Kinetics of the relativistic electron beams emitting hard X– and γ–rays

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Abstract

With increase in the electron energy and the frequency of the external driving forces – undulators of X–FELs, lasers of Compton sources, or cristals of channeling radiation sources – recoils caused by emission of a quanta induce energy losses. These losses are random and affect the energy and angular spreads of the beam distribution in the phase space. We are going to study these effects aimed on evaluation the evolution of the spectrum and the coherency degree of hard radiation.

Background

Intense sources of hard x– and gamma–rays are highly demanding

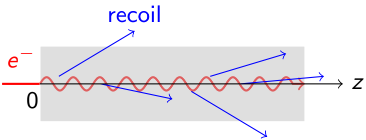
Chronology of our activity:

- Compton sources for medicine (angiography): 40 MeV electrons + 1 eV laser $\rightarrow$ 30 keV photons
- Compton sources of polarised gammas for production of polarised positrons (for lepton colliders ILC, CLIC)
- Compton sources of gamma range for nuclear resonance fluorescentce (isotopes fingerprints)
- Optimisation of the undulator–based polarised gamma sources (ILC)

These tasks and besides require study on self-consistent kinetics of the electron beam interacting with external periodic driving forces

Relativistic electrons + periodic fields $\Rightarrow$ bright x/γ–rays

Spectral brightness $\propto 1/$(width of electron spectrum)



Periodic fields with flat/circular symmetry

- magnetic field of undulators, period $L_p \geq 1$cm
- laser electro–magnetic field, $L_p \geq 400$nm
- electric field of cristal $L_p \geq 1$Å

recoil $\propto \gamma^2/L_p$

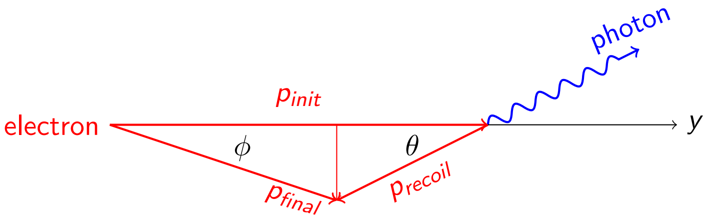
Specific features of hard radiation sources:

- highly populated bunches, up to 10^{10} electrons
- short path length $\Rightarrow$ emitted small number of photons per electron

Problem setup. Photon → particle

Statistics caused by kicks

Momentum plane



w — Probability Density Function (PDF) of recoils

- statistically independent kicks
- the kick is independent of the electron energy
- w – PDF of kicks – has compact support
- $\int w(\omega)\,\mathrm{d}\omega < \infty$

Focus at the beginning of the process

Master Formula, Bulyak, Shul'ga (2016, 2017)

random recoils, not depend on electron's energy

probability density function

$$\hat{f} = \hat{f}_0 \exp[x(\check{w} - 1)]$$

$\hat{(\cdot)}$ Fourier transform, $\check{(\cdot)}$ inverse Fourier transform

moments (suitable at $x \gg 1$)

$$\overline{\gamma}(x) = \overline{\gamma_0} - x\,\overline{\omega}$$
$$\mathrm{Var}[\gamma](x) = \overline{(\gamma - \overline{\gamma})^2}$$
$$= \mathrm{Var}[\gamma_0] + x\,\overline{\omega^2}$$
$$\dots$$

electrons PDF completely determined with:

- recoil spectrum $w(\omega)$
- ensemble average integral number of recoils $x = \int_0^z P(z')\,dz'/\overline{\omega}$

Results 1. Beginning of the emission process

assumption $x \propto t$ (or $x \propto s$ pass length) yields evolution of PDF

- $x \to \infty \Rightarrow f \to$ Gaussian (Central Limit Theorem)
- $x \lesssim (1 \ldots 10) \Rightarrow$ characterised by w

Approximation PDF with α–stable distributions and evolution with (abnormal) diffusion:

- $x \to \infty \Rightarrow \alpha \to 2$ Gaussian, normal diffusion $c \propto \sqrt{x}$
- $x < 1 \Rightarrow \alpha \geq 1$ ballistic diffusion $c \propto x$

c is the scaling parameter (width of spectrum) $c \propto x^{1/\alpha}$

Focus at w

- validation1: agreement with simulations of dipole radiation (E.Bulyak, N.Shul'ga, NIM 2017)
- validation2: ionisation losses (scattering on Coulomb potential) and $f_0 = \delta \Rightarrow$ Landau function for distribution of ionisation losses, $\alpha = 1$ ballistic diffusion
- radiation in the fundamental (dipole) harmonics: for $0 < x < \infty$ $1 \le \alpha \le 2$

Recoil spectrum of ‘multiphoton’ radiation

Classical: higher harmonics, quantum - fusion of n photons into one

quantum model

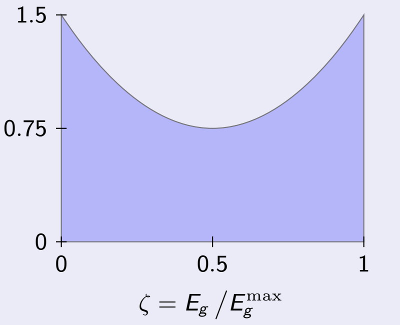
$e + n\gamma \to e' + \gamma'$ comp. with the classical model: n is the harmonics number

Results of our study: n has the Poisson distribution with the parameter $\lambda = K^2$ (helical undulator / circular laser polarisation); accordingly $\lambda = K^2/2$ for the linear polarisation.

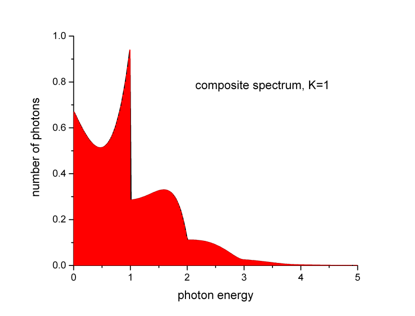
Multiphoton recoil spectrum

w_λ for $\lambda \sim 1$

dipole harmonic, $\lambda \ll 1$



many harmonics, $\lambda = 1$



E.Bulyak, N.Shul'ga, Phys.Rev. AB (2019)

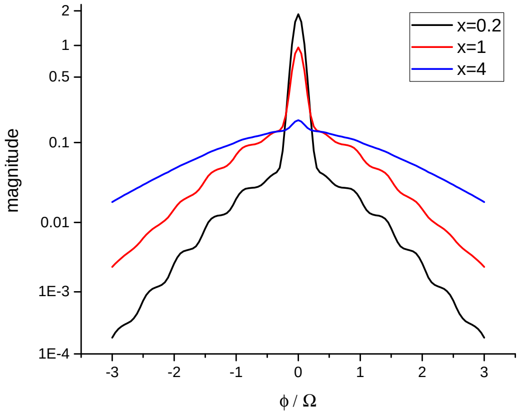
‘Multiphoton’ second moment

$$\overline{(\gamma - \overline{\gamma})^2} = \mathrm{Var}[\gamma_0] + x\,\overline{\omega^2}$$

$$\overline{\omega^2} = \frac{\omega_*^2}{k}\left[1 + \frac{\lambda}{(1+\lambda)^2}\right]$$

where $\lambda = K^2, K^2/2$ for the helical and flat undulators, resp.; $k \approx (2\ldots4)$
Maximal spread at $\lambda = 1$

PDF for angular distribution



PDFfor $\kappa = 1$; $x = 0.2, 1, 4$. Here $\Omega = \frac{\hbar\omega_{\text{undul}}}{m_e c^2} = \frac{\lambda_C}{\lambda_{\text{undul}}}$

Summary and Outlook

Width of electron bunch spectrum increases linearly over the initial section of Free Electron Laser where the coherent micro bunches formed. With increase in energy (aimed at attain higher energy photons) the length of formation increased $\propto \gamma$ while the width of the spectrum $\propto \gamma^2$ that limits the formation of coherent microbunches. To preserve coherence at increase in the energy, $\gamma/\lambda_{\text{undul}} = \mathrm{const}$ should be kept

The transversal kicks are rather small, but essential for storage-ring based Compton sources (laser cooling, E.Bulyak, P.Gladkikh, L.Rinolfi, T.Omori and J.Urakawa, IPAC 2010).

The energy spectrum of the radiating electrons weakly depends of of the driving field magnitude – the higher harmonics composition – while the angular spread strongly depends upon it. Enable to optimise tapering of FELs undulators with adjusting to the mode of PDF.

Citations

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