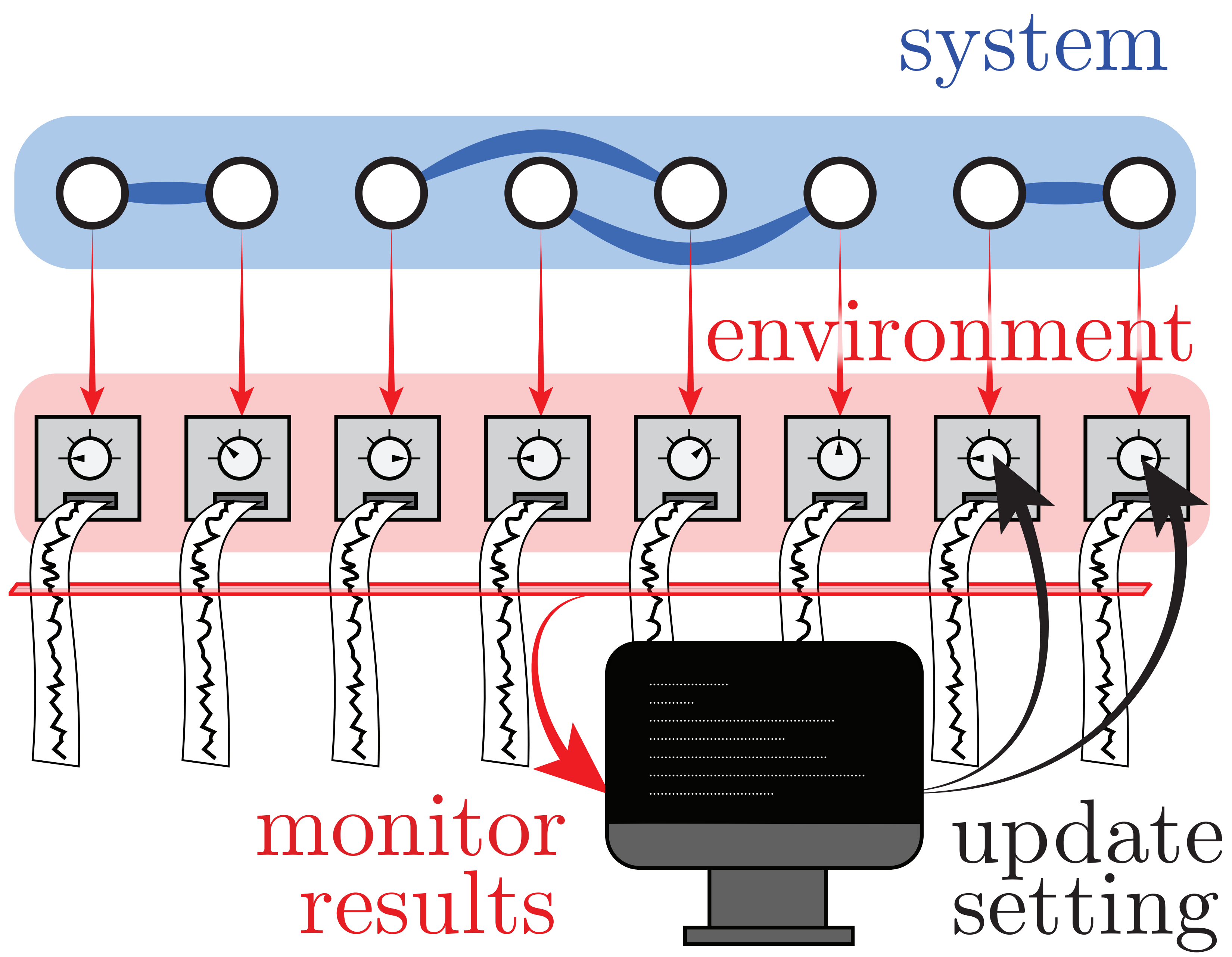
Entanglement-Optimal Trajectories of Many-Body Quantum Markov Processes

Hannes Pichler, IQOQI & University of Innsbruck



Ref: Tatiana Vovk & HP, PRL **128**, 243601 (2022

Noisy quantum many-body systems

Goal: Solve Master equation

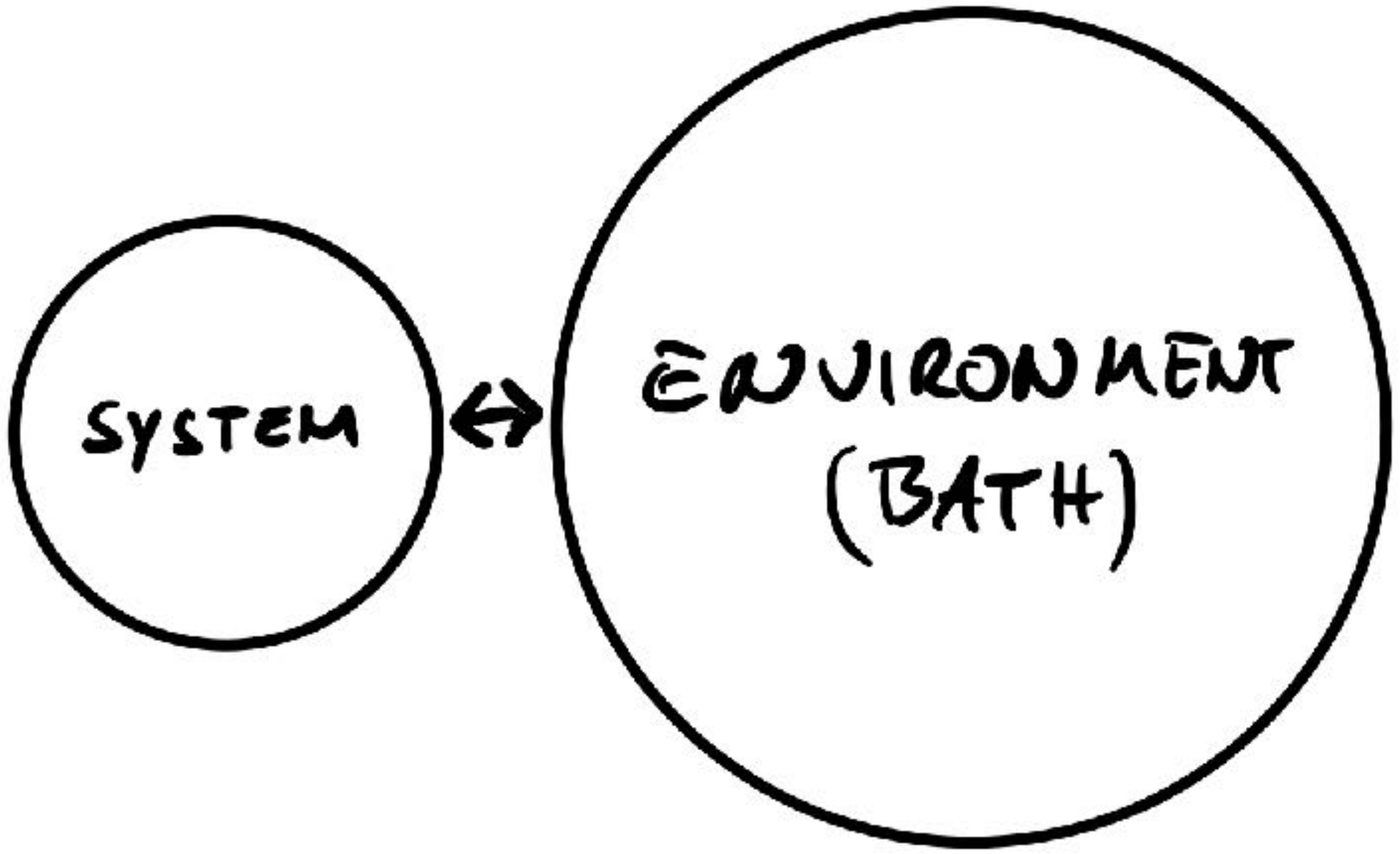
$$\frac{d\rho}{dt} = -i\left[H_{\mathrm{sys}}, \rho\right] + \sum_{j=1}^{m} \gamma_j \left(c_j \rho c_j^\dagger - \frac{1}{2}\left\{\rho, c_j^\dagger c_j\right\}\right)$$

ρ = system density operator of system

H_{sys} = system Hamiltonian

c_i = jump operators

γ_i = decay rates



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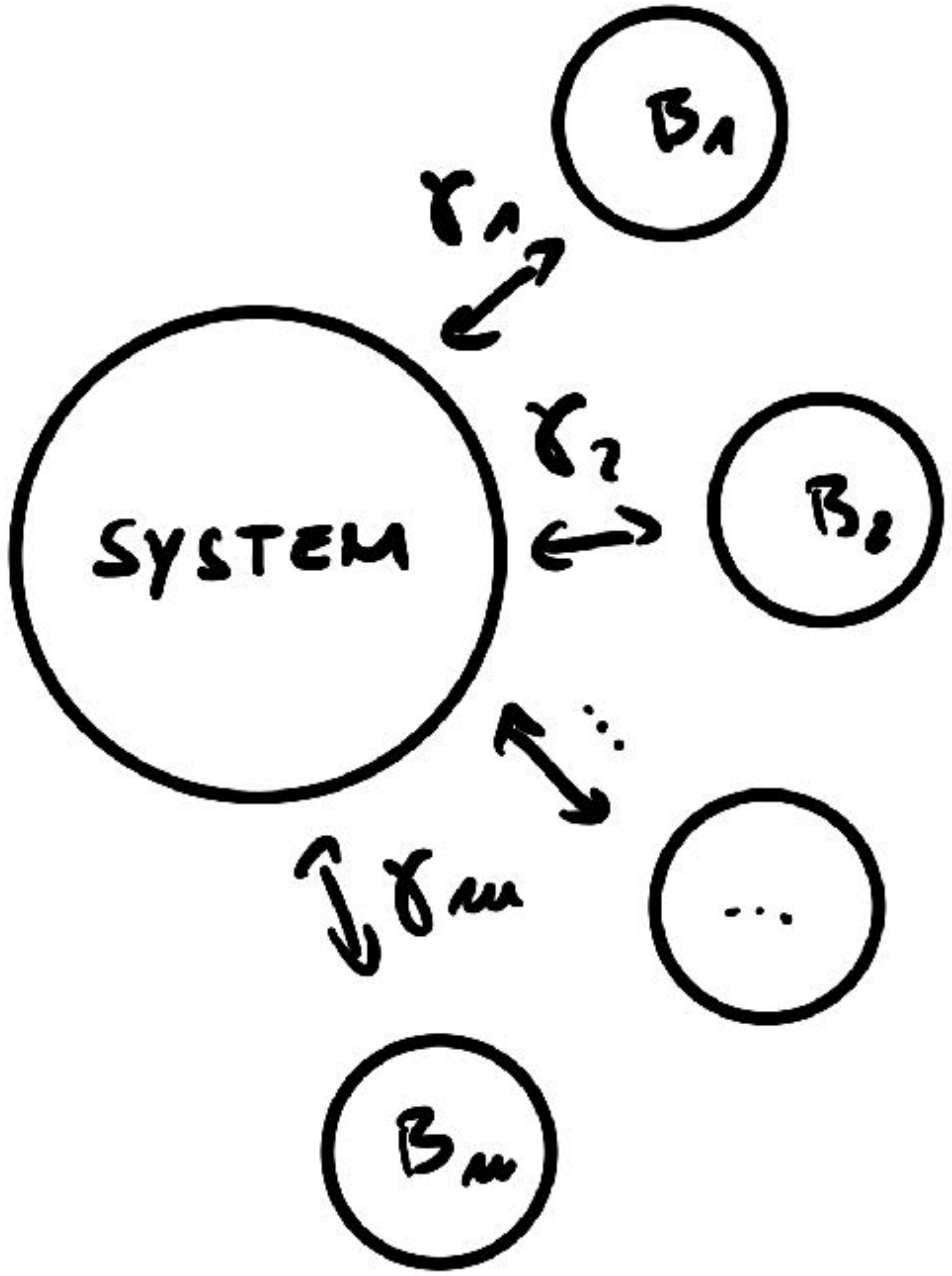
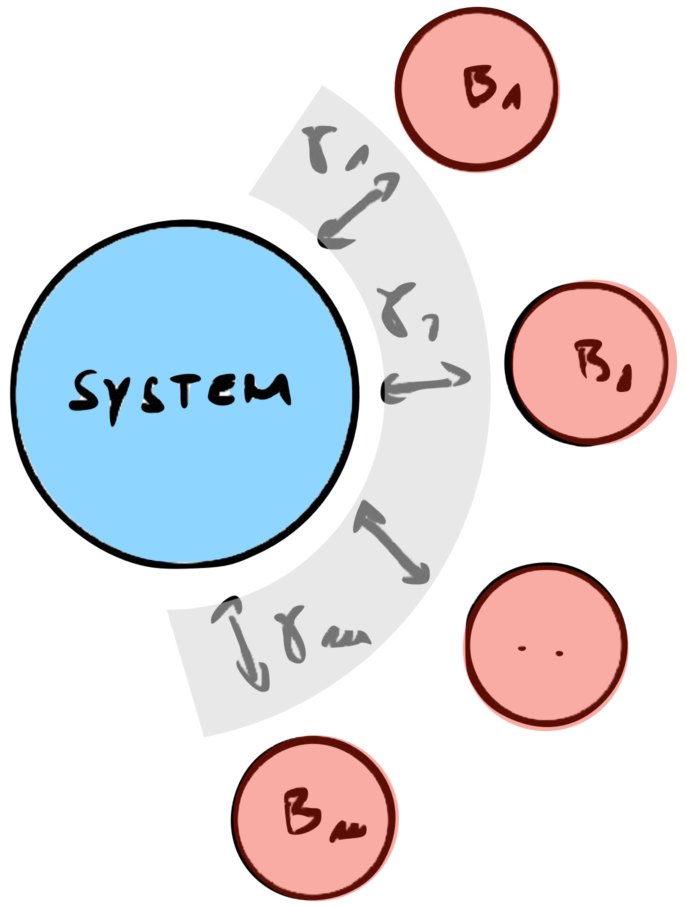


Quantum Optical Model



Schrödinger Equation of system+environment

$$i\partial_t |\Psi(t)\rangle = H|\Psi(t)\rangle$$

Hamiltonian $H = H_{\text{sys}} + H_B + H_{int}$.

$[b_j(\omega), b^\dagger_{j'}(\omega')] = \delta_{j,j'}\delta(\omega - \omega')$

Initial state $b_j(\omega)|\Psi(0)\rangle = 0$

Bath: $H_B = \sum_j \int d\omega \omega b_j^\dagger(\omega) b_j(\omega)$

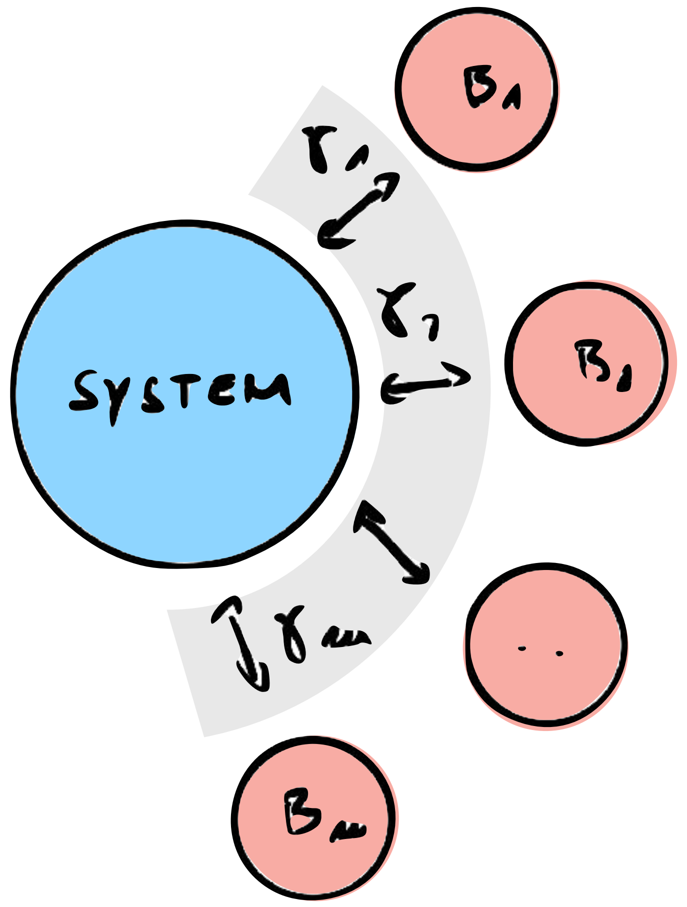
Interaction: $H_{int} = i \sum_j \sqrt{\frac{\gamma_j}{2\pi}} \int d\omega \left[b_j(\omega)^\dagger c_j - c_j^\dagger b_j(\omega) \right]$

Rem: ro

Tracing over bath degrees of freedom gives

$$\Rightarrow \quad \frac{d\rho}{dt} = -i\left[H_{\text{sys}}, \rho\right] + \sum_{j=1}^{m} \gamma_j \left(c_j \rho c_j^\dagger - \frac{1}{2}\left\{\rho, c_j^\dagger c_j\right\}\right)$$

Quantum Optical Model



Schrödinger Equation of system+environment:

$$i\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$$

interaction picture with H_B …

Hamiltonian: $H(t) = H_{\text{sys}} + i\sum_{j=1}^{m}\sqrt{\gamma_j}\left[b_j(t)^\dagger c_j - c_j^\dagger b_j(t)\right].$

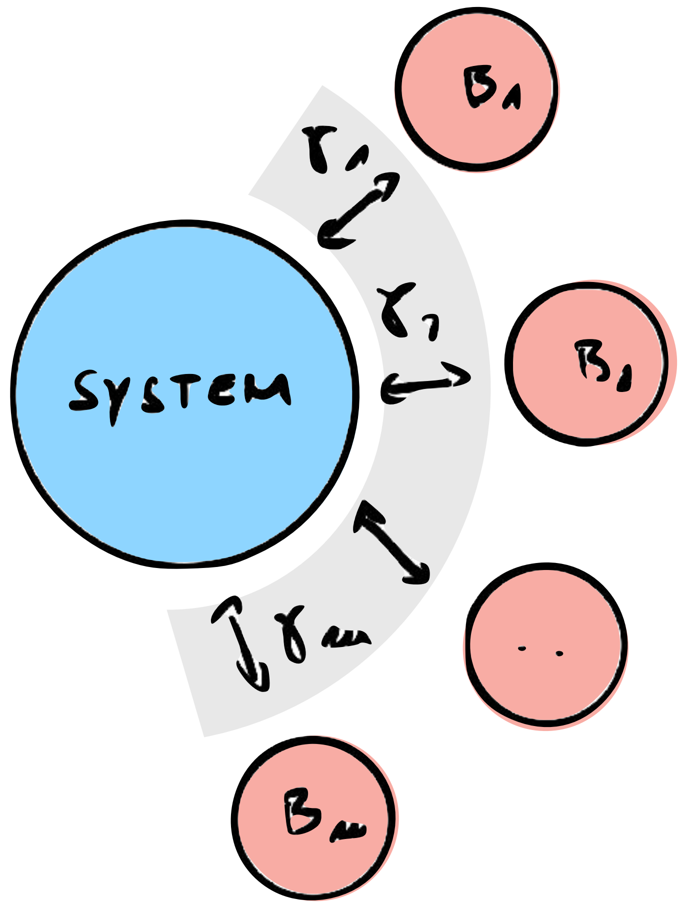
Quantum noise operators:

$b_j(t) = \frac{1}{\sqrt{2\pi}}\int d\omega b_j(\omega)e^{-i\omega t}$,

$[b_j(t), b_{j'}(t')^\dagger] = \delta_{j,j'}\delta(t-t')$

Initial state $b_j(t)|\Psi(0)\rangle = 0$

Quantum Optical Model



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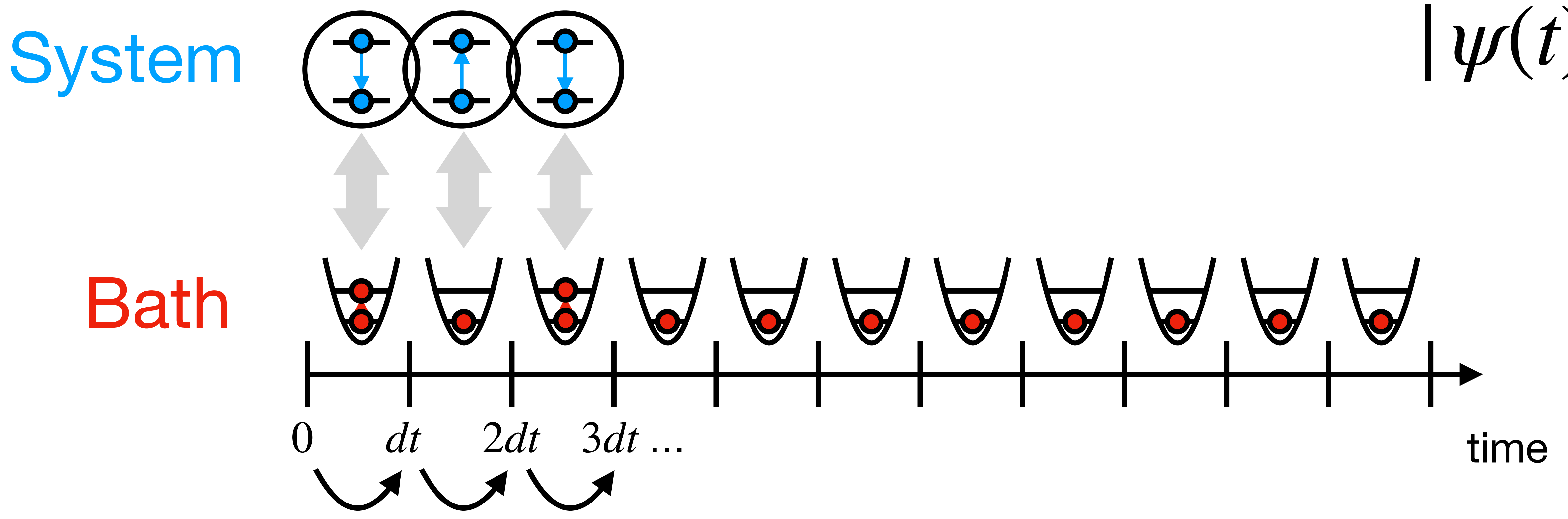
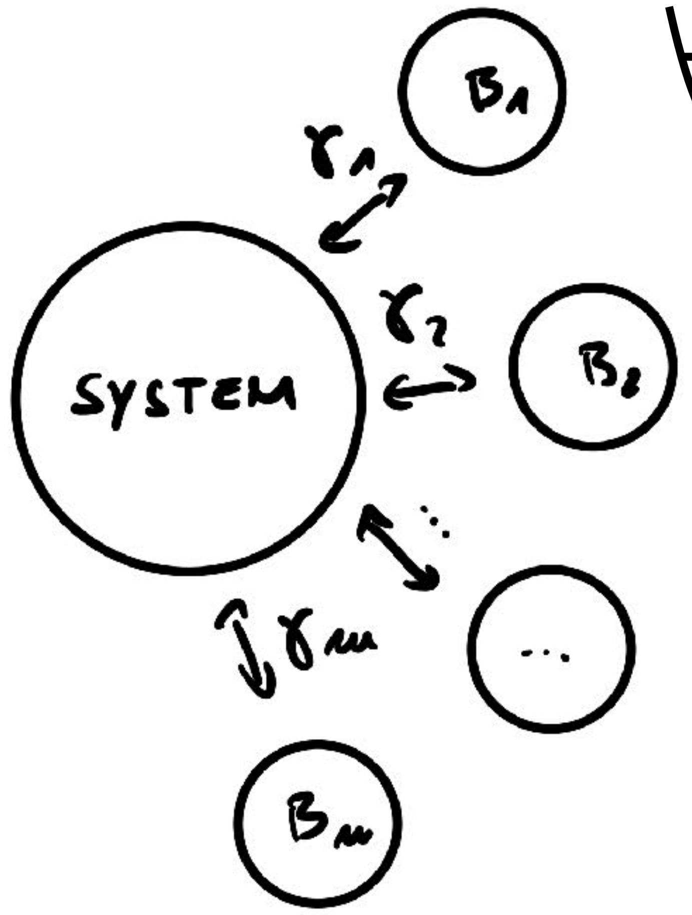
Interpretation:

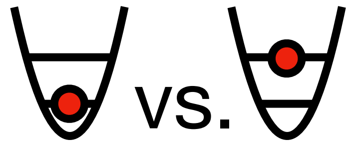


$$|\psi(t)\rangle = |\phi(t)\rangle|\text{vac}\rangle$$
$$+\int dt_1|\phi(t|t_1)\rangle b^\dagger(t_1)|\text{vac}\rangle$$
$$+\int dt_2\int dt_1|\phi(t|t_1,t_2)\rangle b^\dagger(t_2)b^\dagger(t_1)|\text{vac}\rangle$$
$$+\ \dots$$

….Quantum Trajectories

Quantum trajectories





Conditional dynamics of system under continuous measurement of bath degrees of freedom

Example: Quantum jump method: monitor $b_i(t)^\dagger b_i(t)$

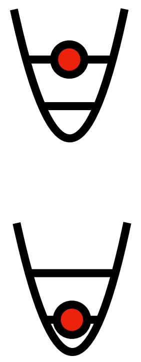
$$|\phi(t)\rangle \rightarrow |\phi(t+dt)\rangle$$

stochastic propagator

$$|\phi^1\rangle = |\phi(t)\rangle$$

for $j = 1,\ldots,m$ $\quad K_j = \sqrt{\gamma_j dt}\, c_j$ with probability $p_j = \gamma_j\, dt \langle \phi^j | c_j^\dagger c_j | \phi^j \rangle$

$|\phi^{j+1}\rangle \sim K_j |\phi^j\rangle$ $\quad K_j = e^{-\gamma_j dt c_j^\dagger c_j/2}$ with probability $1 - p_j$



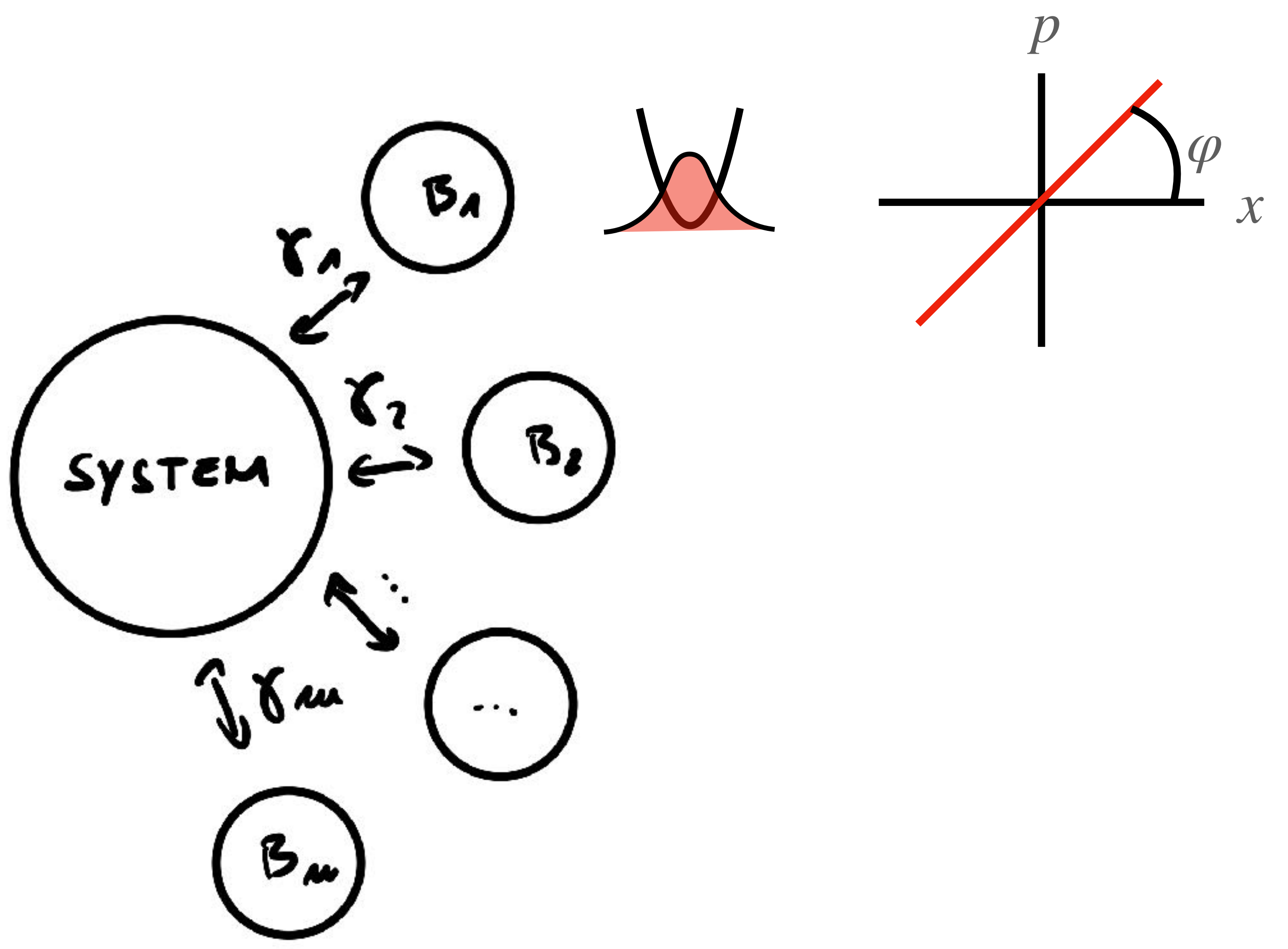
determ. prop.

$$|\phi(t+dt)\rangle = e^{-iH_{\text{sys}}dt}|\phi^{m+1}\rangle$$

repeat N times: $\rho(t) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} |\phi_k(t)\rangle\langle\phi_k(t)|$

R. Dum, P. Zoller, and H. Ritsch, Phys. Rev. A 45, 4879 (1992),
J. Dalibard, Y. Castin, and K. Mølmer, Phys. Rev. Lett. 68, 580 (1992)

Quantum trajectories



Conditional dynamics of system under continuous measurement of bath degrees of freedom

Example: Homodyne method, monitor $b_j(t)^\dagger e^{i\varphi_j} + b_j(t) e^{-i\varphi_j}$

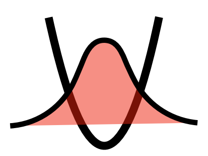
$$|\phi(t)\rangle \to |\phi(t+dt)\rangle$$

stochastic propagator

$$|\phi^1\rangle = |\phi(t)\rangle$$

for $j = 1,\ldots,m$

Wiener increments



$$d\xi_j(t) = \sqrt{\gamma_j}\langle \phi_j^{(k)}(t)|c_j e^{i\varphi_j} + c_j^\dagger e^{-i\varphi_j}|\phi_j^{(k)}(t)\rangle dt + dW_j(t)$$

$$|\phi^{j+1}\rangle \sim K_j|\phi^j\rangle \qquad K_j = e^{-\gamma_j dt c_j^\dagger c_j/2} + \sqrt{\gamma_j} c_j e^{i\varphi_j} d\xi_j(t)$$

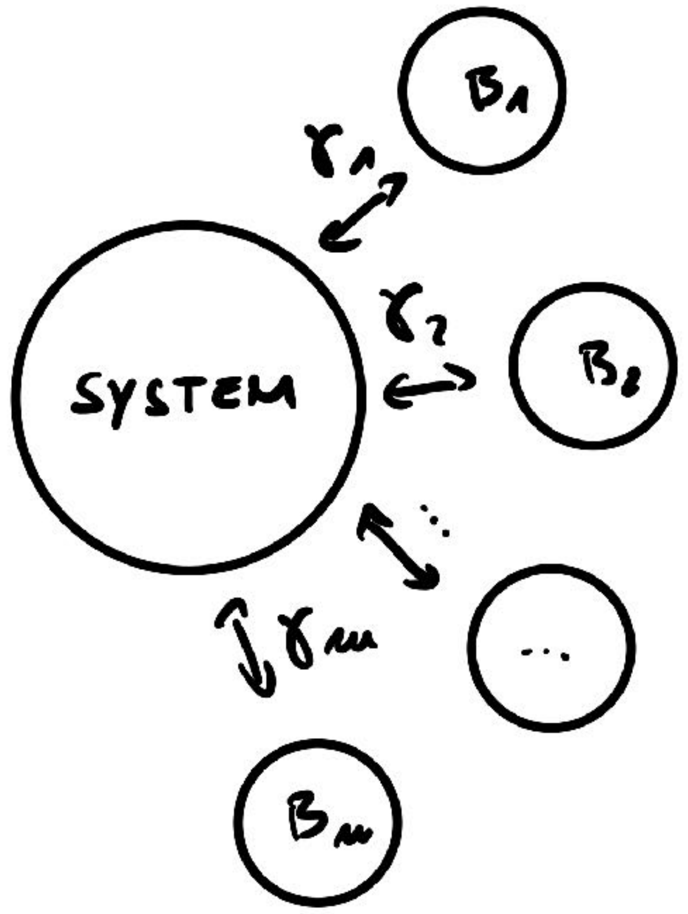
determ. prop.

$$|\phi(t+dt)\rangle = e^{-iH_{\text{sys}}dt}|\phi^{m+1}\rangle$$

repeat N times: $\rho(t) = \lim_{N\to\infty} \frac{1}{N}\sum_{k=1}^{N} |\phi_k(t)\rangle\langle\phi_k(t)|$

L. Tian and H. Carmichael, Phys. Rev. A 46, R6801 (1992).

Quantum trajectories



Conditional dynamics of system under continuous measurement of bath degrees of freedom

Example: Arbitrary local measurement $f(b_j(t), b_j(t)^\dagger, t)$

$$|\phi(t)\rangle \rightarrow |\phi(t+dt)\rangle$$

stochastic propagator

$$|\phi^1\rangle = |\phi(t)\rangle$$

for $j = 1,\ldots,m$

$$|\phi^{j+1}\rangle \sim K_j|\phi^j\rangle \qquad K_j = \ldots$$

determ. prop.

$$|\phi(t+dt)\rangle = e^{-iH_{\text{sys}}dt}|\phi^{m+1}\rangle$$

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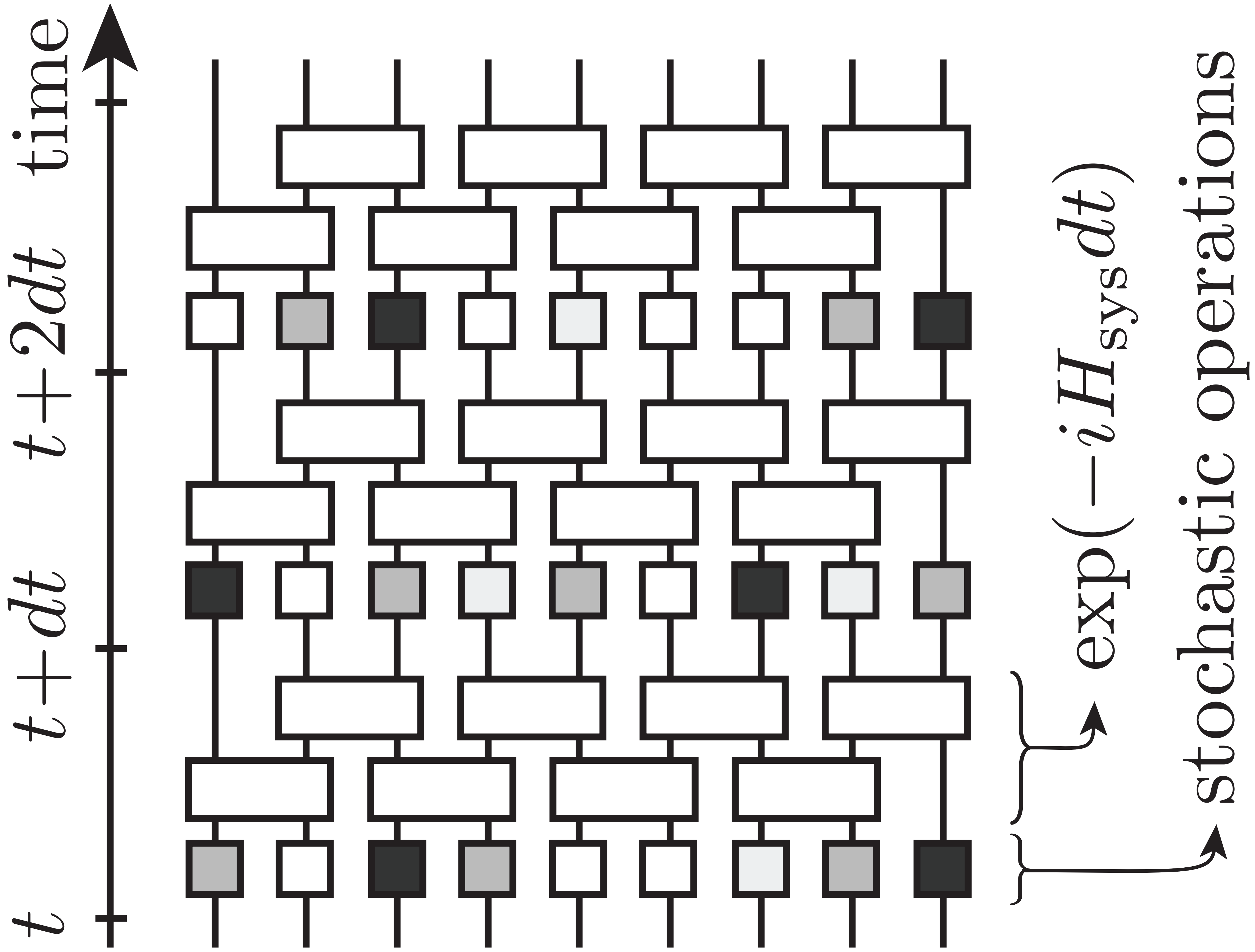
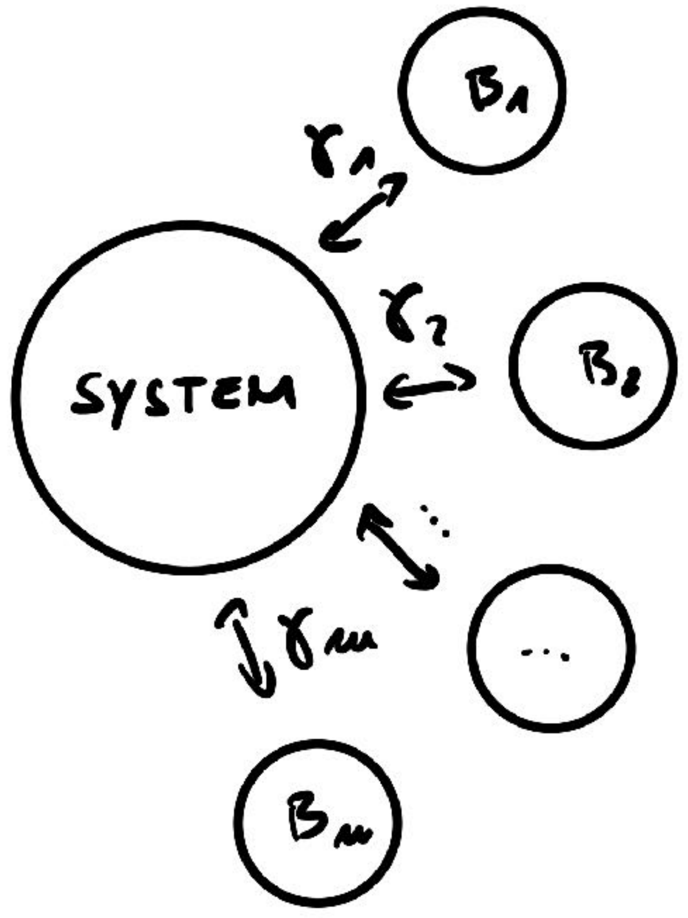
Quantum trajectories



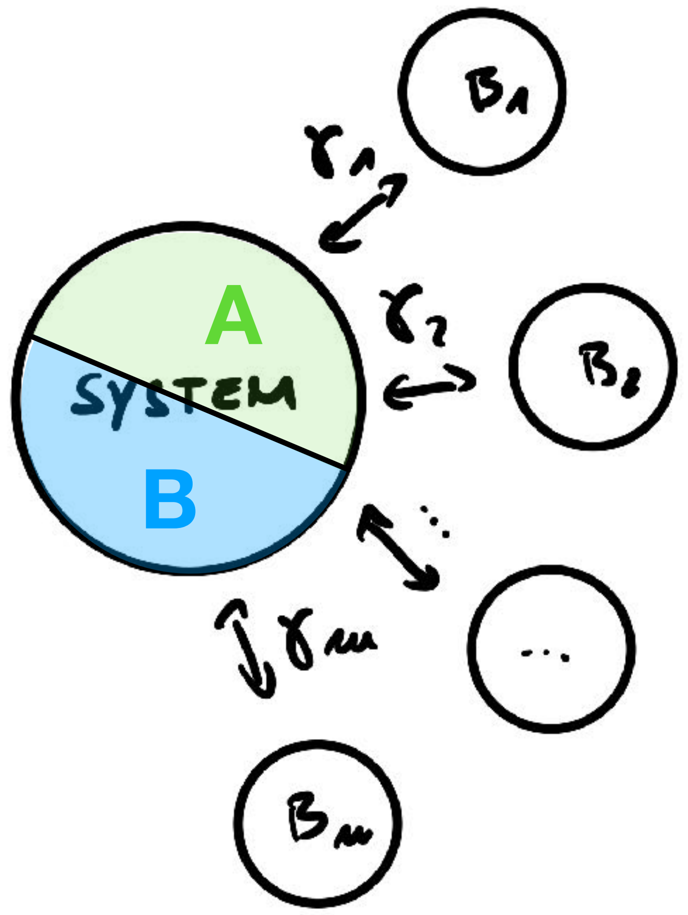


Different stochastic propagators give different ensembles of trajectories

Ensemble averages of linear state functionals coincide

Ensemble averages of non-linear quantities differ in general

Entanglement in trajectories



Bipartite entanglement entropy $E(|\phi\rangle) = S(\phi_A) = S(\phi_B)$

$\phi_A = \mathrm{tr}_B(|\phi\rangle\langle\phi|)$

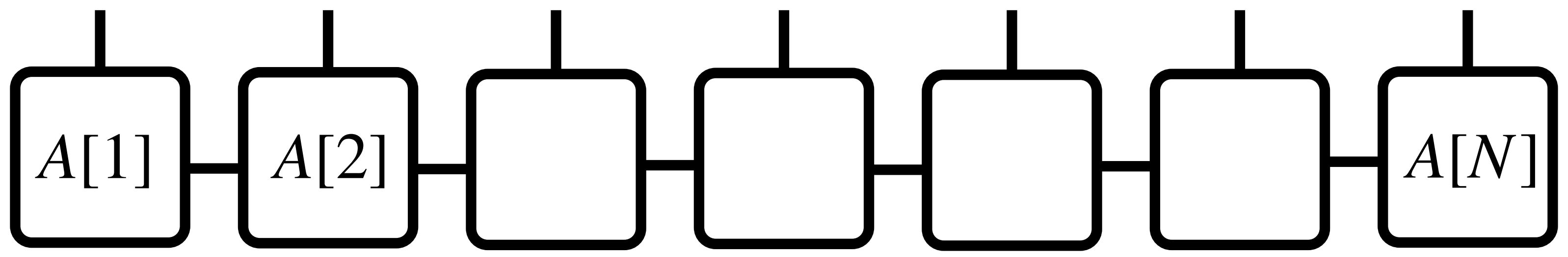
Ensemble averaged ent. entropy (EAEE)

$$\overline{E} = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} E(|\phi_k\rangle)$$

$$E_f(\rho) \leq \overline{E} \leq \min[S(\rho_A), S(\rho_B)]$$

Matrix Product States

$$|\phi(t)\rangle = \sum_{i_1, i_2, \ldots i_N} A[1]^{i_1} A[2]^{i_2} \cdots A[N]^{i_N} |i_1, i_2, \ldots, i_n\rangle$$

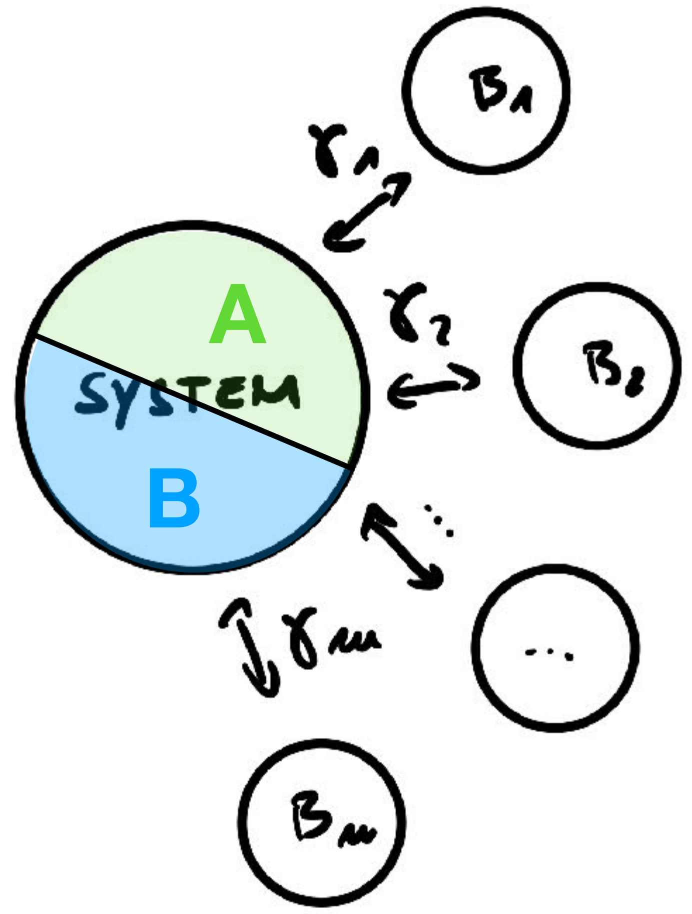


$A[k]^{i_k}$ … $\chi \times \chi$ Matrix (χ… bond dimension)

efficient if entanglement is small

S. R. White, Phys. Rev. Lett. 69, 2863 (1992).
G. Vidal, Phys. Rev. Lett. 91, 147902 (2003).

Entanglement in trajectories



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Ensemble averaged ent. entropy (EAEE)

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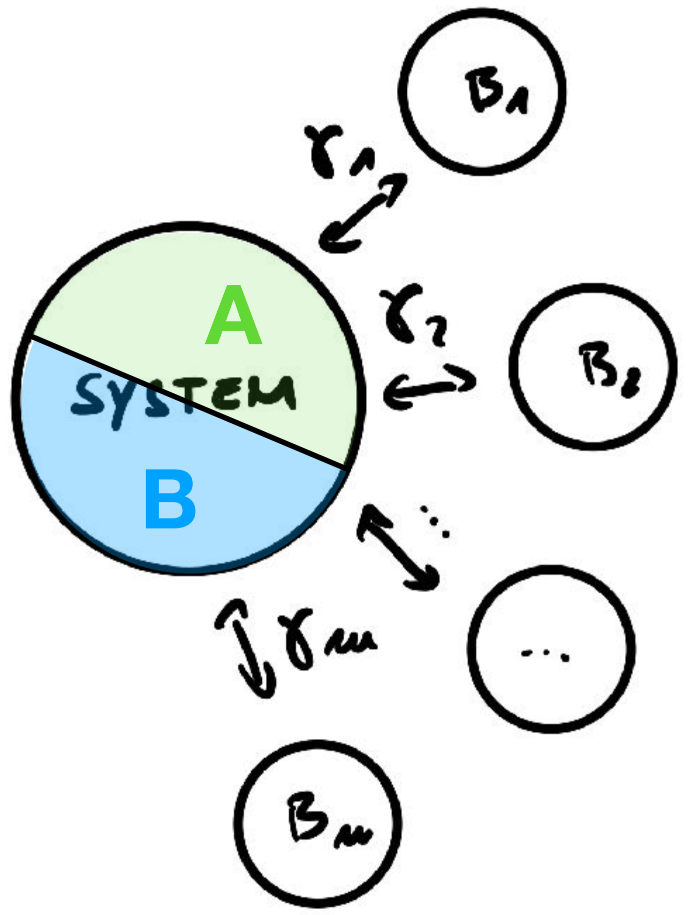
$$E_f(\rho) \leq \overline{E} \leq \min[S(\rho_A), S(\rho_B)]$$

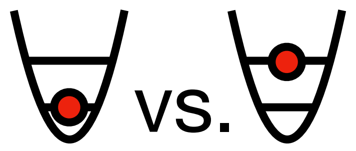
Q: Can I choose the stochastic propagator such that $\overline{E}$ is as small as possible?

Q: Can I choose the stochastic propagator such that $\dot{\overline{E}}$ is as small as possible?

Entanglement in trajectories

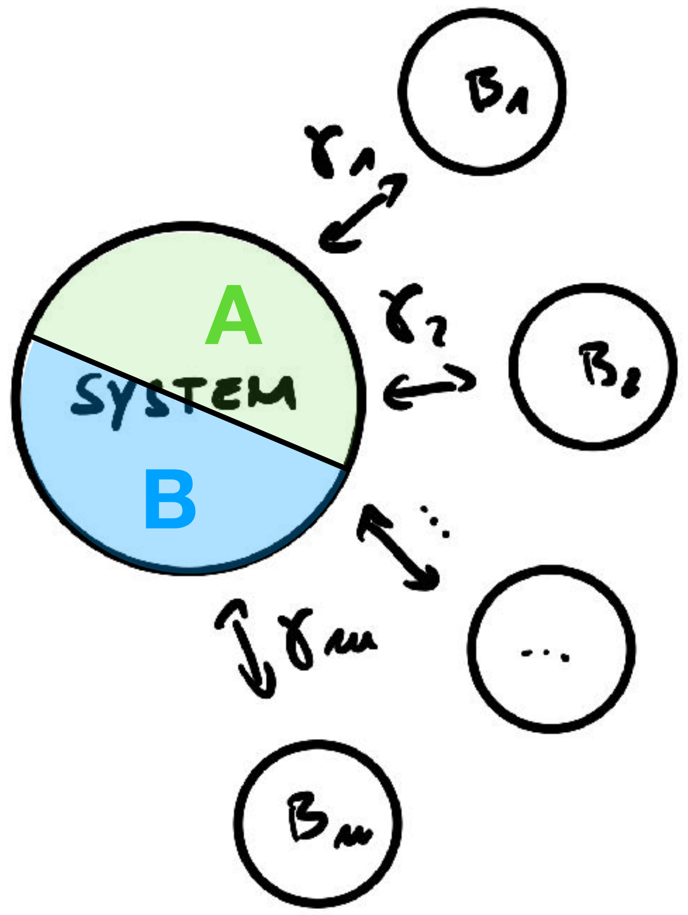


Q: Can I choose the stochastic propagator such that $\dot{\bar{E}}$ is as small as possible?

Example: monitor $b_j(t)^\dagger b_j(t)$ 

$$\dot{\bar{E}}_j^{\mathrm{num}} = \mathrm{tr}(c_j\phi c_j^\dagger)\log_2\left[\mathrm{tr}(c_j\phi c_j^\dagger)\right] + \mathrm{tr}\left(\mathrm{tr}_B(c_j\phi c_j^\dagger)\left\{\log_2\left[\mathrm{tr}_B\phi\right] - \log_2\left[\mathrm{tr}_B c_j\phi c_j^\dagger\right]\right\}\right)$$

Entanglement in trajectories



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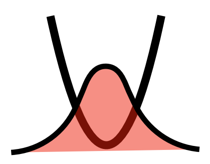
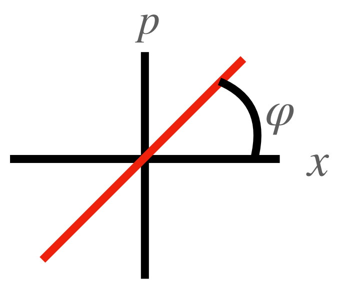
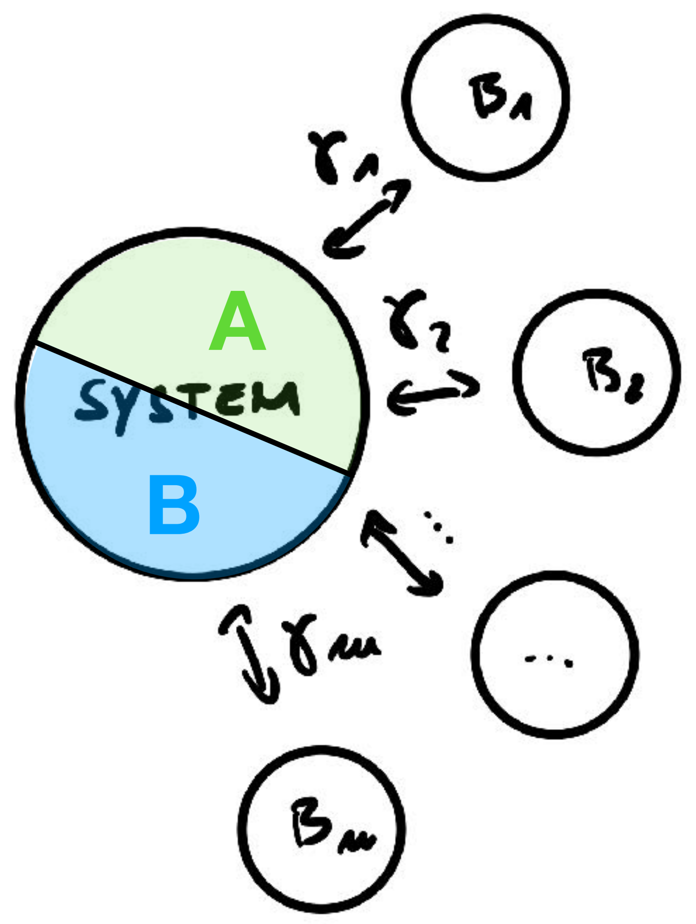
Example: monitor $b_j(t)^\dagger e^{i\varphi_j} + b_i(t) e^{-i\varphi_j}$  

$$\dot{\overline{E}}_j^{\text{hom}} = \frac{1}{2\ln 2}\left[\left| e^{-i\varphi_j}\text{tr}(c_j\phi) + \text{c.c} \right|^2 - \sum_{k,\ell} \frac{\ln(\xi_k) - \ln(\xi_\ell)}{\xi_k - \xi_\ell} \left| e^{-i\varphi_j}\langle \xi_k | \text{tr}_B(c_j\phi) | \xi_\ell \rangle + c.c. \right|^2 \right]$$

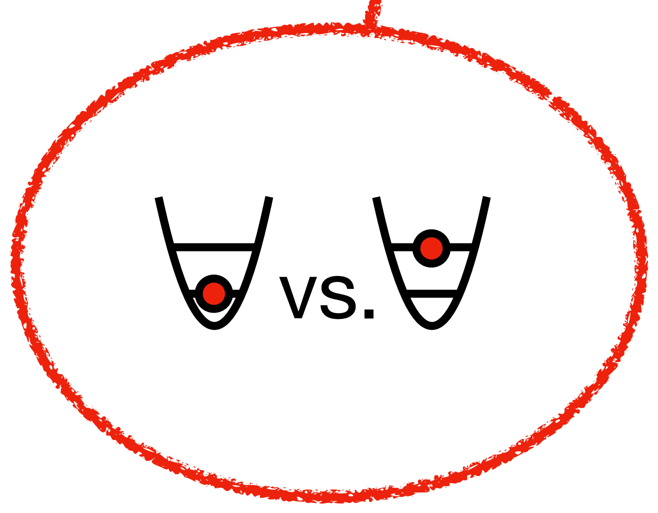
where: $\text{tr}_B(\phi)|\xi_k\rangle = \xi_k|\xi_k\rangle$ and $\phi = |\phi\rangle\langle\phi|$

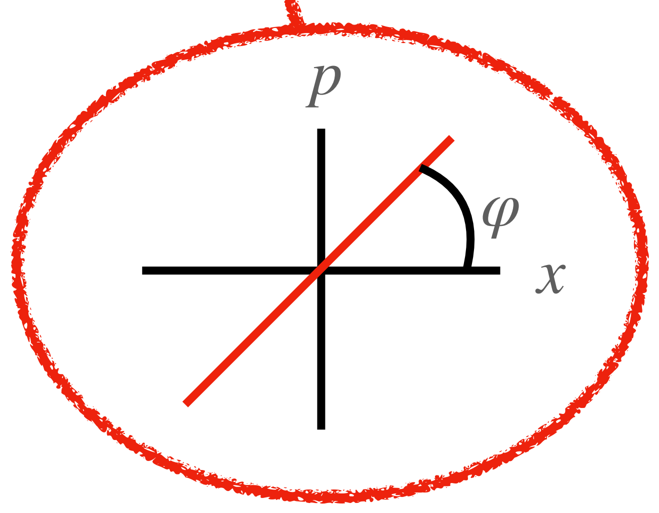
Entanglement in trajectories



Q: Can I choose the stochastic propagator such that $\dot{\bar{E}}$ is as small as possible?

One can show: $$\min_{f(b_j(t),b_j(t)^\dagger)} \dot{\bar{E}}_j = \min\left(\dot{\bar{E}}_j^{\mathrm{num}}, \min_{\varphi_j}\left(\dot{\bar{E}}_j^{\mathrm{hom}}\right)\right)$$





Details: Tatiana Vovk & HP, PRL **128**, 243601 (2022)

Adaptive greedy algorithm

Goal: $|\phi(t)\rangle \to |\phi(t+dt)\rangle$ such that $\dot{\bar{E}}$ minimal

stochastic propagator

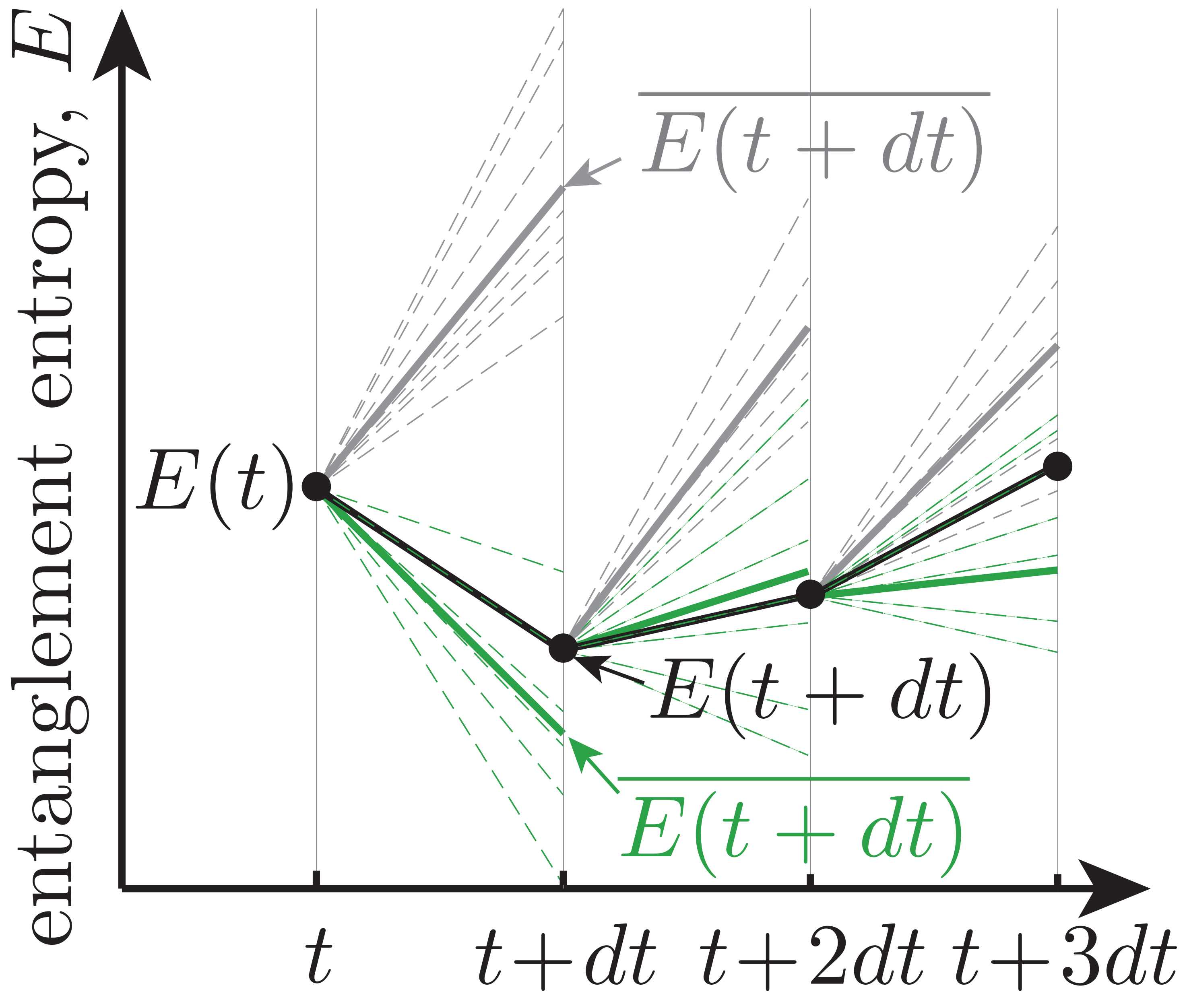
$|\phi^1\rangle = |\phi(t)\rangle$

for $j = 1,\ldots,m$

- calculate $\dot{\bar{E}}_j^{\mathrm{num}}, \min_{\varphi_j}\left(\dot{\bar{E}}_j^{\mathrm{hom}}\right)$ (cost: $\mathcal{O}(\chi^3 d)$) and choose propagator K_j accordingly
- $|\phi^{j+1}\rangle \sim K_j|\phi^j\rangle$

determ. prop.

$|\phi(t+dt)\rangle = e^{-iH_{\mathrm{sys}}dt}|\phi^{m+1}\rangle$ (cost: $\mathcal{O}(\chi^3 d^3)$)



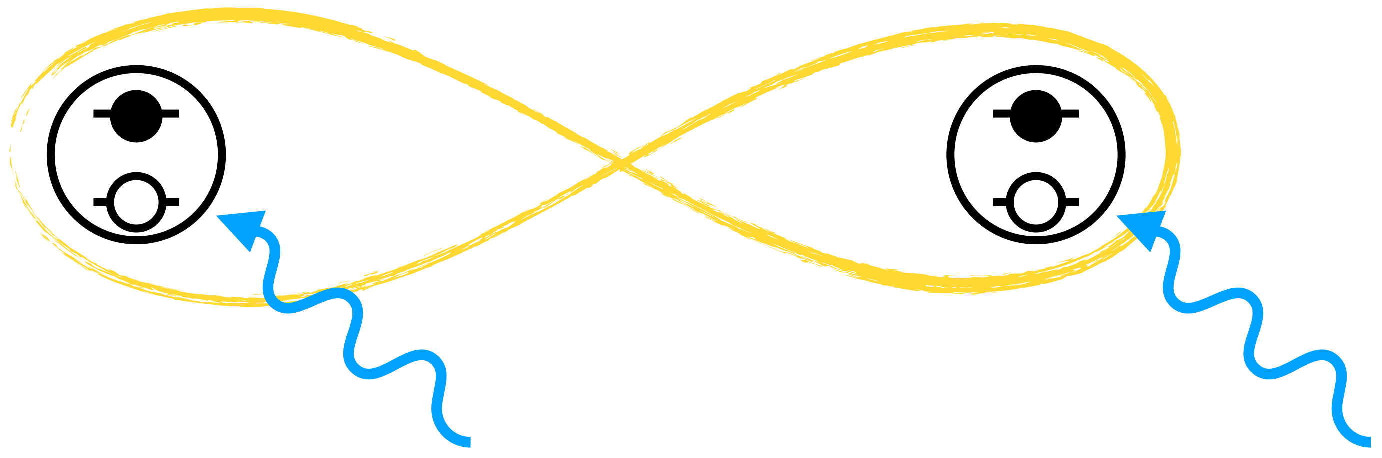
Toy example: Bell Pair

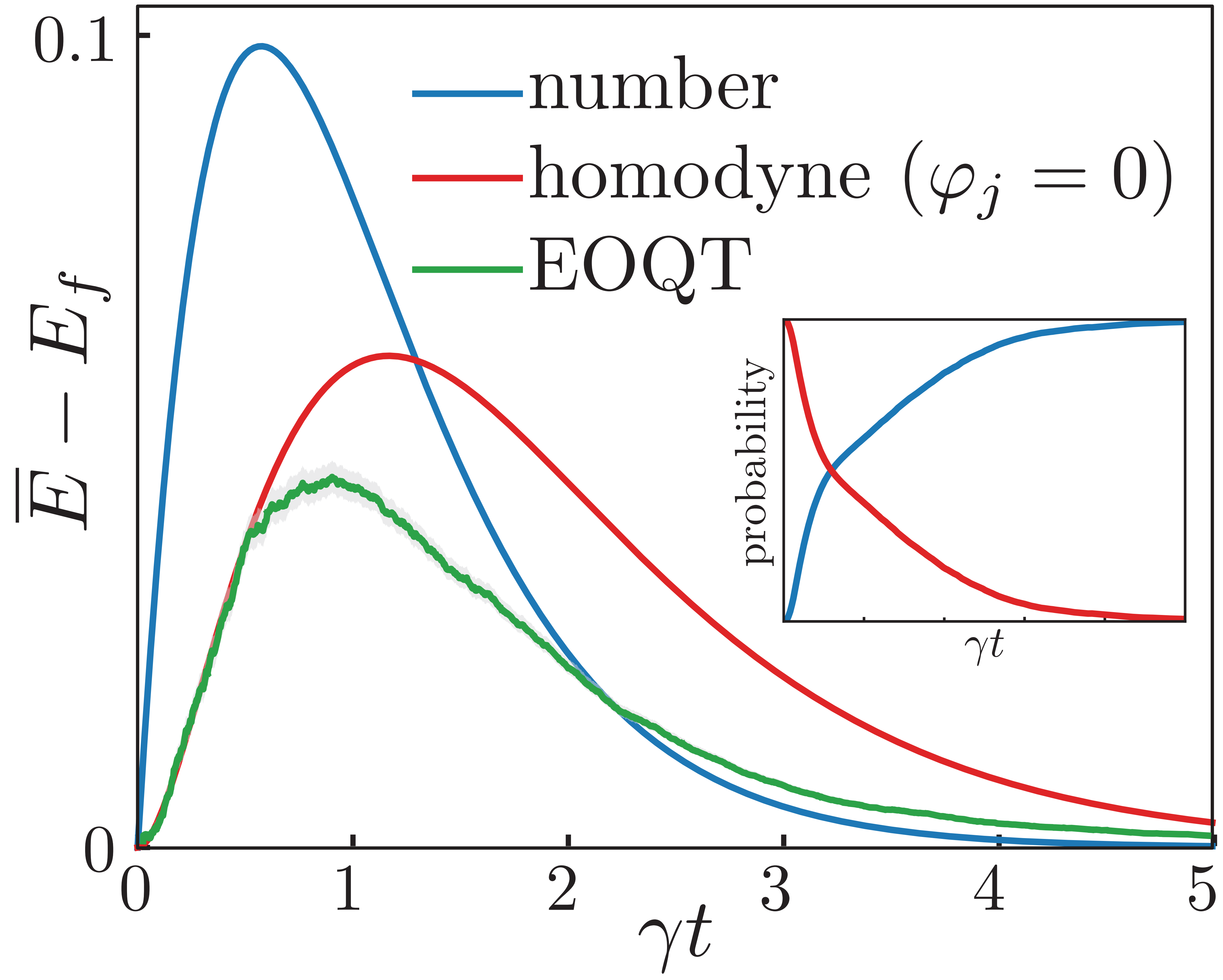
Initial state

$$|\phi(0)\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Master equation

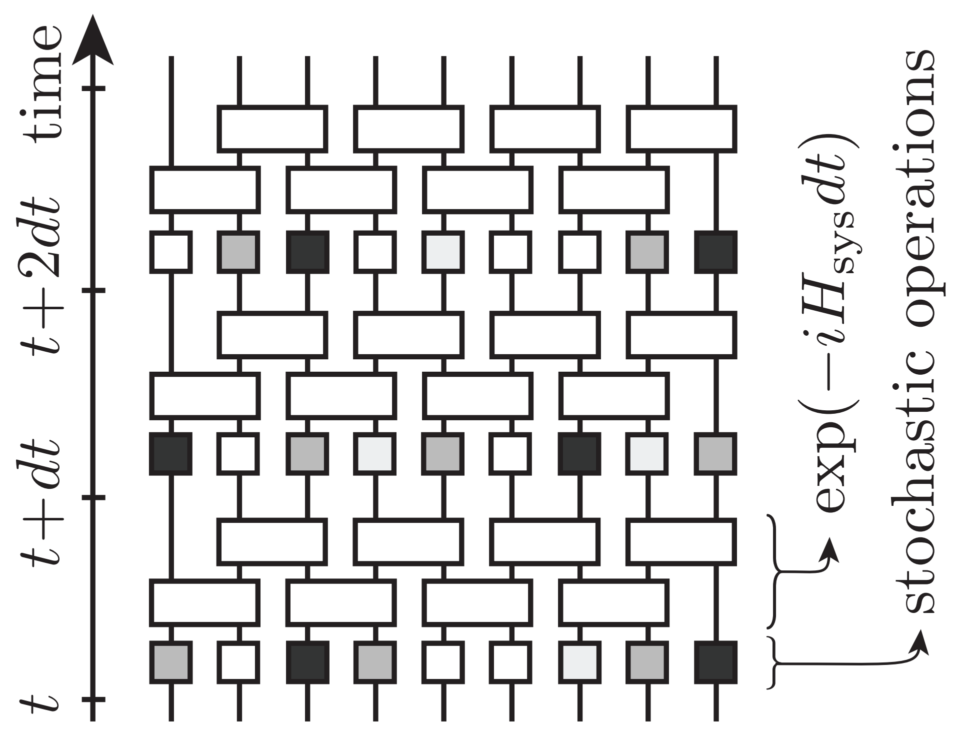
$$\frac{d\rho}{dt} = \sum_{j=1}^{2} \gamma \left(\sigma_j^z \rho \sigma_j^z - \frac{1}{2} \left\{ \rho, \sigma_j^z \sigma_j^z \right\} \right)$$





Example: Open Random Brownian Circuit

Master equation $\frac{d\rho}{dt} = -i\left[H_{\text{sys}}(t), \rho\right] + \sum_{j=1}^{m} \gamma \left(c_j \rho c_j^\dagger - \frac{1}{2}\left\{\rho, c_j^\dagger c_j\right\}\right)$



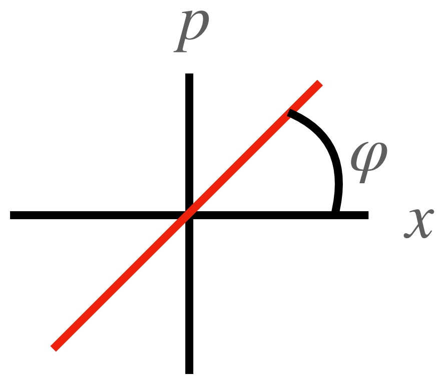
Hamiltonian $H_{\text{sys}}(t) = \sum_{j=1}^{n-1} \sum_{k,\ell=0}^{3} g_j^{k,\ell}(t) \sigma_j^k \otimes \sigma_{j+1}^\ell \, .$

Jump operators $c_j = \sigma_j^z$

Homodyne propagator: $K_j = \exp\left[\cos(\varphi_j)\sqrt{\gamma}\, d\xi_j(t) \sigma_j^z\right]$ $\exp\left[i \sin(\varphi_j)\sqrt{\gamma}\, d\xi_j(t)\sigma_j^z\right]$

non-unitary

unitary

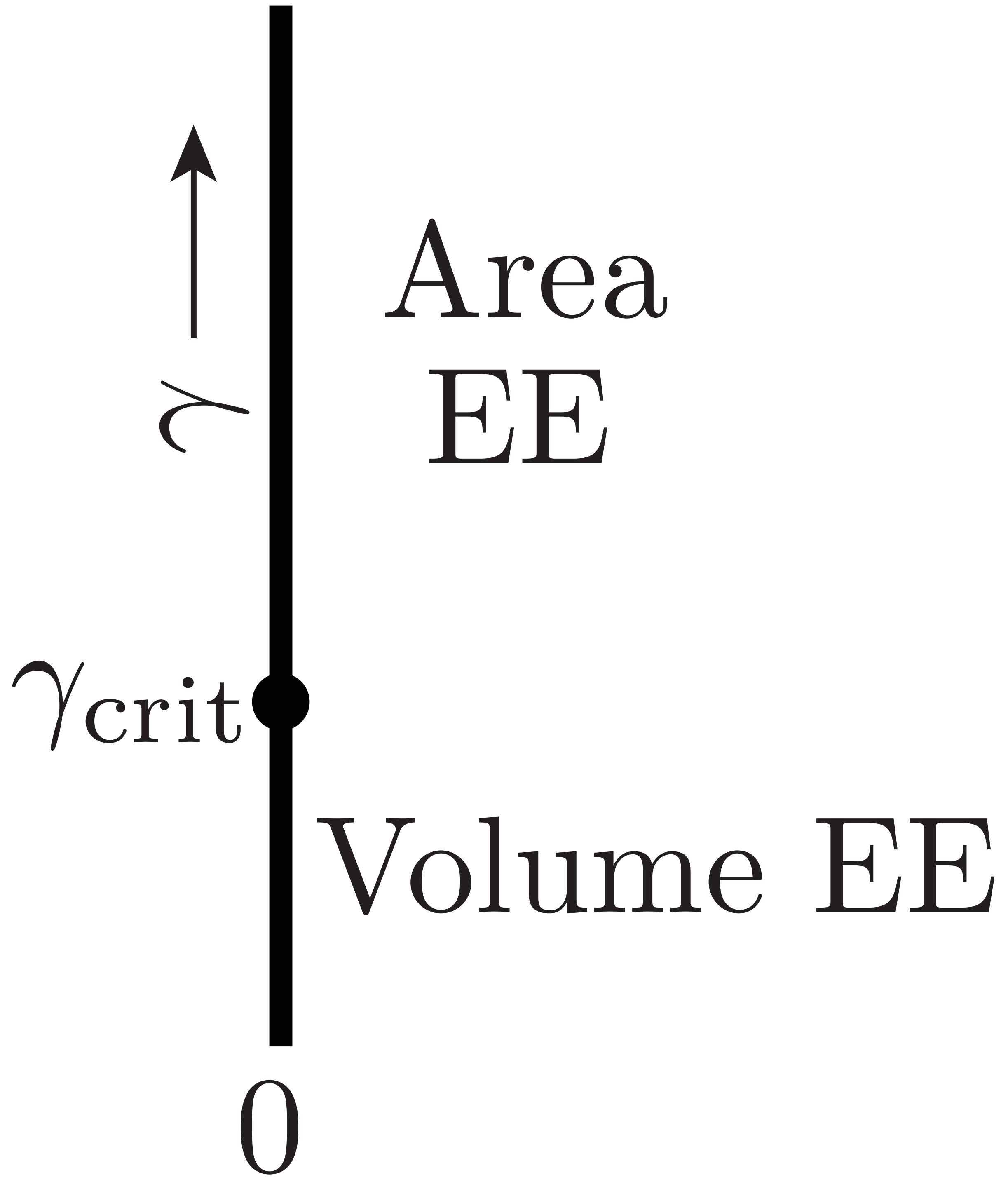


Changing measurement at fixed rate = changing rate at fixed measurement

$$\gamma \cos^2(\varphi) = \gamma_{\text{eff}}$$

Example: Open Random Brownian Circuit

Measurement induced phase transition



Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev. B 98, 205136 (2018). B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X 9, 031009 (2019). A. Chan, R. M. Nandkishore, M. Pretko, and G. Smith, Phys. Rev. B 99, 224307 (2019). M. J. Gullans and D. A. Huse, Phys. Rev. Lett. 125, 070606 (2020) S. Choi, Y. Bao, X.-L. Qi, and E. Altman, Phys. Rev. Lett. 125, 030505 (2020). M. Ippoliti, M. J. Gullans, S. Gopalakrishnan, D. A. Huse, and V. Khemani, Phys. Rev. X 11, 011030 (2021). T. Müller, S. Diehl, and M. Buchhold, arXiv:2105.08076 (2021)

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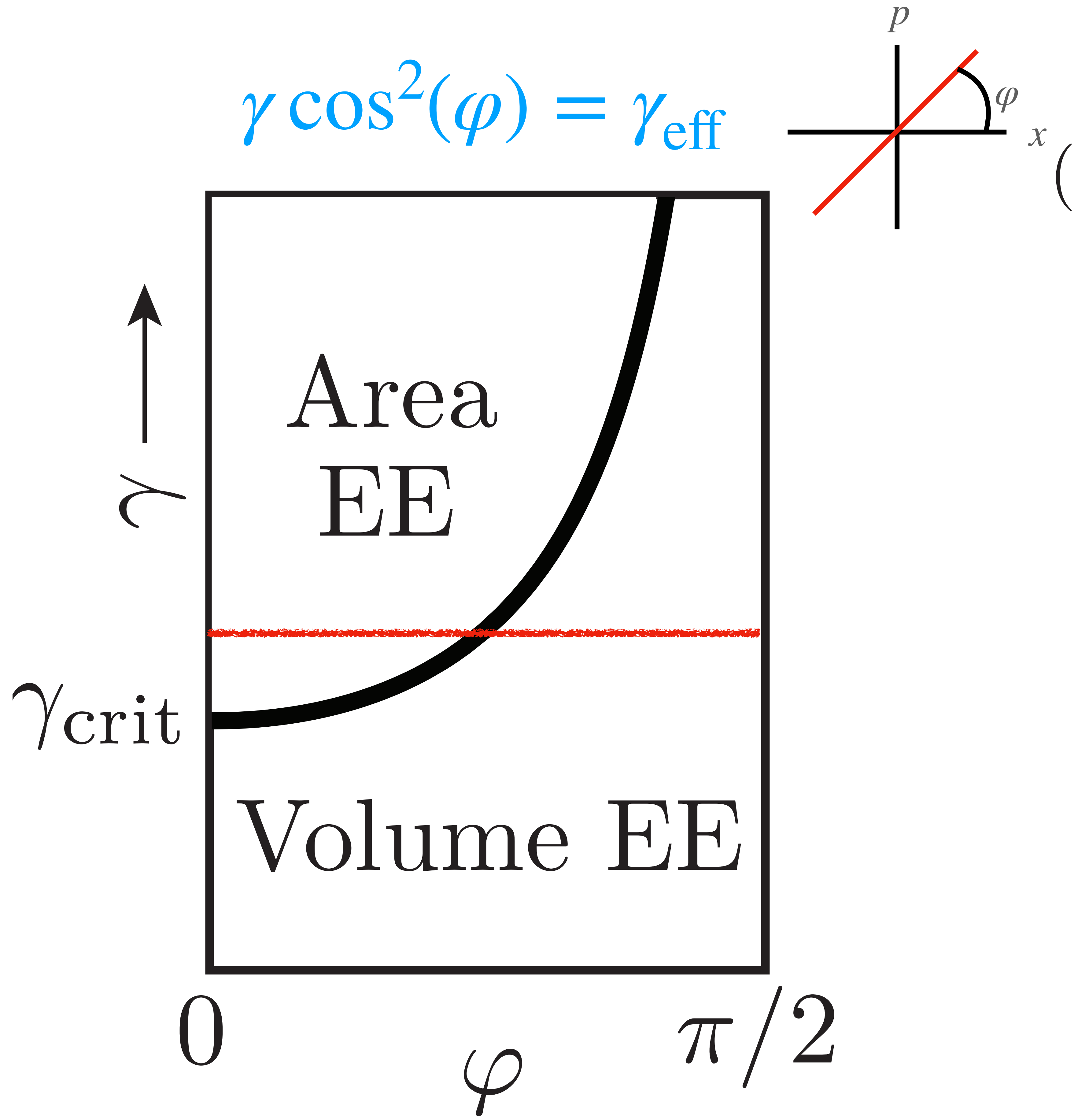

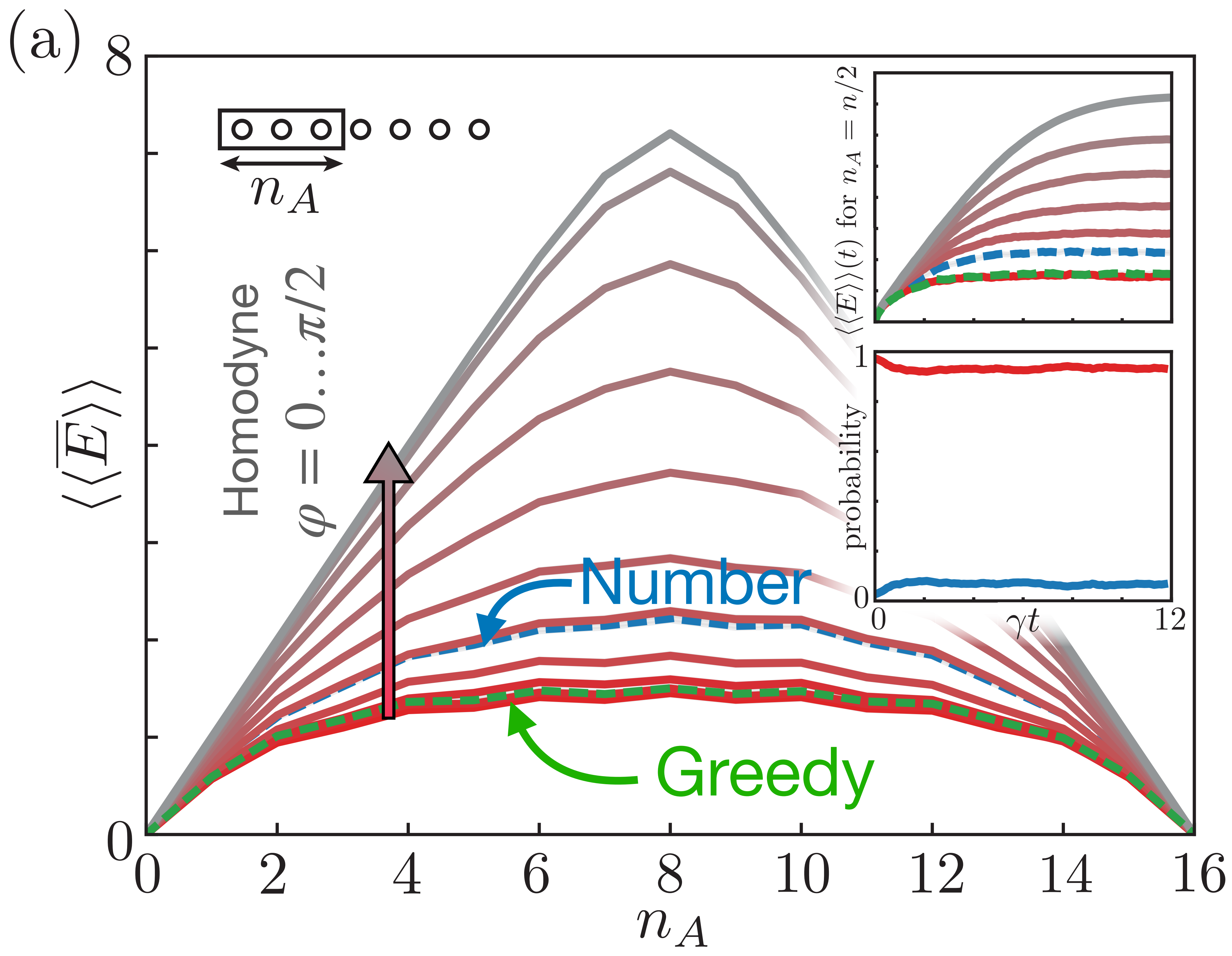

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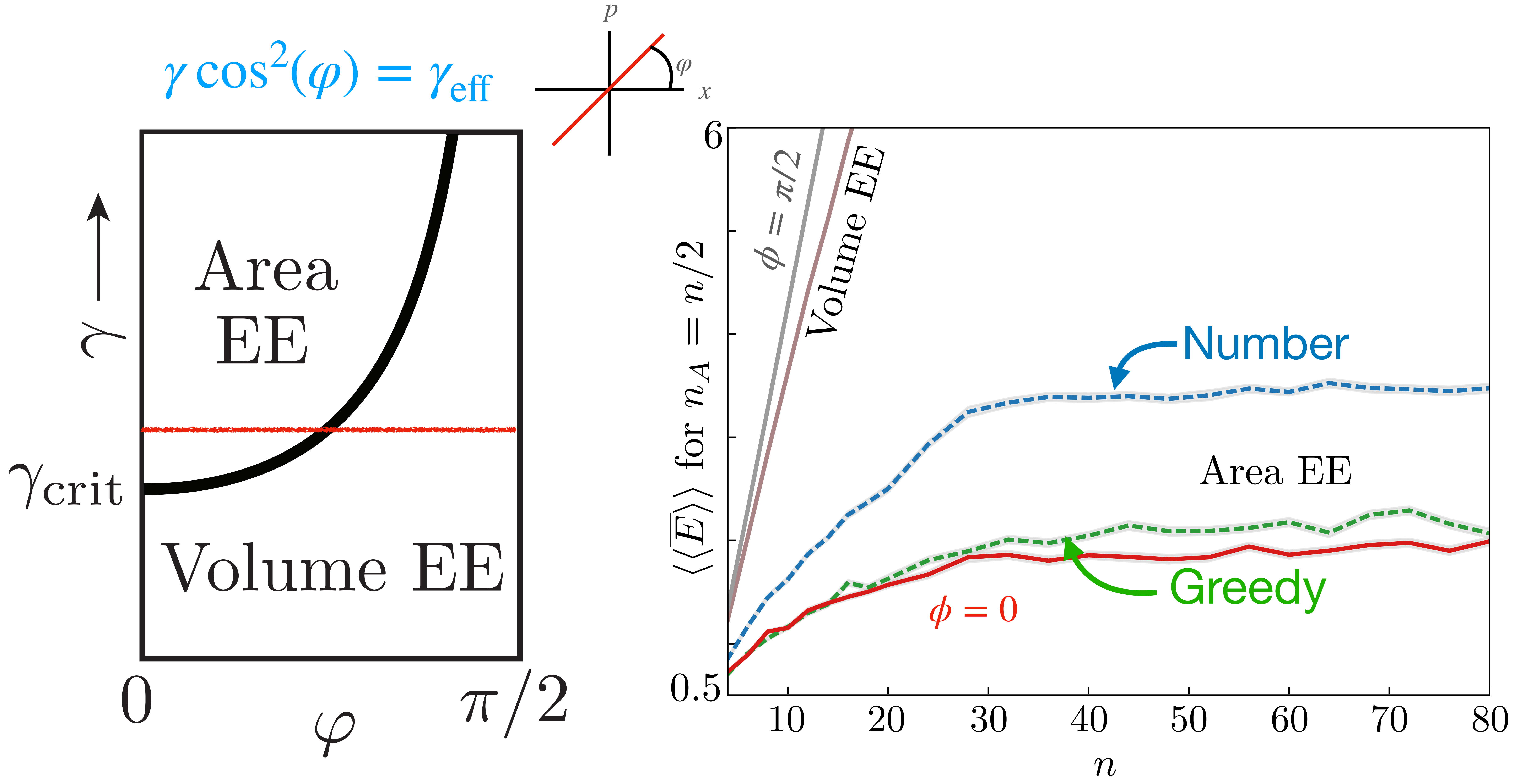
Example: Open Random Brownian Circuit

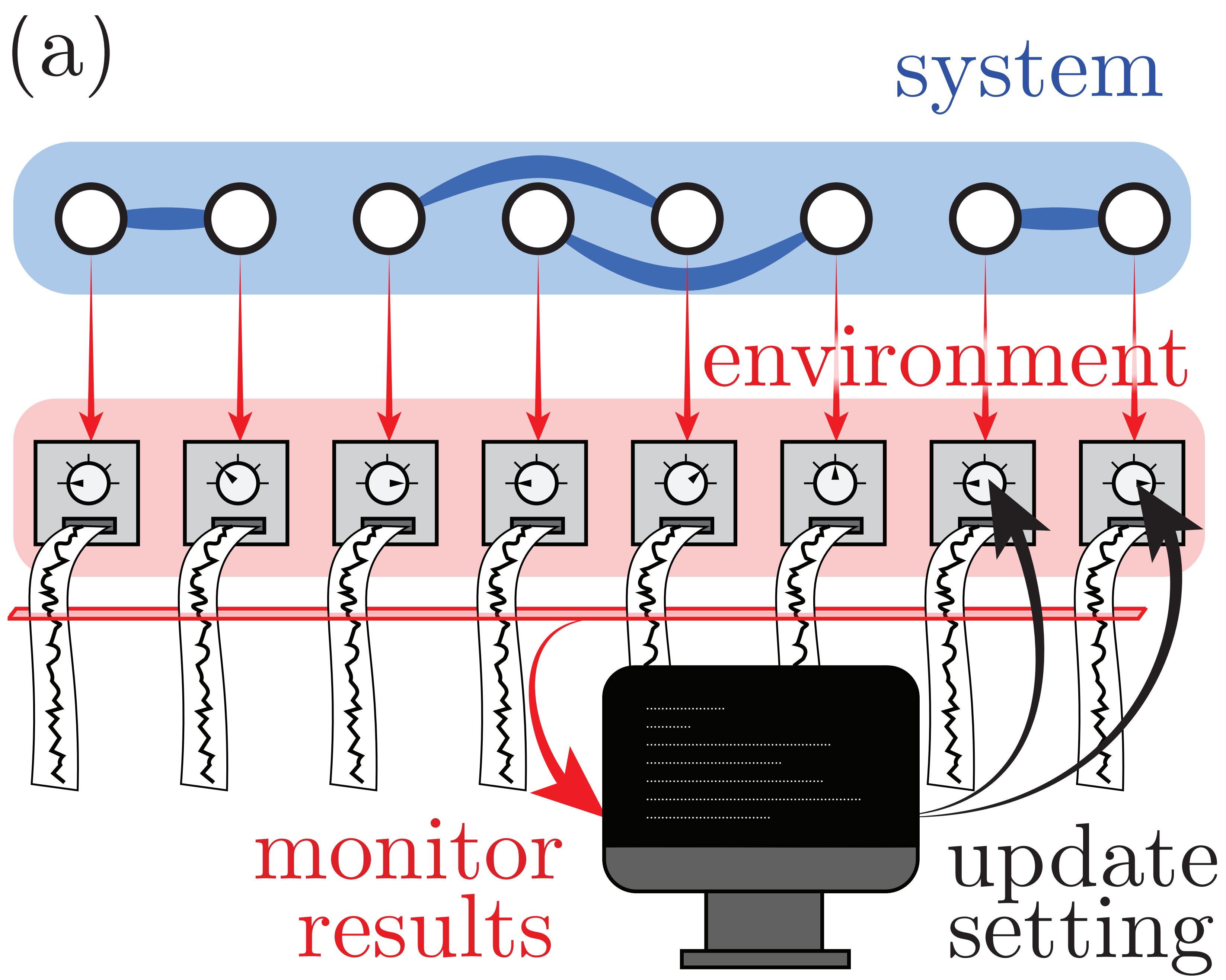


Y. Li, X. Chen, and M. P. A. Fisher, Phys. Rev. B 98, 205136 (2018). B. Skinner, J. Ruhman, and A. Nahum, Phys. Rev. X 9, 031009 (2019). A. Chan, R. M. Nandkishore, M. Pretko, and G. Smith, Phys. Rev. B 99, 224307 (2019). M. J. Gullans and D. A. Huse, Phys. Rev. Lett. 125, 070606 (2020) S. Choi, Y. Bao, X.-L. Qi, and E. Altman, Phys. Rev. Lett. 125, 030505 (2020). M. Ippoliti, M. J. Gullans, S. Gopalakrishnan, D. A. Huse, and V. Khemani, Phys. Rev. X 11, 011030 (2021). T. Müller, S. Diehl, and M. Buchhold, arXiv:2105.08076 (2021)

Summary


Tatiana Vovk

- ☑ Adaptive stochastic propagator to minimize entanglement in trajectories (continuous) time quantum Markov processes
- ☑ Can find area law unravelling (if existent)
- ☐ So far: local measurements only (time and space). Can we do better?
- ☐ Comparison with other methods (MPO,…)?



Details: Tatiana Vovk & HP, PRL **128**, 243601 (2022)