Exercise

Calculate the resonant frequency of the fundamental mode in:

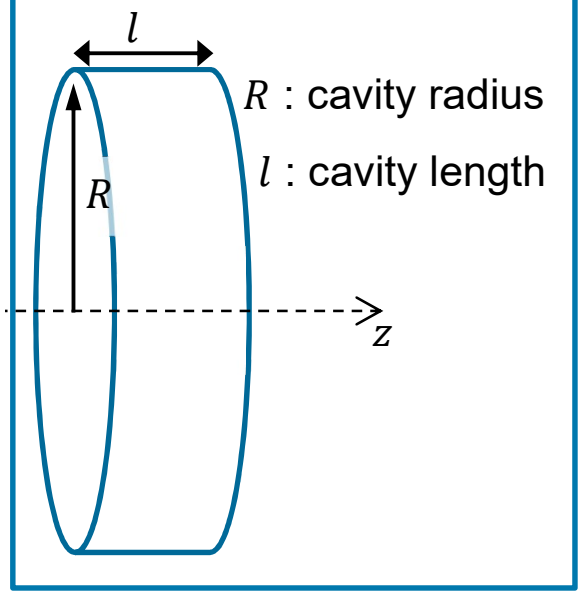
a ‘coca-cola’ tin, assume a cylindrical shape
with a diameter of 6.4 cm and a height of 12.1 cm (350 ml)

a ‘red bull’ tin, assume a cylindrical shape
with a diameter of 5.3 cm and a height of 13.5 cm (250 ml)

Calculate the diameter of the ADONE pill box cavity
for a fundamental frequency of 51 MHz

ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

boundary conditions



fundamental solution with $B_z = 0$ (that is, $\vec{B}$ is transverse)

$$E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$

$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega \frac{R}{x_{01}c^2} E_0 J_1\left(x_{01}\frac{r}{R}\right) e^{j\omega t}$$

$m = 0$: rotation symmetry of the fields

$n = 1$: no zeros of the axial field component in $\vec{r}$

$p = 0$: no variation in z of the fields

J_m : Bessel´s functions

J'_m : derivative of the Bessel´s functions

angular frequency :

$$\omega = c\frac{x_{01}}{R} \qquad x_{01} = 2.405$$

Exercise

Calculate the resonant frequency of the fundamental mode in a ‘coca-cola’ tin

assume a cylindrical shape
with a diameter of 6.4 cm and a height of 12.1 cm

$$\omega = c\frac{x_{01}}{R} = 3 \cdot 10^8 \frac{2.405}{0.032} = 2.25 \cdot 10^{10} \quad rad \cdot s^{-1}$$

$$f = \frac{\omega}{2\pi} = 3.6 \quad GHz$$

Compare it a ‘red bull’ tin: is the frequency lower or higher?
assume a cylindrical shape
with a diameter of 5.3 cm and a height of 13.5 cm (250 ml)

$$\omega = c\frac{x_{01}}{R} = 3 \cdot 10^8 \frac{2.405}{0.0265} = 2.72 \cdot 10^{10} \quad rad \cdot s^{-1}$$

$$f = \frac{\omega}{2\pi} = 4.3 \quad GHz$$

Exercise

Calculate the diameter of the ADONE pill box cavity for a fundamental frequency of 51 MHz

ADONE cavity 51 MHz (pill box)
Frascati lab, Italy

$$2\pi f = \omega = c\frac{x_{01}}{R}$$

$$R = c\frac{x_{01}}{2\pi f} = 3 \cdot 10^8 \frac{2.405}{2\pi\, 51 \cdot 10^6} = 2.25\, m$$

$$d = 2R = 4.5\, m$$