Applications of Exceptional Field Theory to the AdS/CFT correspondence

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with Bobev, Galli, Giambrone, Guarino, Josse, Nicolai, Petrini, Robinson, Samtleben, Sterckx, Trigiante, van Muiden, Waldram

Geometry of Compactification = Physics



E.g.

- $\blacktriangleright Moduli \rightarrow massless \ scalar \ fields$
- $\blacktriangleright \ \ \mathsf{Kaluza}\text{-}\mathsf{Klein} \ \mathsf{modes} \to \mathsf{massive} \ \mathsf{fields}$

Ricci-flat vs flux compactifications

Ricci-flat



- 🕨 Moduli 🗸
- ▶ Uplift 4-d theory \rightarrow Calabi-Yau data \checkmark
- Quantum corrections (some control)

 $\mathsf{SUSY} \to$ "Special holonomy" spaces, Calabi-Yau

Ricci-flat vs flux compactifications

Flux compactification



- 🕨 Moduli 🗡
- ▶ Uplift 4-d theory \rightarrow Geometric data X
- Quantum corrections X

Not "special holonomy" spaces: Fluxes \rightarrow torsion

Exceptional Field Theory

AdS/CFT correspondence

All known AdS vacua of String Theory are flux compactifications!



Crucial quantitative tool for strongly-coupled CFTs

Conformal manifold & anomalous dimensions

 $\mathsf{Moduli} \Leftrightarrow (\mathsf{exactly}) \mathsf{ marginal deformations}$



Kaluza-Klein masses \Leftrightarrow anomalous dimensions

Lower-dimensional "toy models" vs String Theory

Use lower-dimensional gauged supergravity to study AdS

- Only subset of fluctuations and masses
- (Misleading?) subset of conformal manifold





FIG. 2. Mass spectrum of scalars.

Two goals:

Uplift *D*-dim gauged supergravity to String Theory

Spectrum of masses \Leftrightarrow anomalous dimensions

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Uplift *D*-dim gauged supergravity to String Theory

Spectrum of masses \Leftrightarrow anomalous dimensions



Dangers of trusting lower-dimensional supergravity!

Exceptional Field Theory & consistent truncations



Kaluza-Klein spectroscopy



Exceptional field theory & consistent truncations



Which gauged supergravities actually arise from String Theory?



Consistent truncation

No well-controlled AdS vacua of String Theory have scale separation



FIG. 2. Mass spectrum of scalars.

 $\frac{\text{Consistent truncation}}{\text{All solutions of lower-dim. theory}} \rightarrow \text{solutions of } 10\text{-d}/11\text{-d SUGRA}$

Consistent truncation

Non-linear embedding of lower-dimensional supergravity into 10-/11-d supergravity

- ▶ All solutions of lower-d SUGRA \rightarrow solutions of 10-/11-d SUGRA
- Non-linearity: highly non-trivial!
- Symmetry arguments crucial for consistency & construction

Consistent truncation on group manifold





Consistent truncation on group manifold



Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+2}x \sqrt{|g|} \left(R_g - (\nabla \phi)^2 - e^{\alpha \phi} F^2 \right)$$



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Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+3}x \sqrt{|G|} (R_G)$$



Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+3}x \sqrt{|G|} (R_G)$$



Consistent truncations beyond group manifolds





[de Wit, Nicolai '82]

Exceptional Field Theory

[Berman, Perry '10], [Coimbra, Strickland-Constable, Waldram '11], [Hohm, Samtleben '13], ...

11-d SUGRA on $M_4 \times C_7$:

$$\{g, C_{(3)}, C_{(6)}, \ldots\} = \mathcal{M}_{MN} \in \frac{E_{7(7)}}{SU(8)}.$$

 $\begin{array}{rcl} \mbox{Diffeo} + \mbox{gauge transf} & \rightarrow & \mbox{generalised vector field } V^M \in {\bf 56} \mbox{ of } E_{7(7)} \\ & \mbox{Lie derivative } \rightarrow & \mbox{generalised Lie derivative} \end{array}$

 $\mathcal{L}_{V} = V^{M} \partial_{M} - (\partial \times_{adj} V) = \text{diffeo} + \text{gauge transf},$

with $\partial_M = (\partial_i, \partial^{ij}, \partial^{ijklm}, \ldots) = (\partial_i, 0, \ldots, 0).$

Exceptional Field Theory = reformulation of supergravity

Exceptional Field Theory: Reformulation of 10-/11-d supergravity

$$\{g, C_{(3)}, C_{(6)}, \ldots\} = \mathcal{M}_{MN}$$

$$L=R-rac{1}{48}F_{\mu
u\lambda
ho}F^{\mu
u\lambda
ho}+\ldots$$

with $F_{\mu\nu\rho\lambda} = 4\partial_{[\mu}C_{\nu\rho\lambda]}$.

Exceptional Field Theory = reformulation of supergravity

Exceptional Field Theory: Reformulation of 10-/11-d supergravity

$$\{g, C_{(3)}, C_{(6)}, \ldots\} = \mathcal{M}_{MN}$$

$$L = R - \frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} + \dots$$
$$= \mathcal{M}^{MN} \partial_M \mathcal{M}^{PQ} \partial_N \mathcal{M}_{PQ} + \dots$$

Exceptional Field Theory = reformulation of supergravity

Exceptional Field Theory: Reformulation of 10-/11-d supergravity

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Generalised Lie derivative \Rightarrow generalised Ricci scalar

Exceptional Field Theory and consistent truncations

Consistent truncations captured by "generalised group manifolds" in ExFT



$$U_A^M \in E_{7(7)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

Exceptional Field Theory and consistent truncations

$\label{eq:consistent truncations captured by $$ "generalised Leibniz parallelisable manifolds" in ExFT $$$



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Consistent truncations on $S^p,\ H^{p,q},\ S^p\times S^q,\ \ldots$ to maximal gSUGRA

$$\mathcal{M}_{MN}(x,Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$











Warped compactifications with few/no remaining (super-)symmetries





Multiplication by $E_{7(7)}$ matrix!

Less SUSY consistent truncation \longleftrightarrow "generalised special holonomy"

[Cassani, Galli, Josse, EM, Petrini, Samtleben, Vall Camell, Waldram, ...]



String Theory origin \Rightarrow Constraints on lower-dim theories



E.g.

- Symmetric scalar manifolds
- Small number of matter multiplets
- No-go theorems for gauge groups



Consistent truncation:

- Lower-dimensional theory
- Compute subset of masses for any vacuum!



Consistent truncation:

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- Compute subset of masses for any vacuum!



FIG. 2. Mass spectrum of scalars.



Consistent truncation:

- Lower-dimensional theory
- Compute subset of masses for any vacuum!


Kaluza-Klein spectroscopy



Traditional Kaluza-Klein spectroscopy

Traditionally:

- ► Spin-2 fields [Bachas, Estes '11] ✓
- $M_{int} = \frac{G}{H} \checkmark$
- Here: [EM, Samtleben Phys. Rev. Lett. 2020]
 - Warped compactifications with few or no remaining (super-)symmetries!
 - Spectrum along RG flow

KK spectroscopy strategy

First at max symmetric point:

 $\mathcal{M}_{MN}(x,Y) = \delta_{AB}(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$



KK spectroscopy strategy

Then at less symmetric point:

 $\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$



KK spectroscopy strategy

Then at less symmetric point:

 $\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$



Multiplication by $E_{7(7)}$ matrix, $M_{AB}(x)$!





 $\mathcal{M}_{MN}(x,Y)\in E_{7(7)}/\mathrm{SU}(8)$







Immediate mass diagonalisation!

► Lower-dim info:

$$\mathcal{L}_{U_A}U_B = X_{AB}{}^C U_C \,,$$

Higher-dim info:

$$\mathcal{L}_{U_{\mathcal{A}}}\mathcal{Y}_{\Sigma} = \mathcal{L}_{\mathcal{K}_{\mathcal{A}}}\mathcal{Y}_{\Sigma} = \mathcal{T}_{\mathcal{A}\Sigma}{}^{\Omega}\mathcal{Y}_{\Omega}.$$

Algebraic mass matrix:

$$\mathbb{M}_{I\Sigma,J\Omega}^{(\text{scalar})} = \mathbb{M}_{IJ}^{(0)} \,\delta_{\Sigma\Omega} + \delta_{IJ} \,\mathbb{M}_{\Sigma\Omega}^{(\text{spin}-2)} + \mathcal{N}_{IJ}{}^{\mathcal{C}}\mathcal{T}_{\mathcal{C},\Omega\Sigma}$$

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• Lower-dim SUGRA mass matrix $\mathbb{M}_{U}^{(0)}$

$$\mathbb{M}_{IJ}^{(0)} = \frac{1}{7} \left(X_{AE}{}^{F} X_{BE}{}^{F} + X_{EA}{}^{F} X_{EB}{}^{F} + X_{EF}{}^{A} X_{EF}{}^{B} + 7 X_{AE}{}^{F} X_{BF}{}^{E} \right) \mathcal{P}_{AD}{}^{I} \mathcal{P}_{BD}{}^{J}$$

$$+ \frac{2}{7} \left(X_{AC}{}^{E} X_{BD}{}^{E} - X_{AE}{}^{C} X_{BE}{}^{D} - X_{EA}{}^{C} X_{EB}{}^{D} \right) \mathcal{P}_{AB}{}^{I} \mathcal{P}_{CD}{}^{J}$$

$$+ \frac{1}{6} \mathcal{P}_{AB}{}^{I} \mathcal{P}_{CD}{}^{J} X_{FA}{}^{B} X_{FC}{}^{D} .$$

Lower-dim info:

$$\mathcal{L}_{U_A}U_B = X_{AB}{}^C U_C \,,$$

Higher-dim info:

$$\mathcal{L}_{U_{\mathcal{A}}}\mathcal{Y}_{\Sigma} = \mathcal{L}_{\mathcal{K}_{\mathcal{A}}}\mathcal{Y}_{\Sigma} = \mathcal{T}_{\mathcal{A}\Sigma}{}^{\Omega}\mathcal{Y}_{\Omega}.$$

Algebraic mass matrix:

$$\mathbb{M}_{I\Sigma,J\Omega}^{(\mathrm{scalar})} = \mathbb{M}_{IJ}^{(0)} \, \delta_{\Sigma\Omega} + \delta_{IJ} \, \mathbb{M}_{\Sigma\Omega}^{(\mathrm{spin}-2)} + \mathcal{N}_{IJ}{}^{\mathcal{C}} \mathcal{T}_{\mathcal{C},\Omega\Sigma}$$

Lower-dim SUGRA mass matrix M⁽⁰⁾_{IJ}
 Spin-2 mass matrix M^(spin-2)_{ΣΩ} = T_{A,ΣΛ}T_{A,ΛΩ}

Lower-dim info:

$$\mathcal{L}_{U_A}U_B = X_{AB}{}^C U_C \,,$$

Higher-dim info:

$$\mathcal{L}_{U_{A}}\mathcal{Y}_{\Sigma} = L_{K_{A}}\mathcal{Y}_{\Sigma} = \mathcal{T}_{A\Sigma}{}^{\Omega}\mathcal{Y}_{\Omega}.$$

Algebraic mass matrix:

$$\mathbb{M}_{I\Sigma,J\Omega}^{(\mathrm{scalar})} = \mathbb{M}_{IJ}^{(0)} \, \delta_{\Sigma\Omega} + \delta_{IJ} \, \mathbb{M}_{\Sigma\Omega}^{(\mathrm{spin}-2)} + \mathcal{N}_{IJ}{}^{\mathcal{C}} \mathcal{T}_{\mathcal{C},\Omega\Sigma}$$

- Lower-dim SUGRA mass matrix $\mathbb{M}_{II}^{(0)}$
- Spin-2 mass matrix $\mathbb{M}_{\Sigma\Omega}^{(\mathrm{spin}-2)} = \mathcal{T}_{A,\Sigma\Lambda}\mathcal{T}_{A,\Lambda\Omega}$

Key object:

$$\mathcal{N}_{IJ}{}^{C} = -4(X_{CA}{}^{B} + 12X_{AB}{}^{C})\mathcal{P}^{AD}{}_{[I}\mathcal{P}^{BD}{}_{J]}.$$

KK spectroscopy at less symmetric point



 $\mathsf{KK}\;\mathsf{Ansatz}=\mathsf{consistent}\;\mathsf{truncation}\,\otimes\,\mathsf{scalar}\;\mathsf{harmonics}$

KK spectroscopy at less symmetric point



KK Ansatz = consistent truncation \otimes scalar harmonics

Multiplication by $E_{7(7)}$ matrix, $M_{AB}(x)$!

Use same harmonics as for max. symmetric point

KK Spectroscopy Summary

- No need for explicit metric
- Only scalar harmonics of maximally symmetric point (round sphere)
- ▶ ExFT KK Ansatz \implies Differential problem \rightarrow algebraic problem
- Compute full spectrum for any vacuum in consistent truncation
- Spectrum for compactifications with few/no remaining (super-)symmetries

Applications



Applications



- $1. \ {\sf Stability of non-SUSY AdS} \\$
- 2. Global properties of conformal manifold



- Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- Non-SUSY SO(3) \times SO(3) AdS₄ vacuum [Warner '83]







- Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
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Ex 1. Perturbative stability?

4-d "zero-mode" stability enough for 11-d perturbative stability?



FIG. 2. Mass spectrum of scalars.



Modes $\ell \leq 1$: still stable!

[EM, Nicolai, Samtleben '20]



Modes $\ell \leq 2$: tachyons!

[EM, Nicolai, Samtleben '20]







Ex 1. Kaluza-Klein instability

Higher KK modes are tachyonic! [EM, Nicolai, Samtleben '20]

- ▶ Non-SUSY SO(3) \times SO(3) AdS₄ [Warner '83] is perturbatively unstable
- "Zero-mode" stability does not guarantee perturbative stability in higher dimensions
- Related to brane-jet instability [Bena, Pilch, Warner '20]?

 Examples of perturbatively stable non-SUSY AdS₄ vacua in 10-d [Guarino, EM, Samtleben '20]
 Non-SUSY exactly marginal deformation?
 [Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

Ex 2. $\mathcal{N} = 2 \text{ AdS}_4$ family

 $[\mathsf{SO}(6) \times \mathsf{SO}(1,1)] \ltimes \mathbb{R}^{12}$ supergravity

2 moduli $(\varphi, \delta) \in \mathbb{R}^2_{\geq 0}$ in 4-d theory $\Leftrightarrow \mathcal{N} = 2$ conformal manifold [Guarino, Sterckx, Trigiante '20], [Bobev, Gautason, van Muiden '21]



Expected to be compact e.g. [Perlmutter, Rasteli, Vafa, Valenzuela, '20]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold AdS₄ × S⁵ × S¹ KK spectrum along φ direction

[Giambrone, EM, Samtleben, Trigiante '21]



$$arphi \sim arphi + rac{2\pi}{T}$$
, T radius of S^1

Ex 2. Space invaders

Higher KK modes become massless when $\varphi = \frac{p\pi}{T}$, $p \in \mathbb{Z}$ [Giambrone, EM, Samtleben, Trigiante '21]



Spectrum identical for $\varphi = \frac{2 p \pi}{T}$, $p \in \mathbb{Z}$ Spectrum differs for $\varphi = \frac{(2 p+1) \pi}{T}$, $p \in \mathbb{Z}$ Ex 2. KK spectrum along $\mathcal{N} = 2$ conformal manifold

[Giambrone, EM, Samtleben, Trigiante '21]

- $\blacktriangleright \ \varphi \in \mathbb{R}^+$ is a 4-d artefact
- $\varphi \in [0, \frac{2\pi}{T})$ in 10 dimensions
- KK spectrum as fct of φ:

$$\Delta = \frac{1}{2} + \sqrt{\frac{17}{4} + \frac{1}{2}R^2 - J(J+1) - 2k(k+1) + \ell(\ell+4) + 4(\frac{\pi n}{T} - j\varphi)^2}.$$

Lorentz spin: JSU(2) spin: kU(1)_R charge: RU(1) \subset SU(2) Cartan: jS⁵ level: ℓ S¹ level: n

KK spectrum as fct of δ: non-compact? [Bobev, Gautason, van Muiden '21], [Cesàro, Larios, Varela '21]

Ex 2. φ as complex structure deformation [Giambrone, EM, Samtleben, Trigiante '21] • φ -family: AdS₄ \times $S^5 \times S_n^1$: $S^5 \to S^3 \times S^2$ ► S^3 Hopf fibre & S_n^1 : S^2 $\begin{aligned} \tau &= \frac{i}{4\pi} - \frac{\varphi T}{2\pi} \\ \varphi &\to \varphi + \frac{2\pi}{T} \Longrightarrow \tau \to \tau - 1 \end{aligned}$ $\triangleright \varphi$ deformation: locally \rightarrow coordinate transformation Similar in other S-fold vacua [Cesaro, Larios, Varela '22]

Ex 3. Non-SUSY flat deformations

2 new flat directions $\chi_1,\,\chi_2$ of 4-D supergravity [Guarino, Sterckx '21]



Non-supersymmetric conformal manifold?
Ex 3. Evidence for non-SUSY exactly marginal deformations

Non-SUSY exactly marginal deformations not expected to exist

Evidence for a miracle

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

- Perturbative stability
- Non-perturbative stability
- $\frac{1}{N}$ corrections

 χ_1 , χ_2 deformations are locally coordinate transformations!

Ex 3. KK Spectroscopy

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

 $\mathsf{KK} \text{ spectroscopy} \to \mathsf{full} \ \mathsf{KK} \ \mathsf{spectrum}$

Perturbatively stable!

$$\Delta = \frac{3}{2} + a + \frac{1}{2}\sqrt{9 + 2\ell(\ell+4) + 4\ell_1(\ell_1+1) + 4\ell_2(\ell_2+1) + 2\left(\frac{2n\pi}{T} + j_1\chi_1 + j_2\chi_2\right)^2}$$

Position within $\mathcal{N} = 4$ multiplet: *a* SO(4) spin: ℓ_1 , ℓ_2 Charges under U(1) × U(1) Cartan: j_1 , j_2 S^5 level: ℓ S^1 level: *n*

Ex 3. Non-perturbative stability?

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

- Probe-brane analysis: T > Q
 Branes more stable than in SUSY case!
- No Ooguri-Vafa instability [Ooguri, Vafa '16]
- ▶ S^1 and S^5 protected against "bubble of nothing" [Witten '82]
- D3-brane bubble of nothing [Bomans, Cassani, Dibitetto, Petri '21] ??

Ex 3. $\frac{1}{N}$ corrections

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

Flat directions lifted by $\frac{1}{N}$ corrections?

Protection by diffeomorphism symmetry

• $\chi_1, \chi_2 \rightarrow \text{coordinate transformations (locally)}$

• χ_1 , χ_2 do not appear in diffeo-invariant quantities

Also applies to $\mathcal{N}=1$ exactly marginal deformations [Bobev, Gautason, van Muiden '21]

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Corrections from D5-instantons?

 $\textit{vol}_{\mathrm{S}^5 \times \mathrm{S}^1}$ independent of $\chi_1\text{, }\chi_2$

Conclusions

Using ExFT:

- Embedding of lower-dimensional AdS vacua into string theory
- Anomalous dimensions of operators
- ► Higher KK modes crucial for physics, e.g. compactness, stability
- Non-supersymmetric AdS/CFT?
- Non-supersymmetric exactly marginal deformations?

Open questions

- Correlation functions?
- Vacua of less SUSY gSUGRA?
- Spectrum along RG flow without consistent truncation?
- ► Λ ≥ 0?

Thank you!