Transverse Momentum Dependent Fragmentation Functions from recent BELLE data

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UNIVERSITÀ DEGLI STUDI DI TORINO

PB TMD meeting 2022

INFN

Based on :
M. Boglione, J. O. Gonzalez-Hernandez, and A. Simonelli (2022), 2206.08876.

Outlook

- Theoretical framework for e+e- -> h X
- Global fits
- Phenomenological analysis of recent BELLE data

Theoretical framework for e+e- -> h X

M. Boglione, A. Simonelli *JHEP* 02 (2021) 076

$$\frac{d\sigma^{\text{NLO, NLL}}}{dz_h\, dT\, dP_T^2} =$$

$$= -\sigma_B\, \pi\, N_C\, \frac{\alpha_S(Q)}{4\pi}\, C_F\, \frac{3 + 8\log(1-T)}{1-T} \exp\left\{-\frac{\alpha_S(Q)}{4\pi}\, 3C_F\, \left(\log(1-T)\right)^2\right\} \times$$

$$\times \sum_f e_f^2 \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\frac{\vec{P}_T}{z_h}\cdot\vec{b}_T}\, \widetilde{D}^{\text{NLL}}_{1,\,H/f}(z_h,\, b_T,\, Q,\, (1-T)\,Q^2) \left[1 + \mathcal{O}\left(\frac{M_H^2}{Q^2}\right)\right]$$

$$\widetilde{D}_{1,\,H/f}(z,\, b_T;\, \mu,\, \zeta) = \underbrace{\frac{1}{z^2}\sum_k \int_z^1 \frac{d\rho}{\rho}\, d_{H/k}(z/\rho,\, \mu_b)\, \left[\rho^2\, \mathcal{C}_{k/f}\,(\rho,\, \alpha_S(\mu_b))\right]}_{\text{TMD at reference scale}} \times$$

$$\times \underbrace{\exp\left\{\frac{1}{4}\, \widetilde{K}(b_T^\star;\, \mu_b)\, \log\frac{\zeta}{\mu_b^2} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'}\left[\gamma_D(\alpha_S(\mu'),\, 1) - \frac{1}{4}\,\gamma_K(\alpha_S(\mu'))\, \log\frac{\zeta}{\mu'^2}\right]\right\}}_{\text{Perturbative Sudakov Factor}}$$

$$\times \underbrace{(M_D)_{j,\,H}\,(z,\, b_T)\, \exp\left\{-\frac{1}{4}\, g_K(b_T)\, \log\frac{z_h^2\,\zeta}{M_H^2}\right\}}_{\text{Non-Perturbative content}}.$$

Theoretical framework for e+e- -> h X

$$D = D^{\star}\sqrt{M_S}$$

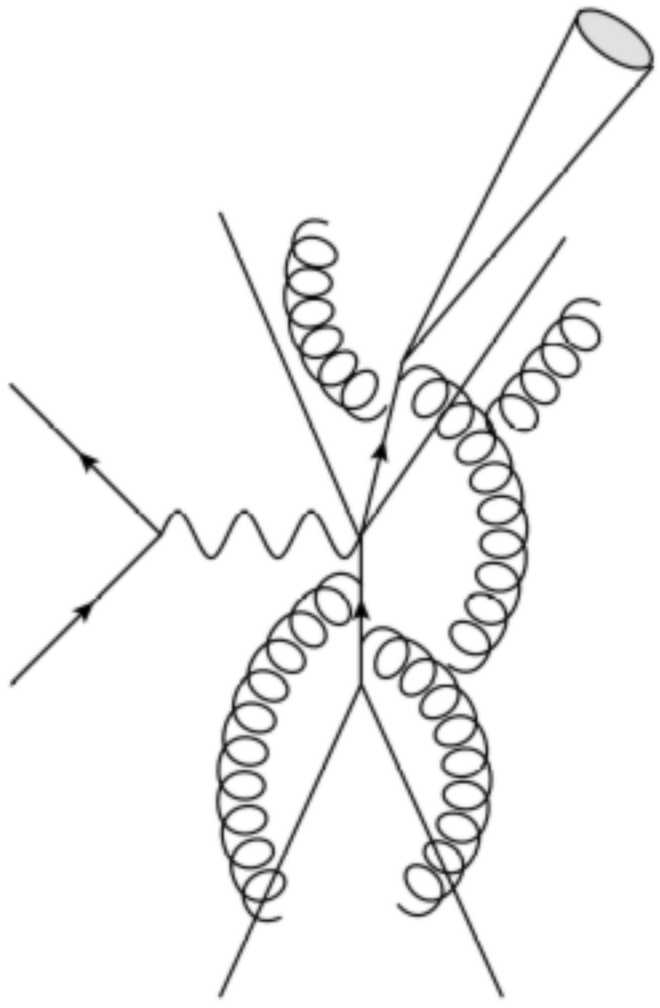
Usual definition TMD FF

For this process TMD FF

Soft non-perturbative Function

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^{\star}(P_T)$$

Theoretical framework for e+e- -> h X

$$D = D^{\star}\sqrt{M_S}$$

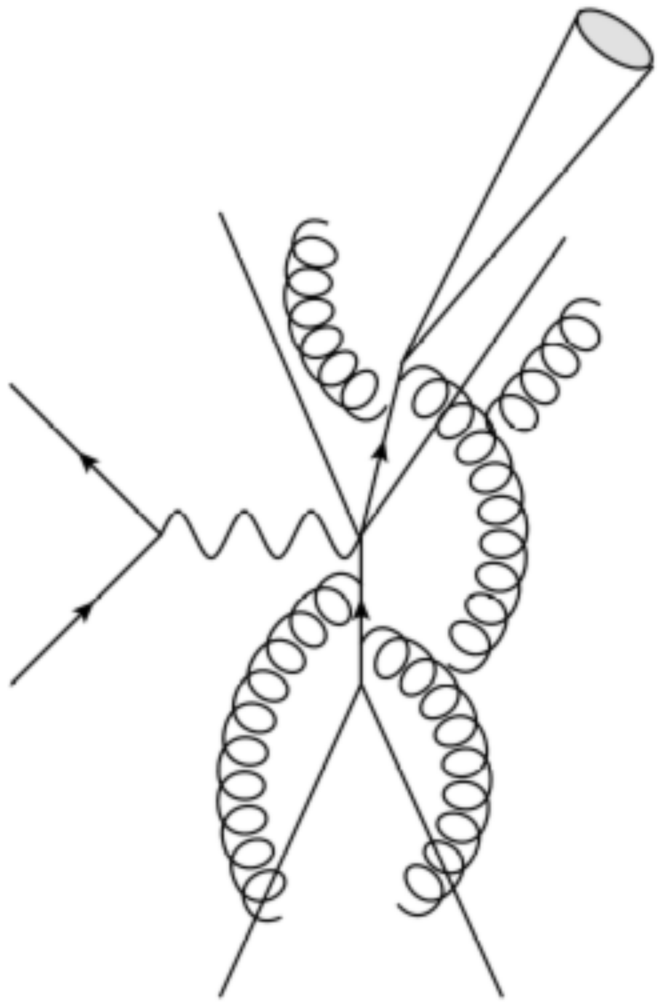
Same constraints to collinear FF

$$g_K(b_T) = \widetilde{K}(b_T^{\star};\, \mu) - \widetilde{K}(b_T;\, \mu)$$

Same function for non-perturbative evolution

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^{\star}(P_T)$$

Theoretical framework for e+e- -> h X

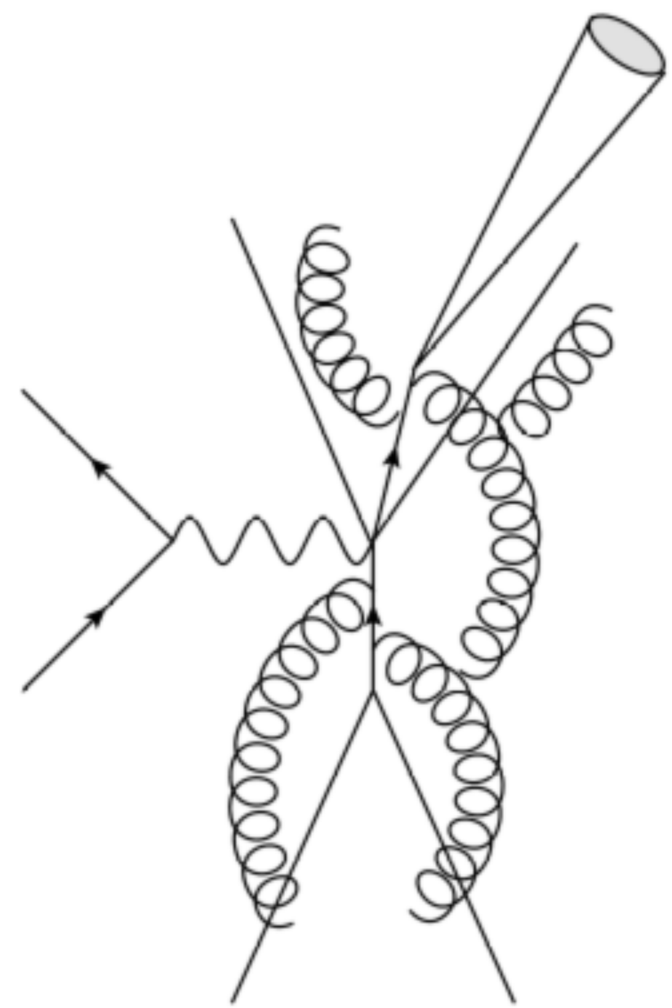
$$D = D^{\star}\,\sqrt{M_S}$$

What is the effect of the collinear FFs (and PDFs in general) ?

Large-bT behaviour of gK ?

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$

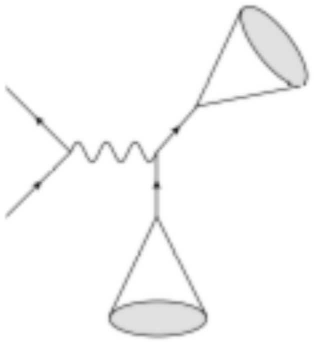


$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^{\star}(P_T)$$

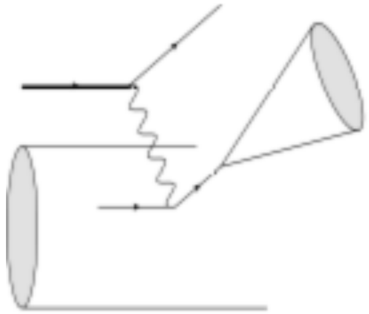
Global Fits

Possible roadmap

1. Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_S



3. SIDIS
   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_S, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, $F^* \neq F$).

Taken from M. Boglione DIS 2021

(Global) Fits

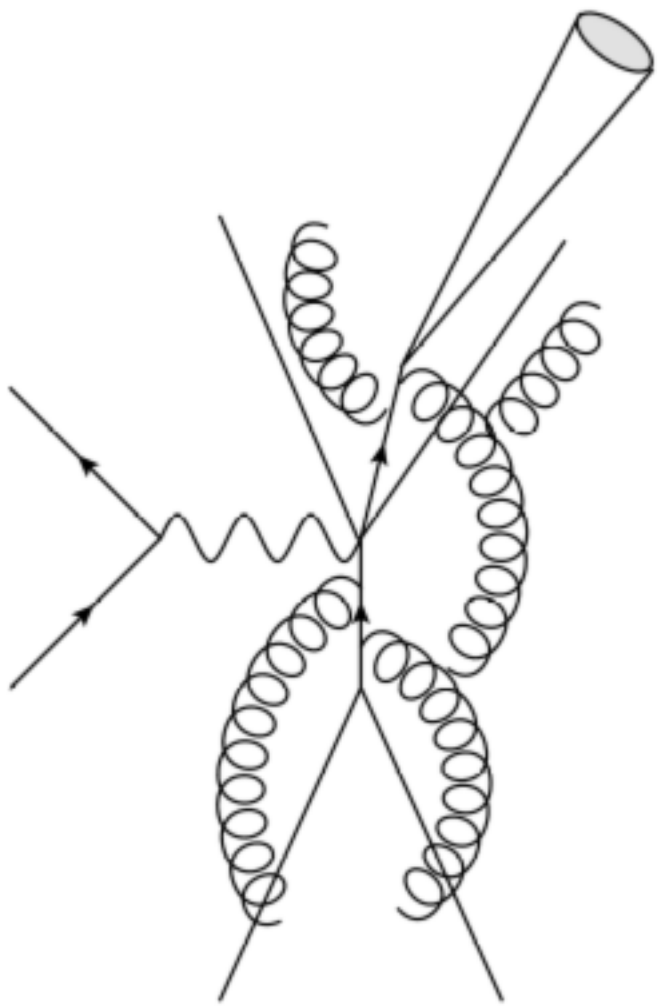
Must consider:

- Which collinear functions are more appropriate?
- Which regions in bT are being mapped by extractions.
- Constraints of bT-behaviour for TMDs.
- Physical pictures/theoretical arguments /models (not parametrizations)
- Non perturbative evolution (gK) should be consistent with SIDIS, DY, e+e- two-hadron production.

Phenomenological analysis of recent BELLE data

BELLE data overview

$e^+e^- \rightarrow hX$ (Charged pions)



Binned in PT, zh and T (thrust)

Q=10.6 GeV

$0.06 < P_T < 2.5$ GeV

$0.125 < z_h < 0.975$ (18 bins)

$0.6 < T < 0.975$. (6 bins)

For our analysis

$P_T/z_h < 0.15\, Q$

$0.375 < z_h < 0.725$. (8 bins)

$0.750 < T < 0.875$. (3 bins)

zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

$$\widetilde{D}_{1,\,H/f}(z,\,b_T;\,\mu,\,\zeta) = \underbrace{\frac{1}{z^2}\sum_k \int_z^1 \frac{d\rho}{\rho}\,\boxed{d_{H/k}(z/\rho,\,\mu_b)}\left[\rho^2\,\mathcal{C}_{k/f}\,(\rho,\,\alpha_S(\mu_b))\right]}_{\text{TMD at reference scale}} \times$$

$$\times \underbrace{\exp\left\{\frac{1}{4}\,\widetilde{K}(b_T^{\star};\,\mu_b)\,\log\frac{\zeta}{\mu_b^2} + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_D(\alpha_S(\mu'),\,1) - \frac{1}{4}\,\gamma_K(\alpha_S(\mu'))\,\log\frac{\zeta}{\mu'^2}\right]\right\}}_{\text{Perturbative Sudakov Factor}}$$

$$\times \underbrace{(M_D)_{j,\,H}\,(z,\,b_T)\,\exp\left\{-\frac{1}{4}\,g_K(b_T)\,\log\frac{z_h^2\,\zeta}{M_H^2}\right\}}_{\text{Non-Perturbative content}}.$$

zh-dependence and choice of collinear FFs

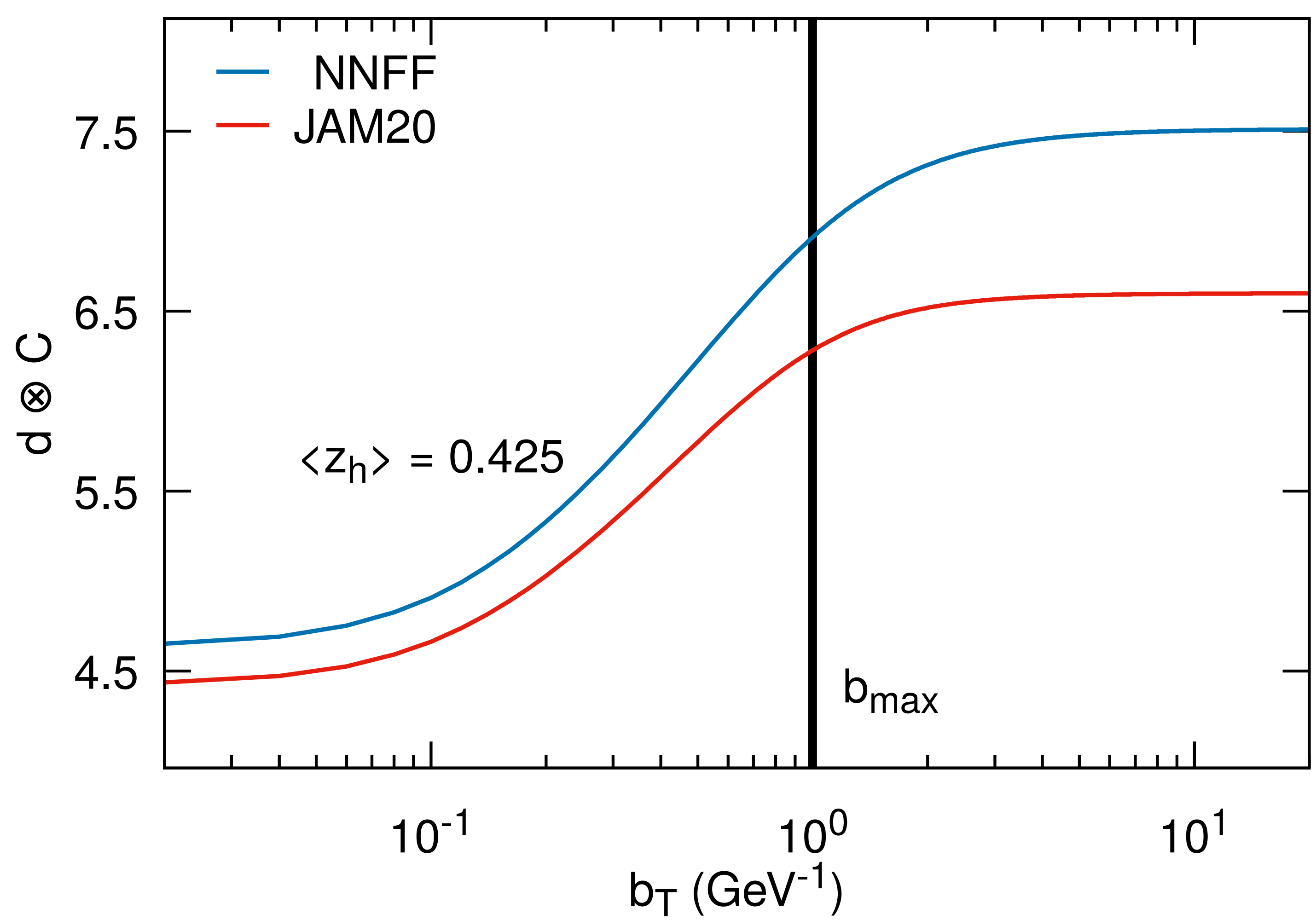
- We compare results obtained with NNFFnlo and JAM20nlo

$$\boxed{\frac{1}{z^2}\sum_k \int_z^1 \frac{d\rho}{\rho}\, d_{H/k}(z/\rho,\, \mu_b)\ \left[\rho^2\, \mathcal{C}_{k/f}\,(\rho,\, \alpha_S(\mu_b))\right]}$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_T^\star(b_T)}$$

$$\vec{b}_T^\star\,(b_T) = \frac{\vec{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

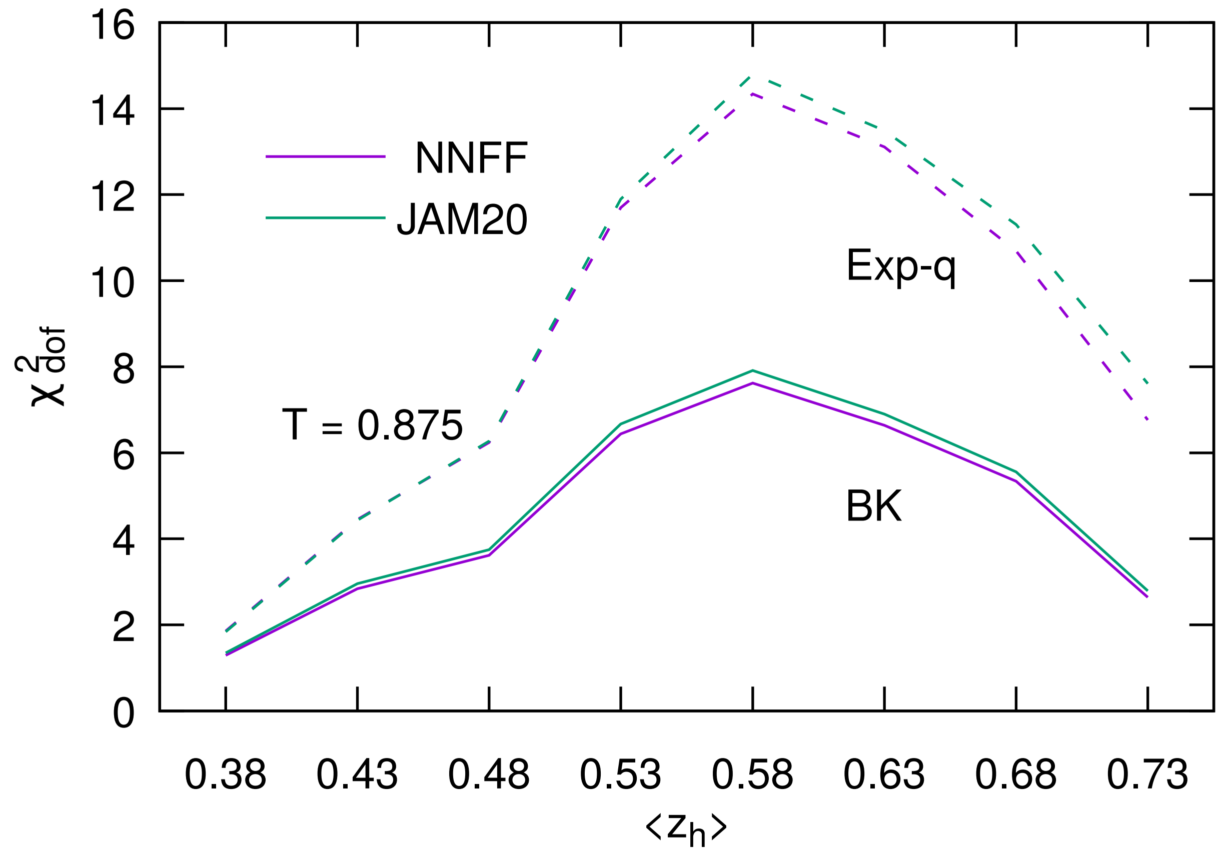
$$\vec{b}_T^\star\,(b_c(b_T)) = \vec{b}_T^\star\left(\sqrt{b_T^2 + b_{\rm min}^2}\right)$$



zh-dependence and choice of collinear FFs

- We compare results obtained with NNFFnlo and JAM20nlo

Nomenclature	M_{D}-model	parameters
z_h-independent models		
1)Exponential-q	$e^{-(M_0 b_{\mathrm{T}})^q}$	$M_0,\ q$
2)Bessel-K	$\dfrac{2^{2-p}(b_{\mathrm{T}}M_0)^{p-1}}{\Gamma(p-1)}K_{p-1}(b_{\mathrm{T}}M_0)$	$M_0,\ p$



**Proxy models: performed fits at fixed T=0.875.
One INDEPENDENT fit for each zh-bin in the range 0.375<zh<0.725.**

JAM20 & NNFF perform equally.

zh-dependence and choice of collinear FFs

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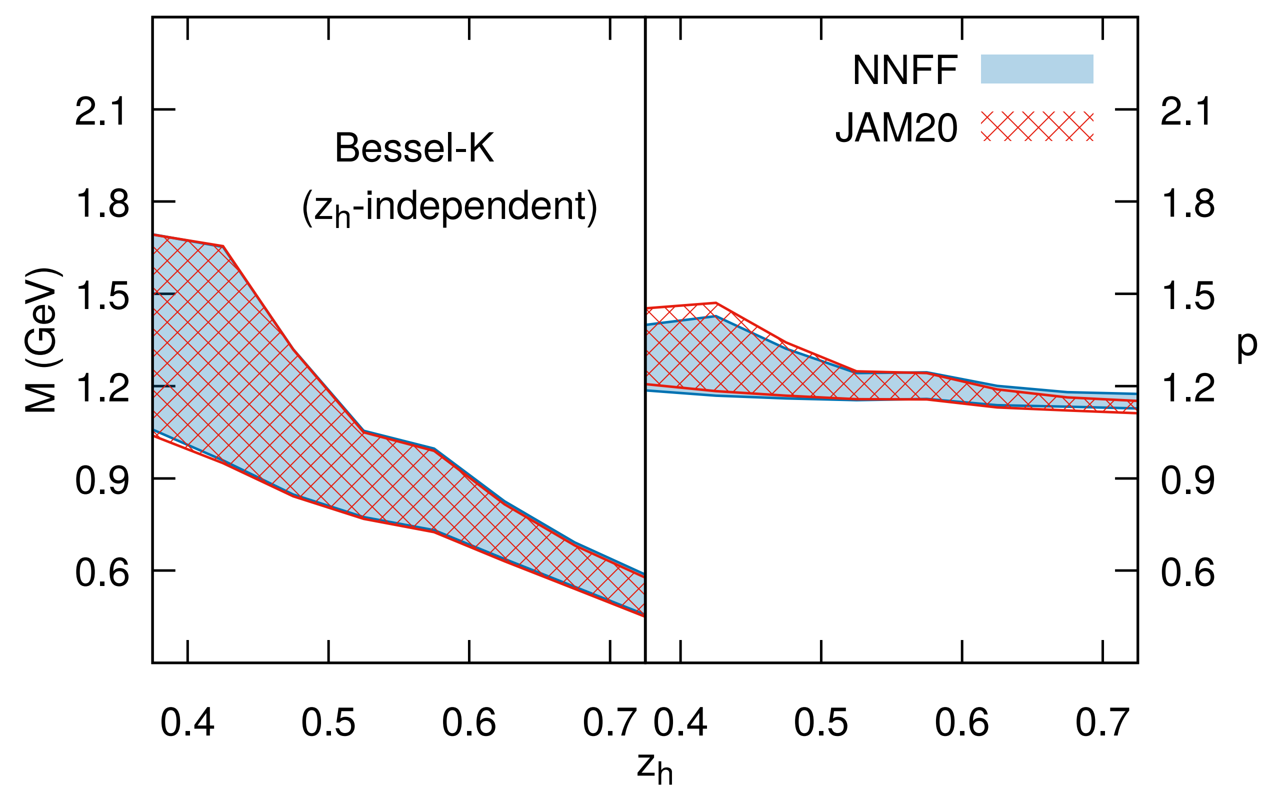

Proxy models: performed fits at fixed T=0.875.
One INDEPENDENT fit for each zh-bin in the range 0.375<zh<0.725.

Stronger zh-dependence in *dimensionfull* parameter

Model for PT-dependence

Example: preliminary fits at fixed T

$\chi^2_{\mathrm{d.o.f.}}$ (fixed-T fits)			
M_{D} model	$T = 0.750$	0.825	0.875
I	1.2	0.38	1.02
IG	1.46	0.47	1.51

$$M_{\mathrm{D}} = \frac{2^{2-p}(b_{\mathrm{T}}M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_{\mathrm{T}}M_0) \times \boxed{F(b_{\mathrm{T}}, z_h)}$$

$$M_z = -M_1 \log(z_h)$$

ID	F-model	parameters
I	$F = \left(\frac{1+\log\left(1+(b_{\mathrm{T}}M_z)^2\right)}{1+((b_{\mathrm{T}}M_z)^2)}\right)^q$	$M_0, M_1, p,\ q = 8$
IG	$F = \exp\left((M_g b_{\mathrm{T}})^2 \log(z_h)\right)$	$M_0,\ M_g,\ p$

zh-dependence Factor

Model for PT-dependence

Very large bT behaviour might not be resolved by data

$\chi^2_{\rm d.o.f.}$ (fixed-T fits)			
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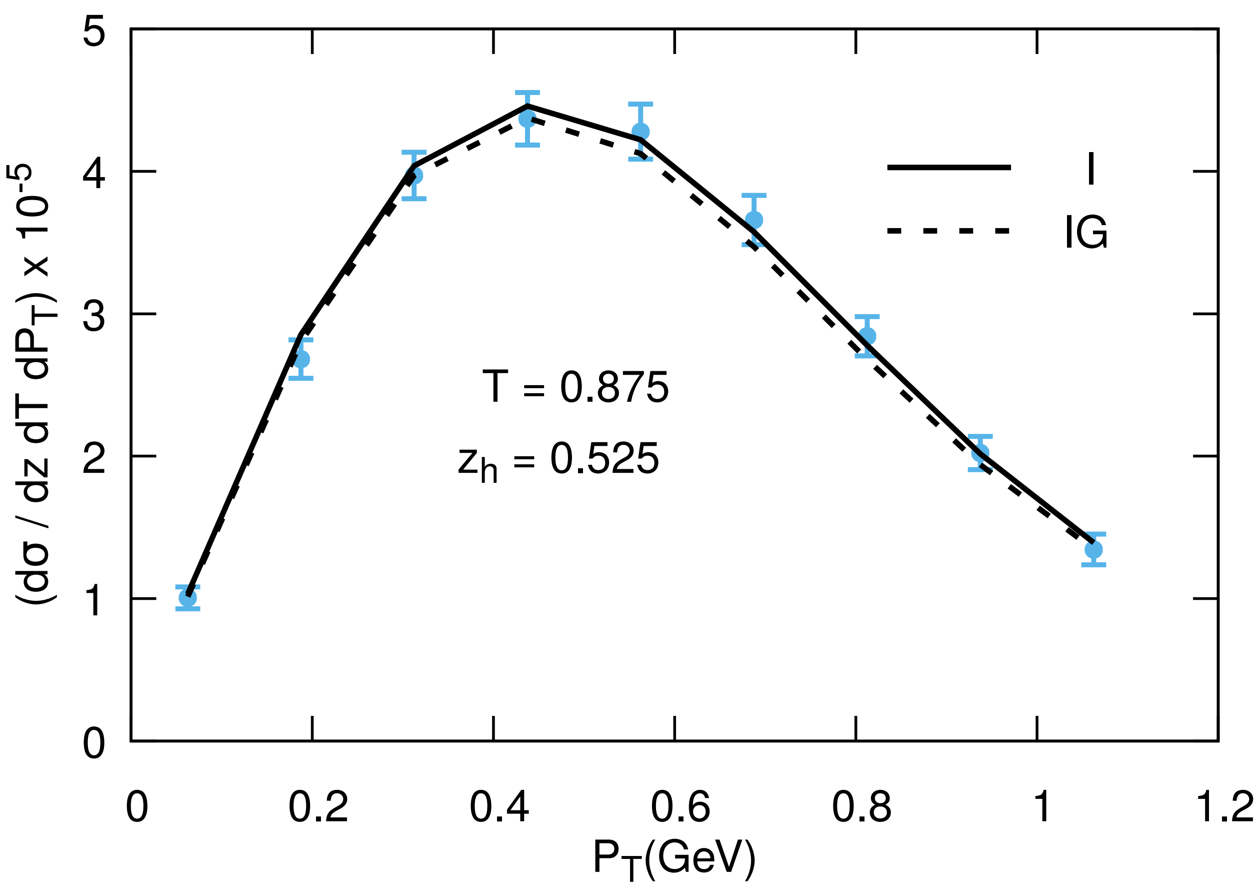

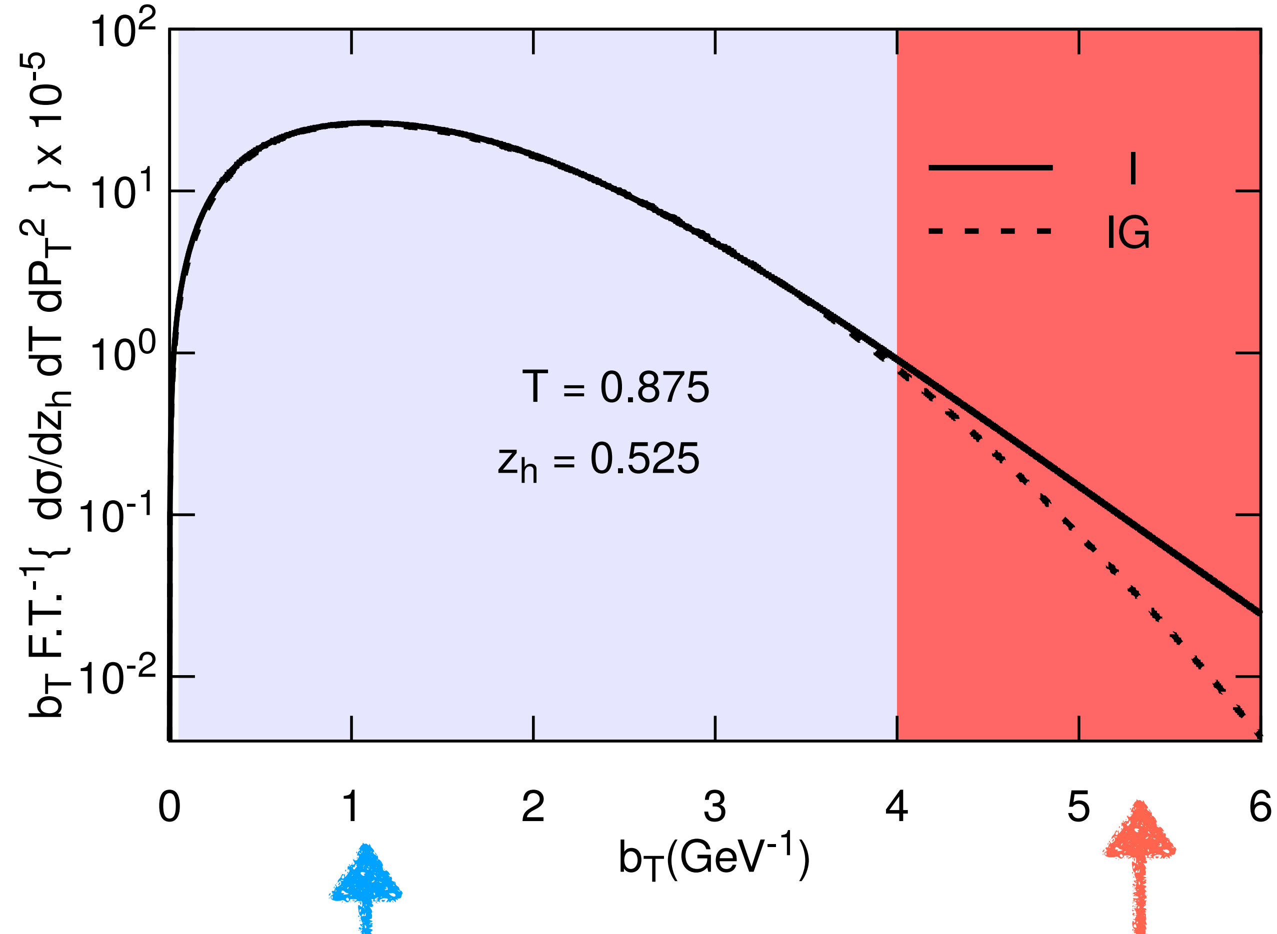

MD —>1 for bT—>0

Different asymptotic behaviour

Model for PT-dependence

Very large bT behaviour
might not be resolved by data

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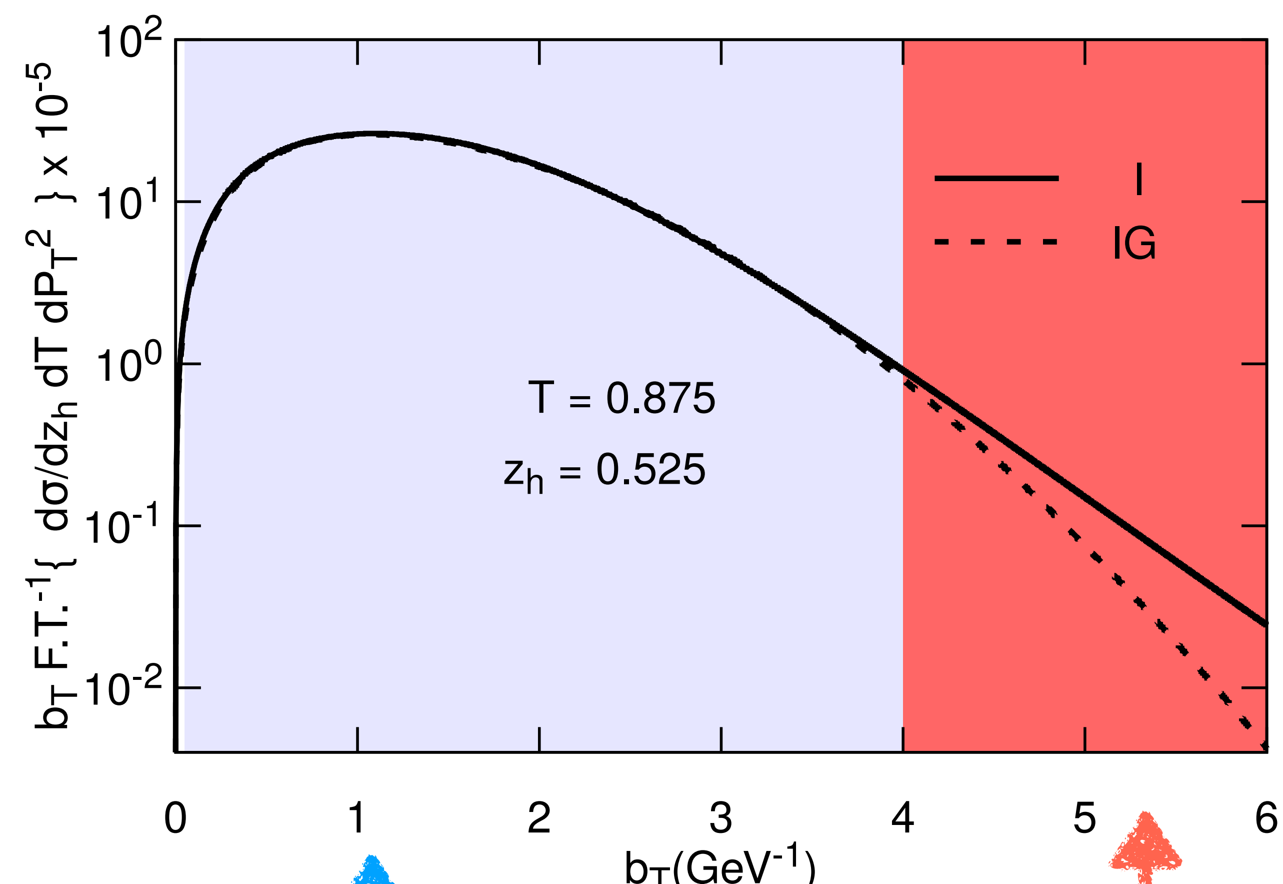
At smaller energies
(say, COMPASS)
one has sensitivity to
larger values of bT.

Need to constraint
very large bT region.



MD —>1 for
bT—>0

Different asymptotic
behaviour

Model for PT-dependence

Which asymptotic behaviour?

P. Schweitzer, M. Strikman, and C. Weiss, JHEP 1301, 163 (2013)

J. Collins and T. Rogers, Phys. Rev. D91 (2015) 074020, [1412.3820].

$$\text{TMDs} \longrightarrow \frac{1}{{b_T}^{\alpha}} e^{-m b_T} \quad \text{as} \quad b_T \to \infty.$$

We consider $$\log(M_D) \underset{b_T \to \infty}{\sim} -C\, b_T + o\,(b_T)$$

Non-perturbative function

Mass parameter

Hypotheses for gK

We consider $$\log(M_{\rm D}) \underset{b_{\rm T}\to\infty}{\sim} -C\, b_{\rm T} + o\,(b_{\rm T})$$ **TMD model**

$$\tilde{D}(b_{\rm T},\zeta) = \tilde{D}(b_{\rm T},\zeta_0)\exp\left\{-\frac{g_{\rm K}}{4}\log\left(\frac{\zeta}{\zeta_0}\right)\right\}(\ldots)$$ **In general**

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$$\log\left(\tilde{D}(b_{\rm T},\zeta)\right) \overset{b_{\rm T}\to\infty}{=} -Cb_{\rm T} - \underbrace{\frac{g_{\rm K}^{\text{large }b_{\rm T}}}{4}\log\left(\frac{\zeta}{\zeta_0}\right)}_{g_{\rm K}(b_{\rm T}) = o\,(b_{\rm T})} + o(b_{\rm T})$$ **With TMD model**

Hypotheses for gK

Asymptotic behaviour of TMD preserved under evolution

$$\tilde{D}(b_{\rm T},\zeta) = \tilde{D}(b_{\rm T},\zeta_0)\exp\left\{-\frac{g_{\rm K}}{4}\log\left(\frac{\zeta}{\zeta_0}\right)\right\}(\dots)$$

In general

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With TMD model

Models for MD

$$M_{\mathrm{D}} = \frac{2^{2-p}(b_{\mathrm{T}} M_0)^{p-1}}{\Gamma(p-1)} K_{p-1}(b_{\mathrm{T}} M_0) \; \times \; F(b_{\mathrm{T}}, z_h)$$

ID	M_{D} model	parameters
I	$F = \left(\frac{1+\log\left(1+(b_{\mathrm{T}} M_z)^2\right)}{1+(b_{\mathrm{T}} M_z)^2} \right)^q$ $M_z = -M_1 \log(z_h)$	$M_0, \; M_1$ $p = 1.51, \; q = 8$
II	$F = 1$ $M_z = M_h \frac{1}{z\, f(z)^2} \sqrt{\frac{3}{1-f(z)}}$ $p_z = 1 + \frac{3}{2} \frac{f(z)}{1-f(z)}$ $f(z) = 1-(1-z)^\beta, \quad \beta = \frac{1-z_0}{z_0}$	z_0

We consider two models; power law as starting point

Two different approaches to model zh-dependence

Models for gK

$g_{\rm K}$ model		
A	$g_{\rm K} = \log\left(1 + (b_{\rm T} M_{\rm K})^{p_{\rm K}}\right)$	$M_{\rm K},\ \ p_{\rm K}$
B	$g_{\rm K} = M_{\rm K} b_{\rm T}^{(1-2p_{\rm K})}$	$M_{\rm K},\ \ p_{\rm K}$

I focus on gK since it can be compared to other extractions. Recall that the TMDFF in this case is differs from the usual definition by a soft factor.

Goodness of Fit

$q_T/Q < 0.15$ (pts = 168)		
	IA	IB
$\chi^2_{\mathrm{d.o.f.}}$	1.25	1.19
$M_0(\mathrm{GeV})$	$0.300^{+0.075}_{-0.062}$	$0.003^{+0.089}_{-0.003}$
$M_1(\mathrm{GeV})$	$0.522^{+0.037}_{-0.041}$	$0.520^{+0.027}_{-0.040}$
p^*	1.51	1.51
q^*	8	8
$M_{\mathrm{K}}(\mathrm{GeV})$	$1.305^{+0.139}_{-0.146}$	$0.904^{+0.037}_{-0.086}$
p^*_{K}	0.609	0.229

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z_0	$0.574^{+0.039}_{-0.041}$	$0.556^{+0.047}_{-0.051}$
$M_{\mathrm{K}}(\mathrm{GeV})$	$1.633^{+0.103}_{-0.105}$	$0.687^{+0.114}_{-0.171}$
p_k	$0.588^{+0.127}_{-0.141}$	$0.293^{+0.047}_{-0.038}$

Comparison of all models similar (within error bands)

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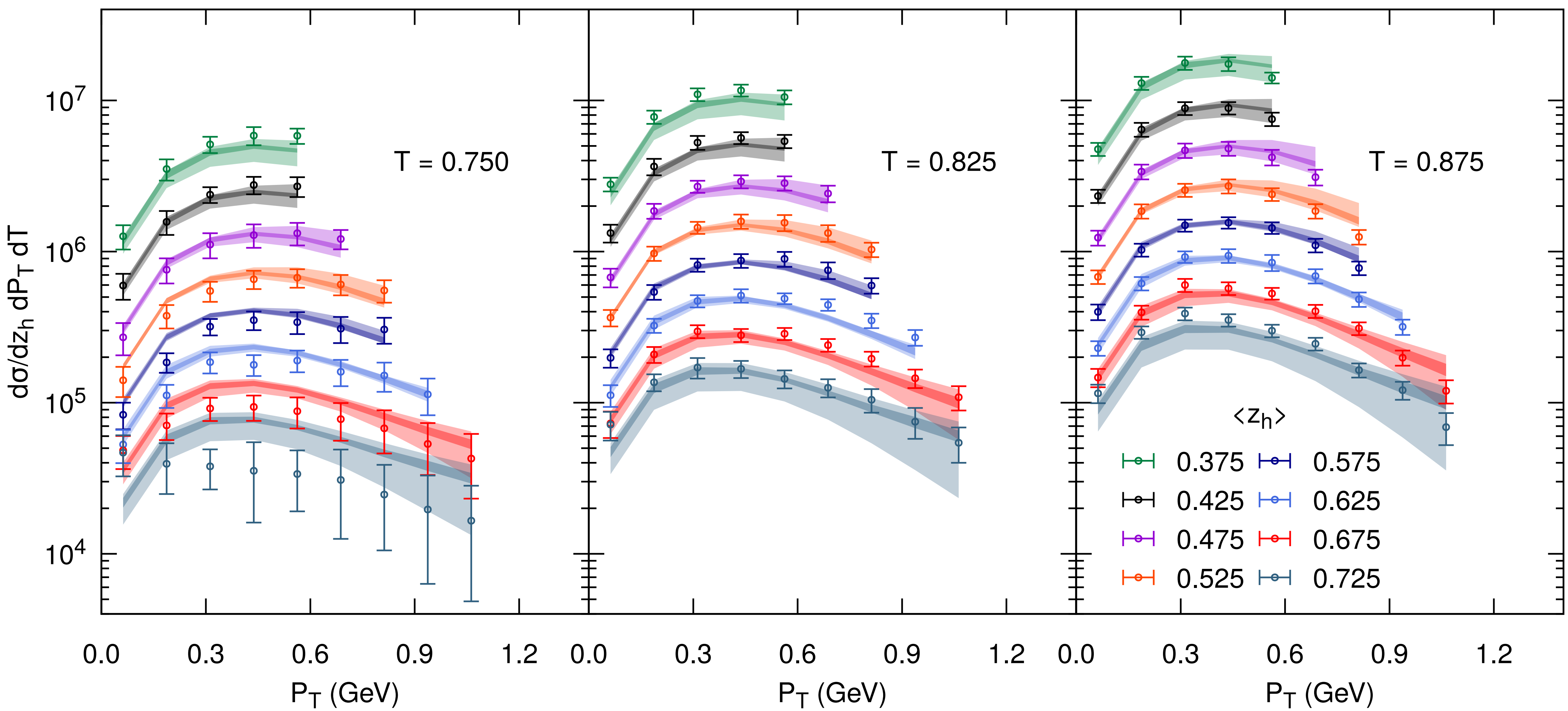
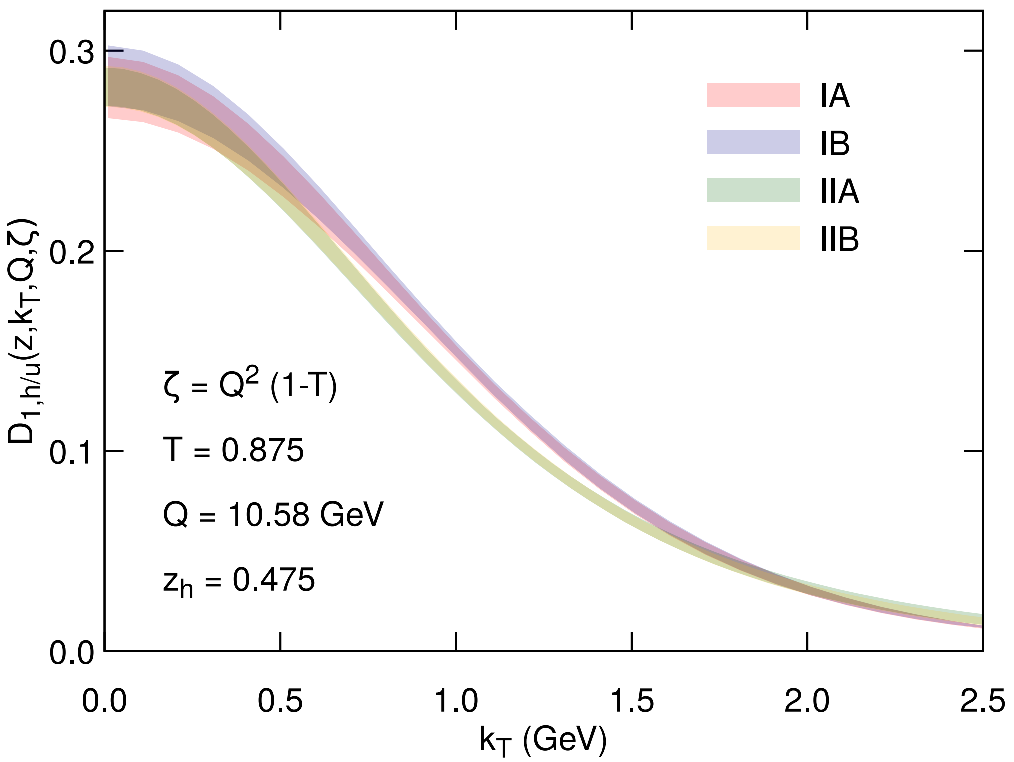
Comparison to data



Dark bands: statistical uncertainties from extraction

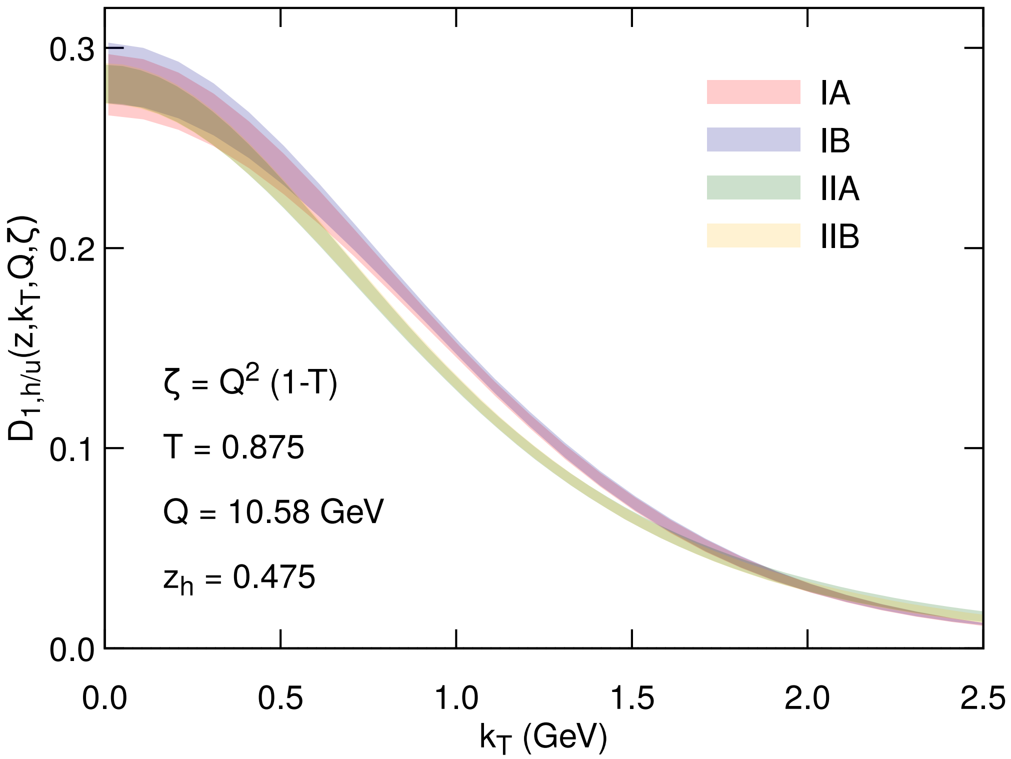
Light bands: rough estimate of uncertainty due to collinear function uncertainties (probably an overestimation)

Extracted functions & correlations

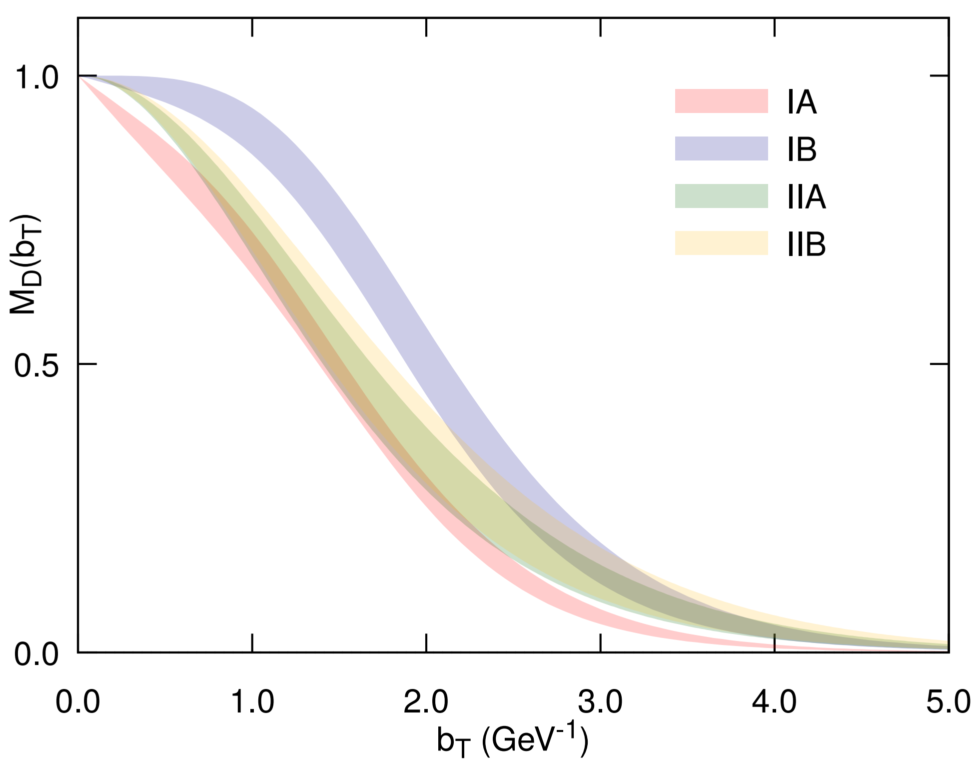


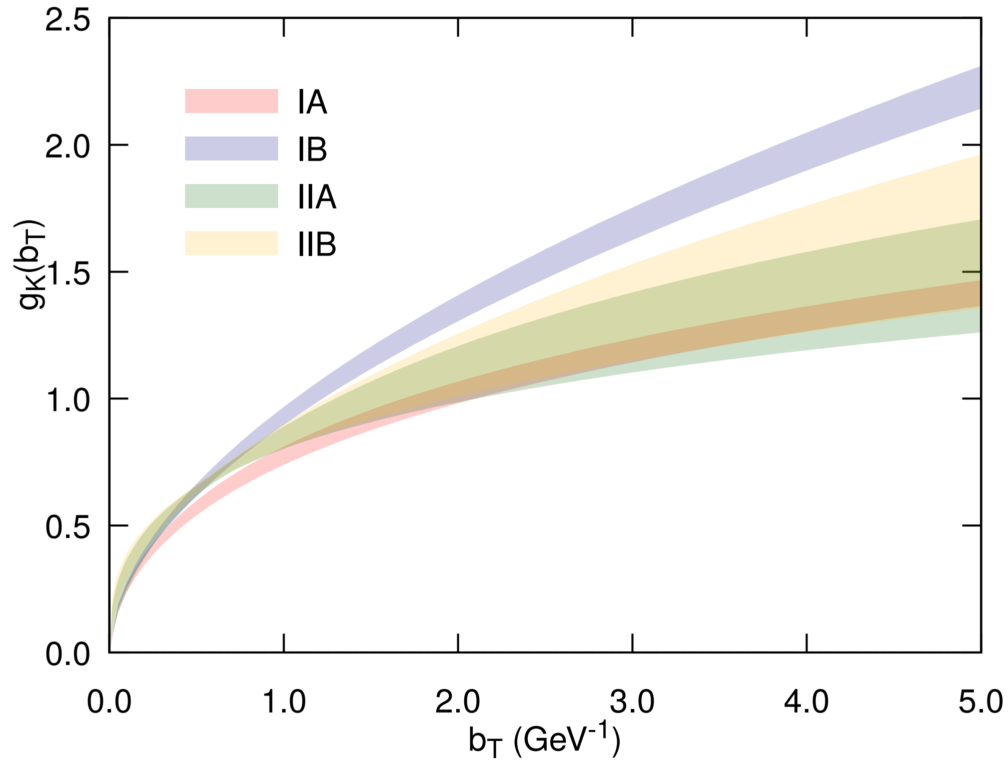
TMDFF:
contains both functions extracted: MD and gK

Extracted functions & correlations

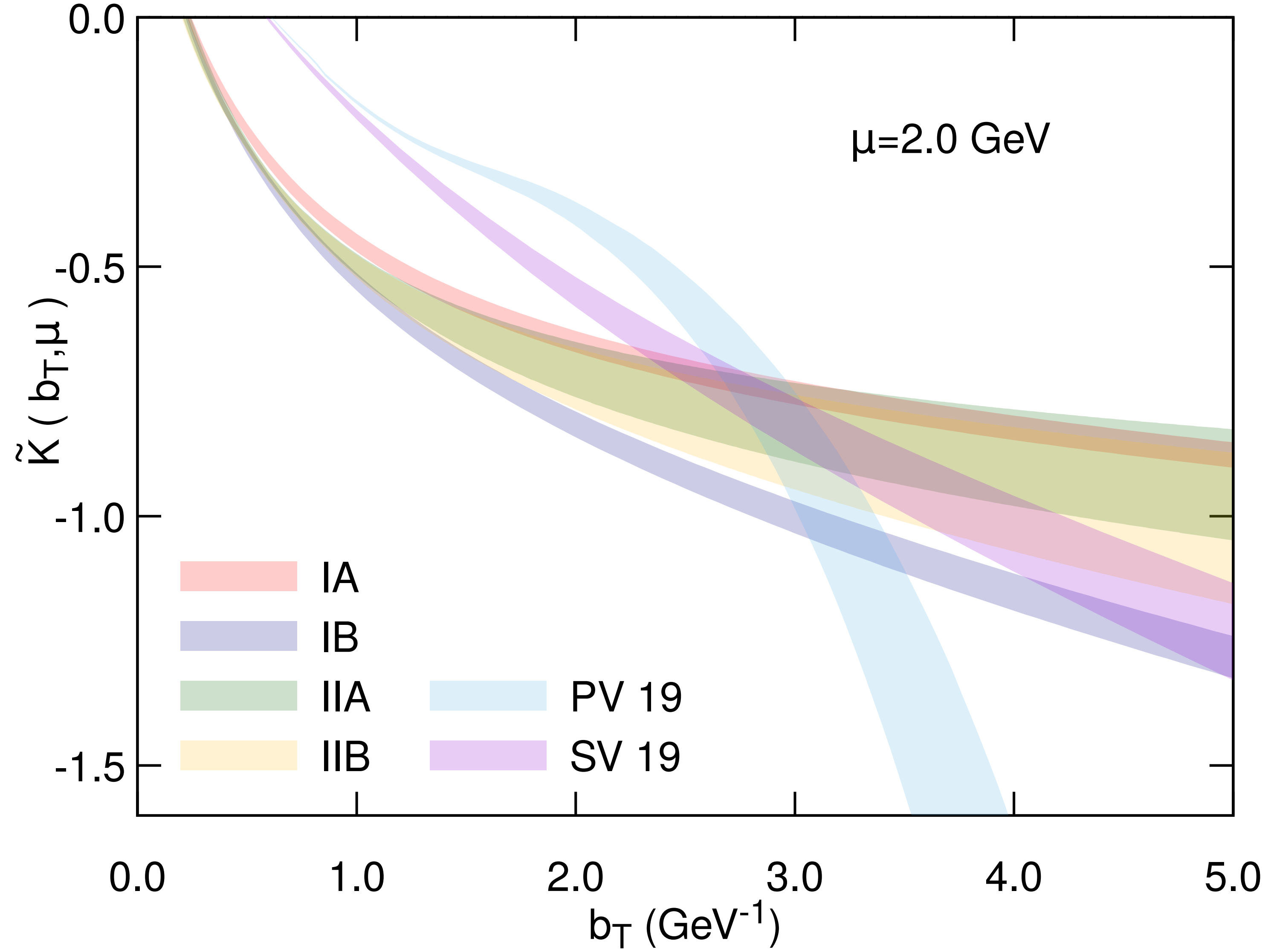


MD and gK differences beyond statistical error bands. Can be thought as a type of theoretical error (model bias) .

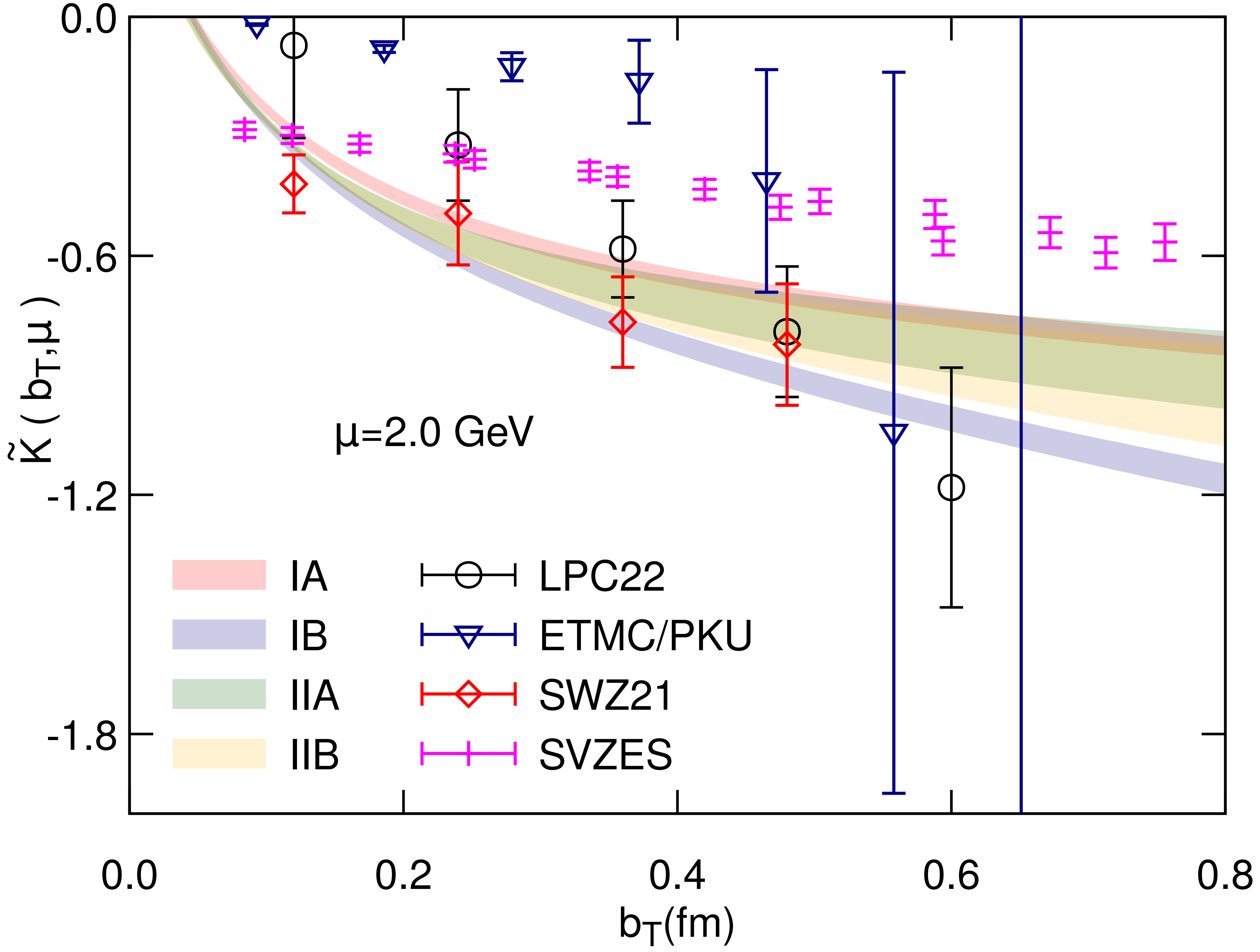




Comparison to other extractions



Comparison to lattice calculations



Thanks.