

QCD

Part 2

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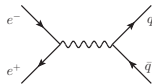
HELMHOLTZ



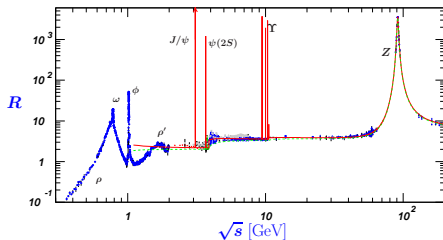
$e^+e^- \rightarrow \text{hadrons}$

$$R = \frac{\sigma(e^+e^- \rightarrow X)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

for $\sqrt{s} \gg$ resonance masses

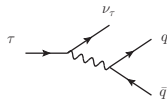


plot: Review of Particle Properties 2021



- ▶ removing electroweak part $\rightsquigarrow \sum_X |\mathcal{A}(\gamma^* \text{ or } Z^* \rightarrow X)|^2$
- ▶ among simplest applications of perturbative QCD
 - fully inclusive final state
 - no hadrons in initial state
- ▶ closely related theory description for

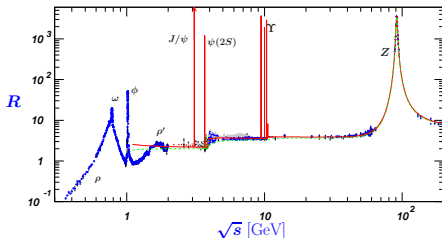
$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + X)}{\Gamma(\tau \rightarrow \nu_\tau + e\nu_e)} \rightsquigarrow \sum_X |\mathcal{A}(W^* \rightarrow X)|^2$$



at lowest order in α_s :

$$R_0 = N_c \sum_q e_q^2$$

from $\gamma^* \rightarrow q\bar{q}$ with $m_q = 0$



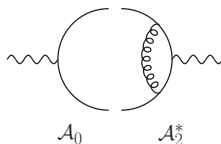
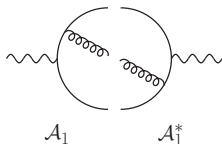
- ▶ expansion known up to $R = R_0 \left[1 + \frac{1}{\pi} \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3 + C_4 \alpha_s^4 \right]$
 - quark mass corrections also partly known
 - same for τ decays
 - suitable observables for α_s determination
- ▶ underlying concept: **parton-hadron duality**:

$$\sum_{X \in \text{partons}} |\mathcal{A}(\gamma^* \rightarrow X)|^2 = \sum_{X \in \text{hadrons}} |\mathcal{A}(\gamma^* \rightarrow X)|^2$$

- $\gamma^* \rightarrow \text{partons}$ valid description for short space-time $\sim 1/\sqrt{s}$
- subsequent dynamics changes **final state**, but **not inclusive rate**

A closer look at the $\mathcal{O}(\alpha_s)$ corrections

- ▶ expand $\mathcal{A}(q\bar{q}g) = g\mathcal{A}_1 + \dots$ and $\mathcal{A}(q\bar{q}) = \mathcal{A}_0 + g^2\mathcal{A}_2 + \dots$

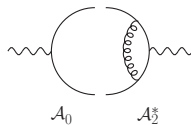
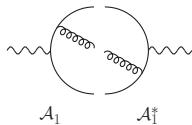


real corrections: extra partons in final state

virtual corrections: loops in \mathcal{A} or \mathcal{A}^*

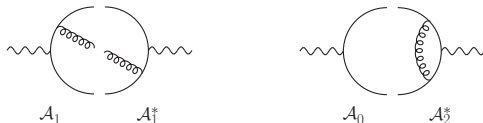
- ▶ virtual corrections have UV divergences
→ standard renormalisation procedure
- ▶ real and virtual corrections: **soft** and **collinear** divergences
 - ▶ regions where gluon momentum $\rightarrow 0$ or \propto momentum of q or \bar{q}
 - ▶ cancel in sum over all graphs

A closer look at soft and collinear divergences



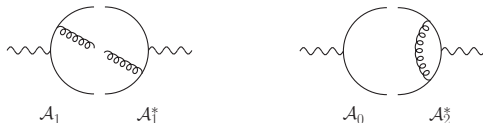
► more detail \rightsquigarrow blackboard

A closer look at soft and collinear divergences



- ▶ have soft (= IR) div. because of massless gluons
same phenomenon in QED: soft photons \rightarrow “IR catastrophe”
- ▶ have collinear (= mass) div. if set quark masses to zero
could formally keep $m_q \neq 0$, but perturbation theory
not trustworthy at scales $\sim m_u, m_d, m_s$
- ▶ divergences cancel, result dominated by large virtualities
otherwise could not use parton-hadron duality
- ▶ technical difficulty: cancellations take place in $D \neq 4$ dimensions
for differential cross sections and numerics want $D = 4$

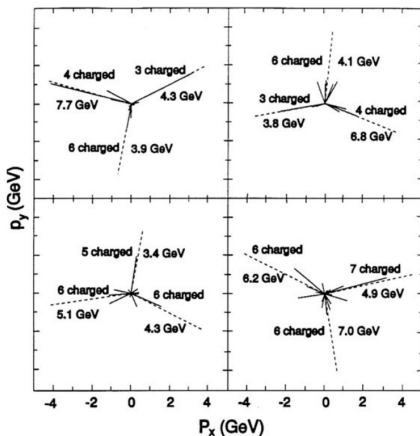
A footprint of divergences: large logarithms



- ▶ both soft and collinear divergences are logarithmic: $\int dE/E \int d\theta/\theta$
- ▶ fixing final-state momenta restricts integration region in real corrections, but not in virtual ones
 - for each emission get double logarithm $\propto \alpha_s \log^2(\dots)$
“Sudakov logarithms”
 - if logarithms are large must sum them to all orders in α_s
“resummation”
 - can be done analytically for certain cases
 - done by “parton showers” in Monte Carlo generators

Beyond inclusive final states: hadronic jets

- ▶ jet = “bunch of hadrons moving approx. in same direction”
- ▶ perhaps the most direct manifestation of quarks or gluons

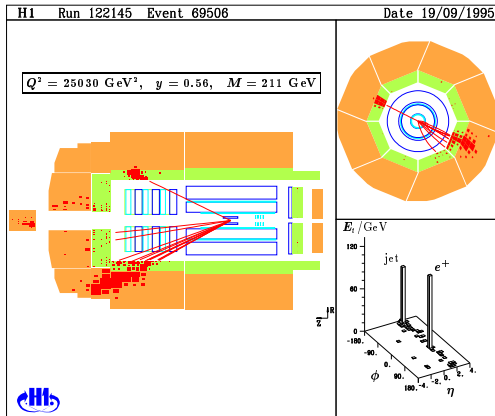


three-jet events in e^+e^-
annihilation at $\sqrt{s} = 27.4$ GeV
TASSO (DESY) 1979

figure from: P Söding,
On the discovery of the gluon
Eur.Phys.J. H 35 (2010) 3

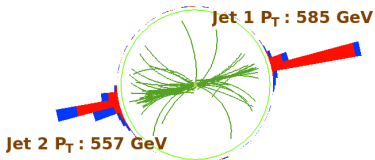
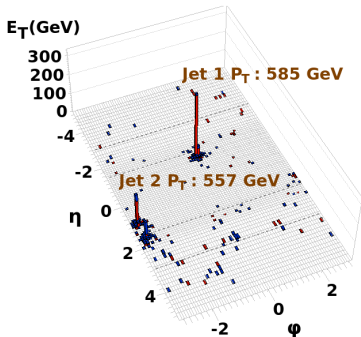
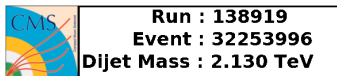
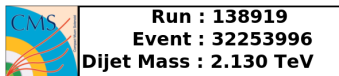
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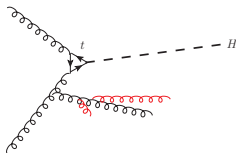
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Beyond inclusive final states: hadronic jets

- ▶ extend idea of parton-hadron duality: dynamics leading from partons (times $\sim 1/Q$) to final-state hadrons (times $\rightarrow \infty$) approx. conserves momentum (hadronisation effects $\sim \text{GeV}$)
- ▶ to minimise theory uncertainties:
 - ▶ define **hadronic jets** using an algorithm that is **not** sensitive to collinear and soft radiation (**beyond perturbative control**)

“collinear and
infrared safe
observables”

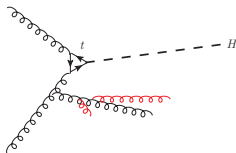


- apply to partons in computation, to hadrons in measurement
- **hadronisation corrections** should then be moderate and typically decrease with jet p_T
estimate using Monte Carlo generators

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“collinear and infrared safe observables”



- different jet definitions are used for different purposes
- **jet substructure** observables as tools to reconstruct underlying parton-level dynamics
 \rightsquigarrow active field of research

Summary of Part 2

- ▶ perturbative calculations beyond tree level only for quantities that are **IR and collinear safe** and hence dominated by large virtualities
- ▶ simplest examples: total cross sections/decay rates for colourless initial states
- ▶ for differential cross sections/distributions: can have large double logarithms from soft and collinear emissions