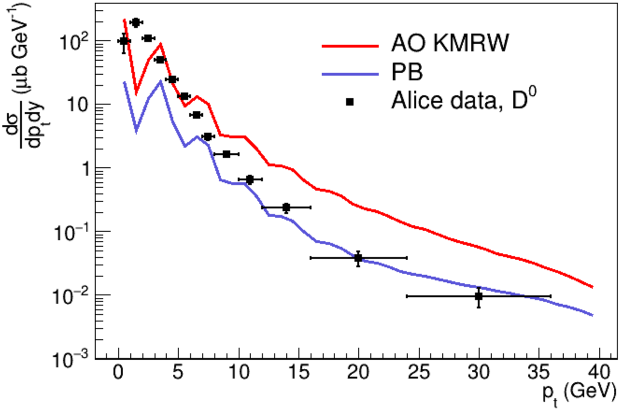
Nefedov and Saleev, Phys. Rev. D 102, 114018 (2020)

J/ψ production (LHCb)



Rafał Maciuła, Antoni Szczurek, and Anna Cisek Phys. Rev. D 99, 054014

D-meson production



cg contribution to D-meson production

Overestimation related to the unconstrained part of the KMRW distribution at $k_t > \mu$.

Modified KMRW UPDFs (angular ordering)

- Issues not observed for UPDFs obeying $\widetilde{f}_a(x,\mu^2) = \int_0^\infty F_a(x,k_t^2;\mu^2)dk_t^2$.

Differential definition:

$$F_a(x,k_t^2;\mu^2) = \frac{\partial}{\partial k_t^2}\left[T_a(k_t,\mu)\widetilde{f}_a(x,k_t \to \mu)\right]$$

- Modification: Avoid $\widetilde{f}_a(x,0)$ and $\widetilde{f}_a(x,\infty)$ after integration.
- Exact normalization if $T_a(\infty,\mu) - T_a(0,\mu) = 1$.

Minimal (?) modification of the Suakov factor:

$$T_a(k_t,\mu) = \exp\left(-\int_{k_t^2}^{\mu^2\to\infty}\frac{dq^2}{q^2}\frac{\alpha_s(q^2)}{2\pi}\sum_b\int_0^{\Delta(q,\mu)}dz\,z\hat{P}_{ba}(z)\right)$$

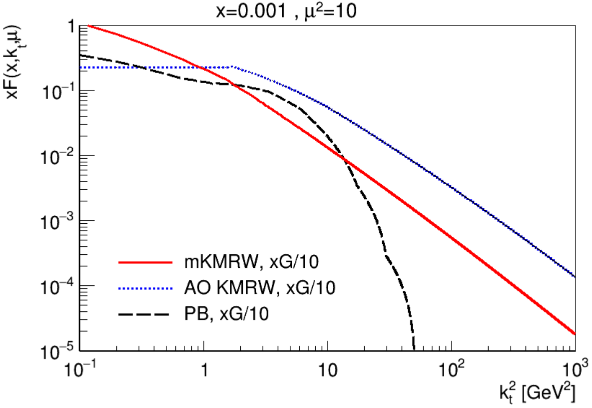
With the usual angular ordering cutoff $\Delta(q,\mu) = \frac{\mu}{\mu+q}$.

Modified KMRW UPDFs (angular ordering)

Integral definition (cutoff-dependent DGLAP eq. not used):

$$F_a(x,k_t^2;\mu^2) = \frac{\alpha_s(k_t^2)}{2\pi k_t^2} T_a(k_t,\mu) \widetilde{f}_a(x,\mu) \sum_b \int_0^{\Delta(k_t,\mu)} dz\, z \hat{P}_{ba}(z)$$

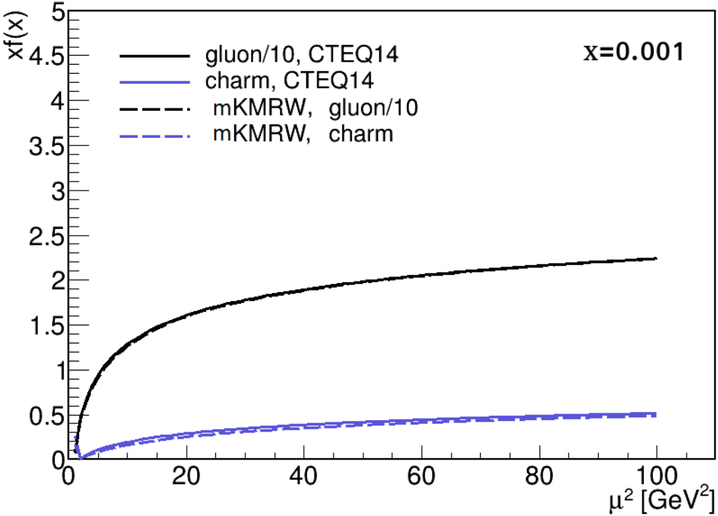
Form simpler than the original KMRW.



Issues solved

- Theoretical issues related to the cutoff in the normalization condition
- Sudakov factor always smaller than one
- Exact normalization (by construction)

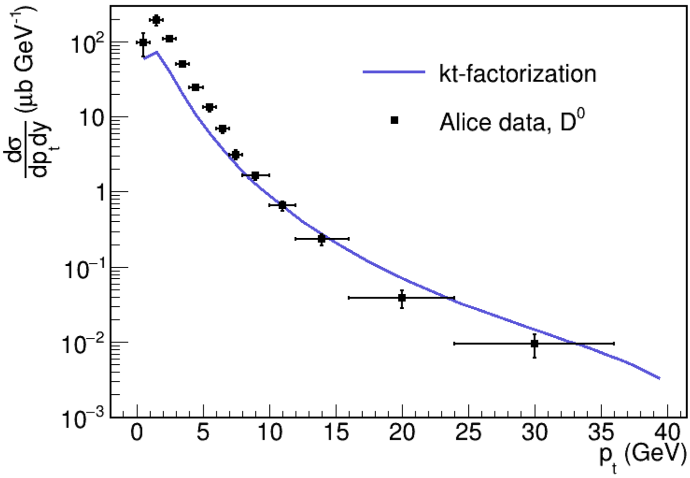
Exact normalization



Issues solved

- Theoretical issues related to the cutoff in the normalization condition
- Sudakov factor always smaller than one
- Exact normalization (by construction)
- Over estimation of the D meson cross section

D meson production

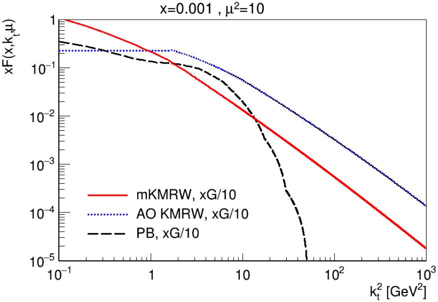


fragmentation parameter $\varepsilon_c = 0.5$

Final remarks

We changed mostly the normalization of the distribution, not the shape.

The shape depends on the evolution eq.



It seems that evolutions based on parton branchings (PB, CCFM, etc...) lead to fast decreasing UPDFs.

What is the evolution eq. of the KMRW UPDFs (never discussed)?