Introduction to Particle Physics Theory

Hyungjin Kim (DESY)

How do we describe nature?

Special Relativity + Quantum Mechanics

How do we describe nature?

Special Relativity + Quantum Mechanics

$$\downarrow$$

Quantum Field Theory (QFT)

Quantum field theory (QFT) provides a very useful tool to

- organize our knowledge

- parametrize our ignorance

Quantum field theory (QFT) provides a very useful tool to

- organize our knowledge
- parametrize our ignorance

QFT plays a crucial role to

- understand/interpret experimental data
- study the evolution of the Universe

Units

we are going to use the natural unit

$$c = \hbar = k_B = 1$$

that is

$$[\text{Energy}] = [\text{Mass}] = [\text{Temperature}] = [\text{Length}]^{-1} = [\text{Time}]^{-1}$$

Units

For instance

$$1\,\mathrm{sec} \simeq 2 \times 10^{15}\,\mathrm{eV}^{-1}$$

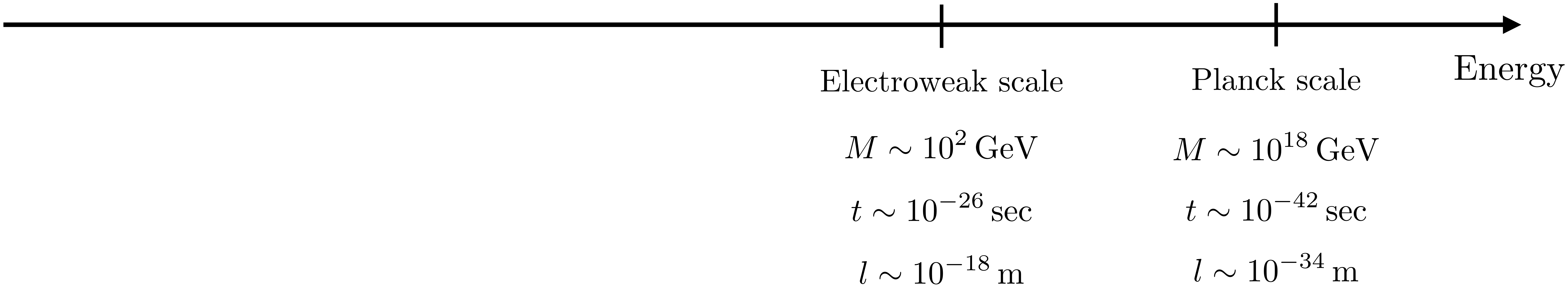
$$1\,\mathrm{meter} \simeq 5 \times 10^{6}\,\mathrm{eV}^{-1}$$

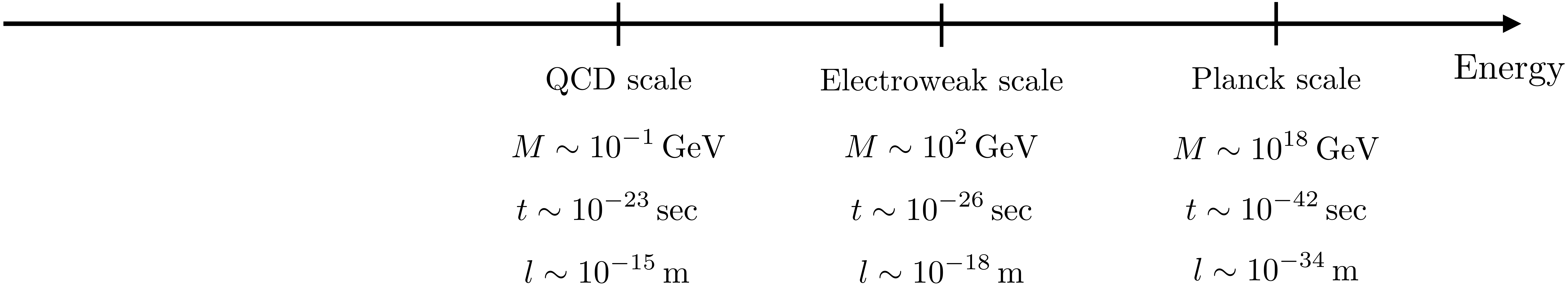
$$1\,\mathrm{gram} \simeq 6 \times 10^{32}\,\mathrm{eV}$$

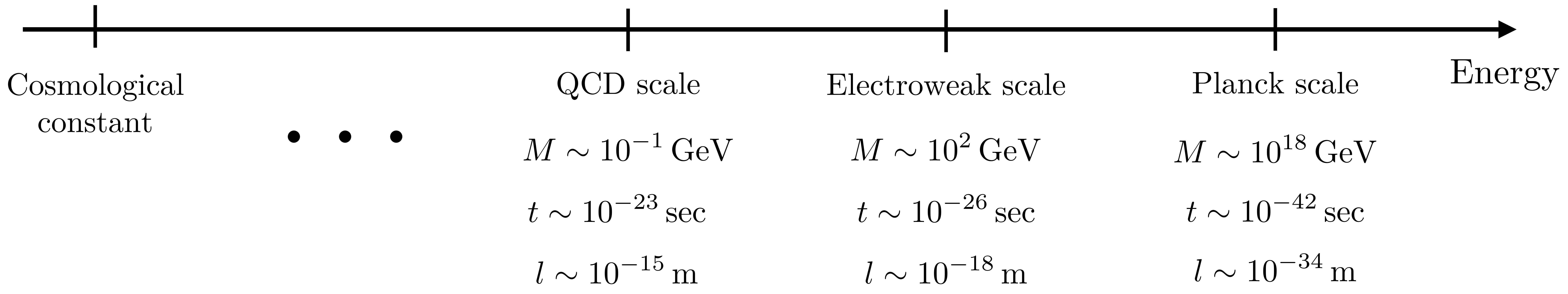
$$1\,\mathrm{Kelvin} \simeq 9 \times 10^{-5}\,\mathrm{eV}$$

we are going to measure every quantity in eV (or GeV) unit


Energy
Planck scale
$M \sim 10^{18}\,\mathrm{GeV}$
$t \sim 10^{-42}\,\mathrm{sec}$
$l \sim 10^{-34}\,\mathrm{m}$

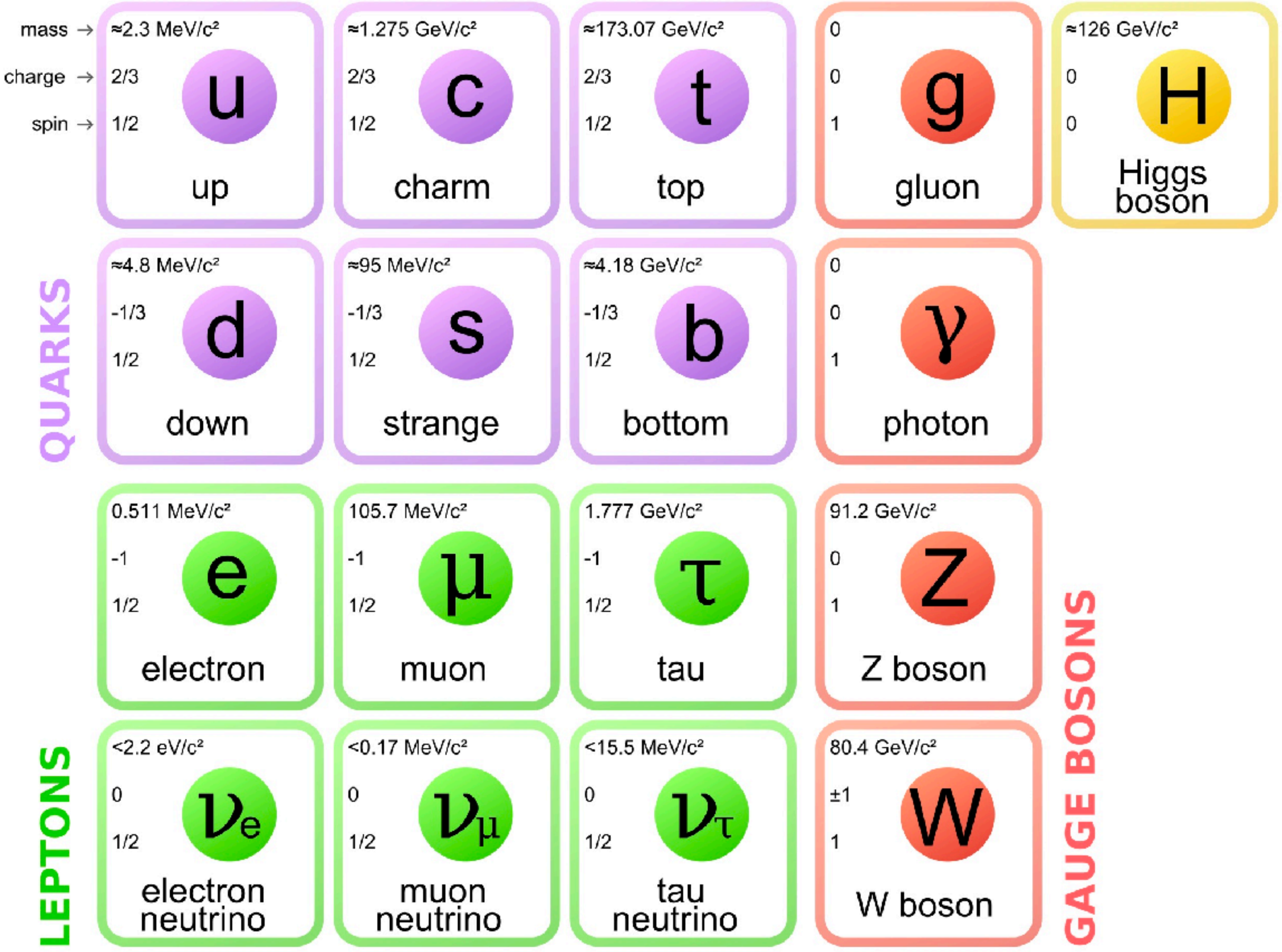

Energy
Electroweak scale
$M \sim 10^2\,\mathrm{GeV}$
$t \sim 10^{-26}\,\mathrm{sec}$
$l \sim 10^{-18}\,\mathrm{m}$
Planck scale
$M \sim 10^{18}\,\mathrm{GeV}$
$t \sim 10^{-42}\,\mathrm{sec}$
$l \sim 10^{-34}\,\mathrm{m}$


Energy
QCD scale
$M \sim 10^{-1}\,\mathrm{GeV}$
$t \sim 10^{-23}\,\mathrm{sec}$
$l \sim 10^{-15}\,\mathrm{m}$
Electroweak scale
$M \sim 10^{2}\,\mathrm{GeV}$
$t \sim 10^{-26}\,\mathrm{sec}$
$l \sim 10^{-18}\,\mathrm{m}$
Planck scale
$M \sim 10^{18}\,\mathrm{GeV}$
$t \sim 10^{-42}\,\mathrm{sec}$
$l \sim 10^{-34}\,\mathrm{m}$



Energy
Cosmological constant
QCD scale
Electroweak scale
Planck scale
$M \sim 10^{-1}\,\mathrm{GeV}$
$t \sim 10^{-23}\,\mathrm{sec}$
$l \sim 10^{-15}\,\mathrm{m}$
$M \sim 10^{2}\,\mathrm{GeV}$
$t \sim 10^{-26}\,\mathrm{sec}$
$l \sim 10^{-18}\,\mathrm{m}$
$M \sim 10^{18}\,\mathrm{GeV}$
$t \sim 10^{-42}\,\mathrm{sec}$
$l \sim 10^{-34}\,\mathrm{m}$

The Standard Model



How do we describe the elementary particles and interactions between them?

Plan

Monday 01.08.22

- Lagrangian
- Lorentz transformation
- Dimensional analysis
- beta decay/muon decay
- Fermi theory

Tuesday 02.08.22

- Gauge theory
- Electroweak interaction
- Chirality
- Spontaneous symmetry breaking
- Higgs mechanism

Lagrangian mechanics

a particle under a potential $\mathrm{V(x)}$

satisfies the equation of motion

$$m\ddot{x} + V'(x) = 0$$

Lagrangian mechanics

a particle under a potential $\mathrm{V(x)}$

satisfies the equation of motion

$$m\ddot{x} + V'(x) = 0$$

Newton's equation can be obtained from the action

$$S = \int dt\, L(x, \dot{x})$$

$$L = \frac{1}{2}m\dot{x}^2 - V(x)$$

From the least action principle

$$0 = \delta S$$

From the least action principle

$$\begin{aligned} 0 &= \delta S \\ &= \delta \int dt\, L(x, \dot{x}) \end{aligned}$$

From the least action principle

$$
\begin{aligned}
0 &= \delta S \\
&= \delta \int dt\, L(x, \dot{x}) \\
&= \int dt \left[\frac{\partial L}{\partial x} \delta x + \frac{\partial L}{\partial \dot{x}} \delta \dot{x} \right]
\end{aligned}
$$

From the least action principle

$$
\begin{aligned}
0 &= \delta S \\
&= \delta \int dt\, L(x, \dot{x}) \\
&= \int dt\, \left[\frac{\partial L}{\partial x} \delta x + \frac{\partial L}{\partial \dot{x}} \delta \dot{x} \right] \\
&= \int dt\, \delta x \left[\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \right] + \text{boundary term}
\end{aligned}
$$

From the least action principle

$$
\begin{aligned}
0 &= \delta S \\
&= \delta \int dt\, L(x, \dot{x}) \\
&= \int dt \left[\frac{\partial L}{\partial x} \delta x + \frac{\partial L}{\partial \dot{x}} \delta \dot{x} \right] \\
&= \int dt\, \delta x \left[\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \right] + \text{boundary term}
\end{aligned}
$$

we find *Euler-Lagrange equation*

$$
\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0
$$

$$L = \frac{1}{2}m\dot{x}^2 - V(x)$$

From Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

the equation of motion is reproduced

$$m\ddot{x} + V' = 0$$

Lagrangian Field Theory

Lagrangian mechanics can be extended to classical field theory

Consider Maxwell's electromagnetic theory (in free space)

$$\nabla \cdot E = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = +\frac{\partial E}{\partial t}$$

Lagrangian Field Theory

Lagrangian mechanics can be extended to classical field theory

Consider Maxwell's electromagnetic theory (in free space)

$$\nabla \cdot E = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = +\frac{\partial E}{\partial t}$$

$$E = -\nabla\phi - \frac{\partial A}{\partial t}$$

$$B = \nabla \times A$$

Lagrangian Field Theory

It can be written in a more compact form

by introducing field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Lagrangian Field Theory

It can be written in a more compact form

by introducing field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A^\mu = (\phi, A)$$

$$x^\mu = (t, \mathbf{x})$$

$$\partial_\mu = \partial/\partial x^\mu$$

$$(\mu = 0,\ 1,\ 2,\ 3)$$

$$(i = 1,\ 2,\ 3)$$

Lagrangian Field Theory

$$A^\mu = (\phi, A)$$

$$x^\mu = (t, \mathbf{x})$$

$$\partial_\mu = \partial / \partial x^\mu$$

$$(\mu = 0,\ 1,\ 2,\ 3)$$

$$(i = 1,\ 2,\ 3)$$

It can be written in a more compact form

by introducing field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

electromagnetic fields are

$$E_i = F_{0i} \qquad B_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}$$

Lagrangian Field Theory

$$A^\mu = (\phi, A)$$

$$x^\mu = (t, \mathbf{x})$$

$$\partial_\mu = \partial/\partial x^\mu$$

$$(\mu = 0,\ 1,\ 2,\ 3)$$

$$(i = 1,\ 2,\ 3)$$

It can be written in a more compact form

by introducing field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

electromagnetic fields are

$$E_i = F_{0i} \qquad B_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}$$

Maxwell's equations can be written as

$$\partial_\mu F^{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} \nabla \cdot E &= 0 \\ \nabla \times B &= +\frac{\partial E}{\partial t} \end{aligned}$$

(repeated indices are contracted)

Lagrangian Field Theory

$$A^\mu = (\phi, A)$$

$$x^\mu = (t, \mathbf{x})$$

$$\partial_\mu = \partial/\partial x^\mu$$

$$(\mu = 0,\ 1,\ 2,\ 3)$$

$$(i = 1,\ 2,\ 3)$$

It can be written in a more compact form

by introducing field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

electromagnetic fields are

$$E_i = F_{0i} \qquad B_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}$$

Maxwell's equations can be written as

$$\partial_\mu F^{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} &\nabla \cdot E = 0 \\ &\nabla \times B = +\frac{\partial E}{\partial t} \end{aligned}$$

(repeated indices are contracted)

the other two equations come from Bianchi identity

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \quad \longleftrightarrow \quad \begin{aligned} &\nabla \cdot B = 0 \\ &\nabla \times E = -\frac{\partial B}{\partial t} \end{aligned}$$

Lagrangian Field Theory

The corresponding Lagrangian is

$$\mathcal{L}(A, \partial A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Lagrangian Field Theory

The corresponding Lagrangian is

$$\mathcal{L}(A, \partial A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

The action is

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

Lagrangian Field Theory

The corresponding Lagrangian is

$$\mathcal{L}(A, \partial A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

The action is

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

Using the least action principle, we find Euler-Lagrange equation

$$0 = \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu A_\nu} - \frac{\partial \mathcal{L}}{\partial A_\nu}$$

which reproduces the Maxwell equation

Lagrangian Field Theory

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

This classical Lagrangian field theory

describes classical electromagnetism

Lagrangian Field Theory

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

This classical Lagrangian field theory

describes classical electromagnetism

When this theory is quantized

it describes a spin-1 gauge boson

which is ***photon***

Lagrangian Field Theory

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

This classical Lagrangian field theory

describes classical electromagnetism

When this theory is quantized

it describes a spin-1 gauge boson

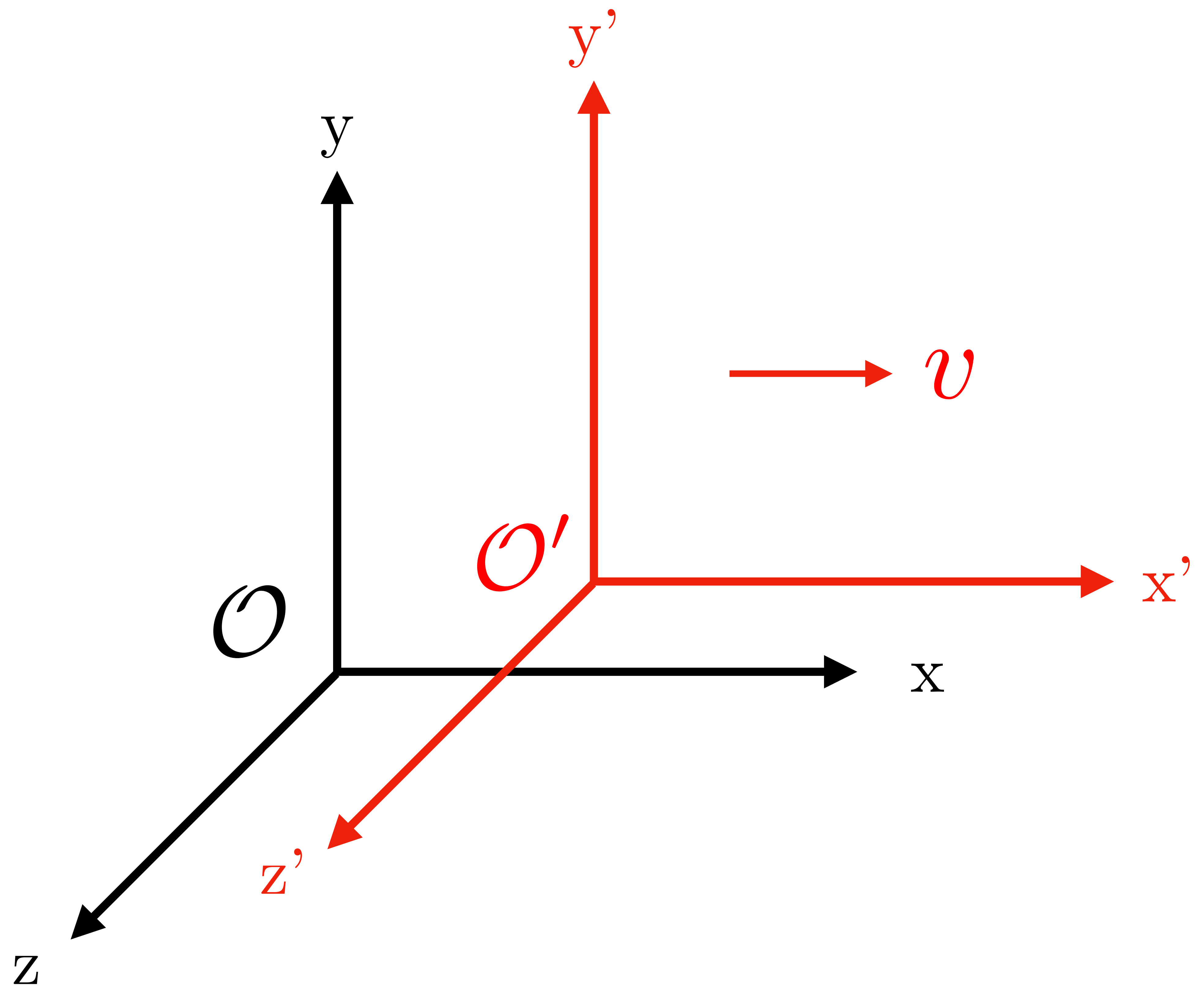
which is ***photon***

The other elementary particles in SM

can also be described by similar Lagrangian field theory

A brief review on Lorentz transformation

Galilean transformation



$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} t \\ x - vt \\ y \\ z \end{pmatrix}$$

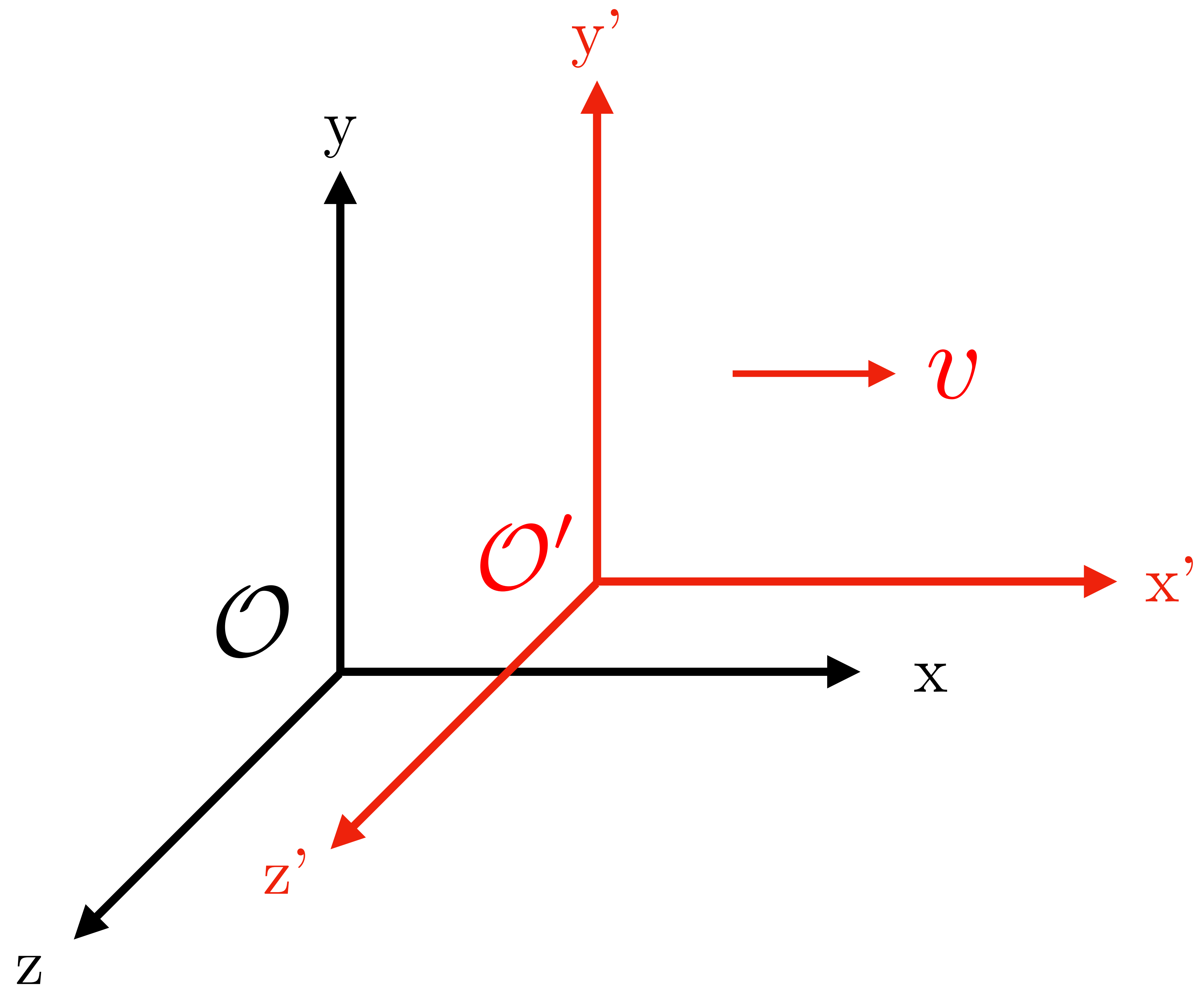
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

(transformation of coordinates in non-relativistic system)

Lorentz transformation

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

boost factor



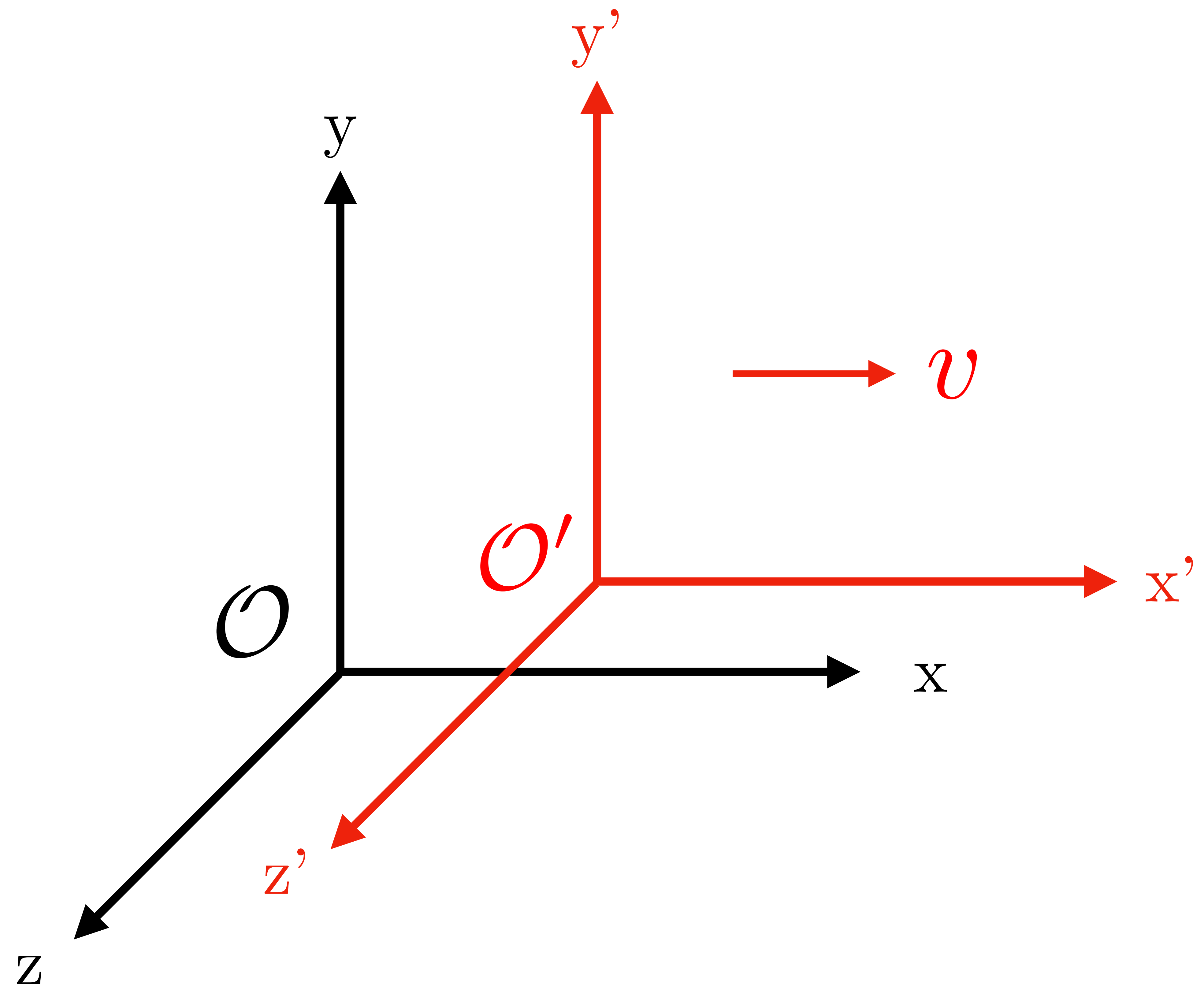
$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} \gamma(t - vx) \\ \gamma(x - vt) \\ y \\ z \end{pmatrix}$$

(time dilation & Length contraction)

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Lorentz transformation



$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma(t - vx) \\ \gamma(x - vt) \\ y \\ z \end{pmatrix}$$

consider now two events

$$E_1 = (t_1, x_1, 0, 0)_{\mathcal{O}} \qquad E_2 = (t_2, x_2, 0, 0)_{\mathcal{O}}$$

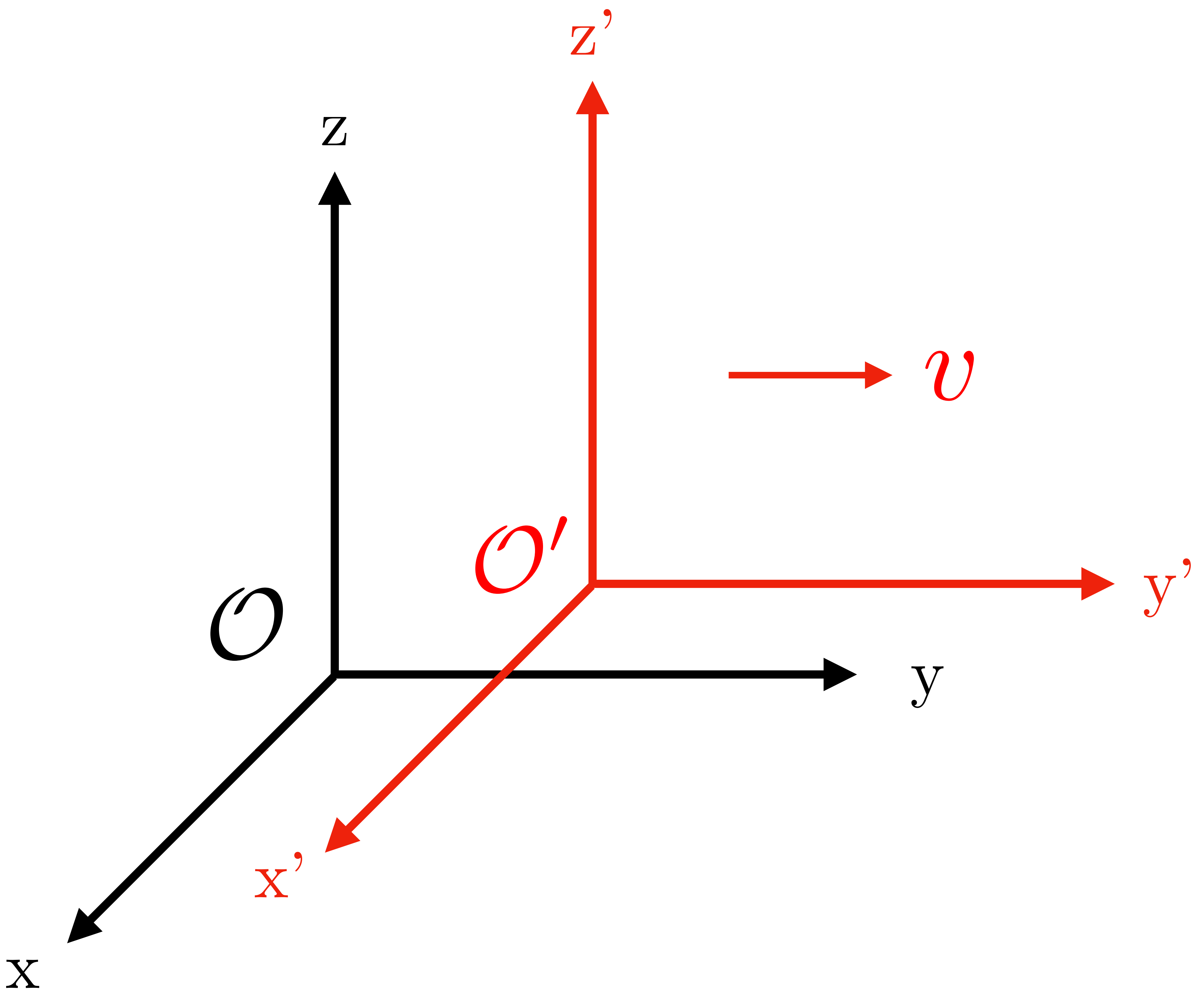
in the other frame

$$E_1 = (t_1', x_1', 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t_2', x_2', 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

Lorentz transformation

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$



consider now two events

$$E_1 = (t_1, x_1, 0, 0)_{\mathcal{O}} \qquad E_2 = (t_2, x_2, 0, 0)_{\mathcal{O}}$$

in the other frame

$$E_1 = (t_1', x_1', 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t_2', x_2', 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\Delta'^2 \equiv (t_2' - t_1')^2 - (x_2' - x_1')^2$$

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

in the other frame

$$E_1 = (t'_1, x'_1, 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t'_2, x'_2, 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\Delta'^2 \equiv (t'_2 - t'_1)^2 - (x'_2 - x'_1)^2$$

in the other frame

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$E_1 = (t'_1, x'_1, 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t'_2, x'_2, 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\Delta'^2 \equiv (t'_2 - t'_1)^2 - (x'_2 - x'_1)^2$$

$$= \gamma^2[(t_2 - t_1) - v(x_2 - x_1)]^2 - \gamma^2[(x_2 - x_1) - v(t_2 - t_1)]^2$$

in the other frame

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$E_1 = (t'_1, x'_1, 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t'_2, x'_2, 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\begin{aligned}
\Delta'^2 &\equiv (t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 \\
&= \gamma^2[(t_2 - t_1) - v(x_2 - x_1)]^2 - \gamma^2[(x_2 - x_1) - v(t_2 - t_1)]^2 \\
&= \gamma^2(1 - v^2)(t_2 - t_1)^2 - \gamma^2(1 - v^2)(x_2 - x_1)^2
\end{aligned}$$

in the other frame

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

$$E_1 = (t'_1, x'_1, 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t'_2, x'_2, 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\begin{aligned}
\Delta'^2 &\equiv (t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 \\
&= \gamma^2[(t_2 - t_1) - v(x_2 - x_1)]^2 - \gamma^2[(x_2 - x_1) - v(t_2 - t_1)]^2 \\
&= \gamma^2(1 - v^2)(t_2 - t_1)^2 - \gamma^2(1 - v^2)(x_2 - x_1)^2 \\
&= (t_2 - t_1)^2 - (x_2 - x_1)^2
\end{aligned}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}}$$

in the other frame

$$E_1 = (t'_1, x'_1, 0, 0)_{\mathcal{O}'} = \gamma(t_1 - vx_1, x_1 - vt_1, 0, 0)_{\mathcal{O}'}$$

$$E_2 = (t'_2, x'_2, 0, 0)_{\mathcal{O}'} = \gamma(t_2 - vx_2, x_2 - vt_2, 0, 0)_{\mathcal{O}'}$$

the *distance* between two events is

$$\Delta^2 \equiv (t_2 - t_1)^2 - (x_2 - x_1)^2$$

$$\begin{aligned}
\Delta'^2 &\equiv (t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 \\
&= \gamma^2[(t_2 - t_1) - v(x_2 - x_1)]^2 - \gamma^2[(x_2 - x_1) - v(t_2 - t_1)]^2 \\
&= \gamma^2(1 - v^2)(t_2 - t_1)^2 - \gamma^2(1 - v^2)(x_2 - x_1)^2 \\
&= (t_2 - t_1)^2 - (x_2 - x_1)^2 \\
&= \Delta^2
\end{aligned}$$

Lorentz-invariant distance between two events

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

quantities with indices μ, ν, ... transforms as Lorentz vector

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

quantities with indices μ, ν, ... transforms as Lorentz vector

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

with a Lorentz transformation matrix

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(repeated indices are contracted)

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu = x_\mu x^\mu$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu = x_\mu x^\mu$$

with a metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu = x_\mu x^\mu$$

with a metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

A quantity with lower index is defined as

$$x_\mu = \eta_{\mu\nu} x^\nu = (t, -x, -y, -z)$$

Lorentz Algebra

$$x^{\mu} = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^{\mu} x^{\nu} = x_{\mu} x^{\mu}$$

it is straightforward to check

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} = \eta_{\mu'\nu'}$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu = x_\mu x^\mu$$

it is straightforward to check

$$\eta_{\mu\nu} \Lambda^\mu{}_{\mu'} \Lambda^\nu{}_{\nu'} = \eta_{\mu'\nu'}$$

$$\begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Lorentz Algebra

$$x^\mu = (t, x, y, z) \qquad \mu = 0, 1, 2, 3$$

the invariant distance (line element) can be written as

$$\Delta^2 = t^2 - x^2 - y^2 - z^2 = \eta_{\mu\nu} x^\mu x^\nu = x_\mu x^\mu$$

it is straightforward to check

$$\eta_{\mu\nu} \Lambda^\mu{}_{\mu'} \Lambda^\nu{}_{\nu'} = \eta_{\mu'\nu'}$$

we will require any field theory to be Lorentz invariant
just like Maxwell's theory

Equations of motion of elementary particles

Schrödiner equation

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-V\right)\psi=0$$

non-relativistic; one-particle QM

Equations of motion of elementary particles

Schrödiner equation

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-V\right)\psi=0$$

non-relativistic; one-particle QM

Klein-Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu+m^2)\Phi=0$$

spin-0 bosons
(when quantized)

Equations of motion of elementary particles

Schrödiner equation

$$\left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-V\right)\psi=0$$

non-relativistic; one-particle QM

Klein-Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu+m^2)\Phi=0$$

spin-0 bosons
(when quantized)

Dirac equation

$$(i\gamma^\mu\partial_\mu-m)\psi=0$$

spin-1/2 fermions
(when quantized)

$$E \leftrightarrow i\frac{\partial}{\partial t}$$

$$p \leftrightarrow -i\frac{\partial}{\partial x}$$

Equations of motion of elementary particles

Schrödiner equation

$$\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - V\right)\psi = 0$$

non-relativistic; one-particle QM

Klein-Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\Phi = 0$$

spin-0 bosons
(when quantized)

Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

spin-1/2 fermions
(when quantized)

$$E \leftrightarrow i\frac{\partial}{\partial t}$$

$$p \leftrightarrow -i\frac{\partial}{\partial x}$$

Equations of motion of elementary particles

Schrödiner equation $\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - V\right)\psi = 0$	non-relativistic; one-particle QM $E = \frac{p^2}{2m} + V$
Klein-Gordon equation $(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\Phi = 0$	spin-0 bosons (when quantized)
Dirac equation $(i\gamma^\mu\partial_\mu - m)\psi = 0$	spin-1/2 fermions (when quantized)

Equations of motion of elementary particles

$$E \leftrightarrow i\frac{\partial}{\partial t}$$

$$p \leftrightarrow -i\frac{\partial}{\partial x}$$

Schrödiner equation

$$\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m} - V\right)\psi = 0$$

non-relativistic; one-particle QM

$$E = \frac{p^2}{2m} + V$$

Klein-Gordon equation

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\Phi = 0$$

spin-0 bosons
(when quantized)

$$E^2 = p^2 + m^2$$

Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

spin-1/2 fermions
(when quantized)

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

the least action principle reproduces Klein-Gordon equation

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

the least action principle reproduces Klein-Gordon equation

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$0 = \delta S$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

the least action principle reproduces Klein-Gordon equation

$$S = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\begin{aligned} 0 &= \delta S \\ &= \int d^4x \, \left[\delta\phi \frac{\partial \mathcal{L}}{\partial \phi} + \delta(\partial_\mu \phi)\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right] \end{aligned}$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

the least action principle reproduces Klein-Gordon equation

$$S = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\begin{aligned} 0 &= \delta S \\ &= \int d^4x \, \left[\delta\phi \frac{\partial \mathcal{L}}{\partial \phi} + \delta(\partial_\mu \phi) \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right] \\ &= \int d^4x \, \delta\phi \left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right] + \text{boundary term} \end{aligned}$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

the least action principle reproduces Klein-Gordon equation

$$S = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\begin{aligned} 0 &= \delta S \\ &= \int d^4x \left[\delta\phi \frac{\partial \mathcal{L}}{\partial \phi} + \delta(\partial_\mu \phi)\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right] \\ &= \int d^4x\, \delta\phi \boxed{\left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi}\right]} + \text{boundary term} \\ &= 0 \end{aligned}$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

Euler-Lagrange equation

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} = 0$$

Klein-Gordon equation is reproduced

$$(\eta^{\mu\nu}\partial_\mu\partial_\nu + m^2)\phi = 0$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

Lorentz transformation

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\phi(x) \to \phi'(x') = \phi(x)$$

$$\partial_\mu \phi \to \partial'_{\mu'} \phi' = \Lambda_\mu{}^\nu \partial_\nu \phi$$

Scalar Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2$$

Lorentz transformation

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\phi(x) \to \phi'(x') = \phi(x)$$

$$\partial_\mu \phi \to \partial'_{\mu'}\phi' = \Lambda_\mu{}^\nu \partial_\nu \phi$$

then

$$\eta^{\mu'\nu'}\partial'_{\mu'}\phi'\partial'_{\nu'}\phi' = \eta^{\mu'\nu'}\Lambda_{\mu'}{}^\mu \Lambda_{\nu'}{}^\nu \partial_\mu\phi\partial_\nu\phi$$

$$= \eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

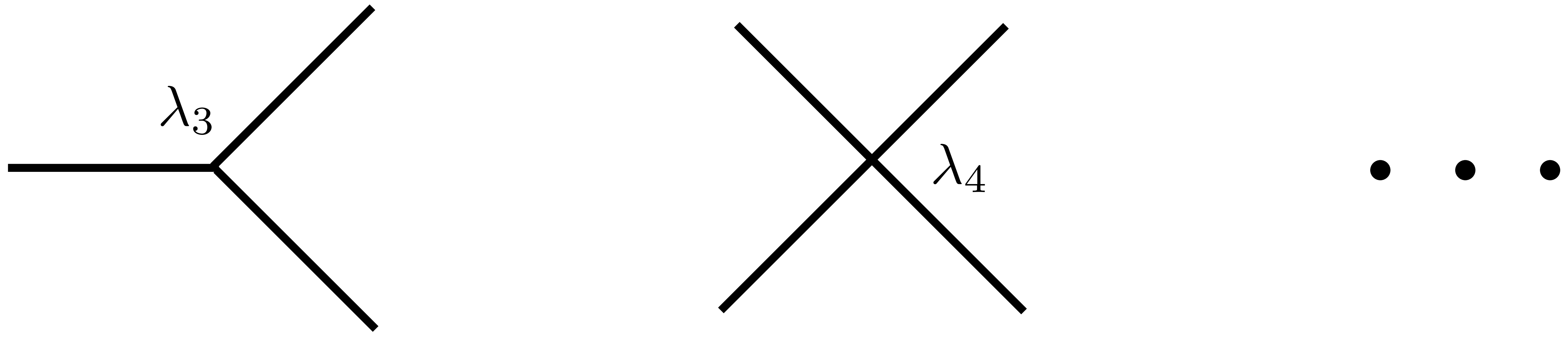
the Lagrangian is Lorentz invariant

Scalar Lagrangian

More generally

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 + \lambda_3\phi^3 + \lambda_4\phi^4 + \cdots$$

terms higher order in ϕ describes interactions among spin-0 bosons



$\bar{\psi} = \psi^\dagger \gamma^0$

Fermion Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

what does each term mean?

ψ :   4-component Dirac spinor that describes spin-1/2 particle

γ^μ :   Dirac matrices (4 by 4 matrix, μ=0, 1, 2, 3) satisfying Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$

Euler-Lagrange equation for $\bar{\psi}$ gives Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Fermion Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

under the Lorentz transformation

$$\bar{\psi}\psi \to \bar{\psi}\psi \qquad \text{(Lorentz scalar)}$$

$$\bar{\psi}\gamma^\mu\psi \to \Lambda^\mu{}_\nu(\bar{\psi}\gamma^\nu\psi) \qquad \text{(Lorentz vector)}$$

the above Lagrangian is invariant under the Lorentz transformation

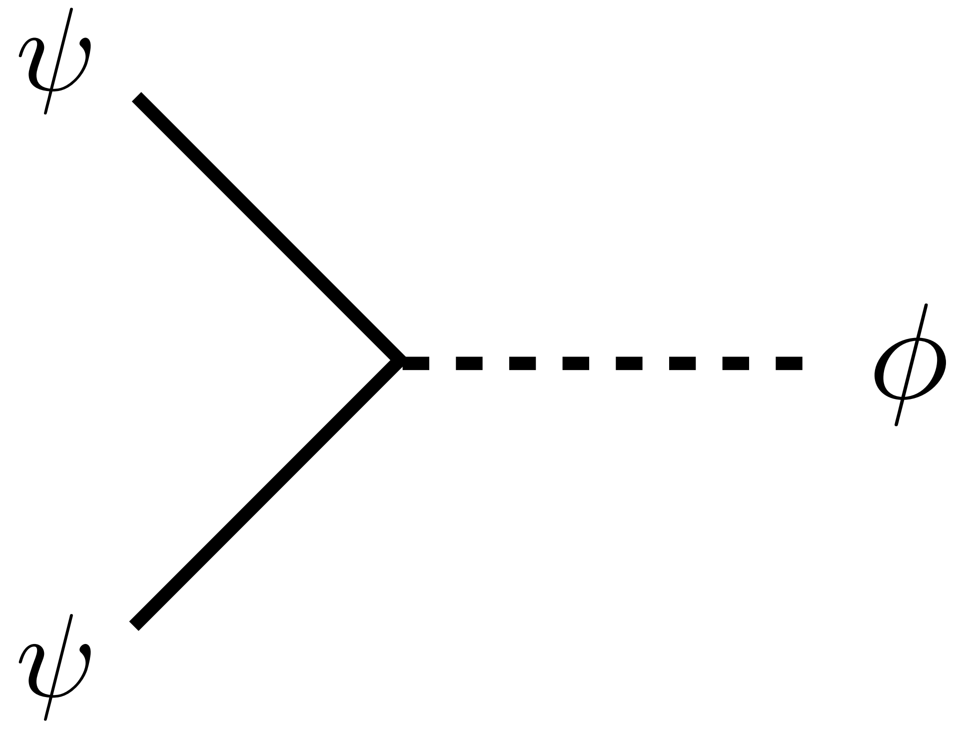
(we do not prove this here)

one can construct a model of scalar and fermion with interactions

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - y\phi\bar{\psi}\psi$$

one can construct a model of scalar and fermion with interactions

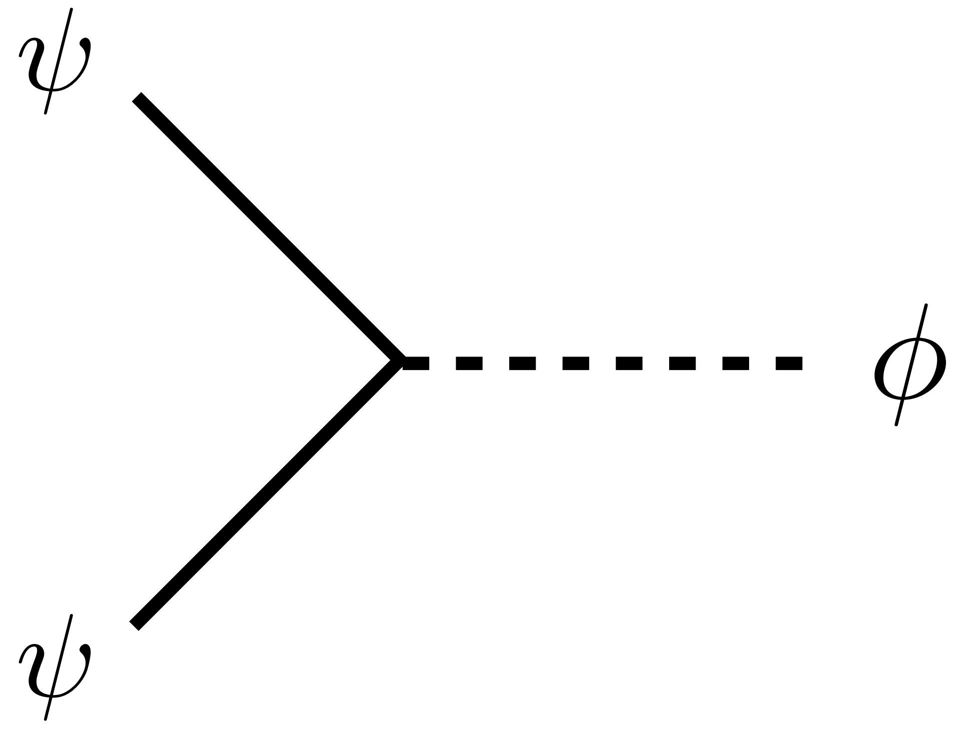
$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \boxed{- y\phi\bar{\psi}\psi}$$



Yukawa interaction (1935)

one can construct a model of scalar and fermion with interactions

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \boxed{- y\phi\bar{\psi}\psi}$$

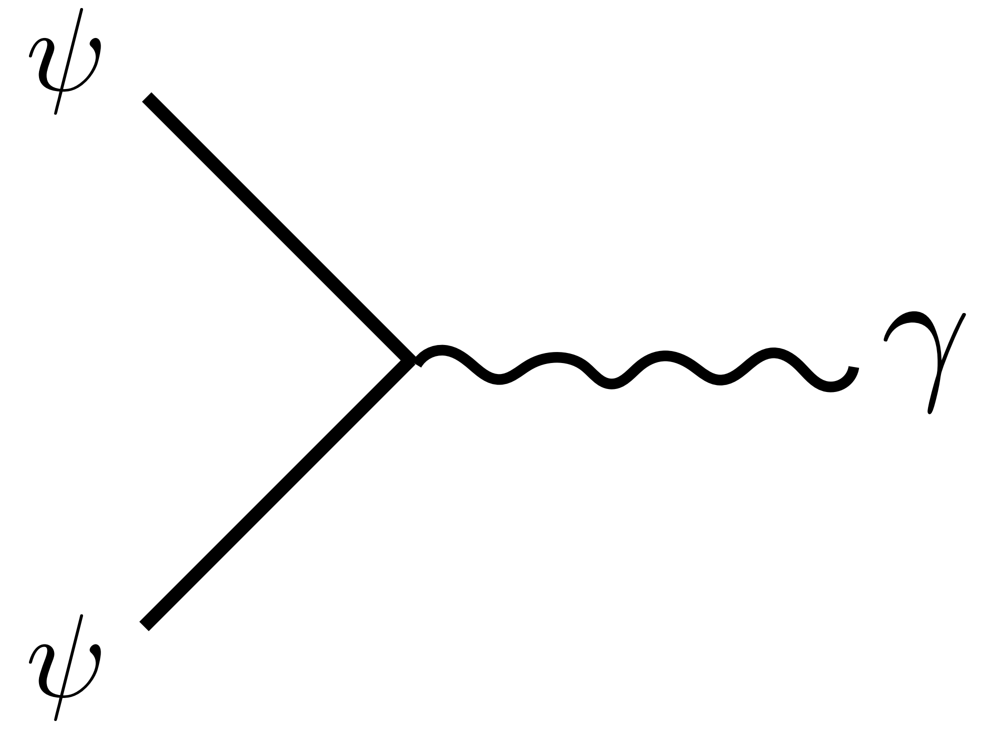


Yukawa interaction (1935)

first introduced to explain the interaction between nucleon

Yukawa interaction is exactly how the SM fermions obtain mass from Higgs one can construct a model of fermion and photon field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

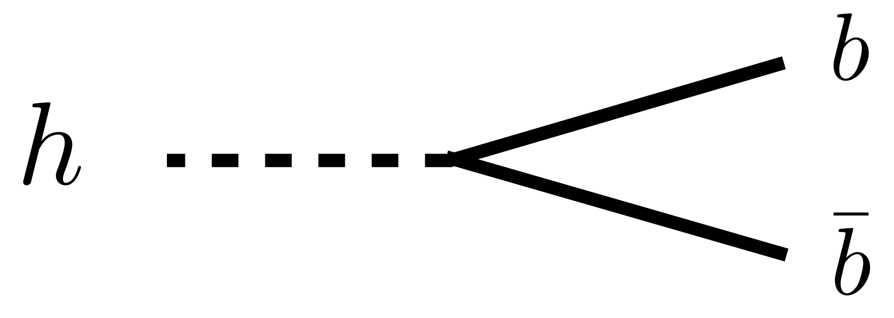


Quantum electrodynamics (QED)

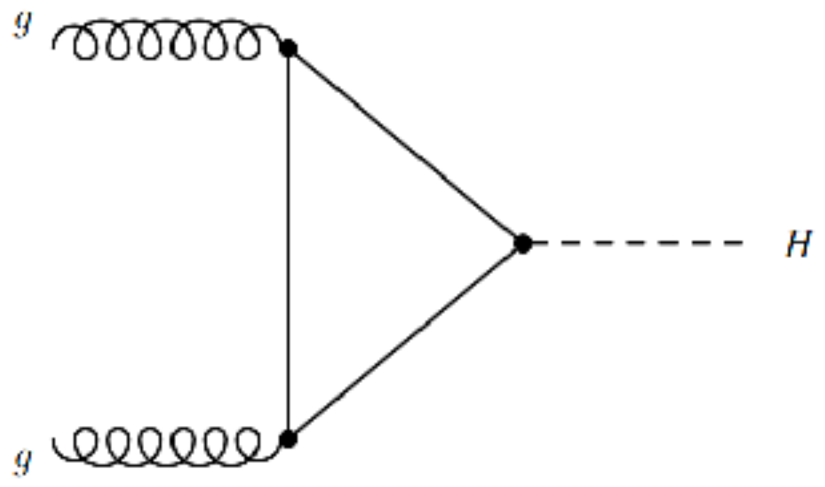
as a warm up

we will make some estimations on physical processes like

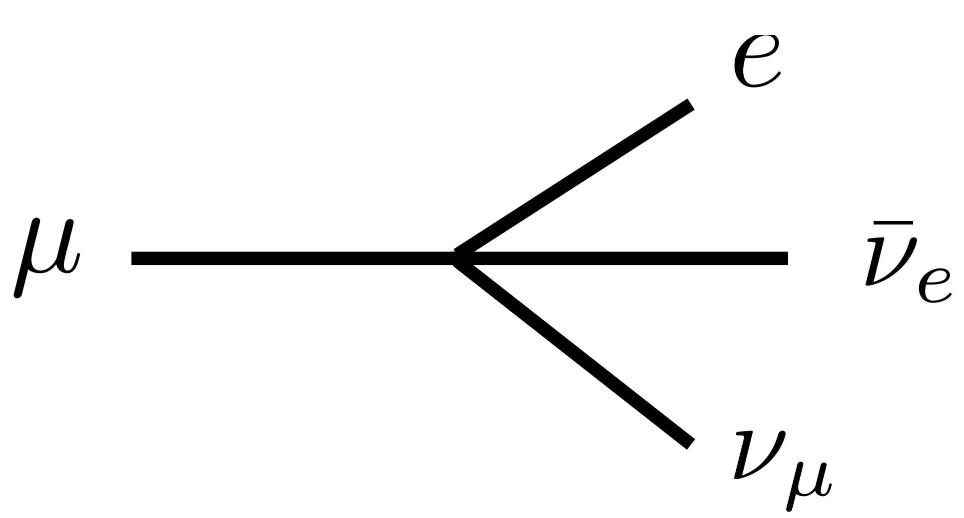
(1) Higgs decay $h \to b\bar{b}$



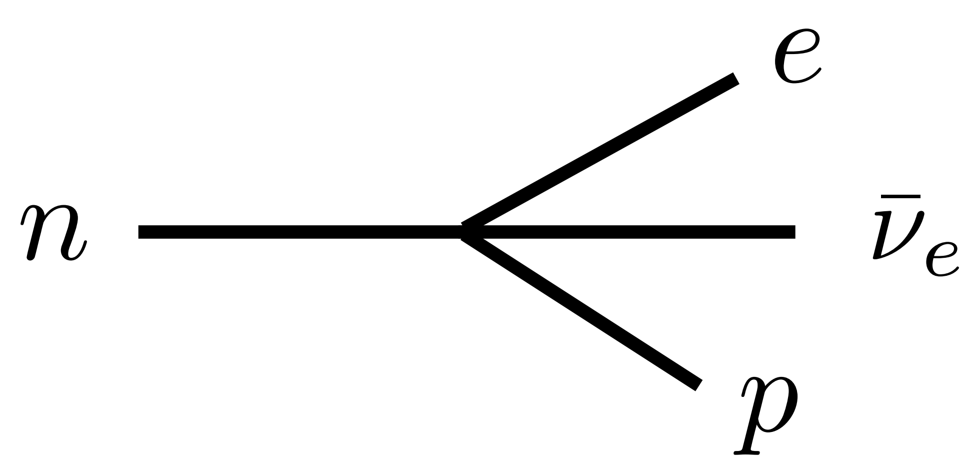
(2) Higgs production $gg \to h$



(3) muon decay $\mu \to e\nu_\mu\bar{\nu}_e$



(4) neutron decay $n \to pe\bar{\nu}_e$



we will estimate the cross-section and decay rate for above processes

Dimensional analysis

$$[S] = 0$$

$$S = \int d^4x \, \mathcal{L}$$

$$[\mathcal{L}] = 4$$

spin-0 $\qquad \mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi) + \cdots \longrightarrow [\phi] = 1$

Dimensional analysis

$$[S] = 0$$

$$S = \int d^4x\, \mathcal{L}$$

$$[\mathcal{L}] = 4$$

spin-0 $\quad \mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi) + \cdots \longrightarrow [\phi] = 1$

spin-1/2 $\quad \mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \cdots \longrightarrow [\psi] = 3/2$

Dimensional analysis

$$[S] = 0$$

$$S = \int d^4x\, \mathcal{L}$$

$$[\mathcal{L}] = 4$$

spin-0 $\quad \mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi) + \cdots \longrightarrow [\phi] = 1$

spin-1/2 $\quad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \cdots \longrightarrow [\psi] = 3/2$

spin-1 $\quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots \longrightarrow [A_\mu] = 1$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Dimensional analysis

$$[S] = 0$$

$$S = \int d^4x \, \mathcal{L}$$

$$[\mathcal{L}] = 4$$

spin-0 $\quad \mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi) + \cdots \longrightarrow [\phi] = 1$

spin-1/2 $\quad \mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + \cdots \longrightarrow [\psi] = 3/2$

spin-1 $\quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \cdots \longrightarrow [A_\mu] = 1$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

cross section & decay rate

$$[\sigma] = [\text{Length}]^2 = -2$$

$$[\Gamma] = [\text{Time}]^{-1} = 1$$

Higgs decay

$$[\mathcal{L}] = 4$$
$$[\phi] = 1$$
$$[\psi] = 3/2$$
$$[y] = 0$$

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

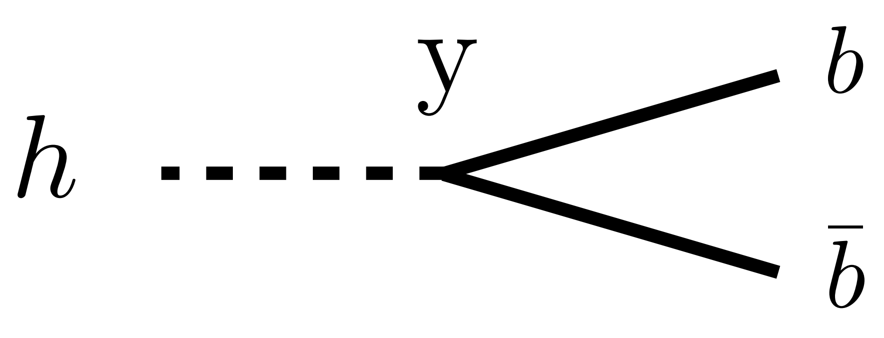
Higgs decay

$$[\mathcal{L}] = 4$$
$$[\phi] = 1$$
$$[\psi] = 3/2$$
$$[y] = 0$$

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$$\Gamma \propto \Bigg|$$



$$\Bigg|^2 \sim \frac{1}{8\pi} y^2 m_h$$

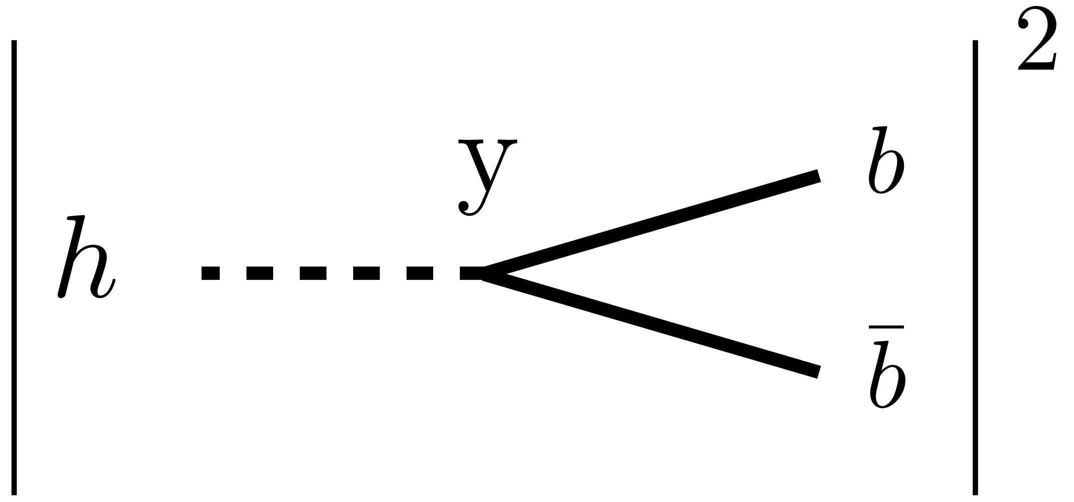
$$[\mathcal{L}] = 4$$
$$[\phi] = 1$$
$$[\psi] = 3/2$$
$$[y] = 0$$

Higgs decay

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$\Gamma \propto$



$\sim \frac{1}{8\pi} y^2 m_h$

Higgs interaction to fermion is proportional to the fermion mass

$$y = \frac{m_f}{v}$$

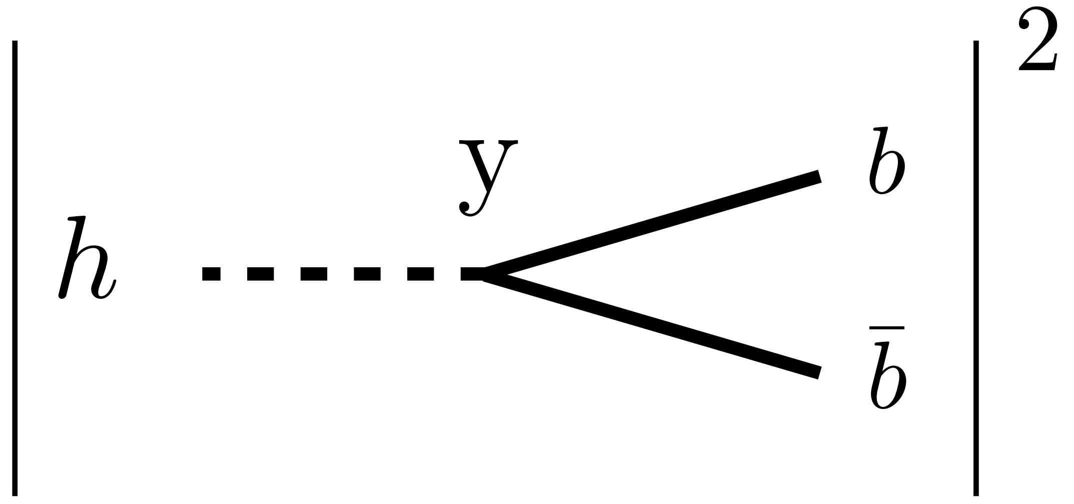
Higgs decays dominantly to heavy fermions, e.g. bottom quark

Higgs decay

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$\Gamma \propto$

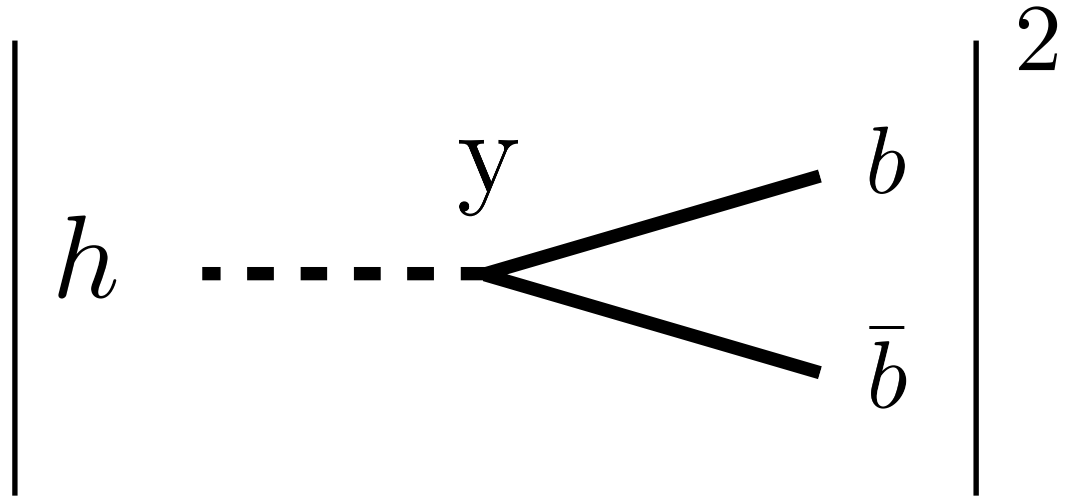


Higgs decay

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$$\Gamma \propto$$



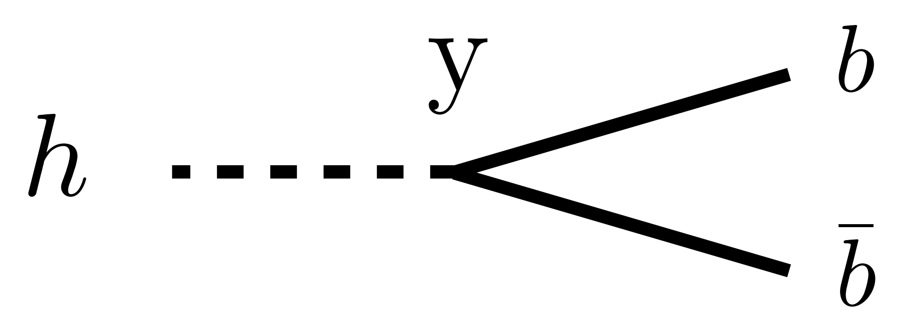
$$\sim \frac{1}{8\pi}\left(\frac{m_b}{v}\right)^2 m_h$$

Higgs decay

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$$\Gamma \propto \left| \quad \right|^2$$



$$\sim \frac{1}{8\pi}\left(\frac{m_b}{v}\right)^2 m_h$$

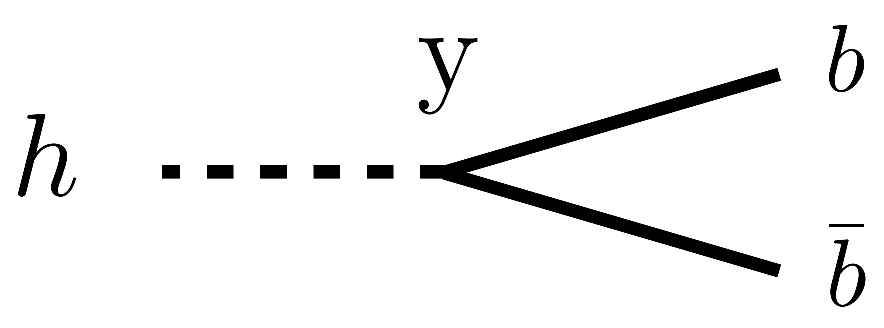
$$\sim \frac{1}{8\pi}\left(\frac{4\,\mathrm{GeV}}{246\,\mathrm{GeV}}\right)^2 125\,\mathrm{GeV}$$

Higgs decay

$$\mathcal{L} = y\phi\bar{\psi}\psi$$

the decay rate is

$$\Gamma \propto \left| \quad \right|^2$$



$$\sim \frac{1}{8\pi}\left(\frac{m_b}{v}\right)^2 m_h$$

$$\sim \frac{1}{8\pi}\left(\frac{4\,\mathrm{GeV}}{246\,\mathrm{GeV}}\right)^2 125\,\mathrm{GeV}$$

$$\sim 1\,\mathrm{MeV} \qquad \leftrightarrow \tau = 10^{-21}\,\mathrm{sec}$$

Higgs production

$$\mathcal{L} \supset g_s G_\mu \bar{t}\gamma^\mu t - y h \bar{t} t$$

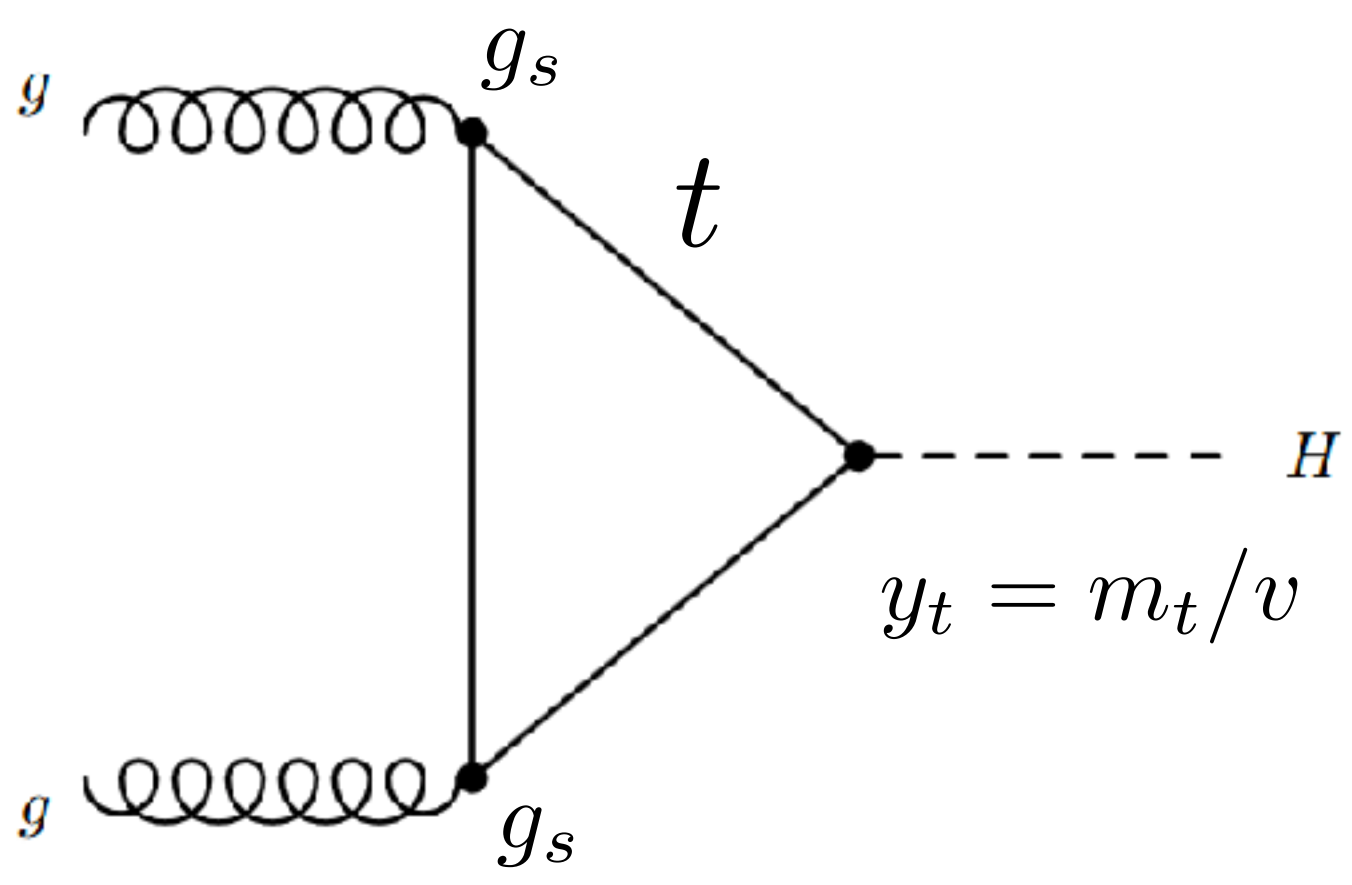
$[\mathcal{L}] = 4$

$[h] = 1$

$[t] = 3/2$

$[G_\mu] = 1$

$[g_s] = 0$



Higgs production

$$[\mathcal{L}] = 4$$

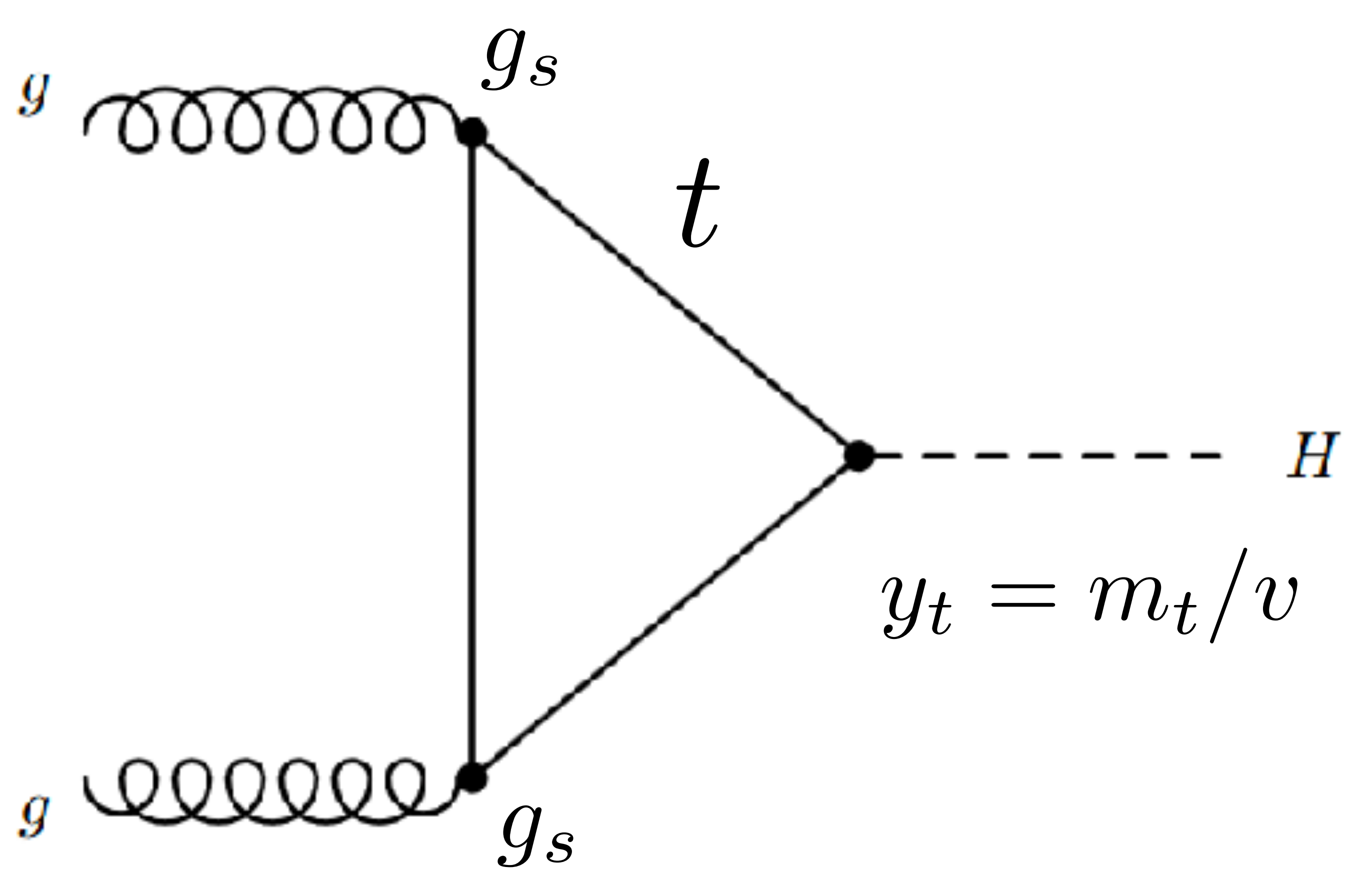
$$[h] = 1$$

$$[t] = 3/2$$

$$[G_\mu] = 1$$

$$[g_s] = 0$$

$$\mathcal{L} \supset g_s G_\mu \bar{t} \gamma^\mu t - y h \bar{t} t$$



$$\sigma \sim \frac{g_s^4}{16\pi^2} \frac{m_t^2}{v^2} \frac{1}{m_t^2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$$

$$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$$

Higgs production

$$[\mathcal{L}] = 4$$

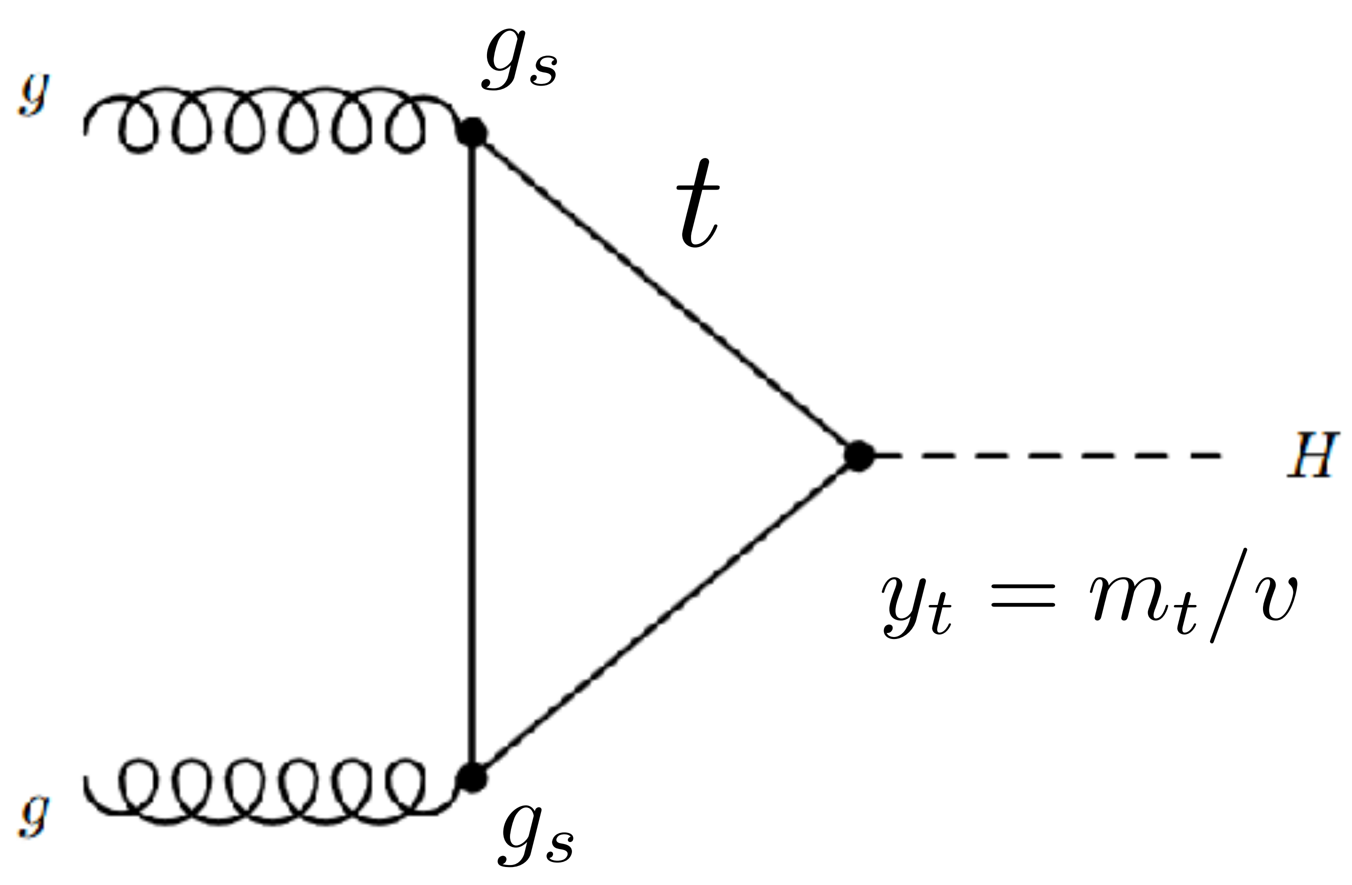
$$[h] = 1$$

$$[t] = 3/2$$

$$[G_\mu] = 1$$

$$[g_s] = 0$$

$$\mathcal{L} \supset g_s G_\mu \bar{t}\gamma^\mu t - y h \bar{t} t$$



$$\sigma \sim \boxed{\frac{g_s^4}{16\pi^2}} \frac{m_t^2}{v^2} \frac{1}{m_t^2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$$

loop + gauge coupling

$$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$$

Higgs production

$$[\mathcal{L}] = 4$$

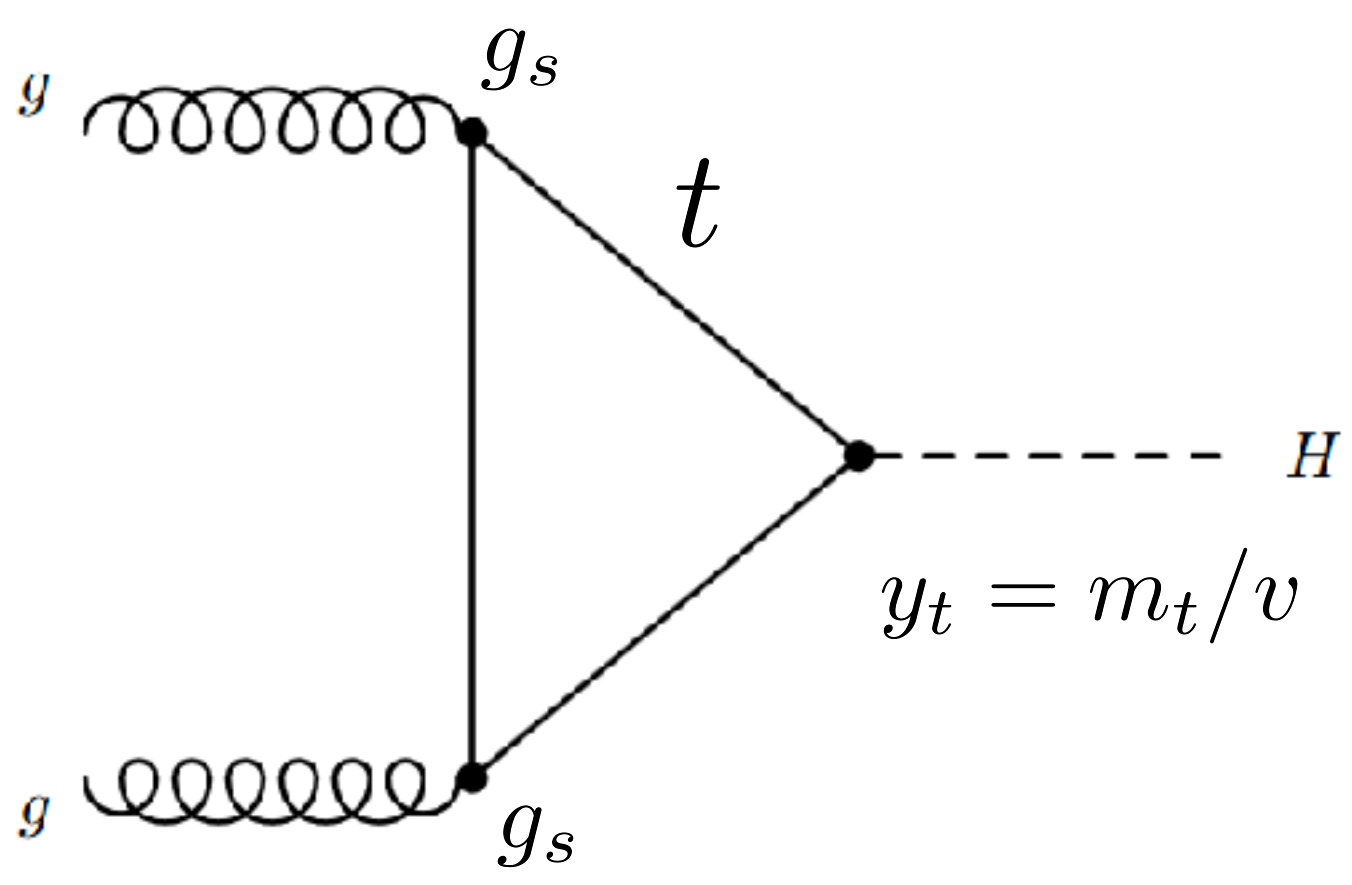
$$[h] = 1$$

$$[t] = 3/2$$

$$[G_\mu] = 1$$

$$[g_s] = 0$$

$$\mathcal{L} \supset g_s G_\mu \bar{t}\gamma^\mu t - yh\bar{t}t$$



Yukawa

$$\sigma \sim \boxed{\frac{g_s^4}{16\pi^2}} \boxed{\frac{m_t^2}{v^2}} \frac{1}{m_t^2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$$

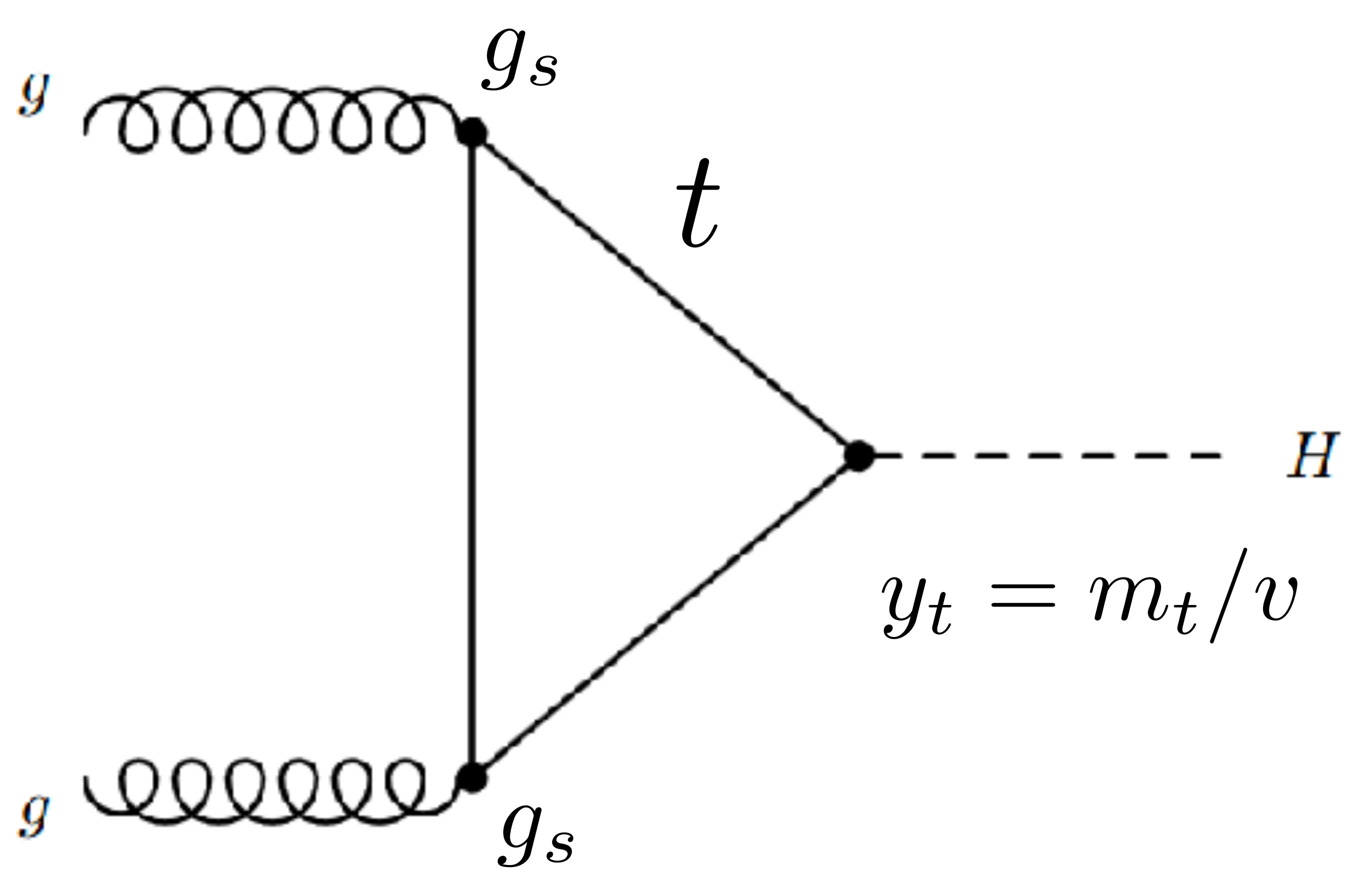
loop + gauge coupling

$$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$$

Higgs production

$$[\mathcal{L}] = 4$$
$$[h] = 1$$
$$[t] = 3/2$$
$$[G_\mu] = 1$$
$$[g_s] = 0$$

$$\mathcal{L} \supset g_s G_\mu \bar{t}\gamma^\mu t - y h \bar{t} t$$



Yukawa dimension

$$\sigma \sim \frac{g_s^4}{16\pi^2}\frac{m_t^2}{v^2}\frac{1}{m_t^2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$$

loop + gauge coupling

$$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$$

Higgs production

$[\mathcal{L}] = 4$

$[h] = 1$

$[t] = 3/2$

$[G_\mu] = 1$

$[g_s] = 0$

$$\mathcal{L} \supset g_s G_\mu \bar{t}\gamma^\mu t - yh\bar{t}t$$

$$\sigma \sim \frac{g_s^4}{16\pi^2}\frac{m_t^2}{v^2}\frac{1}{m_t^2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$$

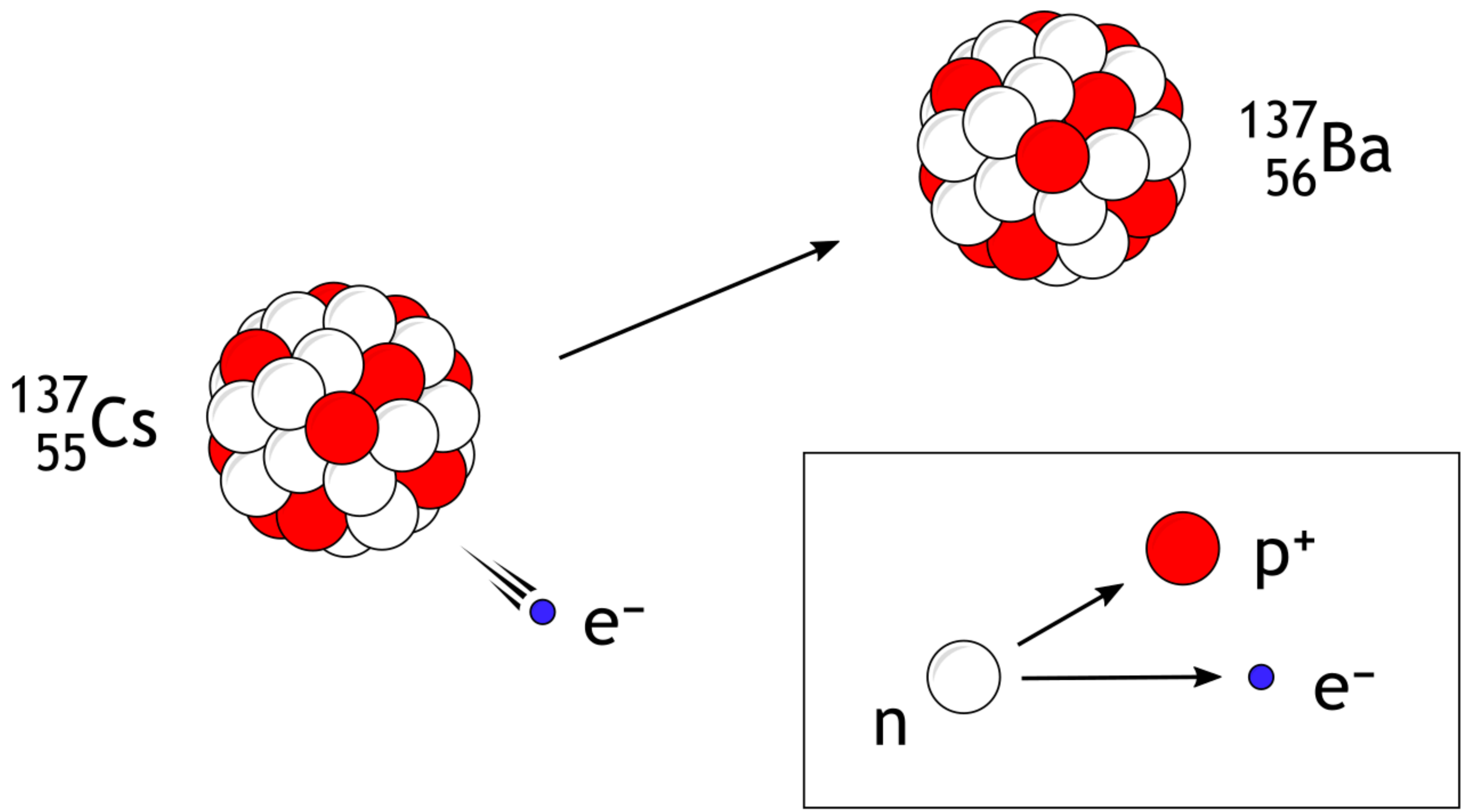
at LHC

$$N = \sigma \int dtL \sim 10\,\mathrm{pb} \times 100\,\mathrm{fb}^{-1} = 10^6$$

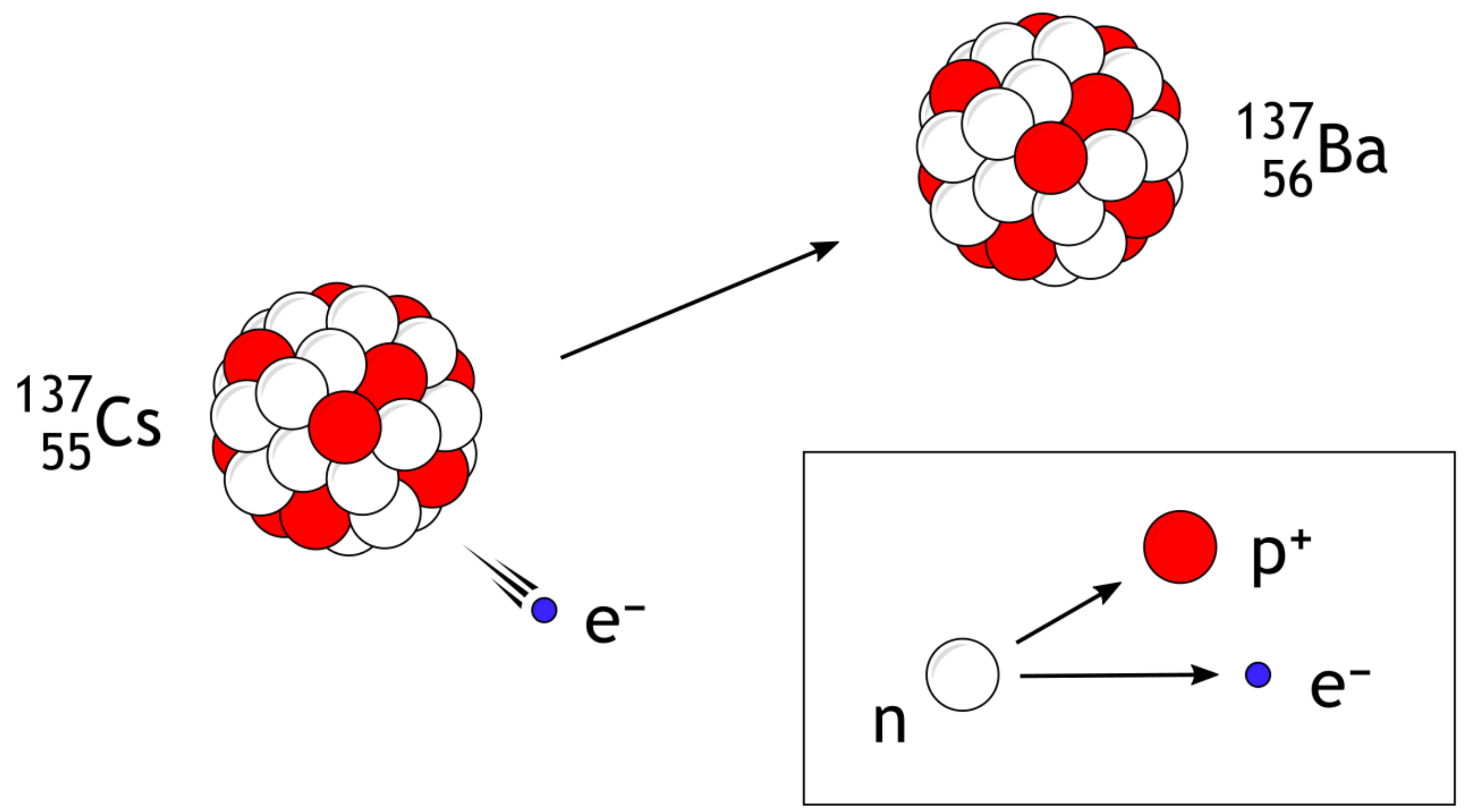
Fermi Theory

beta decay

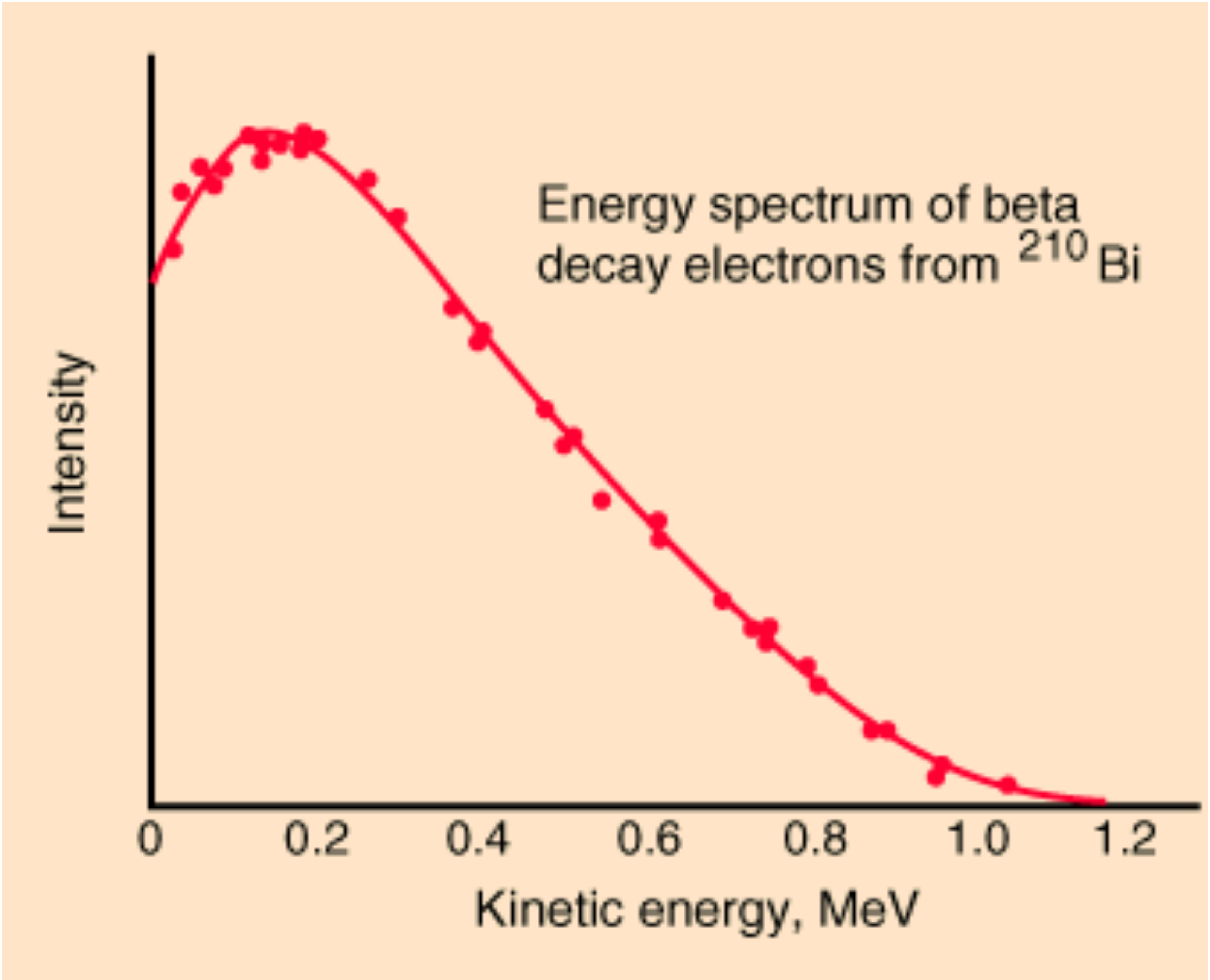
beta decay



beta decay

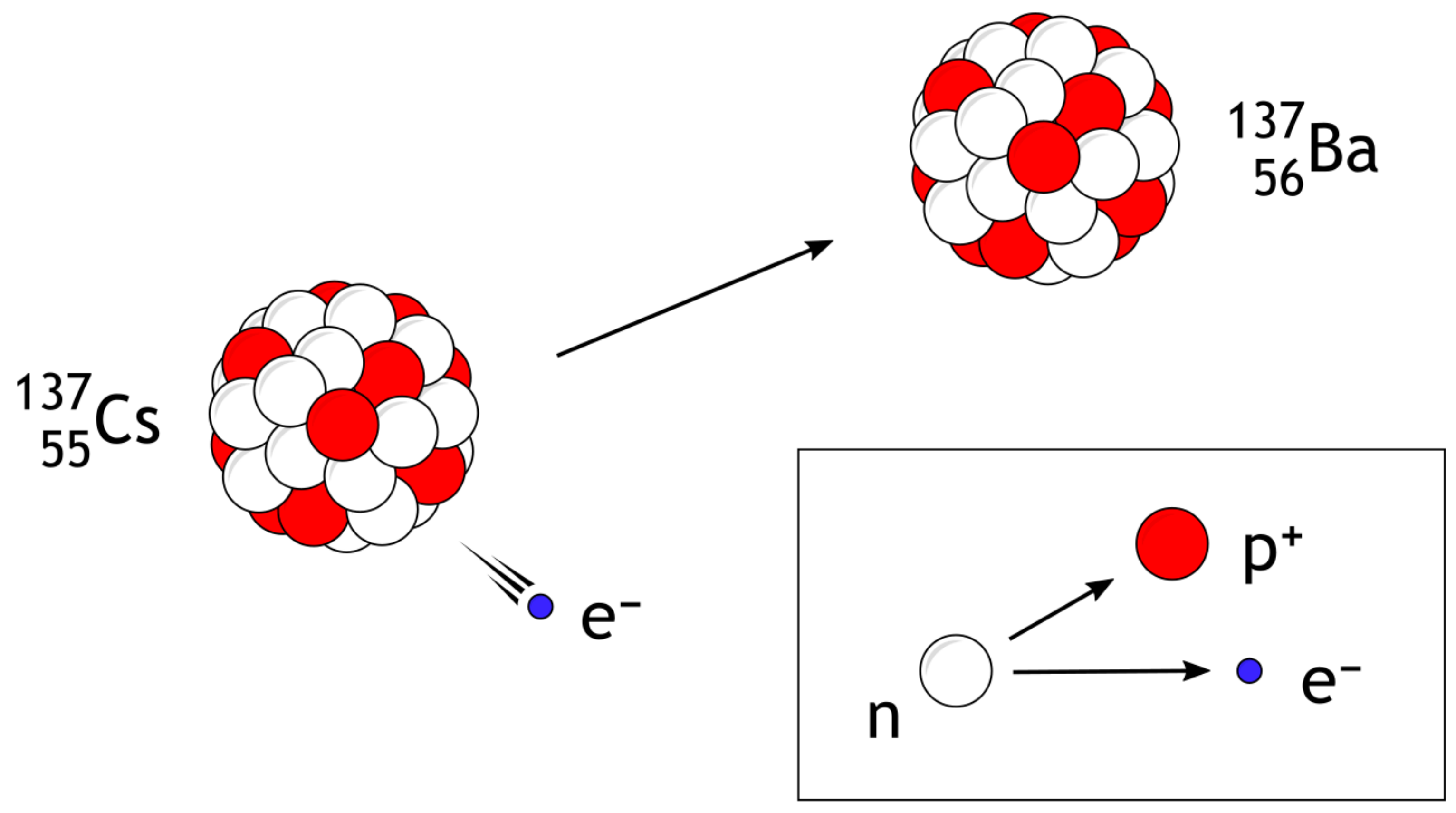


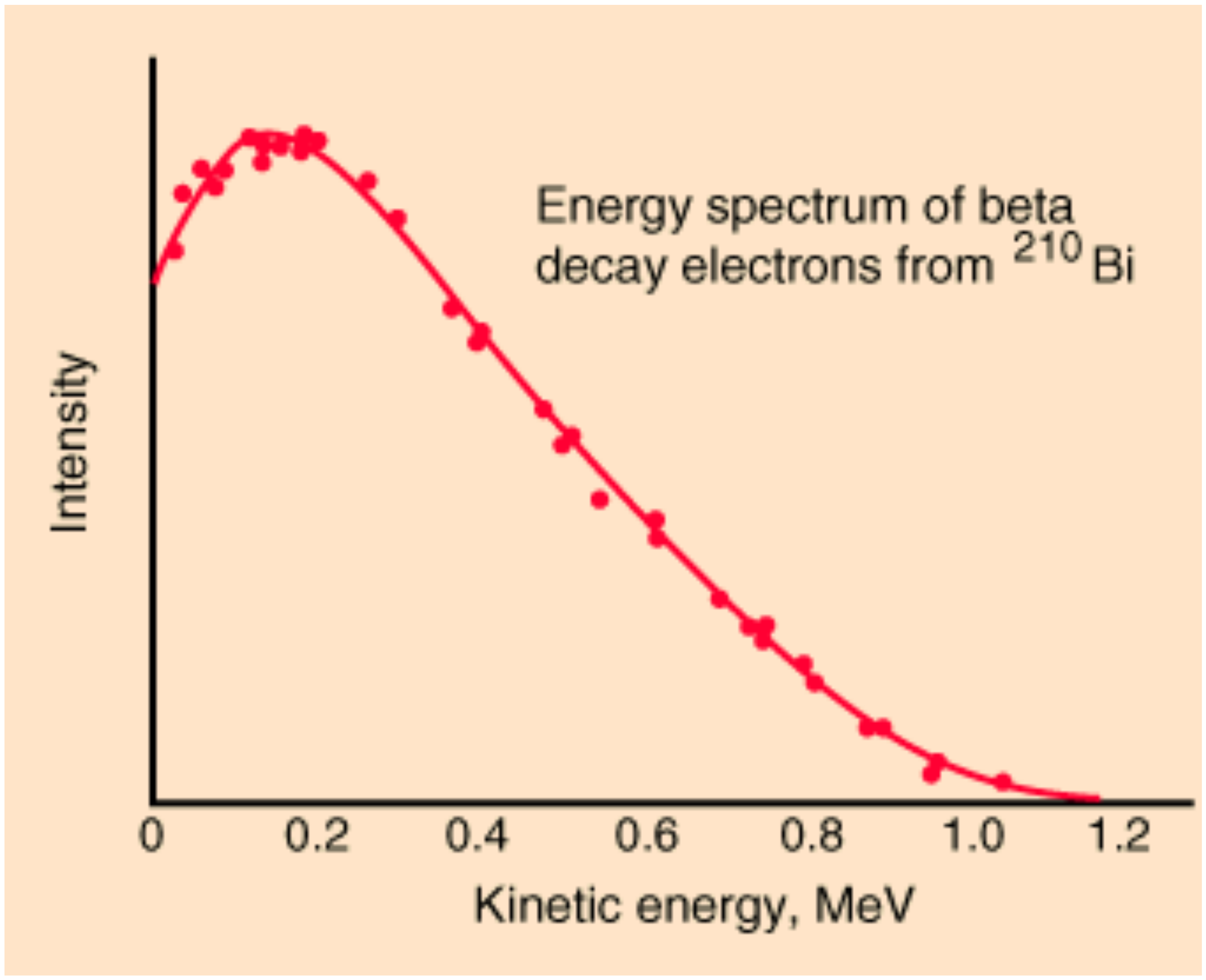
137 55 Cs
137 56 Ba
e⁻
p⁺
n
e⁻



Energy spectrum of beta decay electrons from 210 Bi
Intensity
0
0.2
0.4
0.6
0.8
1.0
1.2
Kinetic energy, MeV

beta decay





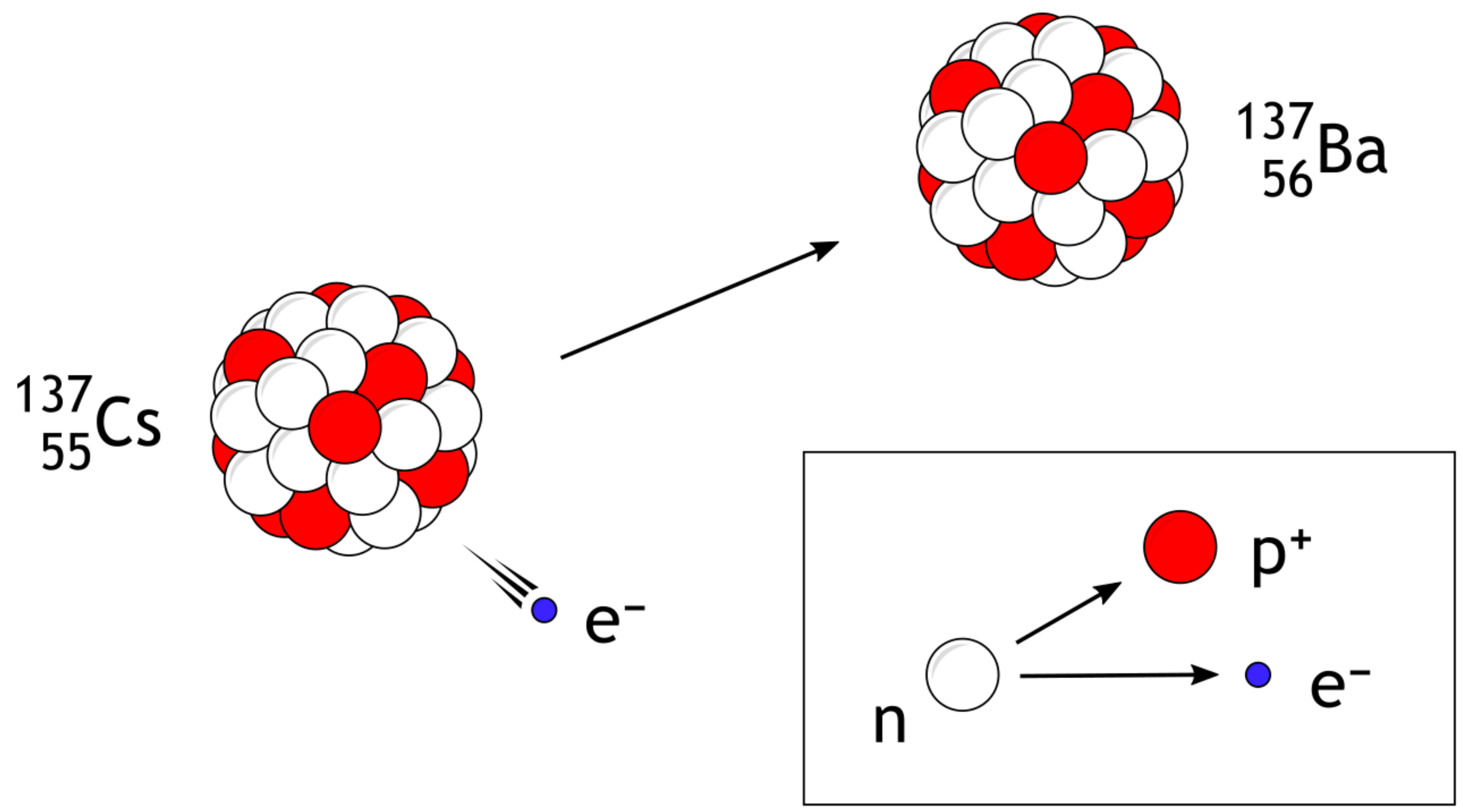
If it is 2-body decay $A \to B + e$

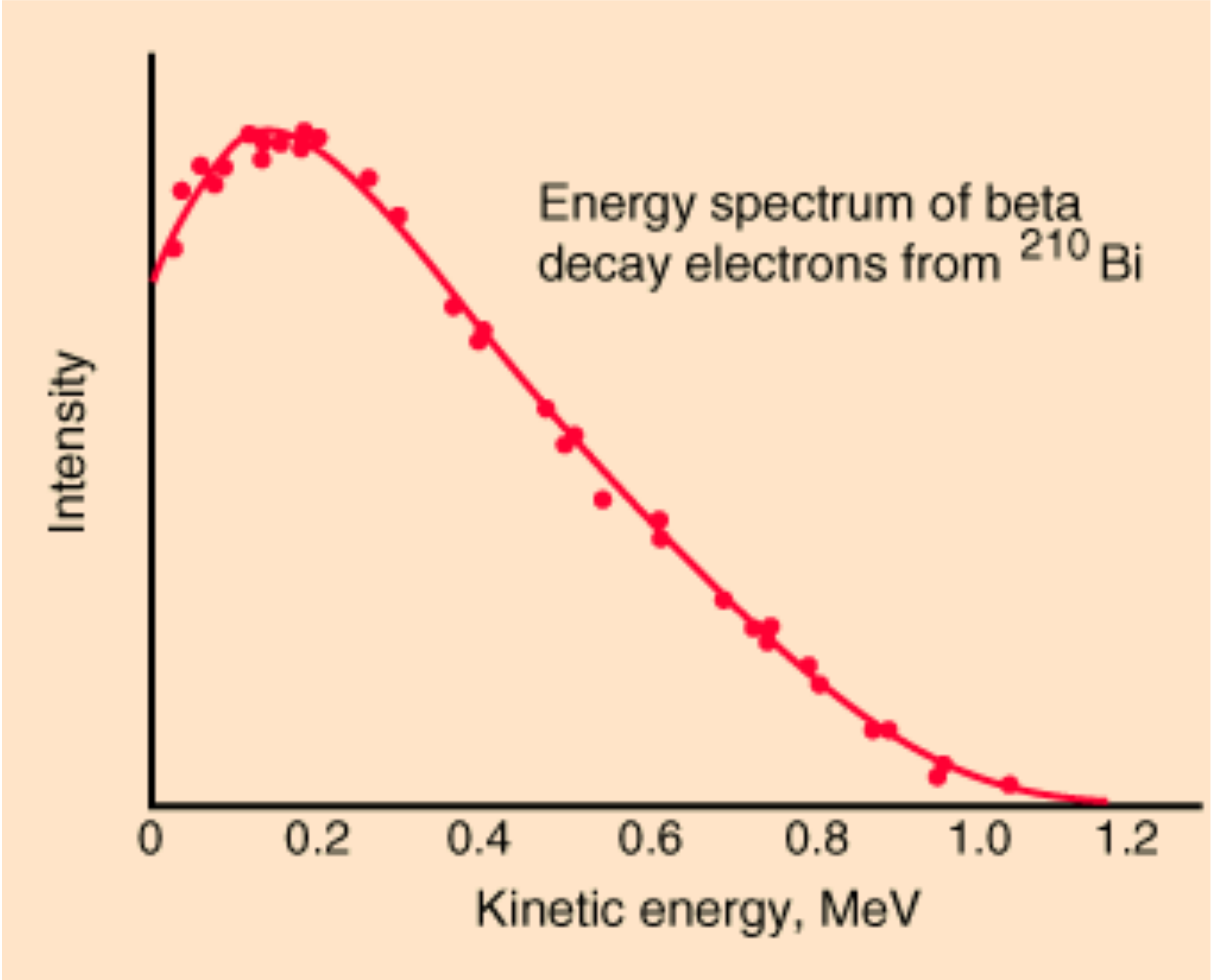
electron spectrum cannot be continuous

Instead

$$E_{\text{electron}} = \frac{m_A^2 + m_e^2 - m_B^2}{2m_A}$$

beta decay





If it is 2-body decay $A \to B + e$

electron spectrum cannot be continuous

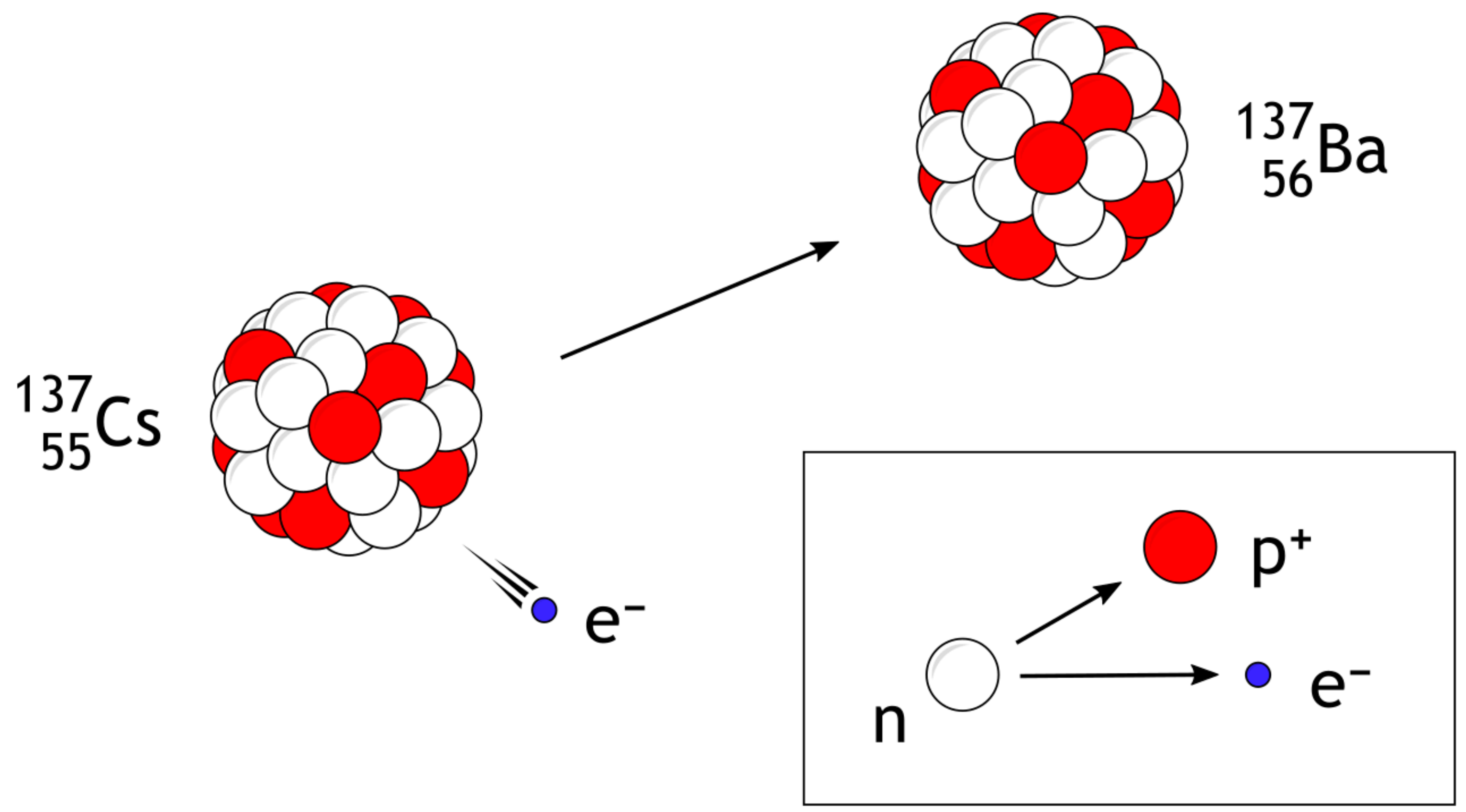
Instead

$$E_{\text{electron}} = \frac{m_A^2 + m_e^2 - m_B^2}{2m_A}$$

If it is N-body decay (N>2) $A \to B_1 + B_2 + \cdots + B_{N-1} + e$

electron spectrum can be continuous

beta decay

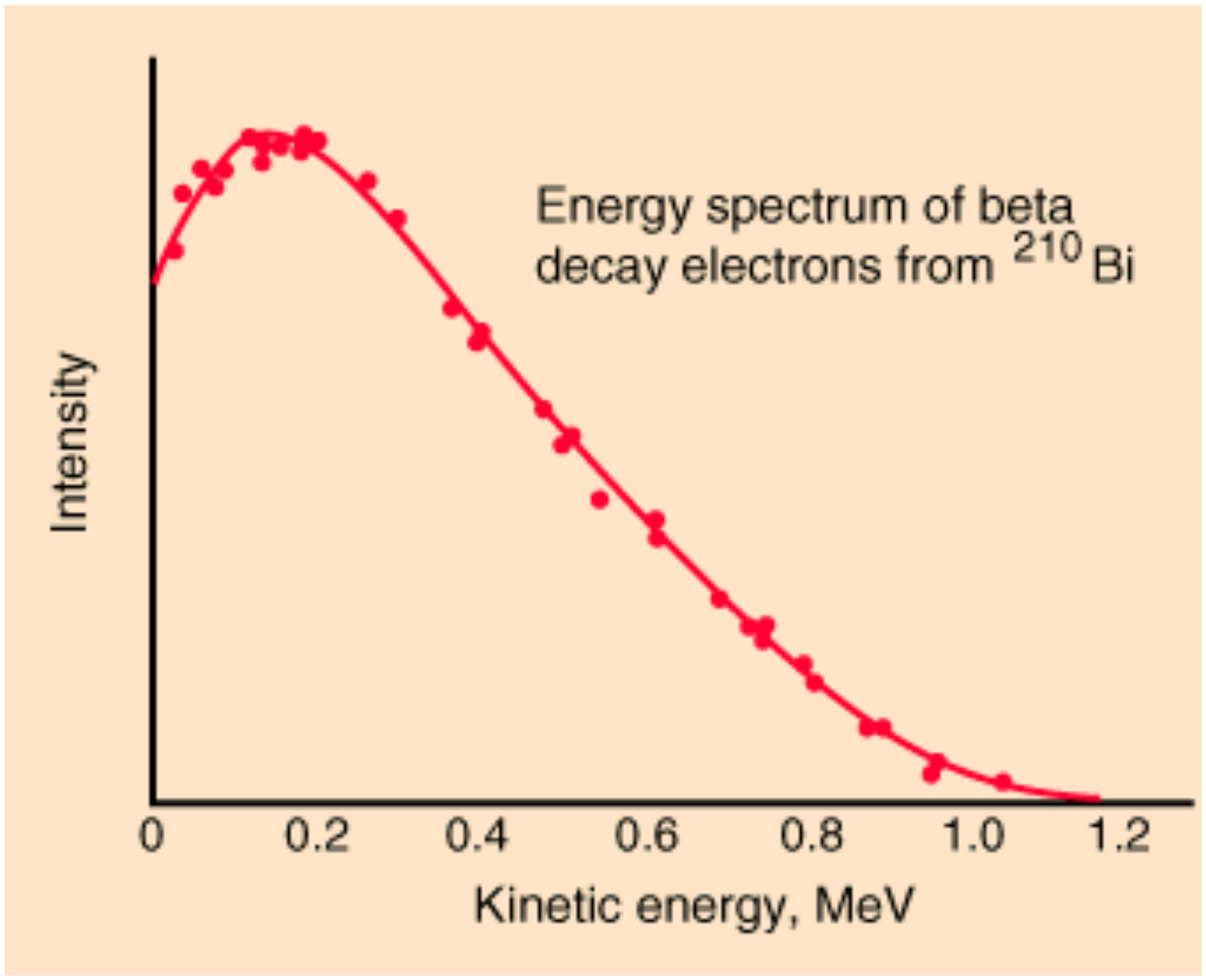


If it is 2-body decay $A \to B + e$

electron spectrum cannot be continuous

Instead

$$E_{\text{electron}} = \frac{m_A^2 + m_e^2 - m_B^2}{2m_A}$$

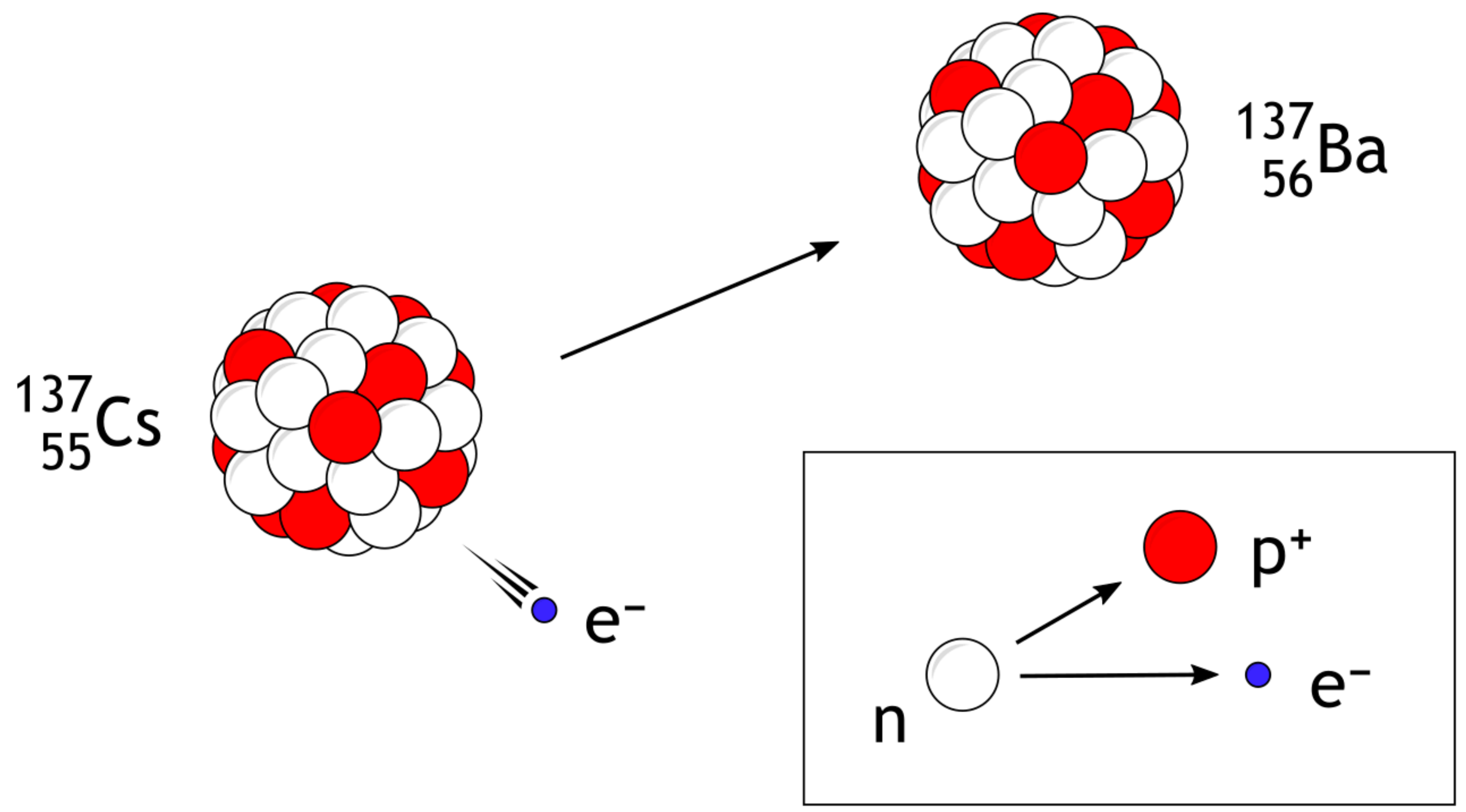


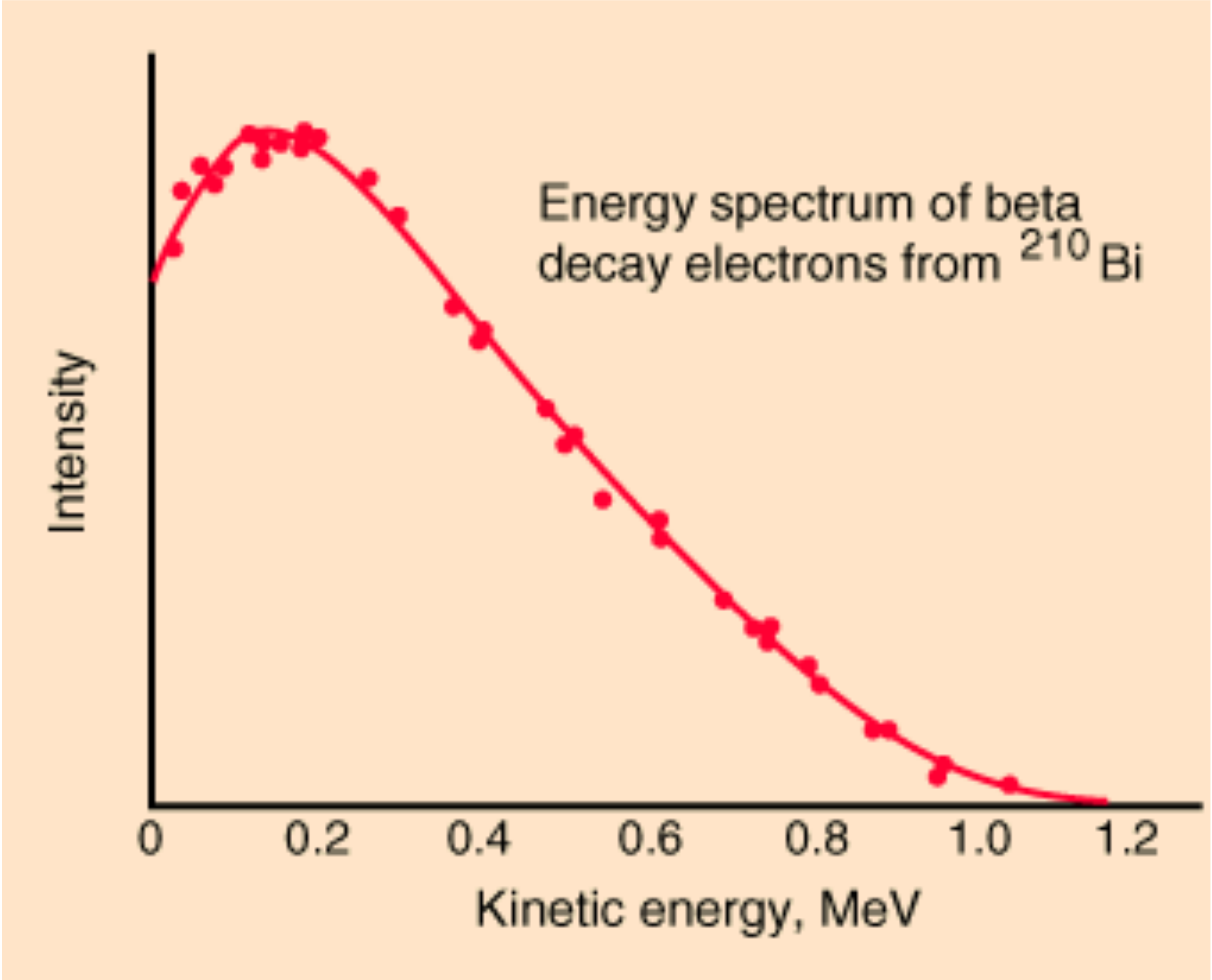
If it is N-body decay (N>2) $A \to B_1 + B_2 + \cdots + B_{N-1} + e$

electron spectrum can be continuous

- Pauli (1930) if a light neutral particle (*neutrino*) is emitted along with electron, the spectrum can be explained

beta decay





If it is 2-body decay $A \to B + e$

electron spectrum cannot be continuous

Instead

$$E_{\text{electron}} = \frac{m_A^2 + m_e^2 - m_B^2}{2m_A}$$

If it is N-body decay (N>2) $A \to B_1 + B_2 + \cdots + B_{N-1} + e$

electron spectrum can be continuous

- Pauli (1930) if a light neutral particle (*neutrino*) is emitted along with electron, the spectrum can be explained
- Fermi (1933) $\mathcal{L} = G_F(\bar{n}\gamma_\mu p)(\bar{\nu}_e\gamma^\mu e)$

Fermi theory

$$[\mathcal{L}] = 4$$

$$[G_F] = -2$$

$$[\Gamma] = 1$$

$$\mathcal{L} = G_F(\bar{n}\gamma_\mu p)(\bar{\nu}_e\gamma^\mu e)$$

$$G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$$

(Fermi constant)

Fermi theory successfully explains β-decay as well as muon decay

$$\mu \to e\nu_\mu\bar{\nu}_e$$

$$n \to pe\bar{\nu}_e$$

Fermi theory

$[\mathcal{L}] = 4$

$[G_F] = -2$

$[\Gamma] = 1$

$G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$

(Fermi constant)

$$\mathcal{L} = G_F(\bar{n}\gamma_\mu p)(\bar{\nu}_e\gamma^\mu e)$$

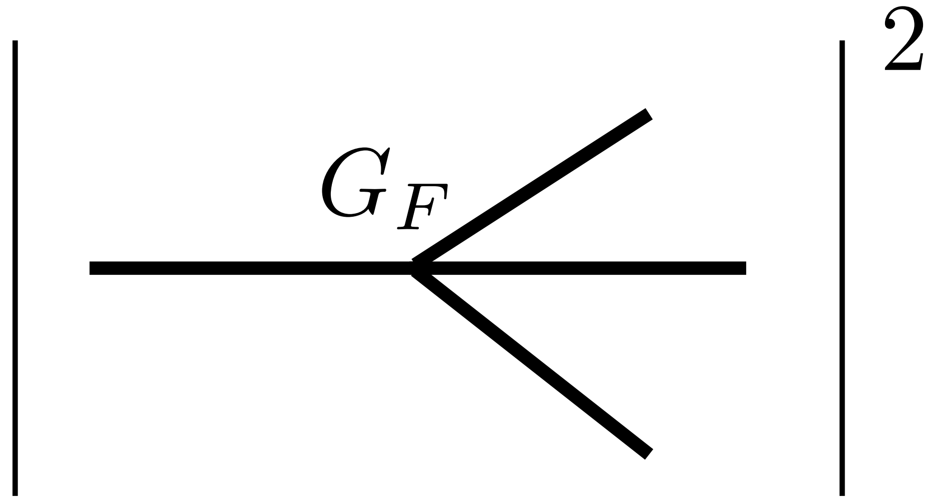
Fermi theory successfully explains β-decay as well as muon decay

$$\mu \to e\nu_\mu\bar{\nu}_e$$

$$n \to pe\bar{\nu}_e$$

using dimensional analysis

$\Gamma \propto$

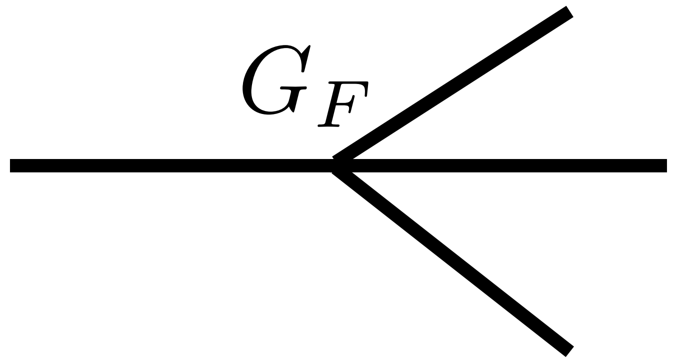


$\propto G_F^2 m^5$

$G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$

$m_\mu \sim 0.1\,\mathrm{GeV}$

using dimensional analysis

$\Gamma \propto \Bigg|$ 

$\Bigg|^2 \propto G_F^2 m^5$

for muon

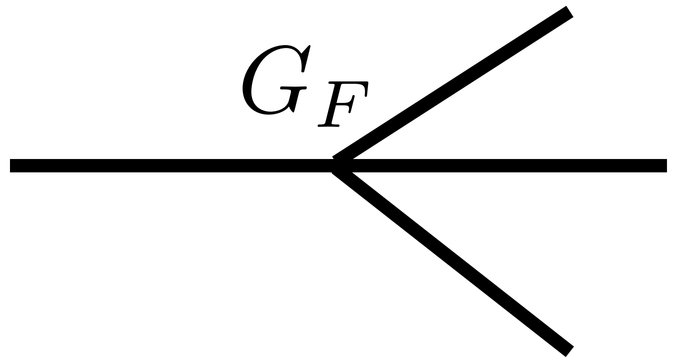
$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} \simeq (2.2\mu\,\mathrm{sec})^{-1}$$

$$G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$$

$$m_\mu \sim 0.1\,\mathrm{GeV}$$

using dimensional analysis

$$\Gamma \propto \left| \quad \right|^2 \propto G_F^2 m^5$$



for muon

$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} \simeq (2.2\mu\,\mathrm{sec})^{-1}$$

[Particle Data Group]

μ MEAN LIFE τ

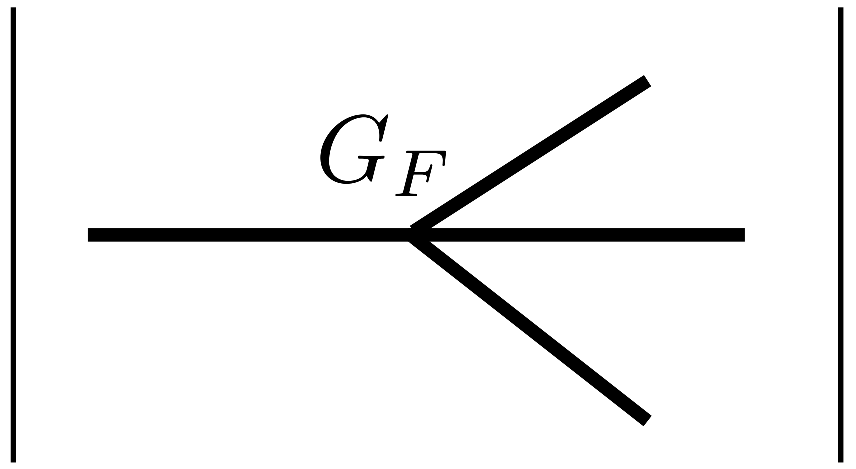
Measurements with an error $> 0.001 \times 10^{-6}$ s have been omitted.

VALUE (10^{-6} s)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
2.1969811±0.0000022 OUR AVERAGE					
2.1969803±0.0000021±0.0000007 [1]	TISHCHENKO	13	CNTR	+	Surface μ^+ at PSI
2.197083 ±0.000032 ±0.000015	BARCZYK	08	CNTR	+	Muons from π^+ decay at rest
2.197013 ±0.000021 ±0.000011	CHITWOOD	07	CNTR	+	Surface μ^+ at PSI
2.197078 ±0.000073	BARDIN	84	CNTR	+	
2.197025 ±0.000155	BARDIN	84	CNTR	−	
2.19695 ±0.00006	GIOVANETTI	84	CNTR	+	
2.19711 ±0.00008	BALANDIN	74	CNTR	+	
2.1973 ±0.0003	DUCLOS	73	CNTR	+	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
2.1969803±0.0000022	WEBBER	11	CNTR	+	Surface μ^+ at PSI

[1] TISHCHENKO 13 uses 1.6×10^{12} μ^+ events and supersedes WEBBER 11.

using dimensional analysis

$\Gamma \propto$



$^{2} \propto G_F^2 m^5$

for muon

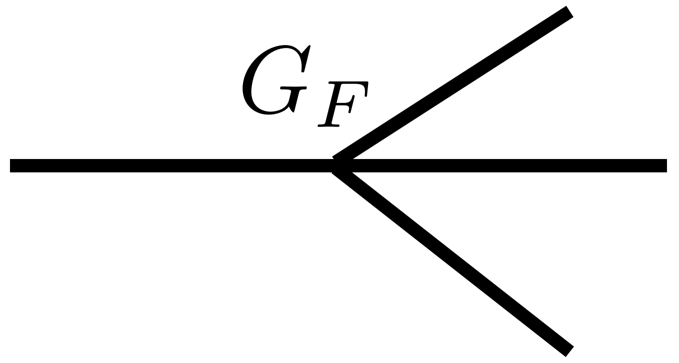
$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} \simeq (2.2\mu\,\mathrm{sec})^{-1}$$

for neutron

$$\Gamma(n \to p e \bar{\nu}_e) \sim \frac{G_F^2 \Delta m^5}{\pi^3} \sim (10^3\,\mathrm{sec})^{-1}$$

$$\Delta m = m_n - m_p \simeq 1.3\,\mathrm{MeV}$$

using dimensional analysis

$\Gamma \propto \Big|$  $\Big|^2 \propto G_F^2 m^5$

for muon

$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2 m_\mu^5}{192\pi^3} \;\simeq (2.2\mu\,\mathrm{sec})^{-1}$$

for neutron

$$\Gamma(n \to p e \bar{\nu}_e) \sim \frac{G_F^2 \Delta m^5}{\pi^3} \;\sim (10^3\,\mathrm{sec})^{-1}$$

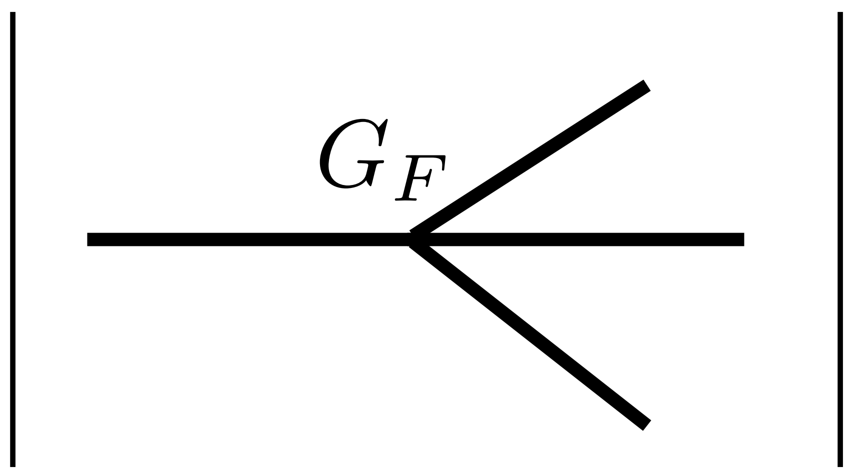
$$\Delta m = m_n - m_p \simeq 1.3\,\mathrm{MeV}$$

VALUE (s)

878.4 ± 0.5 OUR AVERAGE

below. [879.4 ± 0.6 s OUR 202

using dimensional analysis

$\Gamma \propto$ 

$^2 \propto G_F^2 m^5$

(dimensional analysis)

with the same G_F

neutron decay/muon decay can be explained

neutron decay/muon decay proceed

through *the same weak interaction*

4-Fermi interaction can be viewed as a current-current interaction (like EM)

$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

the cross terms generate neutron/muon decay

Problems of Fermi Theory

$$[\sigma] = -2$$

$$[G_F] = -2$$

From the same current-current interaction

$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

$$[\sigma] = -2$$

$$[G_F] = -2$$

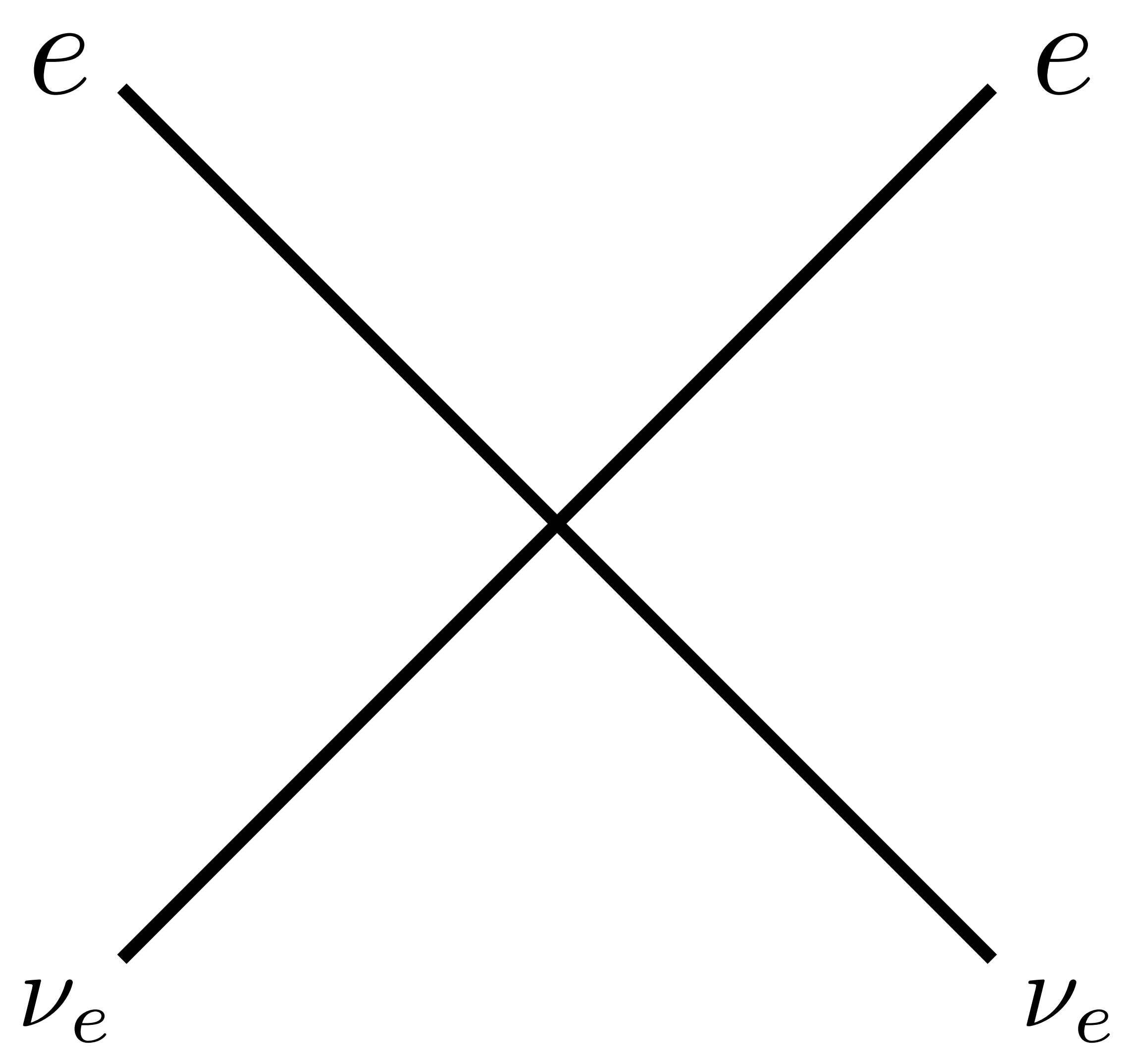
From the same current-current interaction

$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

this also generates electron-neutrino scattering



$$[\sigma] = -2$$

$$[G_F] = -2$$

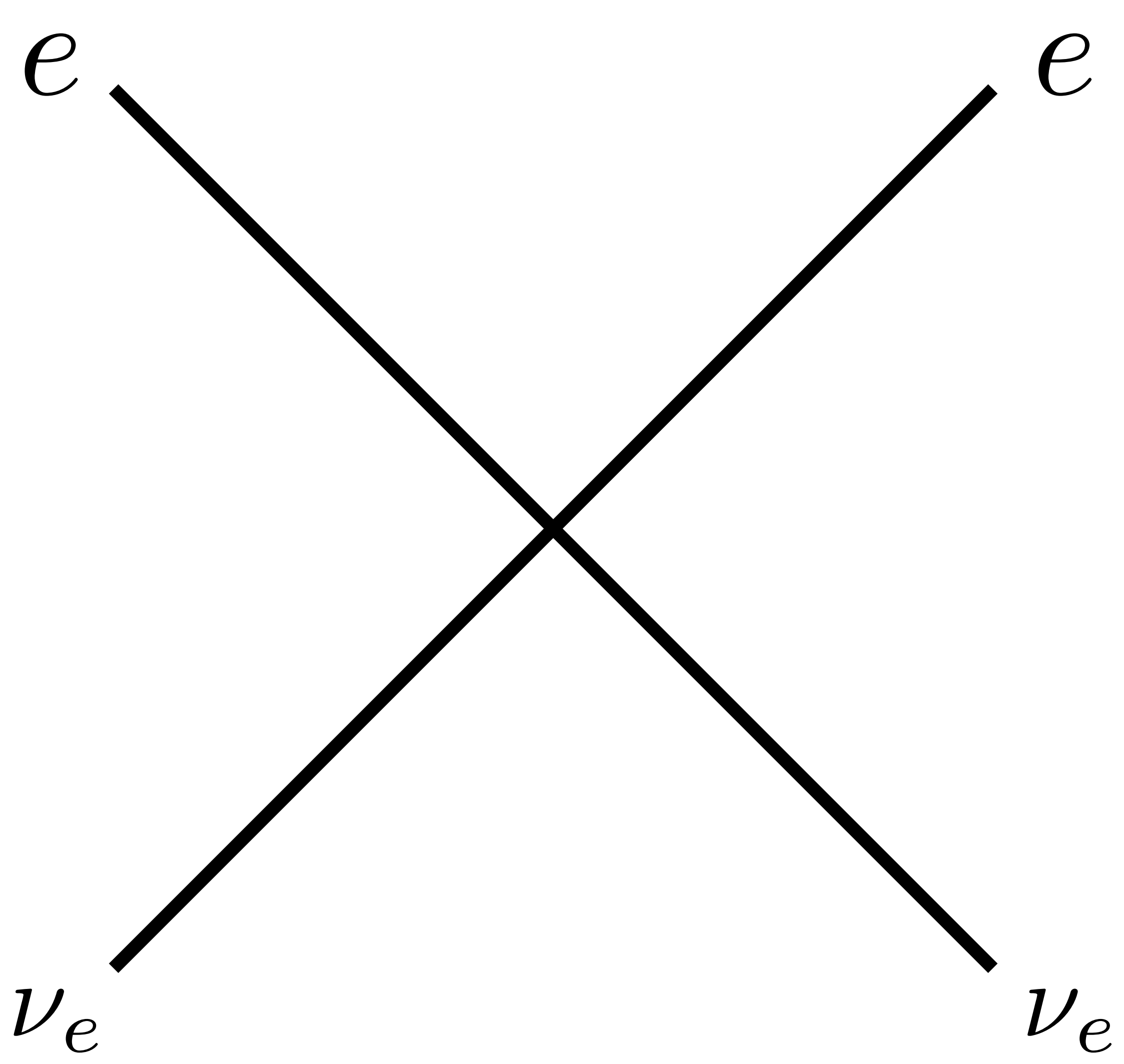
From the same current-current interaction

$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

this also generates electron-neutrino scattering



$$\sigma \propto G_F^2 E^2$$

cross-section cannot grow arbitrarily

this 4-Fermi theory becomes inconsistent at some high energy scale

$$[\sigma] = -2$$

$$[G_F] = -2$$

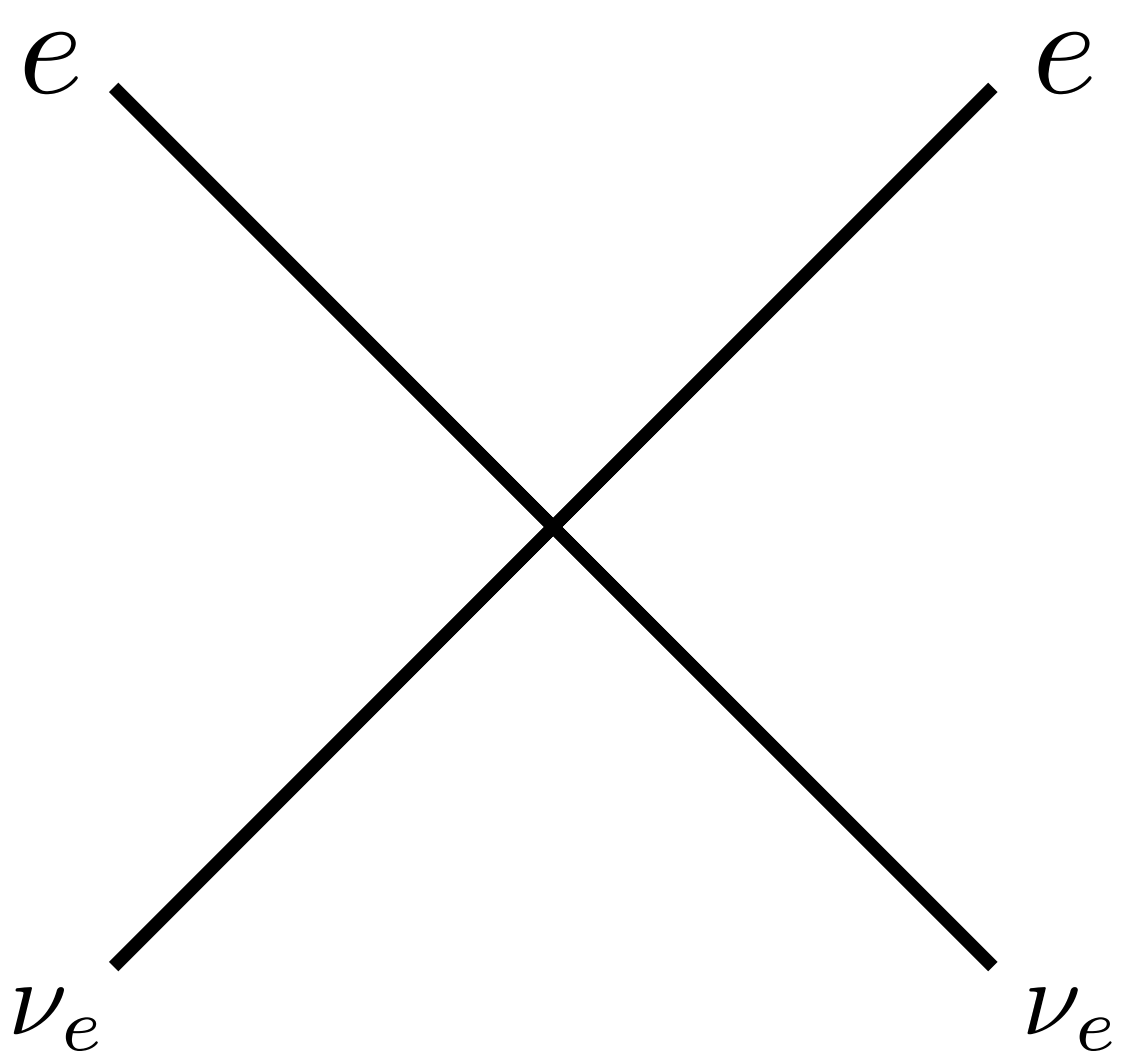
From the same current-current interaction

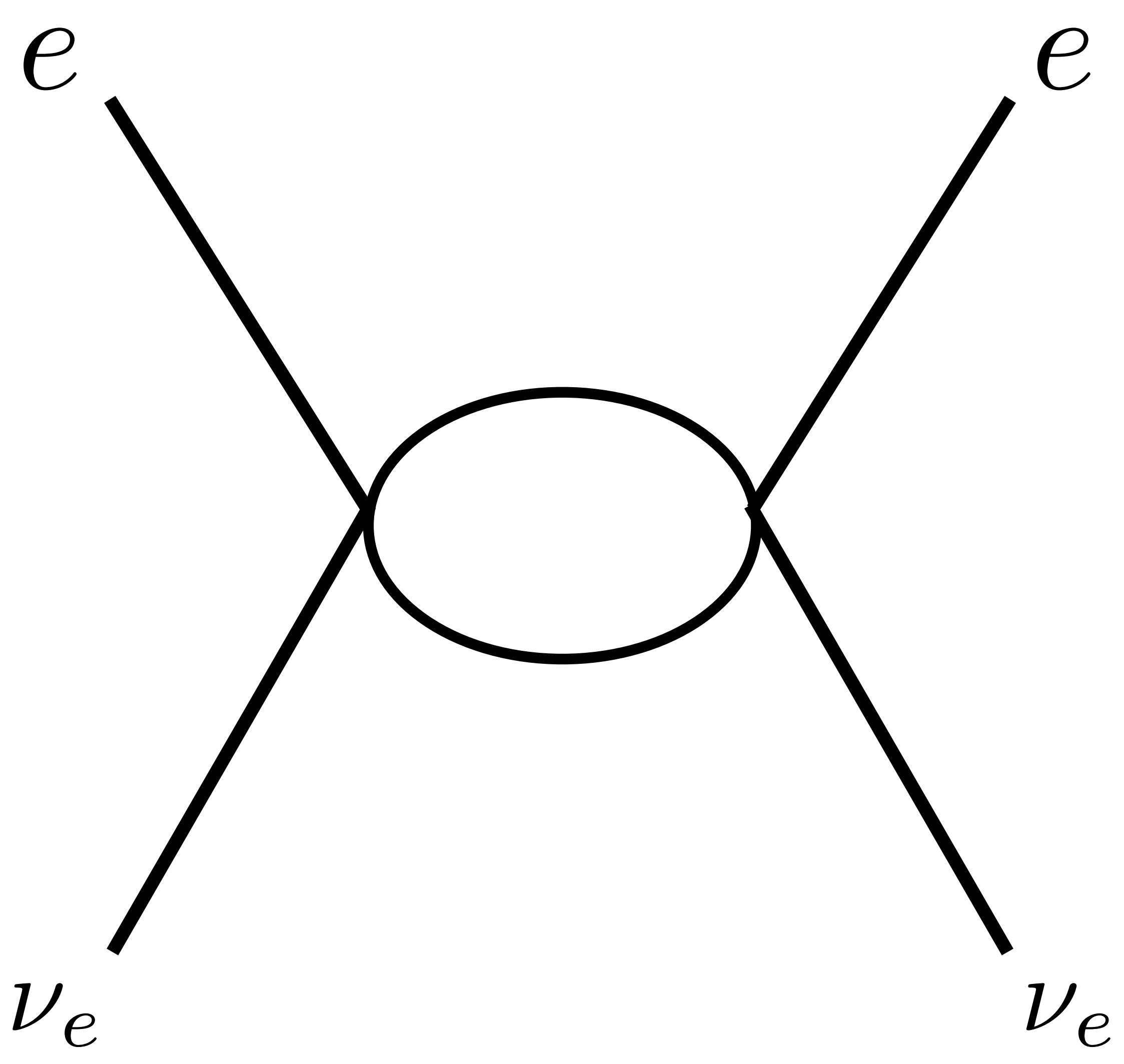
$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

this also generates electron-neutrino scattering





(quantum correction)

$$[\sigma] = -2$$

$$[G_F] = -2$$

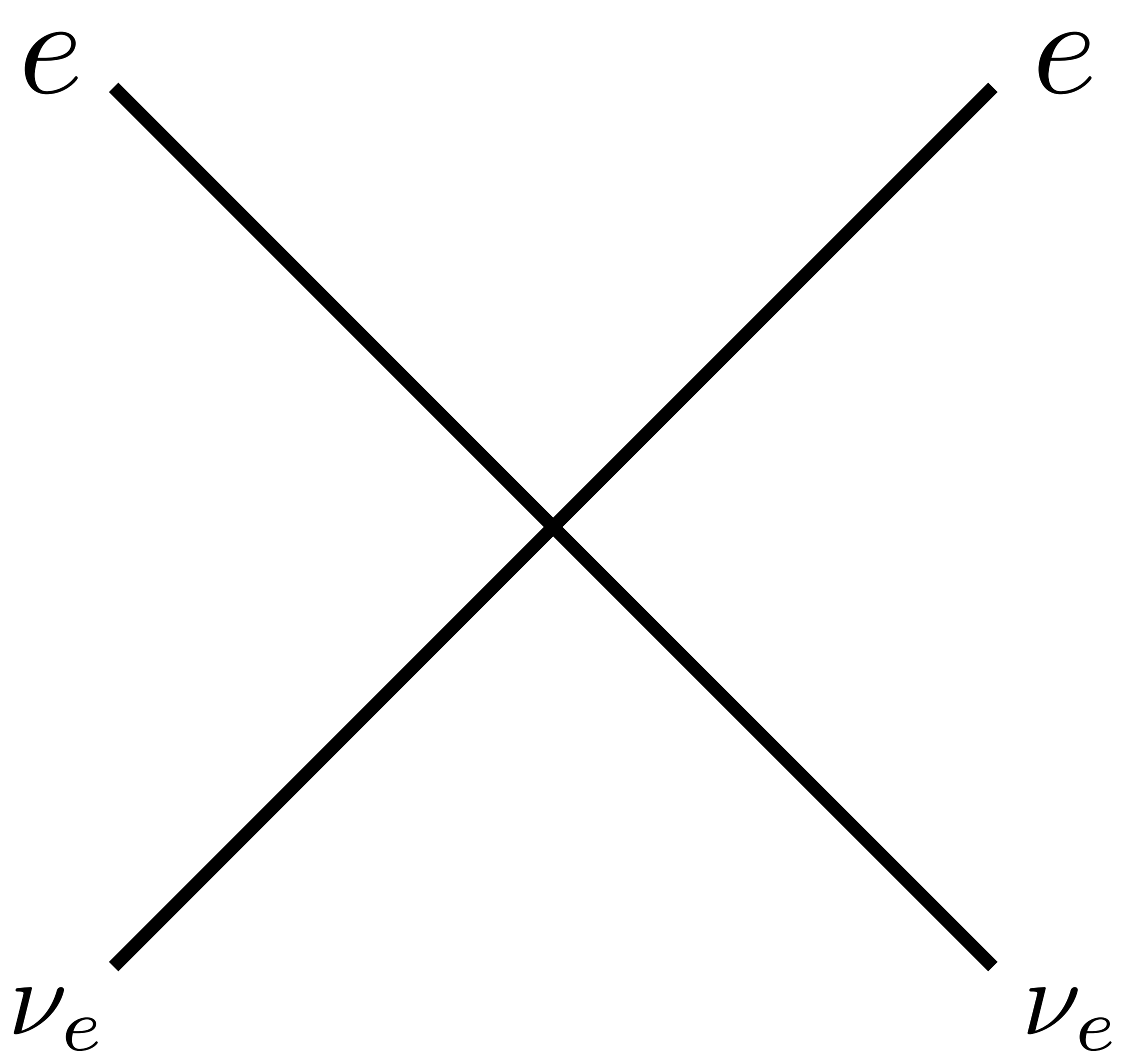
From the same current-current interaction

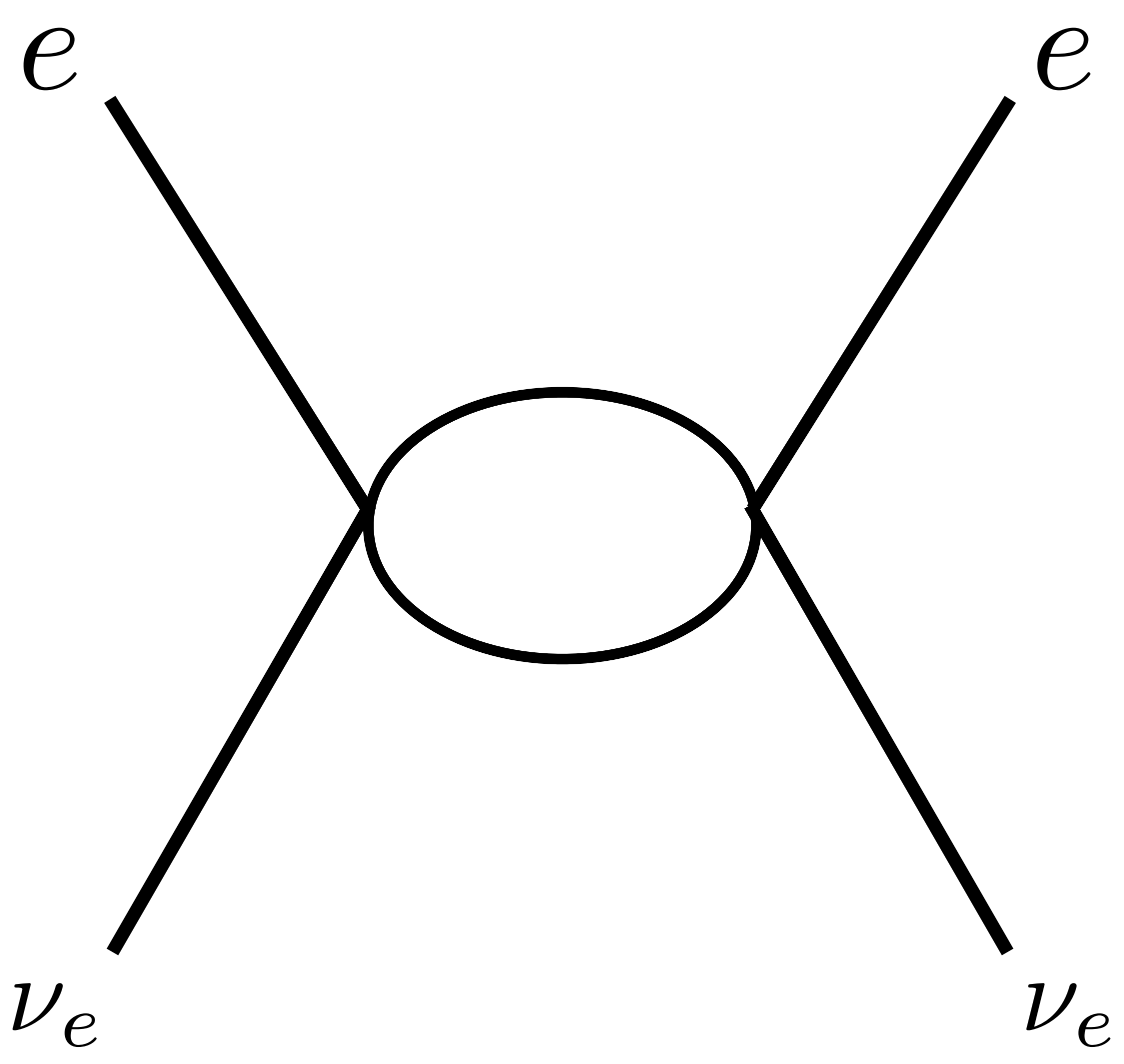
$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

this also generates electron-neutrino scattering





(quantum correction)

$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6$$

$$[\sigma] = -2$$

$$[G_F] = -2$$

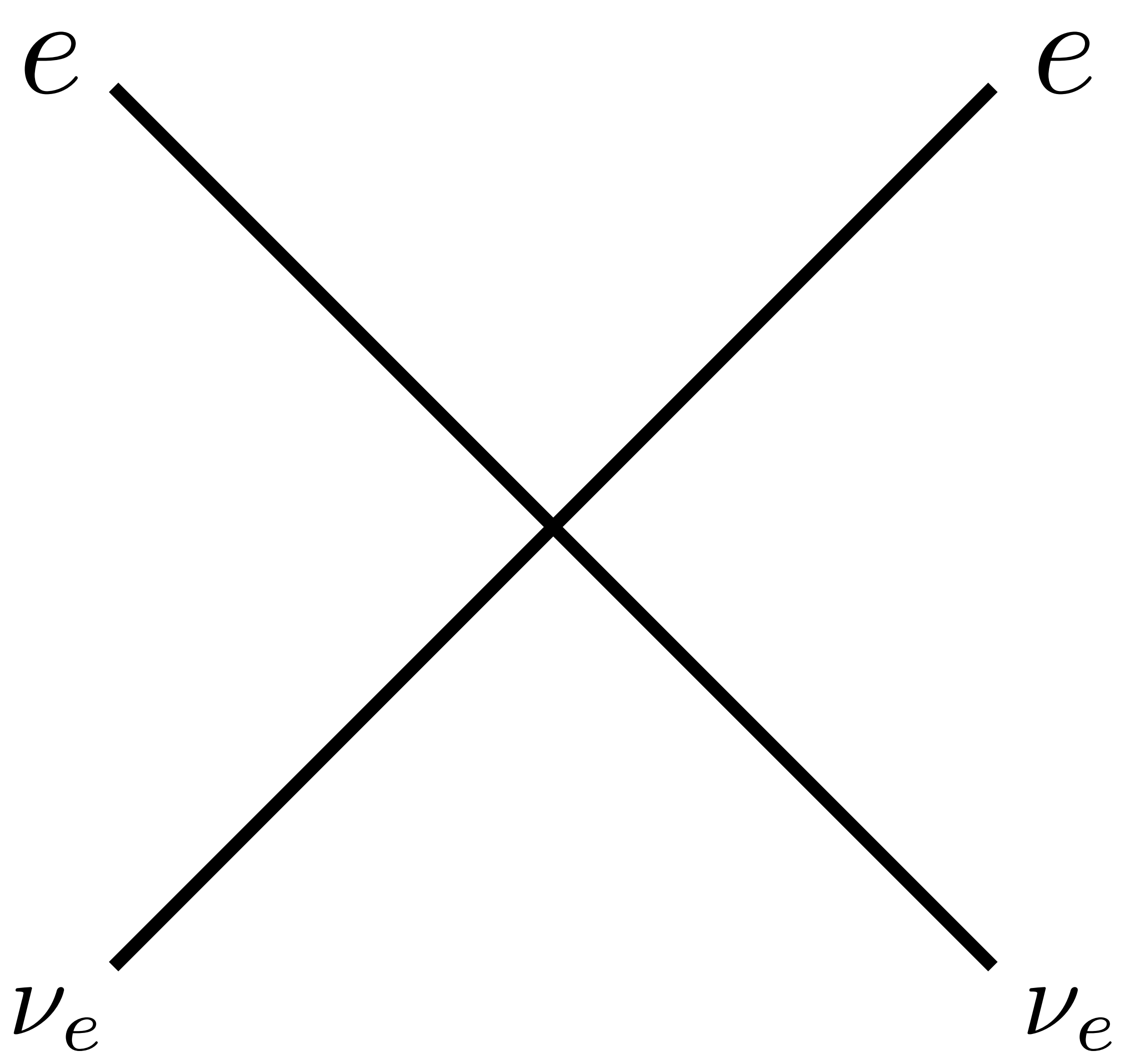
From the same current-current interaction

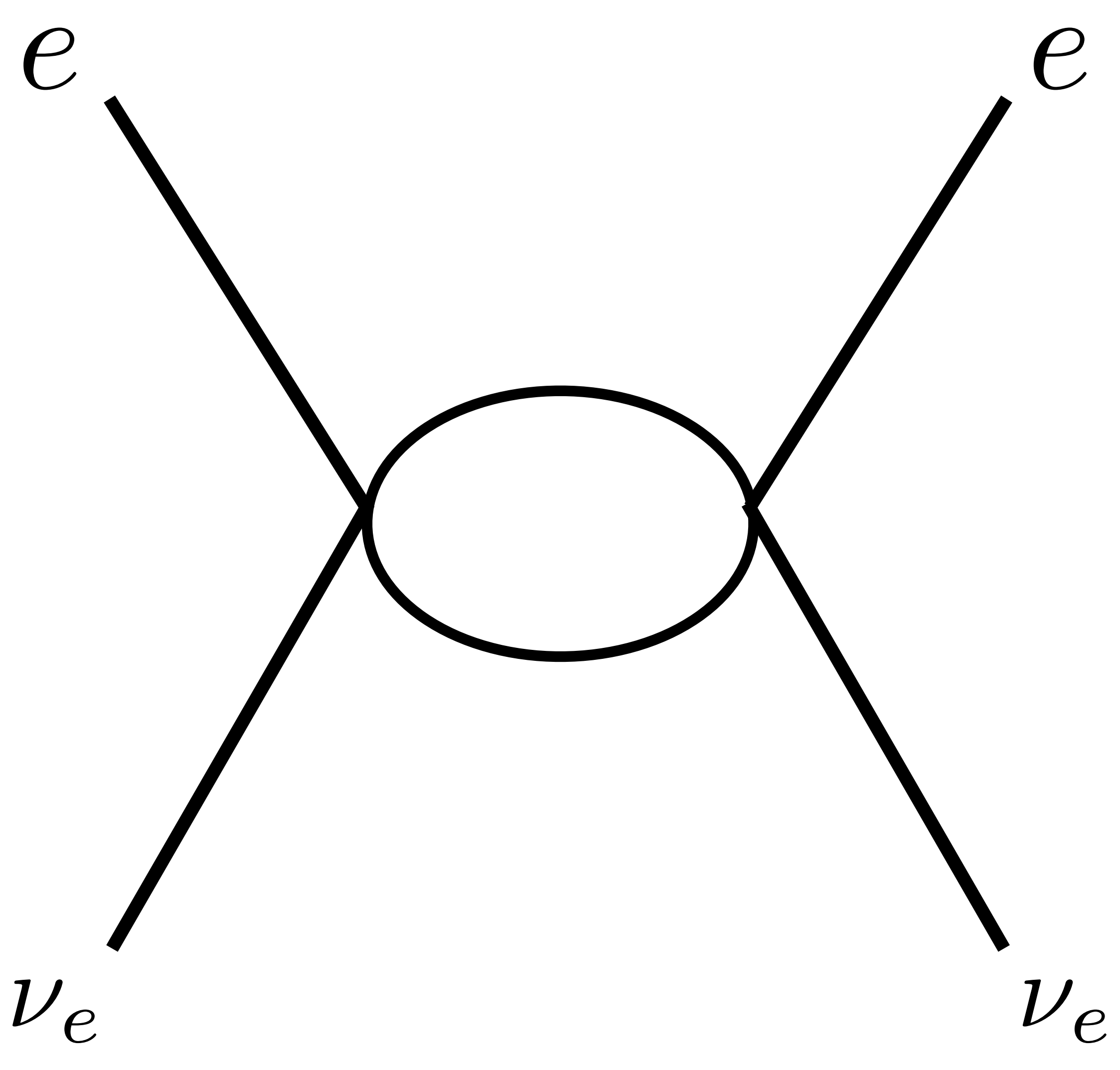
$$\mathcal{L} = G_F J^+_\mu J^{-\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

this also generates electron-neutrino scattering





(quantum correction)

$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6$$

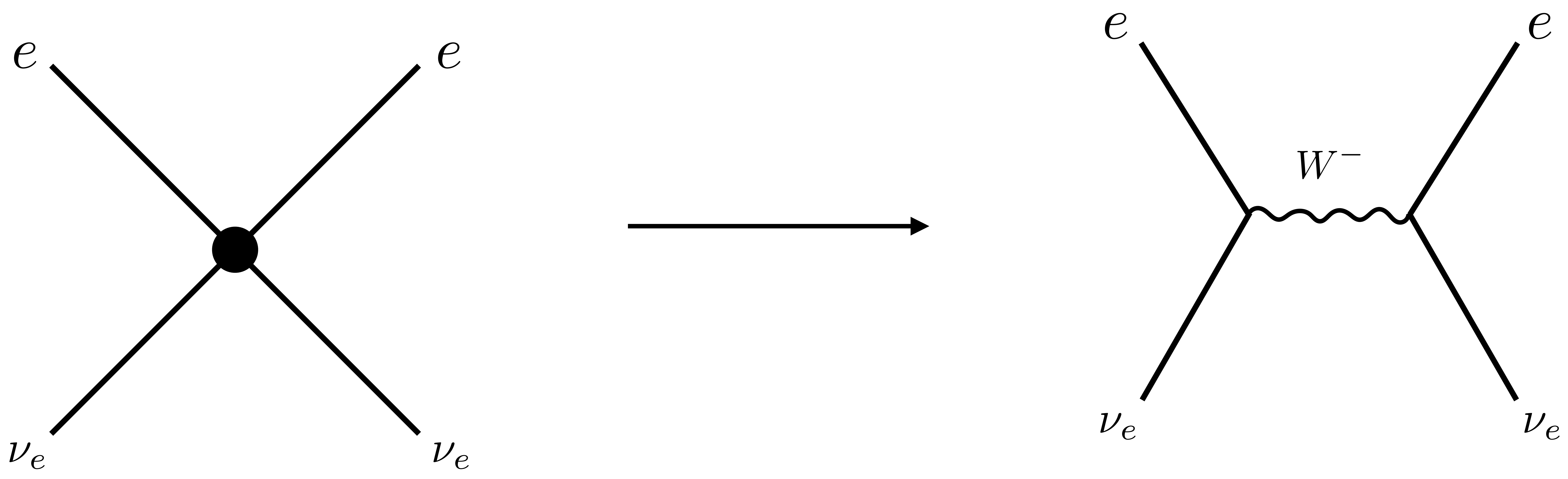
when

$$E = E_{\max} = 2\sqrt{\frac{\pi}{G_F}}$$

perturbation theory breaks down

the current-current interaction seems to suggest

weak interaction might be mediated by spin-1 bosons



before we present resolution to the problem of Fermi theory

let us discuss *gauge symmetry*

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

this theory has a U(1) *global symmetry*

i.e. the Lagrangian is invariant under

$$\psi \to e^{i\alpha}\psi \qquad (\alpha = \text{const.})$$

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

this theory has a U(1) *global symmetry*

i.e. the Lagrangian is invariant under

$$\psi \to e^{i\alpha}\psi \qquad (\alpha = \text{const.})$$

it is straightforward to check

$$\bar{\psi}\psi \to \bar{\psi}\psi$$

$$\bar{\psi}\gamma^\mu\psi \to \bar{\psi}\gamma^\mu\psi$$

so that whole Lagrangian is invariant under the global U(1) transformation

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

Actually, the theory contains a larger symmetry, *U(1) gauge symmetry*

$$\psi \to e^{i\alpha(x)}\psi \qquad\qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$$

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

Actually, the theory contains a larger symmetry, *U(1) gauge symmetry*

$$\psi \to e^{i\alpha(x)}\psi \qquad\qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$$

since $\alpha = \alpha(\mathrm{x})$ we find

$$\bar{\psi}(i\gamma^\mu\partial_\mu)\psi \to \bar{\psi}(i\gamma^\mu\partial_\mu)\psi - \bar{\psi}\gamma^\mu\psi(\partial_\mu\alpha)$$

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

Actually, the theory contains a larger symmetry, *U(1) gauge symmetry*

$$\psi \to e^{i\alpha(x)}\psi \qquad\qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha(x)$$

since $\alpha = \alpha(\mathrm{x})$ we find

$$\bar{\psi}(i\gamma^\mu\partial_\mu)\psi \to \bar{\psi}(i\gamma^\mu\partial_\mu)\psi - \bar{\psi}\gamma^\mu\psi(\partial_\mu\alpha)$$

$$eA_\mu\bar{\psi}\gamma^\mu\psi \to eA_\mu\bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\psi(\partial_\mu\alpha)$$

so the theory is invariant under local gauge transformation

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

local gauge invariance plays a fundamental role in modern particle physics

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

local gauge invariance plays a fundamental role in modern particle physics

in QED [U(1) gauge theory]

- gauge invariance allows A_μ to contain only 2 polarization states
- guarantee that the photon remains massless

$(m^2 A_\mu A^\mu \text{ is not gauge invariant})$

- required for the consistency of the theory

Let us go back to *Quantum Electrodynamics (QED)*

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi$$

local gauge invariance plays a fundamental role in modern particle physics

in QED [U(1) gauge theory]

- gauge invariance allows A_μ to contain only 2 polarization states
- guarantee that the photon remains massless

$(m^2 A_\mu A^\mu \text{ is not gauge invariant})$

- required for the consistency of the theory

gauge invariance is often taken as a starting point for building a consistent theory

U(1) gauge symmetry, again

let us now take the local gauge invariance as a starting point

and require a theory to be invariant under the local gauge transformation

consider a theory of a complex scalar field

$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - m^2 |\phi|^2$$

U(1) gauge symmetry, again

let us now take the local gauge invariance as a starting point

and require a theory to be invariant under the local gauge transformation

consider a theory of a complex scalar field

$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - m^2 |\phi|^2$$

under the gauge transformation

$$\phi \to e^{i\alpha(x)} \phi$$

U(1) gauge symmetry, again

let us now take the local gauge invariance as a starting point

and require a theory to be invariant under the local gauge transformation

consider a theory of a complex scalar field

$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - m^2 |\phi|^2$$

under the gauge transformation

$$\phi \to e^{i\alpha(x)} \phi$$

the derivative transforms

$$\partial_\mu \phi \to e^{i\alpha(x)} (\partial_\mu + i \partial_\mu \alpha) \phi$$

to compensate this we need to introduce a gauge field

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$$

U(1) gauge symmetry, again

consider a theory of a complex scalar field

$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - m^2 |\phi|^2$$

under the gauge transformation

$$\phi \to e^{i\alpha(x)} \phi$$

the derivative transforms

$$\partial_\mu \phi \to e^{i\alpha(x)} (\partial_\mu + i\partial_\mu \alpha)\phi$$

a convenient way to introduce a gauge field is through *covariant derivative*

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$$

$$D_\mu \phi \to e^{i\alpha(x)} D_\mu \phi$$

$$(D_\mu \phi)^* (D_\mu \phi) \to (D_\mu \phi)^* (D_\mu \phi)$$

$$D_\mu = \partial_\mu + ieA_\mu$$

U(1) gauge symmetry, again

a theory of complex scalar field invariant under U(1) gauge symmetry

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - m^2 |\phi|^2$$

(scalar quantum electrodynamics)

repeating the same exercise for the Dirac theory leads to

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_\mu \bar{\psi}\gamma^\mu \psi\end{aligned}$$

$D_\mu = \partial_\mu + ieA_\mu$

U(1) gauge symmetry, again

a theory of complex scalar field invariant under U(1) gauge symmetry

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - m^2 |\phi|^2$$

(scalar quantum electrodynamics)

repeating the same exercise for the Dirac theory leads to

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \\ &= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eA_\mu \underbrace{\bar{\psi}\gamma^\mu\psi}_{J^\mu_{\text{em}}}\end{aligned}$$

Non-Abelian gauge symmetry

we can generalize U(1) gauge symmetry by considering a more general transformation

let us consider for simplicity SU(2) transformation

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad (\psi_{1,2}\text{: Dirac fields})$$

$$\psi \to U(x)\psi = e^{i\alpha^a(x)T^a}\psi$$

Non-Abelian gauge symmetry

we can generalize U(1) gauge symmetry by considering a more general transformation

let us consider for simplicity SU(2) transformation

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad (\psi_{1,2}\text{: Dirac fields})$$

$$\psi \to U(x)\psi = e^{i\alpha^a(x)T^a}\psi$$

introduce gauge field through covariant derivative

$$D_\mu\psi = (\partial_\mu + igA_\mu)\psi \to U(x)D_\mu\psi$$

$$A_\mu \to UA_\mu U^\dagger + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$A_\mu = A_\mu^a T^a$$

$$T^a = \sigma^a/2$$

for SU(2)

Non-Abelian gauge symmetry

we can generalize U(1) gauge symmetry by considering a more general transformation

let us consider for simplicity SU(2) transformation

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad (\psi_{1,2}\text{: Dirac fields})$$

$$\psi \to U(x)\psi = e^{i\alpha^a(x)T^a}\psi$$

introduce gauge field through covariant derivative

$$D_\mu\psi = (\partial_\mu + igA_\mu)\psi \to U(x)D_\mu\psi$$

$$A_\mu \to UA_\mu U^\dagger + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$A_\mu = A_\mu^a T^a$$

with a field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

$$T^a = \sigma^a/2$$
for SU(2)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Non-Abelian gauge symmetry

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\supset g(\partial A)AA + g^2 AAAA$$

(self-interaction between gauge fields unlike U(1) gauge theory)

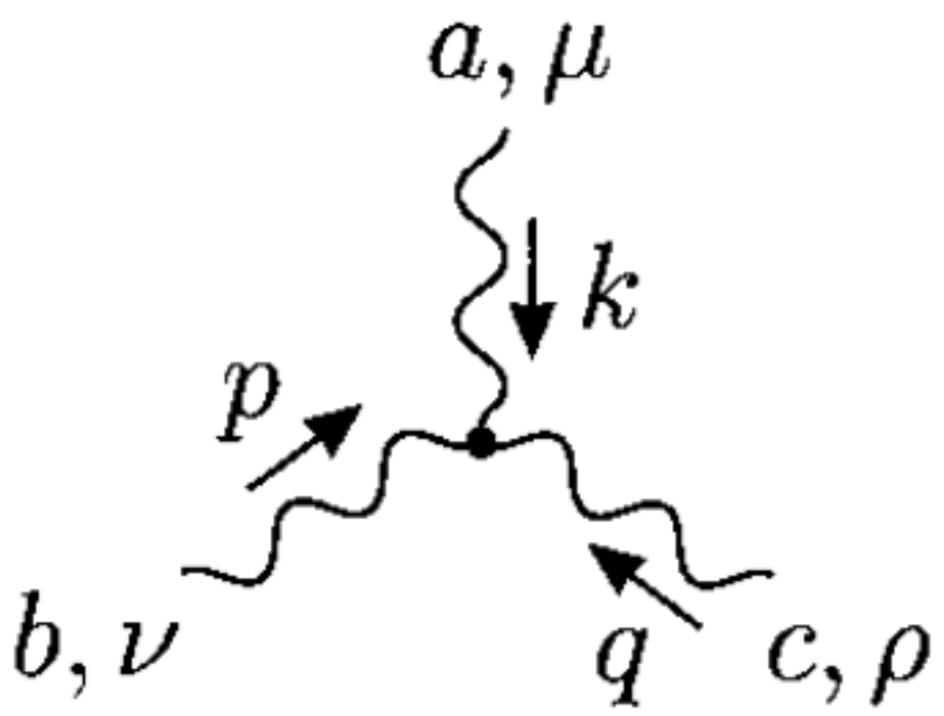
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

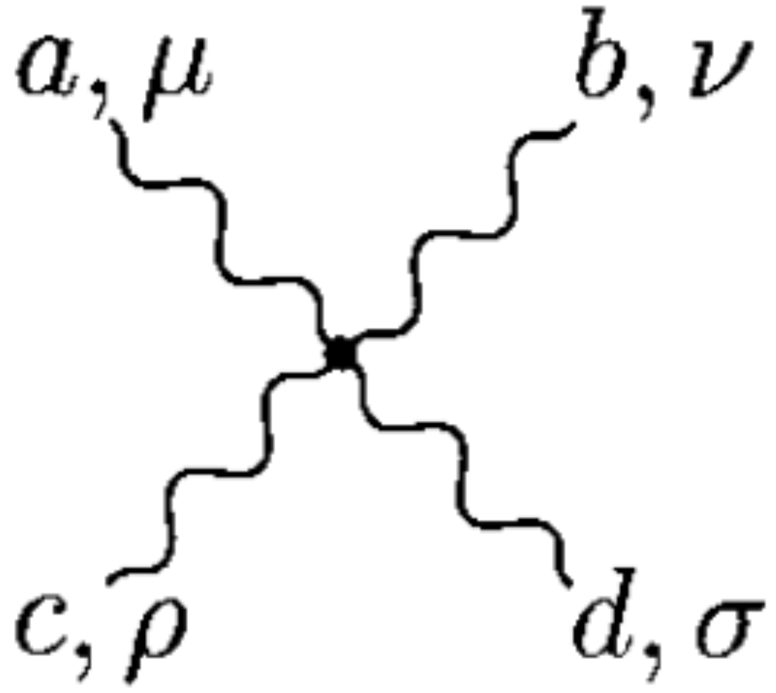
Non-Abelian gauge symmetry

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\supset g(\partial A)AA + g^2 AAAA$$

(self-interaction between gauge fields unlike U(1) gauge theory)





Non-Abelian gauge symmetry

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$D_\mu = \partial_\mu + igA_\mu$$

$$A_\mu = A_\mu^a T^a$$

$$T^a = \sigma^a/2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Non-Abelian gauge symmetry

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

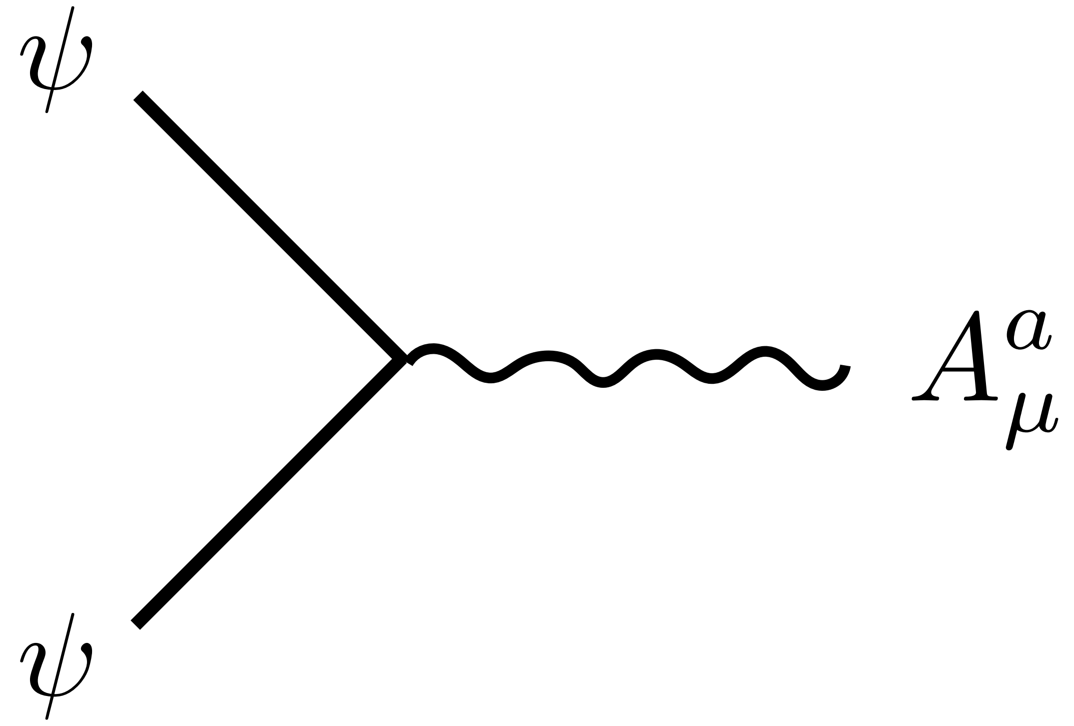
$$D_\mu = \partial_\mu + igA_\mu$$

$$A_\mu = A_\mu^a T^a$$

$$T^a = \sigma^a/2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\supset g\bar{\psi}\gamma^\mu A_\mu \psi$$



$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Non-Abelian gauge symmetry

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

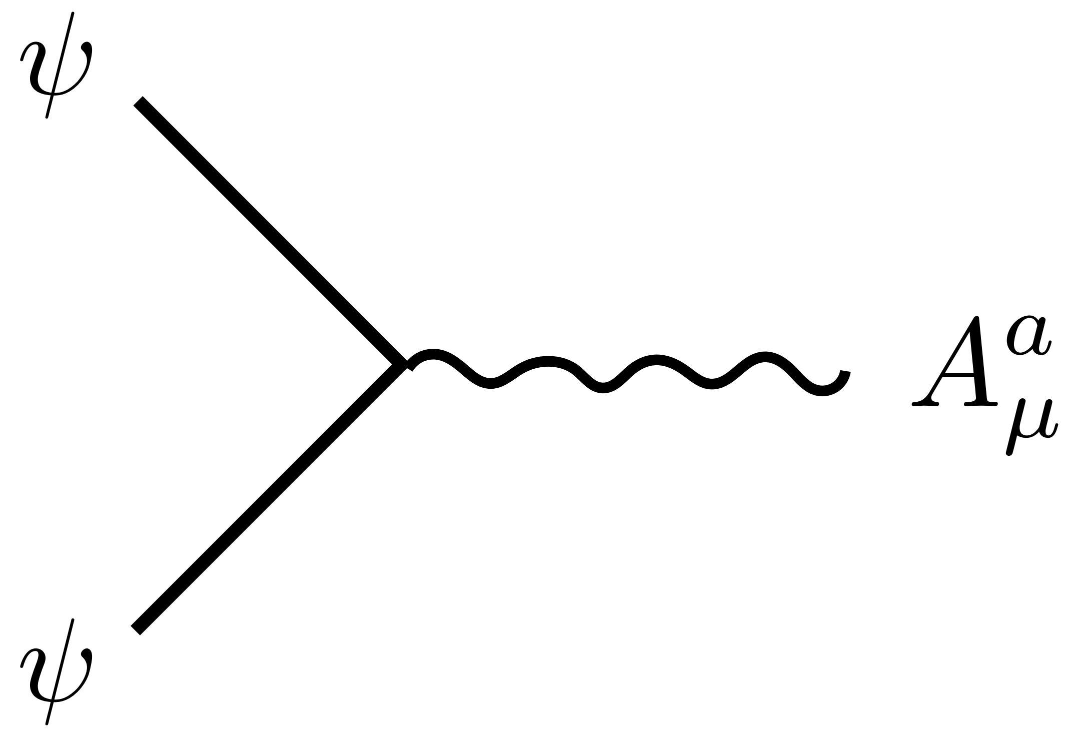
$$D_\mu = \partial_\mu + igA_\mu$$

$$A_\mu = A_\mu^a T^a$$

$$T^a = \sigma^a/2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\supset g\bar{\psi}\gamma^\mu A_\mu \psi$$



$$= g\bar{\psi}\gamma^\mu T^a \psi A_\mu^a \quad = J^{a\mu} A_\mu^a$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

Non-Abelian gauge symmetry

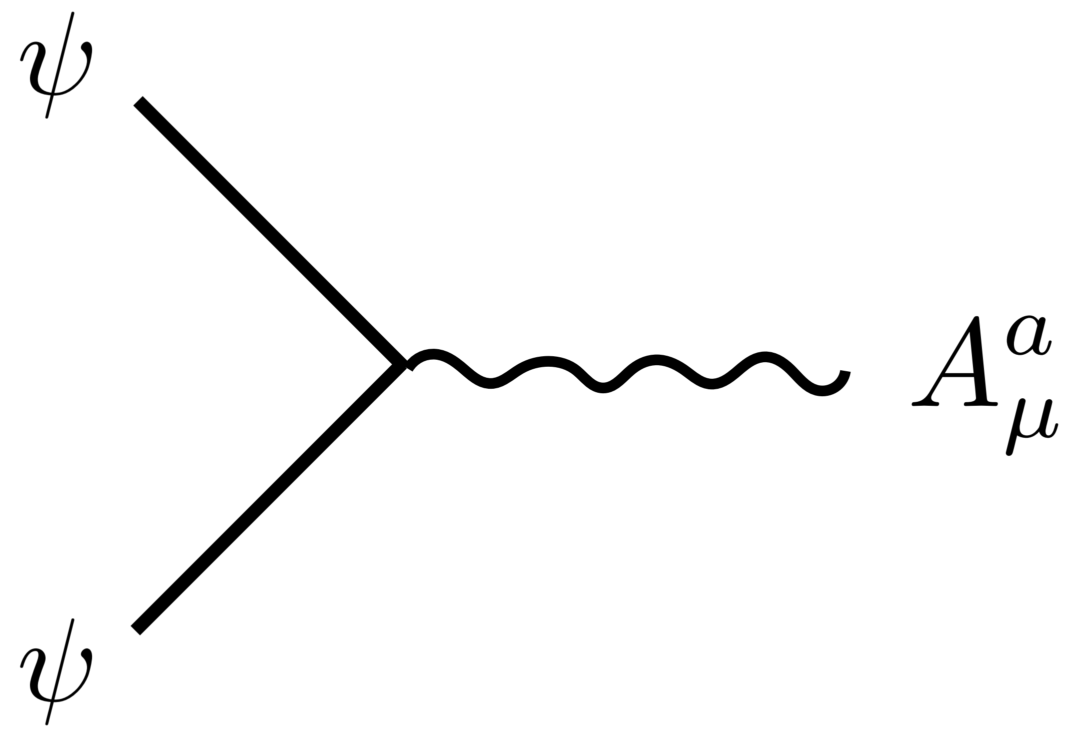
$$D_\mu = \partial_\mu + igA_\mu$$

$$A_\mu = A_\mu^a T^a$$

$$T^a = \sigma^a/2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$\supset g\bar{\psi}\gamma^\mu A_\mu \psi$$



$$= g\bar{\psi}\gamma^\mu T^a \psi A_\mu^a \quad = J^{a\mu} A_\mu^a$$

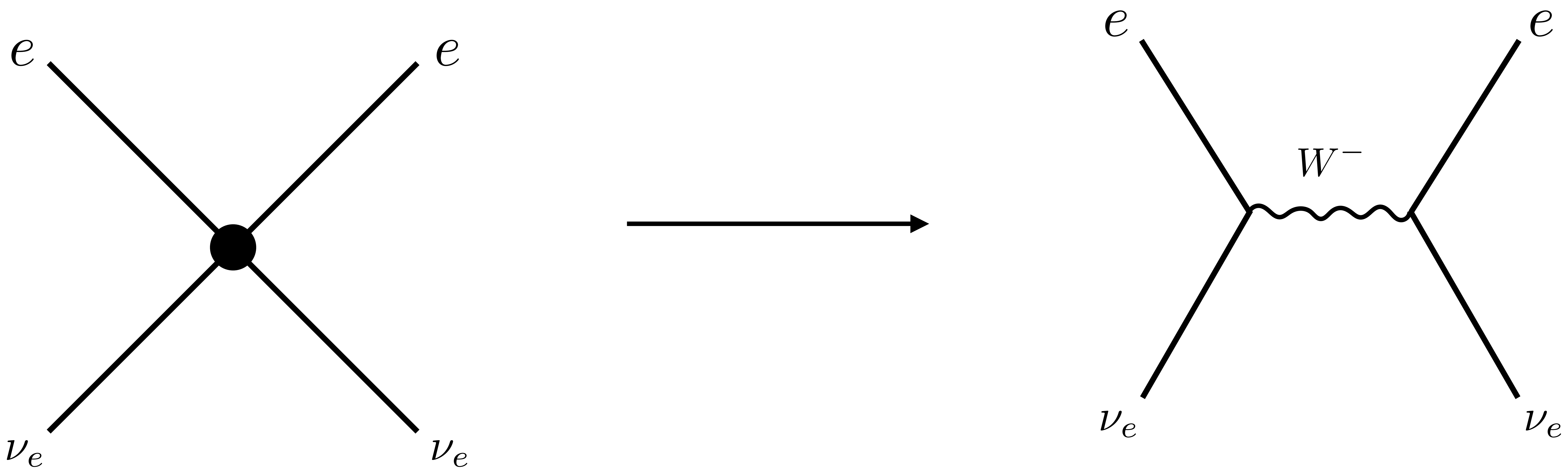
$$= \frac{g}{2}\bar{\psi}\gamma^\mu \begin{pmatrix} A_\mu^3 & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 \end{pmatrix} \psi$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$T^a = \sigma^a/2$
$Y = -1/2$

The form of current-current interaction

suggests that weak interaction might be mediated by spin-1 particles



introduce $SU(2) \times U(1)$ gauge theory

$T^a = \sigma^a/2$

$Y = -1/2$

introduce $SU(2) \times U(1)$ gauge theory

$$\psi = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$T^a = \sigma^a/2$

$Y = -1/2$

introduce $SU(2) \times U(1)$ gauge theory

$$\psi = \left(\begin{array}{c} \nu_e \\ e \end{array} \right)$$

$$= \bar{\nu}_e \gamma^\mu e W_\mu^+ + \bar{e} \gamma^\mu \nu_e W_\mu^-$$

$$T^a = \sigma^a/2$$

$$Y = -1/2$$

introduce $SU(2) \times U(1)$ gauge theory

$$\psi = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

under gauge transformation

$$\psi \to e^{i\alpha^a(x)T^a} e^{iY\beta(x)} \psi$$

$$= \bar{\nu}_e \gamma^\mu e W^+_\mu + \bar{e}\gamma^\mu \nu_e W^-_\mu$$

$T^a = \sigma^a/2$

$Y = -1/2$

introduce $SU(2) \times U(1)$ gauge theory

$$\psi = \left(\begin{array}{c} \nu_e \\ e \end{array} \right)$$

under gauge transformation

$$\psi \to e^{i\alpha^a(x)T^a} e^{iY\beta(x)} \psi$$

the Lagrangian is

$$\mathcal{L} = \bar{\psi} i \gamma^\mu (\partial_\mu - igA_\mu^a T^a - ig' Y B_\mu) \psi$$

$$= \bar{\nu}_e \gamma^\mu e W_\mu^+ + \bar{e} \gamma^\mu \nu_e W_\mu^-$$

$T^a = \sigma^a/2$

$Y = -1/2$

introduce $SU(2) \times U(1)$ gauge theory

$$\psi = \left(\begin{array}{c} \nu_e \\ e \end{array} \right)$$

under gauge transformation

$$\psi \to e^{i\alpha^a(x) T^a} e^{iY\beta(x)} \psi$$

the Lagrangian is

$$\mathcal{L} = \bar{\psi} i\gamma^\mu (\partial_\mu - igA_\mu^a T^a - ig' Y B_\mu)\psi$$

$$\supset \left(\begin{array}{cc} \bar{\nu}_e & \bar{e} \end{array} \right) \gamma^\mu \left(\begin{array}{cc} gA_\mu^3 - g'B_\mu & \sqrt{2} g W_\mu^+ \\ \sqrt{2} g W_\mu^- & gA_\mu^3 + g'B_\mu \end{array} \right) \left(\begin{array}{c} \nu_e \\ e \end{array} \right)$$

$$= \bar{\nu}_e \gamma^\mu e W_\mu^+ + \bar{e} \gamma^\mu \nu_e W_\mu^-$$

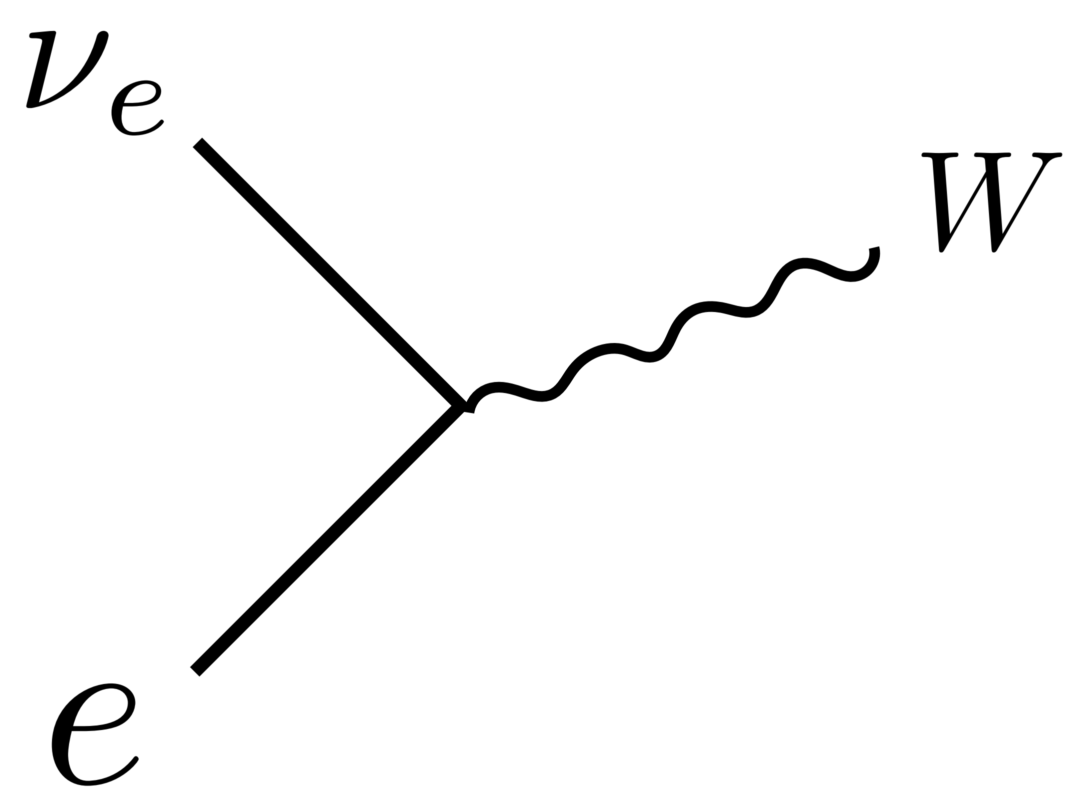
$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu^2)$$

the Lagrangian is

$$\mathcal{L} = \bar{\psi} i\gamma^\mu(\partial_\mu - igA^a_\mu T^a - ig'YB_\mu)\psi$$

$$\supset \begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix} \gamma^\mu \begin{pmatrix} gA^3_\mu - g'B_\mu & \sqrt{2}gW^+_\mu \\ \sqrt{2}gW^-_\mu & gA^3_\mu + g'B_\mu \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$\propto \bar{\nu}_e\gamma^\mu e W^+_\mu + \bar{e}\gamma^\mu \nu_e W^-_\mu$$



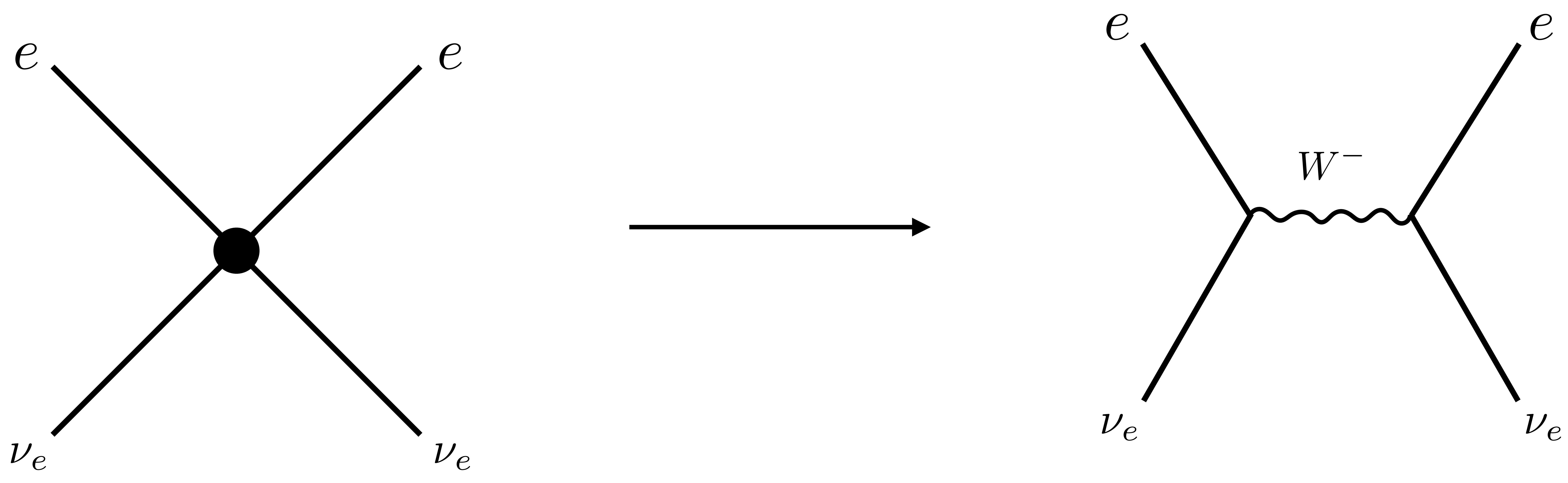
$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)$$

$$Z_\mu = \frac{gA^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}}$$

$$A_\mu = \frac{g'A^3_\mu + gB_\mu}{\sqrt{g^2 + g'^2}}$$

The form of current-current interaction

suggests that weak interaction might be mediated by spin-1 particles

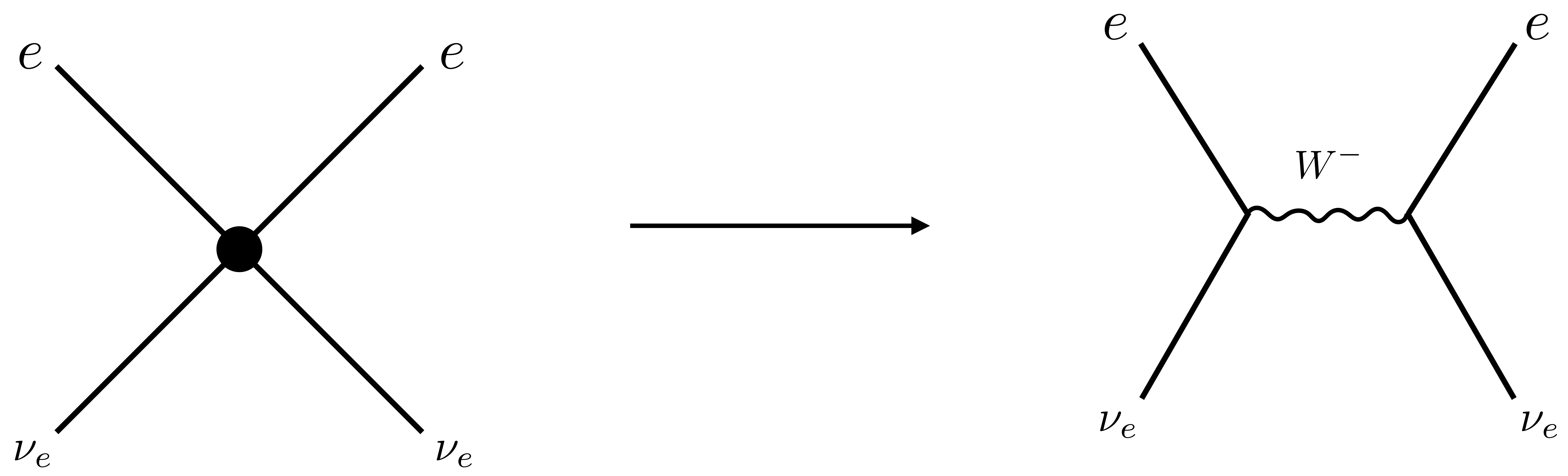


$$\sigma \propto G_F^2 E^2$$

$$\sigma \propto \frac{g^4}{(E^2 + m_W^2)^2} E^2$$

The form of current-current interaction

suggests that weak interaction might be mediated by spin-1 particles



$\sigma \propto G_F^2 E^2$

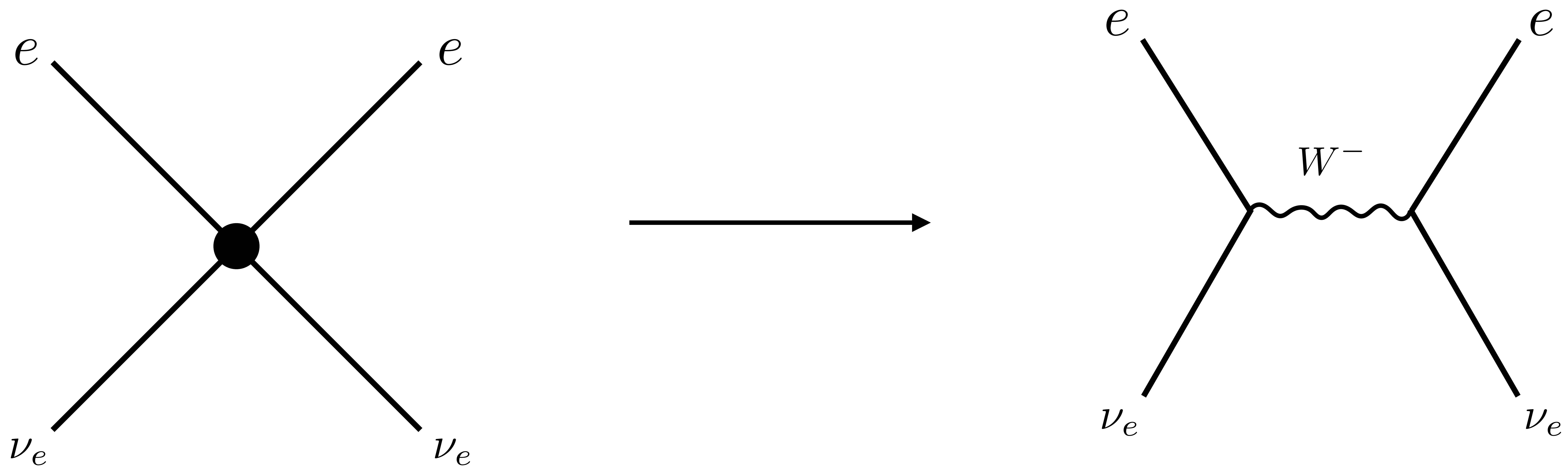
$$\sigma \propto \frac{g^4}{(E^2 + m_W^2)^2} E^2$$

in a low energy limit, the left and right is the same provided

$$G_F \propto \frac{g^2}{m_W^2}$$

The form of current-current interaction

suggests that weak interaction might be mediated by spin-1 particles



$$\mathcal{L} = -m_W^2 W_\mu^+ W^{-\mu} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

$$\mathcal{L} = -m_W^2 W_\mu^+ W^{-\mu} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

In low energy limit

we can ‘integrate out’ heavy gauge boson by using the equation of motion

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \longrightarrow W_\mu^- = \frac{g}{m_W^2} J_\mu^-$$

$$\mathcal{L} = -m_W^2 W_\mu^+ W^{-\mu} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

In low energy limit

we can 'integrate out' heavy gauge boson by using the equation of motion

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \longrightarrow \quad W_\mu^- = \frac{g}{m_W^2} J_\mu^-$$

the same current-current interaction of Fermi theory !

$$\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J^{-\mu}$$

$$\mathcal{L} = -m_W^2 W_\mu^+ W^{-\mu} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

$$J^{+\mu} = (\bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \cdots)$$

$$J^{-\mu} = (\bar{p}\gamma^\mu n + \bar{\nu}_e\gamma^\mu e + \bar{\nu}_\mu\gamma^\mu \mu + \cdots)$$

In low energy limit

we can ‘integrate out’ heavy gauge boson by using the equation of motion

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \longrightarrow W_\mu^- = \frac{g}{m_W^2} J_\mu^-$$

the same current-current interaction of Fermi theory !

$$\mathcal{L} = \boxed{\frac{g^2}{m_W^2}} J_\mu^+ J^{-\mu}$$

$$= G_F$$

While the parity symmetry (x-> -x) is a good symmetry of QED

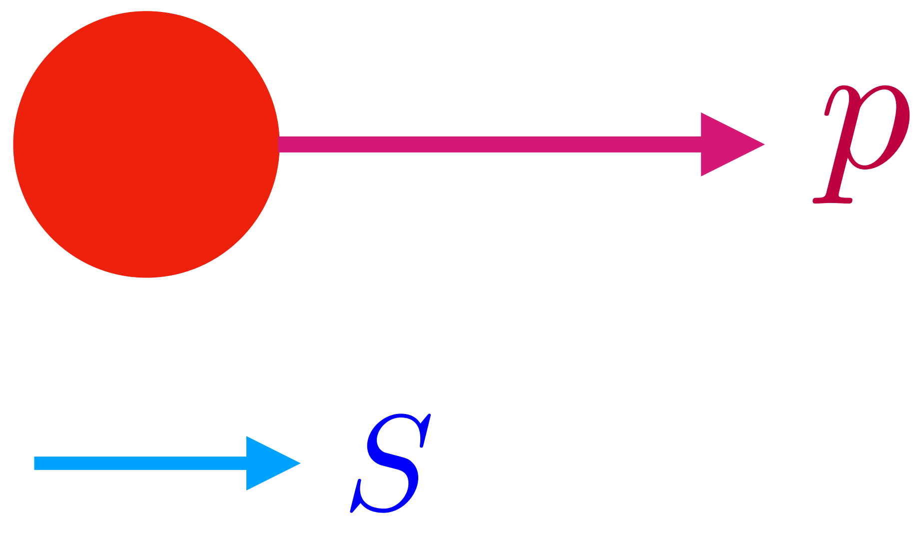
it is maximally broken by weak interaction

which is confirmed by a series of experiments in 50’s [e.g. Wu et al (57)]

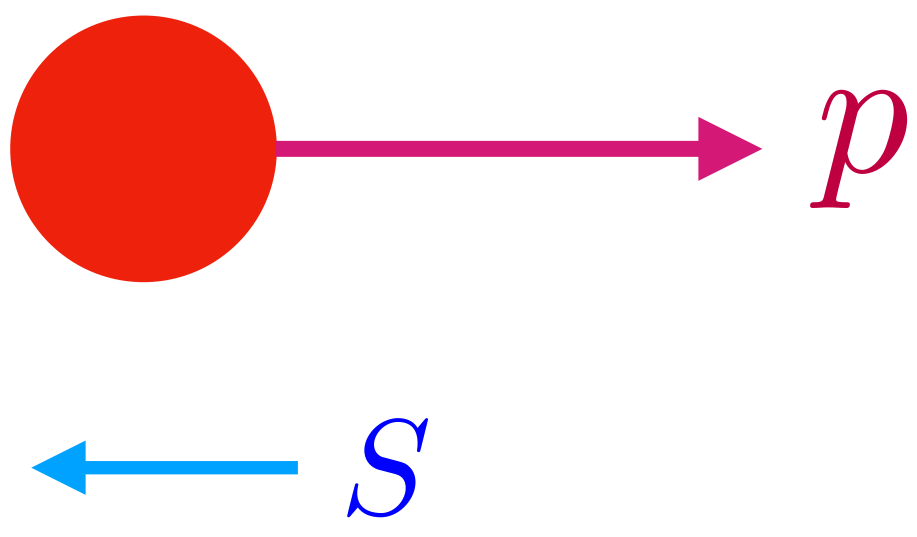
Chirality / Helicity

particles of spin *s* has *(2s+1)* independent states (QM)

electron has two states



right-handed
(helicity = +1/2)

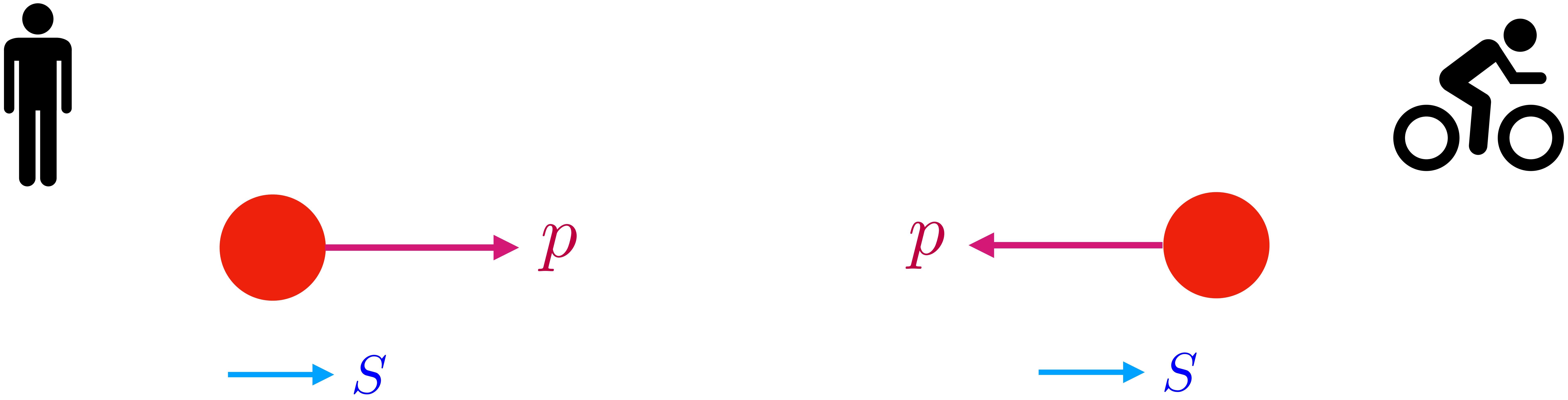


left-handed
(helicity = -1/2)

$$h = \frac{1}{2}\hat{p} \cdot S$$

Chirality / Helicity

consider a right-handed *massive* electron



right-handed

left-handed

helicity is not Lorentz invariant

it changes depending on the choice of frame

Chirality / Helicity

consider a right-handed *massless* electron

right-handed

right-handed

since the particle is massless

helicity becomes invariant under Lorentz transformtion

Chirality / Helicity

consider a right-handed massless electron

right-handed

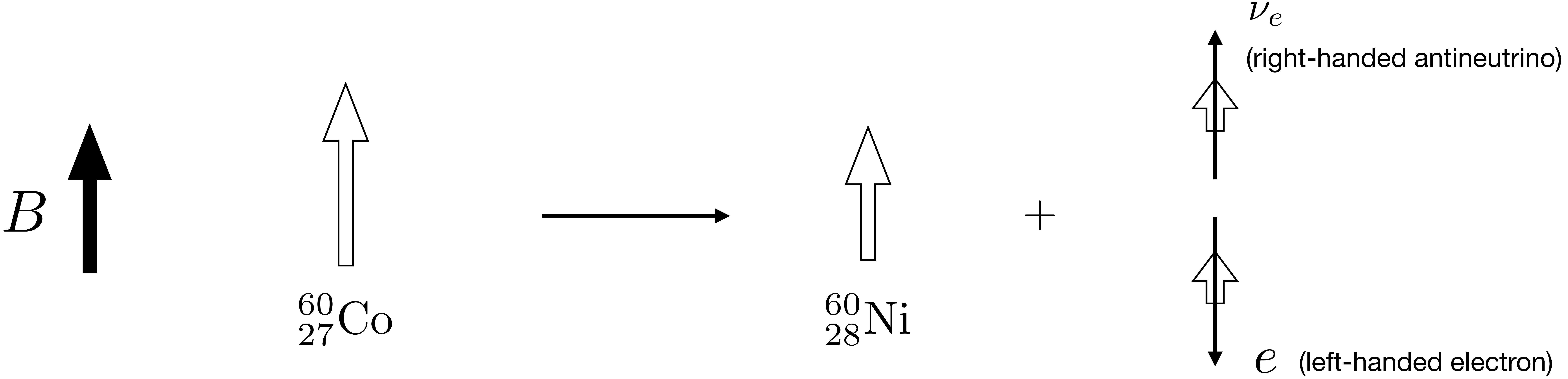
right-handed

left-handed (e_L) and right-handed (e_R) particle

are fundamentally different

Experiment by Wu

$$ {}^{60}_{27}\mathrm{Co} \to {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e $$



$J_z = 5$

$J_z = 4$

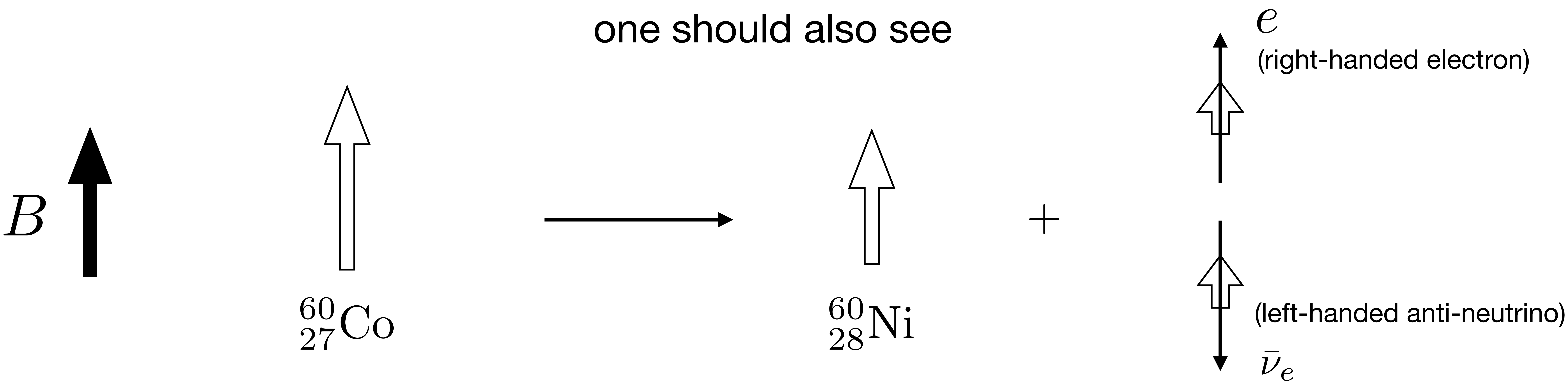
$J_z = J_{z,\bar{\nu}} + J_{z,e} = 1$

Experiment by Wu

$$^{60}_{27}\mathrm{Co} \rightarrow\ ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$$

if parity were symmetry of weak interaction

one should also see



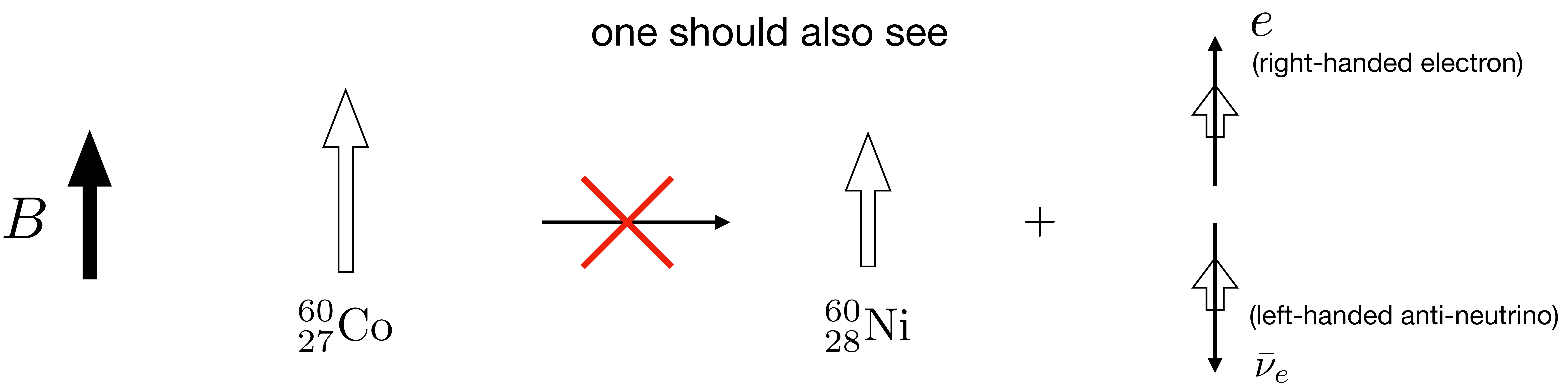
$J_z = 5$ $\qquad$ $J_z = 4$ $\qquad$ $J_z = J_{z,\bar{\nu}} + J_{z,e} = 1$

Experiment by Wu

$$^{60}_{27}\mathrm{Co} \to \, ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$$

if parity were symmetry of weak interaction

one should also see



$J_z = 5$ $\qquad$ $J_z = 4$ $\qquad$ $J_z = J_{z,\bar{\nu}} + J_{z,e} = 1$

parity is maximally broken; only left-handed particles participates in weak interaction

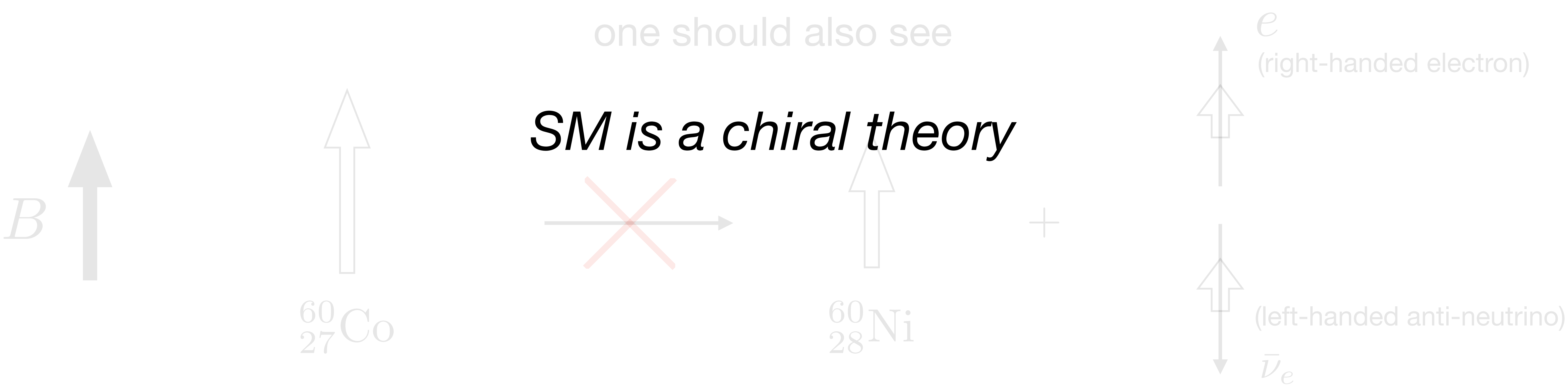
Experiment by Wu

$$^{60}_{27}\mathrm{Co} \to \,^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e$$

if parity were symmetry of weak interaction

one should also see

SM is a chiral theory



$J_z = 5$ $\quad$ $J_z = 4$ $\quad$ $J_z = J_{z,\bar{\nu}} + J_{z,e} = 1$

parity is maximally broken; only left-handed particles participates in weak interaction

Fermion mass term

only *left-handed particle* participates in the weak interaction

since weak interaction is gauge interaction it would mean that

left-handed and right-handed leptons transform differently

under *SU(2)xU(1)* gauge symmetry

$$e_R \to e^{-i\beta} e_R \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to e^{i\vec{\alpha}(x)\cdot\vec{\sigma}/2} e^{-i\beta/2} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \mathrm{h.c.}$$

which is not gauge invariant

The SM Lagrangian should NOT contain fermion mass term

Fermion masses are emergent in SM (from Higgs)

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \mathrm{h.c.}$$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$$Y = -1$$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$$Y = 1/2 \qquad\qquad Y = -1$$

(should also be a part of doublet w. neutrino) $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$$Y = 1/2 \qquad\qquad Y = -1$$

(should also be a part of doublet w. neutrino) $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

introduce Higgs doublet and write

$$\mathcal{L} = -y \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R$$

$$Y = 1/2$$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$$Y = 1/2 \qquad Y = -1$$

(should also be a part of doublet w. neutrino) $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

introduce Higgs doublet and write

$$\mathcal{L} = -y \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R = -\frac{yv}{\sqrt{2}}(1 + \frac{h}{v})\bar{e}_L e_R$$

$$Y = 1/2$$

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$Y = 1/2$ $\quad$ $Y = -1$

(should also be a part of doublet w. neutrino) $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

introduce Higgs doublet and write

$$\mathcal{L} = -y \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R = -\underbrace{\boxed{\frac{yv}{\sqrt{2}}}}_{m_e}(1 + \frac{h}{v})\bar{e}_L e_R$$

$Y = 1/2$

$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$

Fermion mass term

On the other hand

fermion mass term is

$$\mathcal{L} = -m\bar{e}_L e_R + \text{h.c.}$$

$$Y = 1/2 \qquad Y = -1$$

(should also be a part of doublet w. neutrino) $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

introduce Higgs doublet and write

$$\mathcal{L} = -y \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix} \cdot \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R = -\overbrace{\boxed{\frac{yv}{\sqrt{2}}}}^{m_e} (1 + \frac{h}{v}) \bar{e}_L e_R \supset -\frac{m_e}{v} h \bar{e}_L e_R$$

$$Y = 1/2$$

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

Spontaneous symmetry breaking

previously we consider 4-Fermi theory and discuss
that 4-Fermi theory originates from SU(2)xU(1) electroweak gauge theory

$$\mathcal{L} = G_F J_\mu^+ J^{-\mu}$$

$$\downarrow$$

$$\mathcal{L} = -m_W^2 W_\mu^+ W^{-\mu} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

Spontaneous symmetry breaking

we have also discussed that if we require *gauge invariance*

gauge bosons are necessarily **massless**

while from the observation weak bosons are massive

$$\mathcal{L} = \boxed{-m_W^2 W_\mu^+ W^{-\mu}} + g W_\mu^+ J^{-\mu} + g W_\mu^- J^{+\mu}$$

Spontaneous symmetry breaking

we have also discussed that if we require *gauge invariance*

gauge bosons are necessarily **massless**

while from the observation weak bosons are massive

$$\mathcal{L} = \boxed{-m_W^2 W_\mu^+ W^{-\mu}} + gW_\mu^+ J^{-\mu} + gW_\mu^- J^{+\mu}$$

how do we provide mass to weak gauge bosons?

Through spontaneous symmetry breaking by Higgs

$$D_\mu = \partial_\mu + ieA_\mu$$

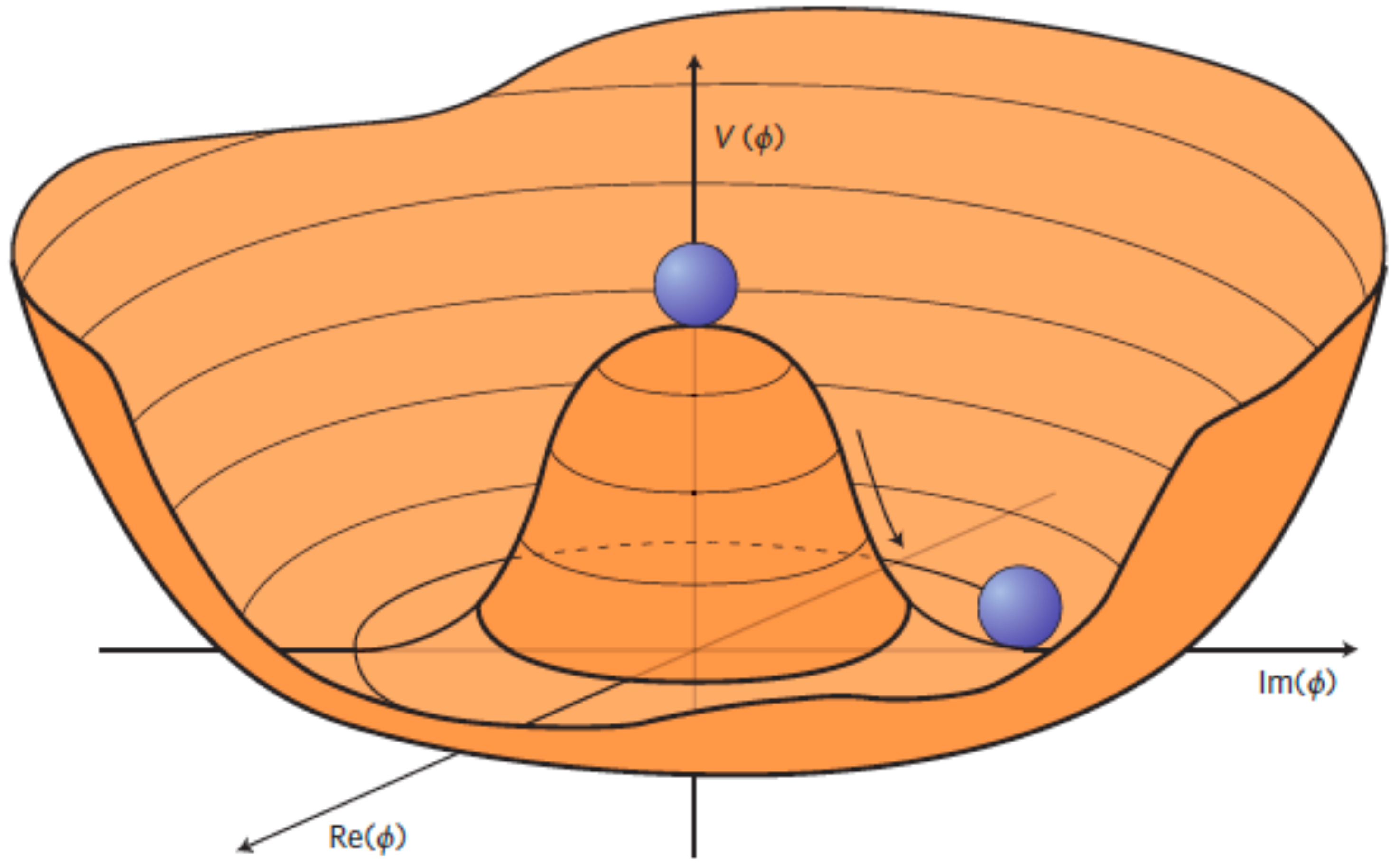
Spontaneous symmetry breaking

let us consider U(1) gauge theory

particularly, scalar electrodynamics

$$\mathcal{L} = (D^\mu\phi)^*(D_\mu\phi) - V(|\phi|^2)$$

$$V(|\phi|^2) = -m^2|\phi|^2 + \lambda|\phi|^4$$



$$\langle\phi\rangle = v = \sqrt{m^2/2\lambda}$$

scalar field obtains *vacuum expectation value (VEV)*

$$D_\mu = \partial_\mu + ieA_\mu$$

Higgs mechanism

let us consider U(1) gauge theory

particularly, scalar electrodynamics

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - V(|\phi|^2)$$

$$\supset e^2 A_\mu A^\mu |\phi|^2$$

$$D_\mu = \partial_\mu + ieA_\mu$$

Higgs mechanism

let us consider U(1) gauge theory

particularly, scalar electrodynamics

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - V(|\phi|^2)$$

$$\supset e^2 A_\mu A^\mu |\phi|^2$$

due to scalar VEV <ϕ> = v

the gauge boson obtains mass

$$m_A^2 = e^2 \langle |\phi|^2 \rangle = e^2 v^2$$

$$D_\mu = \partial_\mu + ieA_\mu$$

Higgs mechanism

let us consider U(1) gauge theory

particularly, scalar electrodynamics

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - V(|\phi|^2)$$

$$\supset e^2 A_\mu A^\mu |\phi|^2$$

due to scalar VEV $\langle\phi\rangle = v$

the gauge boson obtains mass

$$m_A^2 = e^2 \langle |\phi|^2 \rangle = e^2 v^2$$

in this way (spontaneous symmetry breaking)

gauge boson can obtain mass without breaking gauge symmetry

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

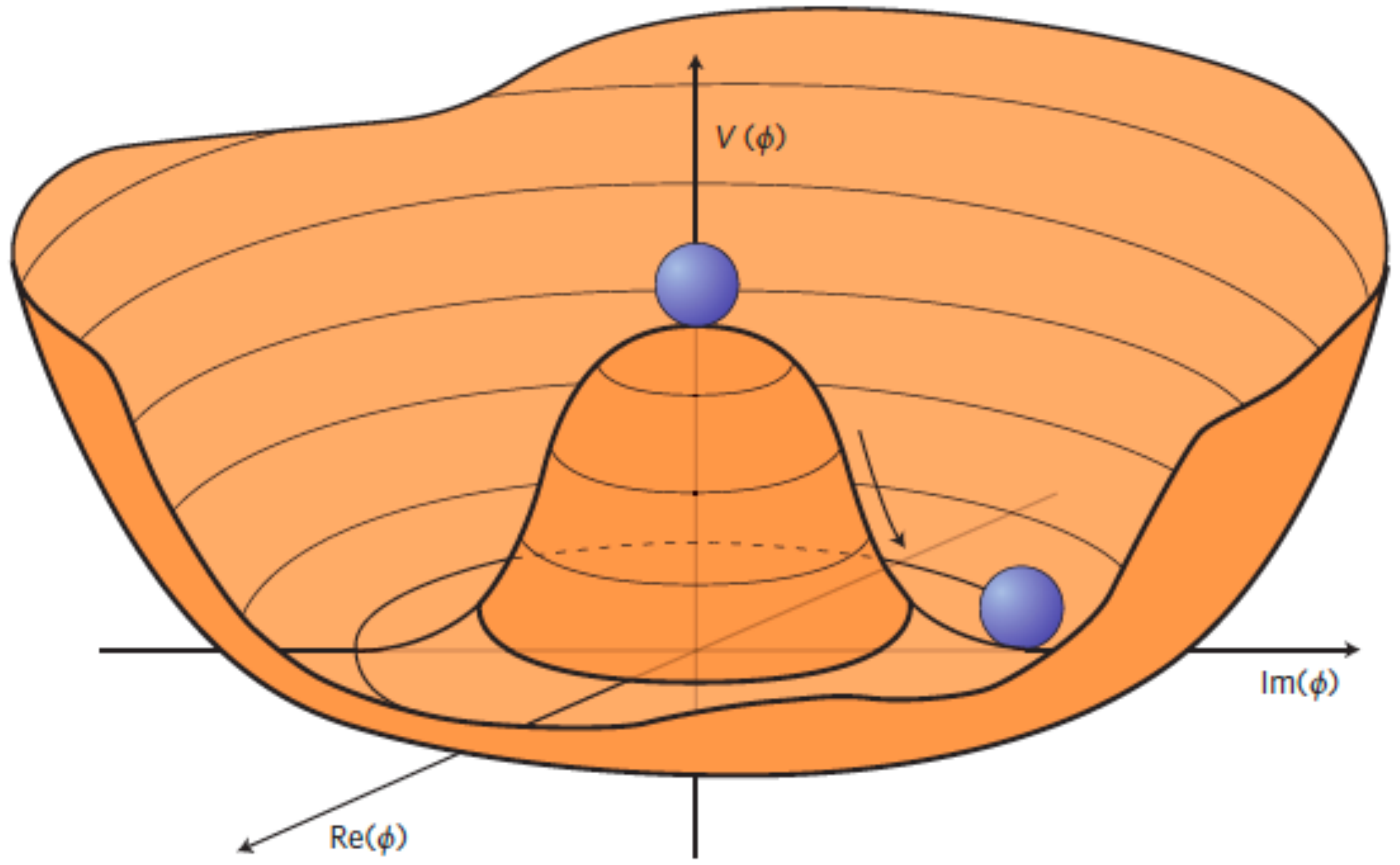
Application to the SM

$$\mathcal{L} = (D^\mu H)^* (D_\mu H) - V(|H|^2)$$

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$

Symmetry of SM

$$SU(2)_L \times U(1)_Y$$



$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

Symmetry of vacuum

$$U(1)_{\rm em}$$

Application to the SM

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu)H$$

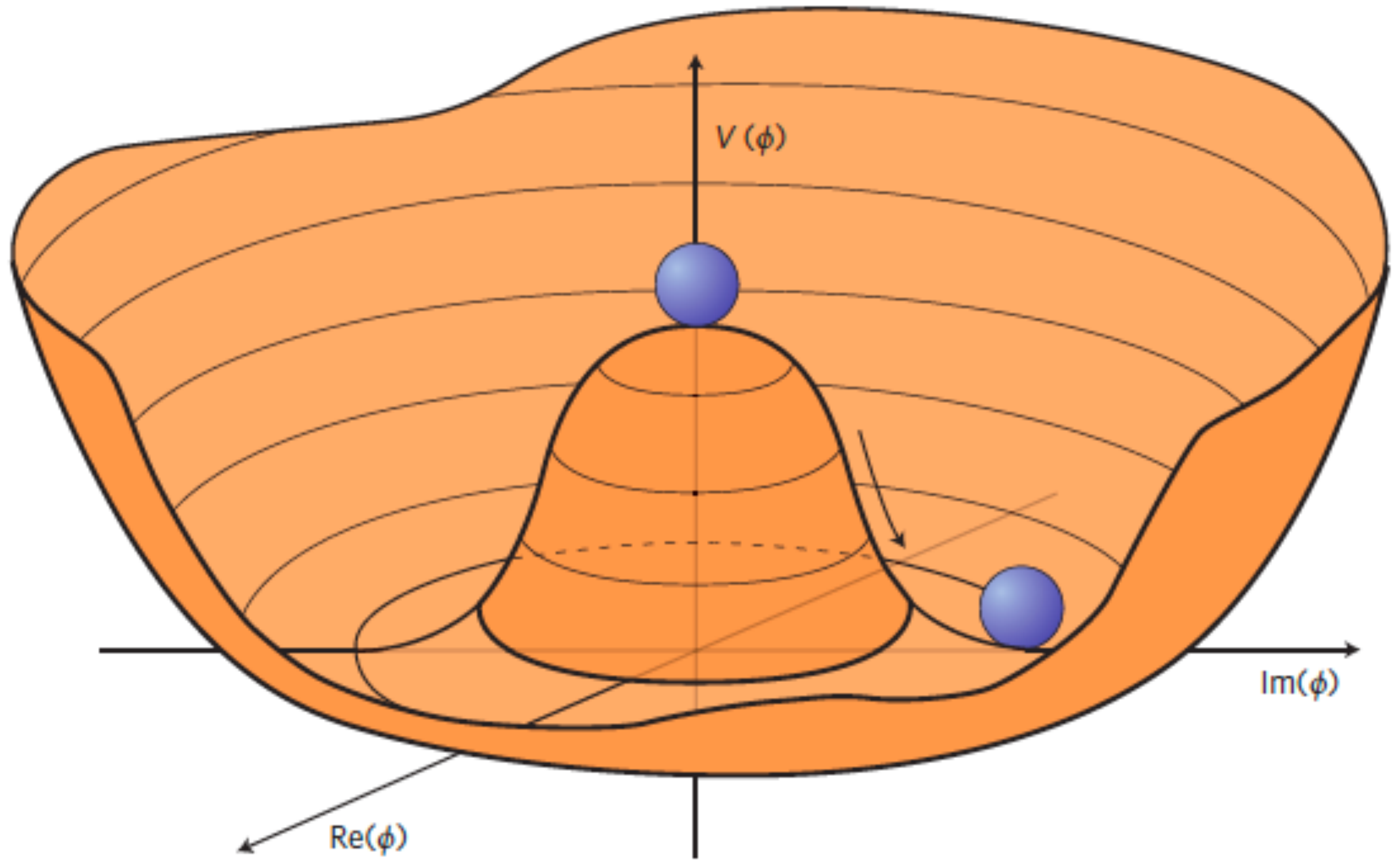
$$Y_H = 1/2$$

$$\mathcal{L} = (D^\mu H)^*(D_\mu H) - V(|H|^2)$$

Symmetry of SM

$$SU(2)_L \times U(1)_Y$$

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$



Symmetry of vacuum

$$U(1)_{\rm em}$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+ W^- + \frac{v^2}{8}(-gW^3 + g' B)^2$$

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig' Y_H B_\mu)H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Application to the SM

$$\mathcal{L} = (D^\mu H)^* (D_\mu H) - V(|H|^2)$$

Symmetry of SM

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$

$$SU(2)_L \times U(1)_Y$$

$$\langle D_\mu H \rangle = -\frac{i}{2\sqrt{2}} \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Symmetry of vacuum

$$U(1)_{\text{em}}$$

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu)H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Application to the SM

$$\mathcal{L} = (D^\mu H)^*(D_\mu H) - V(|H|^2)$$

Symmetry of SM

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$

$$SU(2)_L \times U(1)_Y$$

$$\langle D_\mu H \rangle = -\frac{i}{2\sqrt{2}} \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Symmetry of vacuum

$$|D_\mu H|^2 = \frac{1}{8}(0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$U(1)_{\text{em}}$$

Application to the SM

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = (D^\mu H)^* (D_\mu H) - V(|H|^2)$$

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$

Symmetry of SM

$$SU(2)_L \times U(1)_Y$$

$$\langle D_\mu H \rangle = -\frac{i}{2\sqrt{2}} \begin{pmatrix} gW^3 - g'B & \sqrt{2} g W^+ \\ \sqrt{2} g W^- & g'B - gW^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Symmetry of vacuum

$$U(1)_{\mathrm{em}}$$

$$|D_\mu H|^2 = \frac{1}{8} (0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2} g W^+ \\ \sqrt{2} g W^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{8} (0\ v) \begin{pmatrix} & \\ & 2g^2 W^+ W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Application to the SM

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu)H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = (D^\mu H)^*(D_\mu H) - V(|H|^2)$$

$$V(|H|)^2 = \lambda(|H|^2 - \frac{v^2}{2})^2$$

Symmetry of SM

$$SU(2)_L \times U(1)_Y$$

$$\langle D_\mu H \rangle = -\frac{i}{2\sqrt{2}} \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Symmetry of vacuum

$$U(1)_{\mathrm{em}}$$

$$|D_\mu H|^2 = \frac{1}{8}(0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{8}(0\ v) \begin{pmatrix} & \\ & 2g^2W^+W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{g^2v^2}{4} W^+W^- + \frac{v^2}{8}(gW^3 - g'B)^2$$

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$|D_\mu H|^2 = \frac{1}{8}(0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{8}(0\ v) \begin{pmatrix} & \\ & 2g^2W^+W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{g^2v^2}{4} W^+W^- + \frac{v^2}{8}(gW^3 - g'B)^2$$

one direction remains massless: photon!

$$A = \frac{g'W^3 + gB}{\sqrt{g^2 + g'^2}}$$

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$|D_\mu H|^2 = \frac{1}{8}(0 \ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2} g W^+ \\ \sqrt{2} g W^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{8}(0 \ v) \begin{pmatrix} & \\ & 2g^2 W^+ W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \underbrace{\frac{g^2 v^2}{4}}_{m_W^2} W^+ W^- + \frac{v^2}{8}(gW^3 - g'B)^2$$

one direction remains massless: photon!

$$A = \frac{g' W^3 + gB}{\sqrt{g^2 + g'^2}}$$

$$D_\mu H = (\partial_\mu - igW^a_\mu T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$|D_\mu H|^2 = \frac{1}{8}(0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2}gW^+ \\ \sqrt{2}gW^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{1}{8}(0\ v) \begin{pmatrix} & \\ & 2g^2W^+W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \underbrace{\frac{g^2v^2}{4}}_{m_W^2} W^+W^- + \frac{v^2}{8}(gW^3 - g'B)^2$$

$$Z = \frac{gW^3 - g'B}{\sqrt{g^2 + g'^2}}$$

one direction remains massless: photon!

$$A = \frac{g'W^3 + gB}{\sqrt{g^2 + g'^2}}$$

$$D_\mu H = (\partial_\mu - igW_\mu^a T^a - ig' Y_H B_\mu) H$$

$$Y_H = 1/2$$

$$|D_\mu H|^2 = \frac{1}{8}(0\ v) \begin{pmatrix} gW^3 - g'B & \sqrt{2} g W^+ \\ \sqrt{2} g W^- & g'B - gW^3 \end{pmatrix}^2 \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{8}(0\ v) \begin{pmatrix} & \\ & 2g^2 W^+ W^- + (gW^3 - g'B)^2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \underbrace{\frac{g^2 v^2}{4}}_{m_W^2} W^+ W^- + \frac{v^2}{8}(gW^3 - g'B)^2$$

$$Z = \frac{gW^3 - g'B}{\sqrt{g^2 + g'^2}}$$

$$= m_W^2 W^+ W^- + \frac{1}{2} m_Z^2 Z^2$$

one direction remains massless: photon!

$$A = \frac{g'W^3 + gB}{\sqrt{g^2 + g'^2}}$$

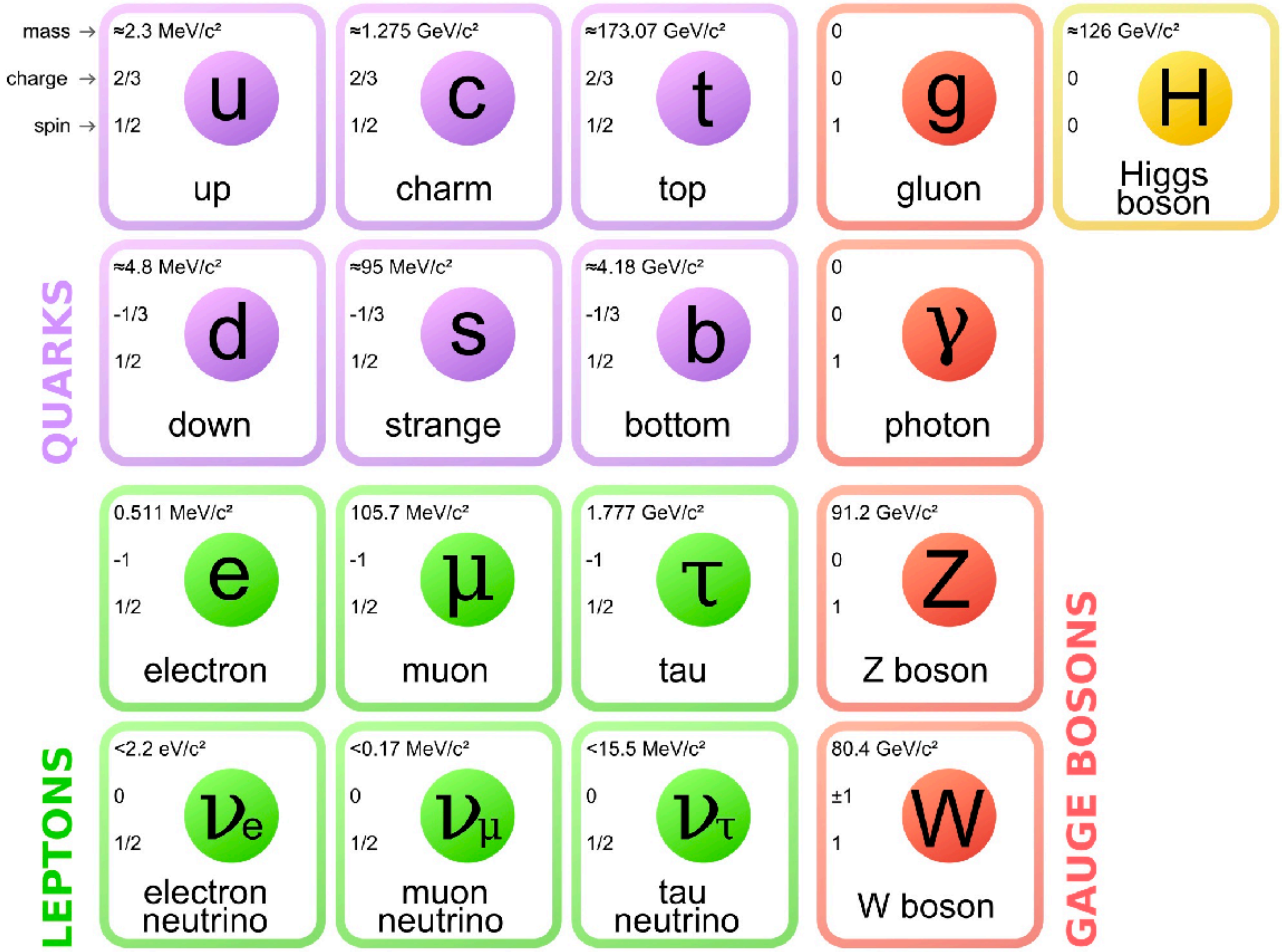
$$D_\mu H = (\partial_\mu - igW_\mu - i\frac{g'}{2}B_\mu)H$$

Application to the SM

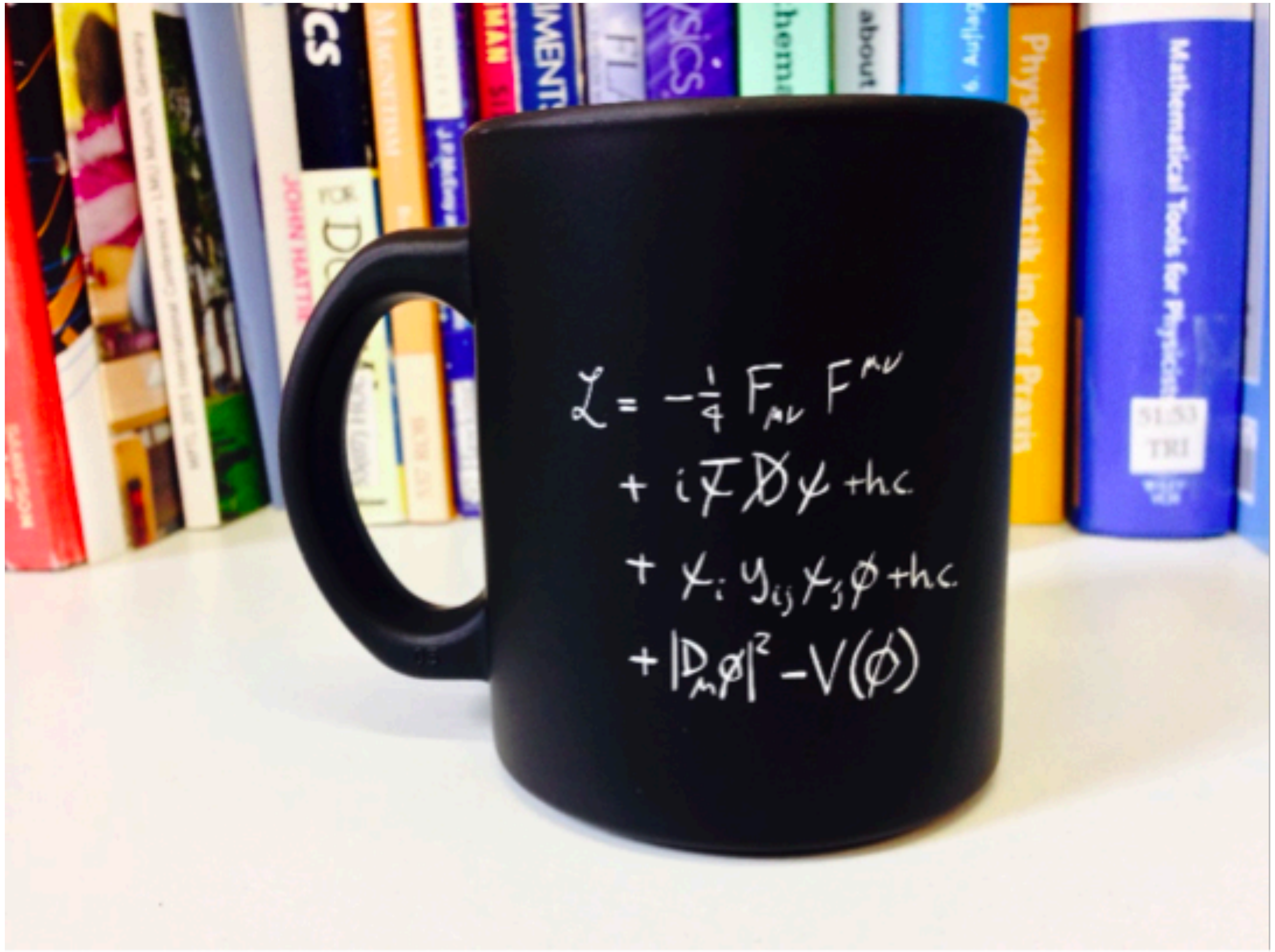
$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+ W^- + \frac{v^2}{8}(-gW^3 + g' B)^2$$

$W_\mu^\pm$	$\frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$	$m_W = \frac{gv}{2}$
Z_μ	$\frac{gW_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$	$m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}$
A_μ	$\frac{g' W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}}$	$m = 0$

Summary


mass →
charge →
spin →
≈2.3 MeV/c²
2/3
1/2
u
up
≈1.275 GeV/c²
2/3
1/2
c
charm
≈173.07 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≈126 GeV/c²
0
0
H
Higgs boson
≈4.8 MeV/c²
-1/3
1/2
d
down
≈95 MeV/c²
-1/3
1/2
s
strange
≈4.18 GeV/c²
-1/3
1/2
b
bottom
0
0
1
γ
photon
0.511 MeV/c²
-1
1/2
e
electron
105.7 MeV/c²
-1
1/2
μ
muon
1.777 GeV/c²
-1
1/2
τ
tau
91.2 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
νe
electron neutrino
<0.17 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
80.4 GeV/c²
±1
1
W
W boson
QUARKS
LEPTONS
GAUGE BOSONS

Summary

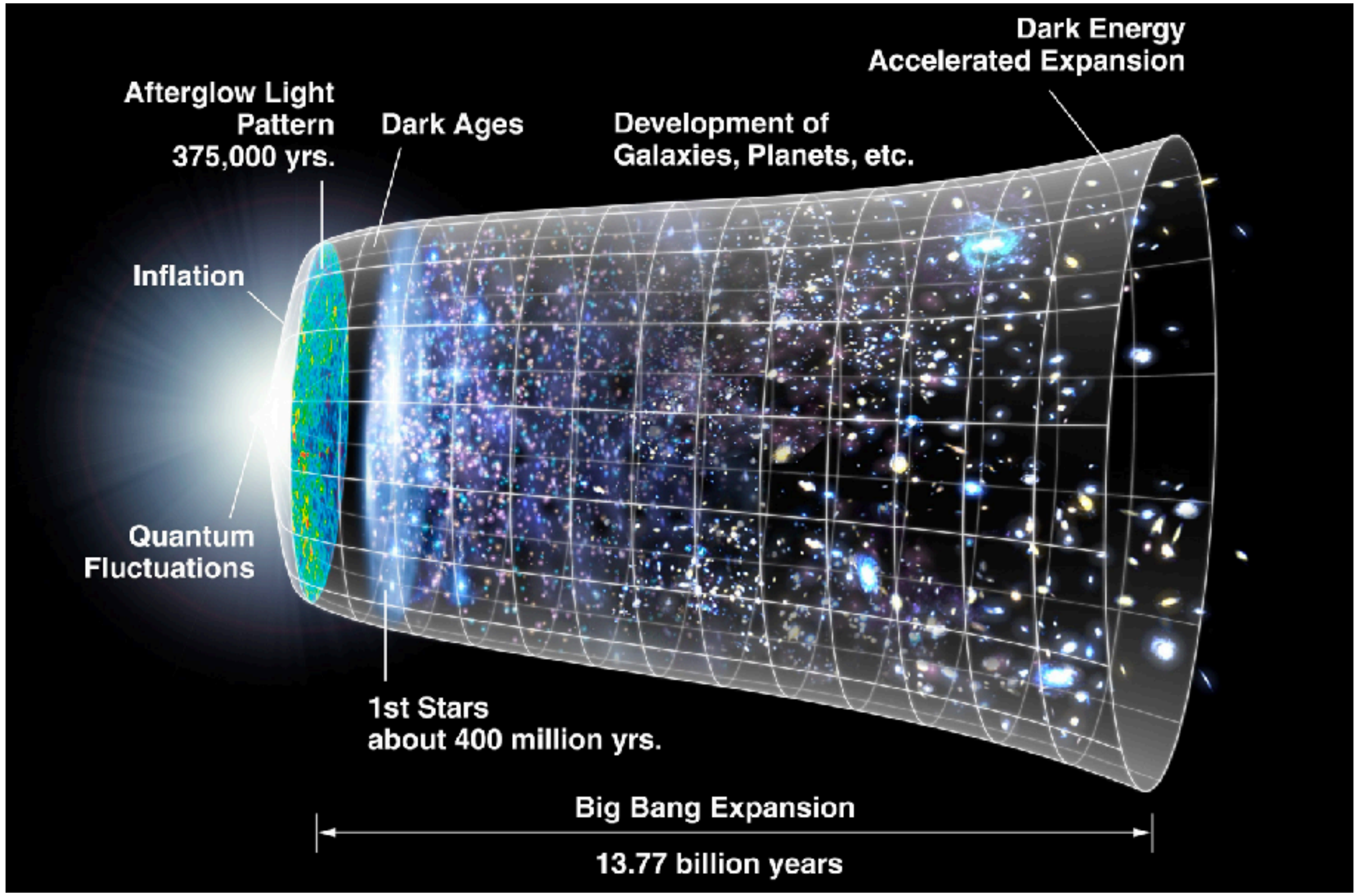

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$
Mathematical Tools for Physicists

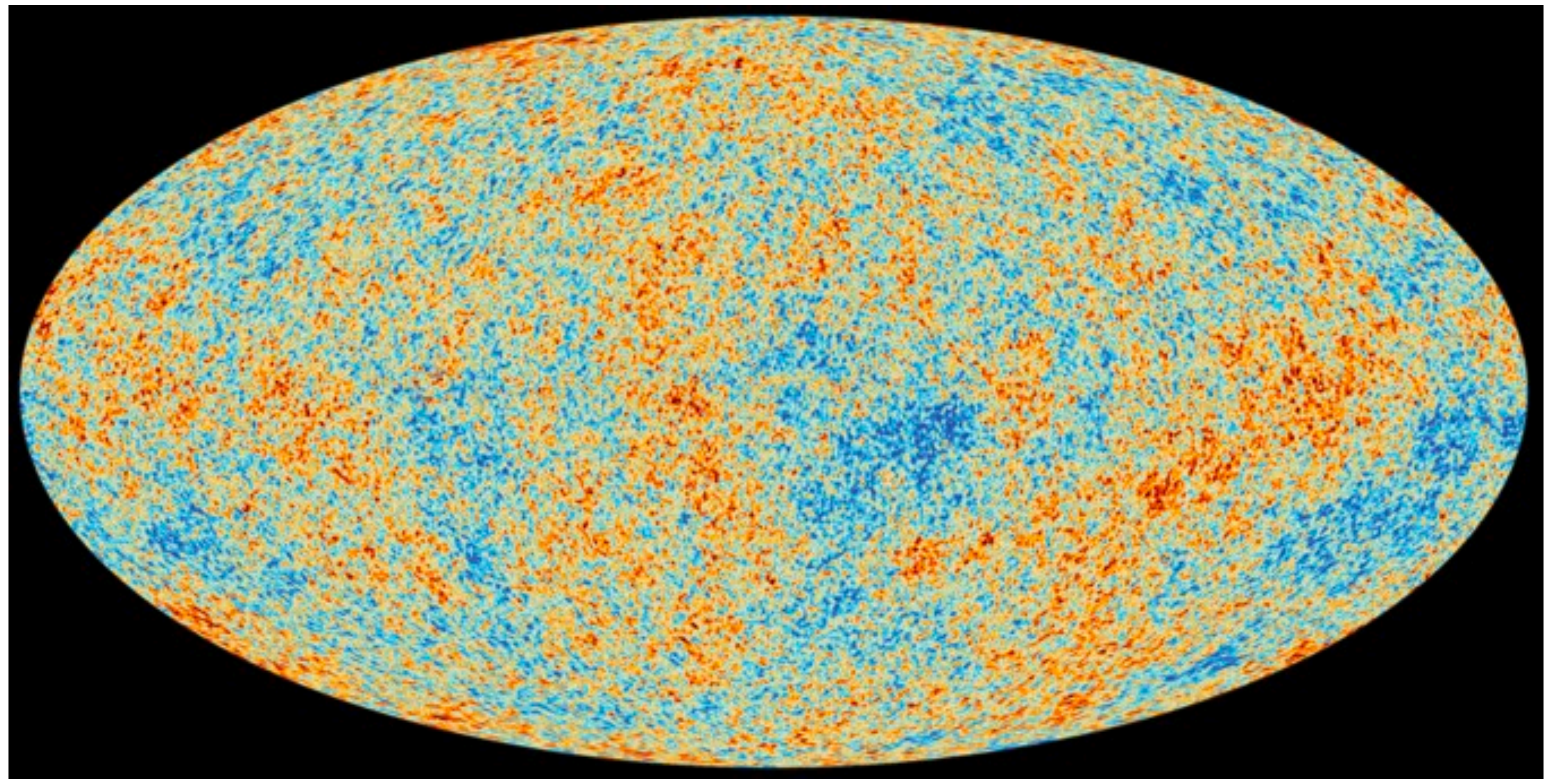
Summary

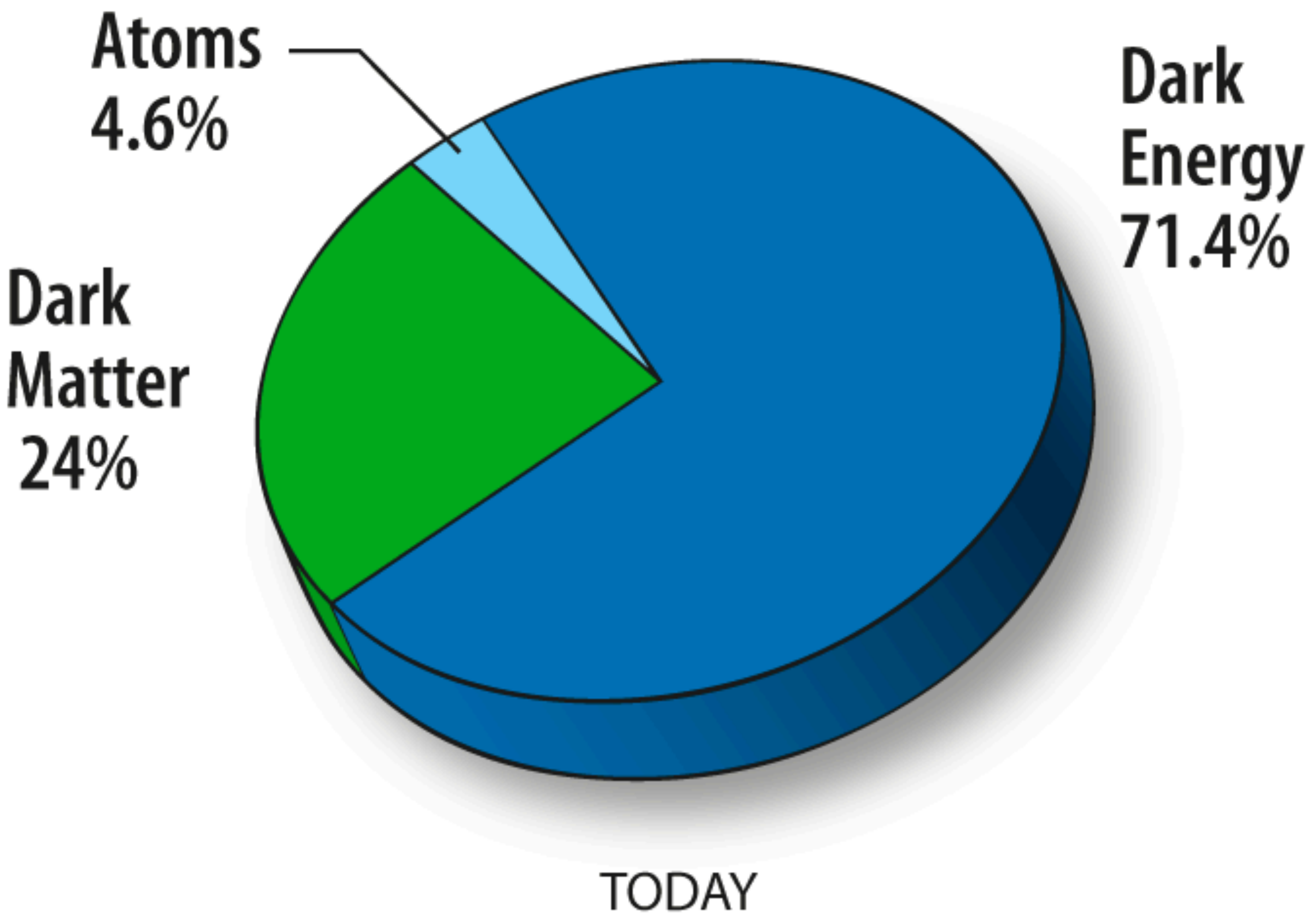
	SPIN	PARTICLES	color $SU(3)_c$ ×	chirality $SU(2)_L$ ×	hypercharge $U(1)_Y$	weak isospin T_{3_L}	electric charge $Q = T_{3_L} + Y$	effective coupling to Z boson g_{eff}	MEANING
LEPTONS	1/2	$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 + \sin^2\theta_W \end{pmatrix}$	doublet under SU(2), singlet under SU(3)
		e_R	1	1	-1	0	-1	$\sin^2\theta_W$	singlet under SU(2) and SU(3)
QUARKS		$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\begin{pmatrix} 1/6 \\ 1/6 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 - \frac{2}{3}\sin^2\theta_W \\ -1/2 + \frac{1}{3}\sin^2\theta_W \end{pmatrix}$	doublet under SU(2), triplet under SU(3)
		u_R	3	1	2/3	0	2/3	$-\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
		d_R	3	1	$-1/3$	0	$-1/3$	$\frac{1}{3}\sin^2\theta_W$	singlet under SU(2), triplet under SU(3)
HIGGS	0	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	1	2	$\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	×	doublet under SU(2), singlet under SU(3)

[taken from C. Grojean's lecture slides]

Cosmology


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years




Atoms
4.6%
Dark
Energy
71.4%
Dark
Matter
24%
TODAY