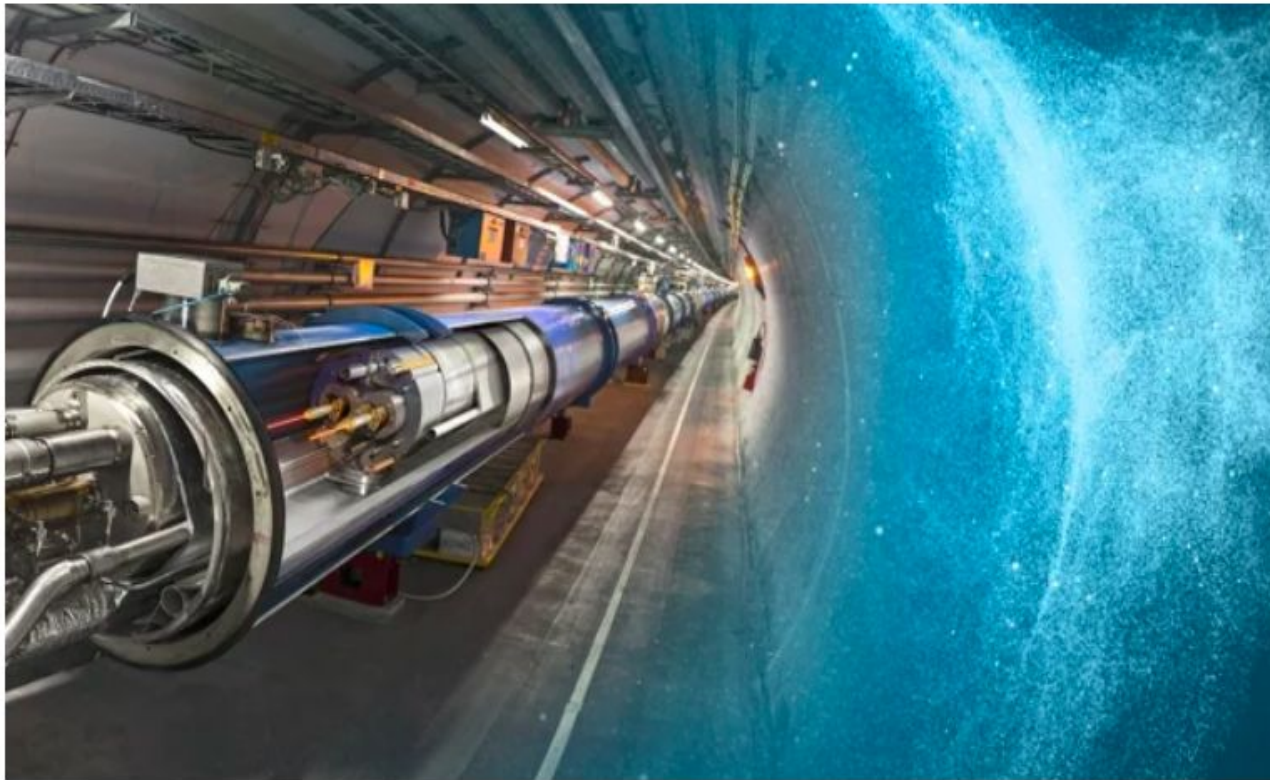


Introduction to the Standard Model

Summer Student Lecture 2022 – Part III



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Deutsches
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Synchrotron

21st-25th July 2022

Content

>0) Introduction

- What is the Standard Model?
- Coupling constants, masses and charges
- Units and scales

>1) Interactions

- Relativistic kinematics
- Symmetries and conserved quantities
- Feynman diagrams
- Running couplings and masses

>2) Quantum electrodynamics

- Test of QED: Magnetic momentum of the muon
- Test of QED: High energy colliders



Content

>3) Strong Interaction: Quantum-Chromodynamics

- A short history of hadrons and quarks
- DIS and gluons
- QCD and its properties

>4) Electroweak interactions

- **History of the weak interaction : β -decays**
- **Parity violation**
- **CP-violation**
- **GSW mechanism and CKM**
- **Experimental verification**

>5) The Higgs

- **Why is it necessary ?**
- **How was it found ?**

> Beyond the Standard Model (brief)



Weak interactions: Electroweak unification

A little bit of history: the β^- decays and radiation

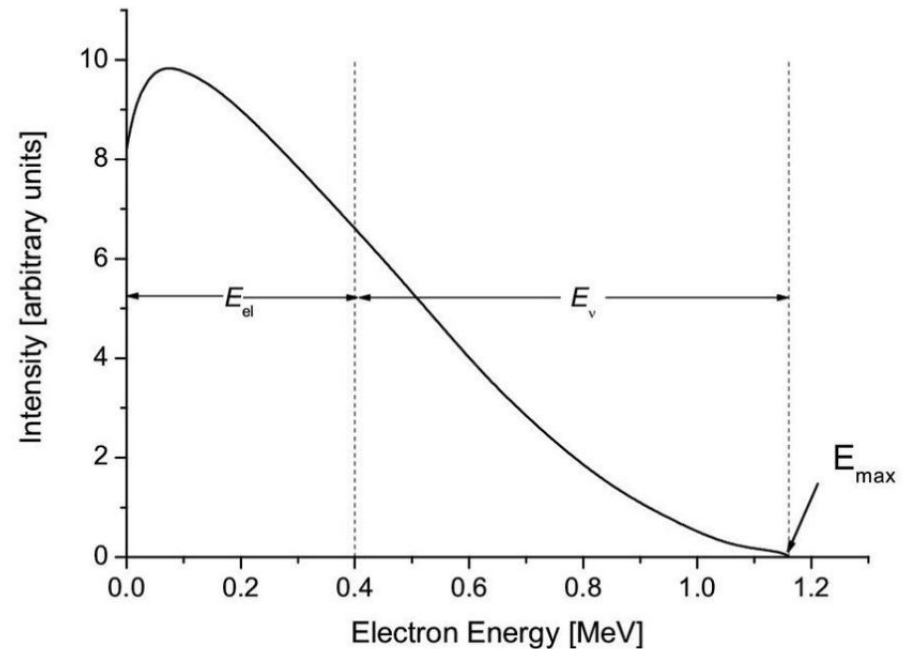
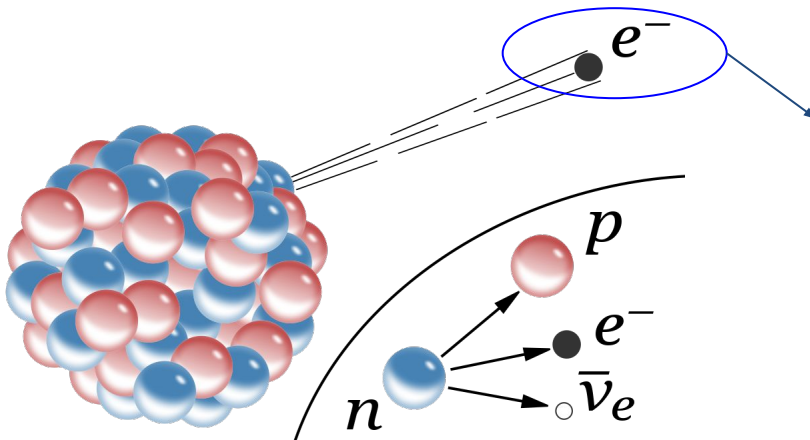
> Discovered with radioactivity. Initially, only observed that nuclei emitted one electron and the atomic number was unchanged.

> Puzzling at the time:

- Spin of the nuclei unchanged or integer change \rightarrow electron with spin $\frac{1}{2}$. How ?
- Energy conservation: if only electron is emitted, energy should have a defined value \rightarrow But continuous

> Pauli's proposal: neutrino

Detected in 1956 ([Cowan,Reines](#))

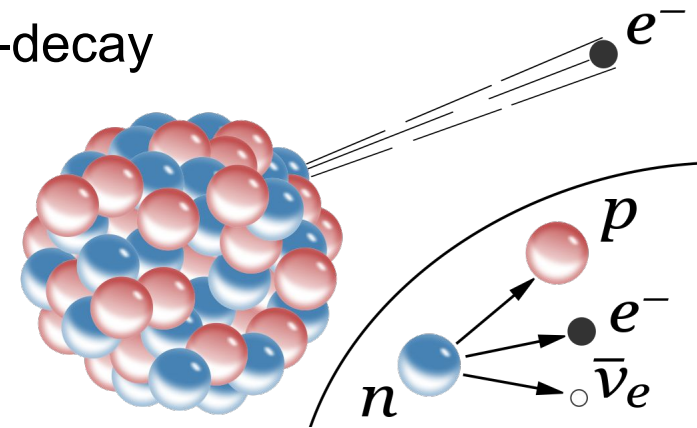


A little bit of history: Weak interactions

- > In addition, β^- decays usually have a long lifetime (e.g. isolated neutron having a half life of 10 mins)
 - Lifetime depends on interaction's strength \rightarrow **Weak interaction !**

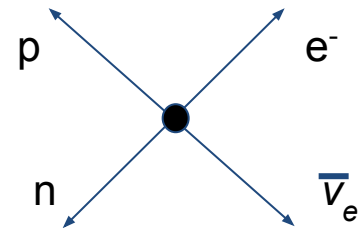
- > Fermi theory proposed as explanation of beta-decay \rightarrow four point interaction

- > Coupling constant G_F measured from lifetime of muon: $1.6637 \times 10^{-5} \text{ GeV}^{-2}$



- > Suggested generic four point interaction (a la QED)

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} (\bar{\psi}_p \gamma^\mu \psi_n) (\bar{\psi}_e \gamma_\mu \psi_{\bar{\nu}}),$$



- > Fermi's theory successfully described decays but incomplete.

Weak interaction decays started to show strange behaviours w.r.t electromagnetic and strong interactions

Let's go back to the properties of an interaction

Helicity

Particles whose momentum direction aligns with spin → right-handed particles

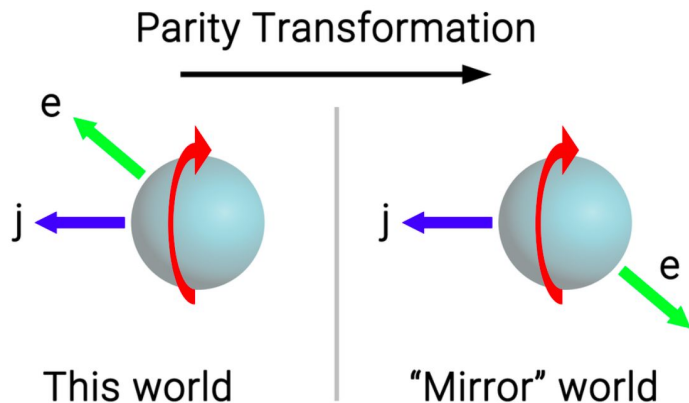
Particles whose momentum direction aligns with spin → left-handed particles

Dependent on the reference frame.



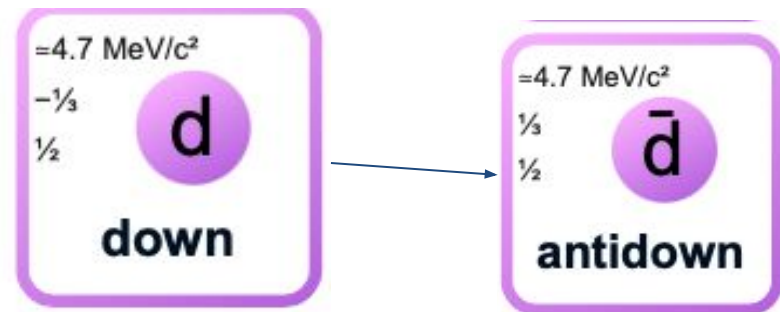
Parity

Mirror the coordinates of the particle.
Changes sign of momentum, coordinates
Spin doesn't change sign.



Charge conjugation

Change a particle by its anti-particle



Let's go back to the properties of an interaction

Chirality

Identical to helicity in the massless case but something more complicated

It tells how two separate components of a fermionic field change under Lorentz boost (space-time change) → Weyl spinors. Each fermion has a left-handed component and a right-handed one

Parity transformations change chirality
General Lorentz transformation

$$S = \exp \begin{bmatrix} \frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi} & 0 \\ 0 & \frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi} \end{bmatrix}$$

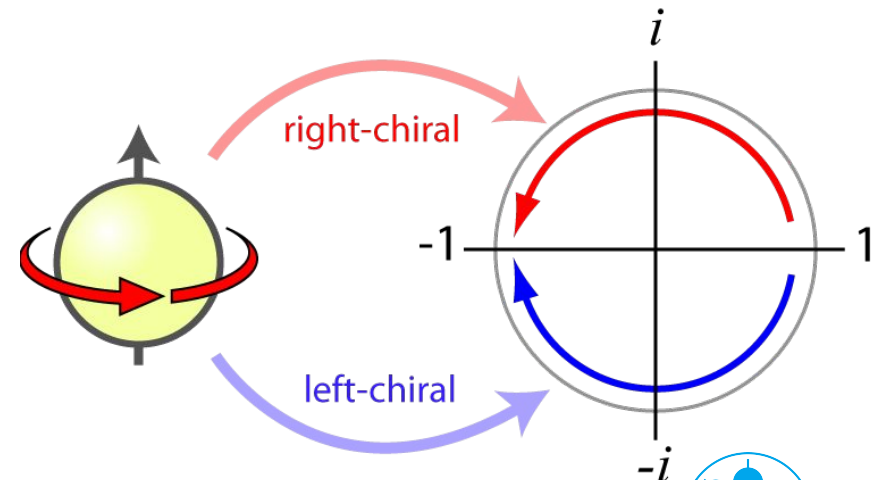
Spinor of fermion: 2 terms (Weyl spinor) with 2 components

$$\Psi = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix} \quad \begin{aligned} \psi'_R &= \exp\left(\frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi}\right) \psi_R \\ \psi'_L &= \exp\left(\frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi}\right) \psi_L \end{aligned}$$

Mass terms in Lagrangian

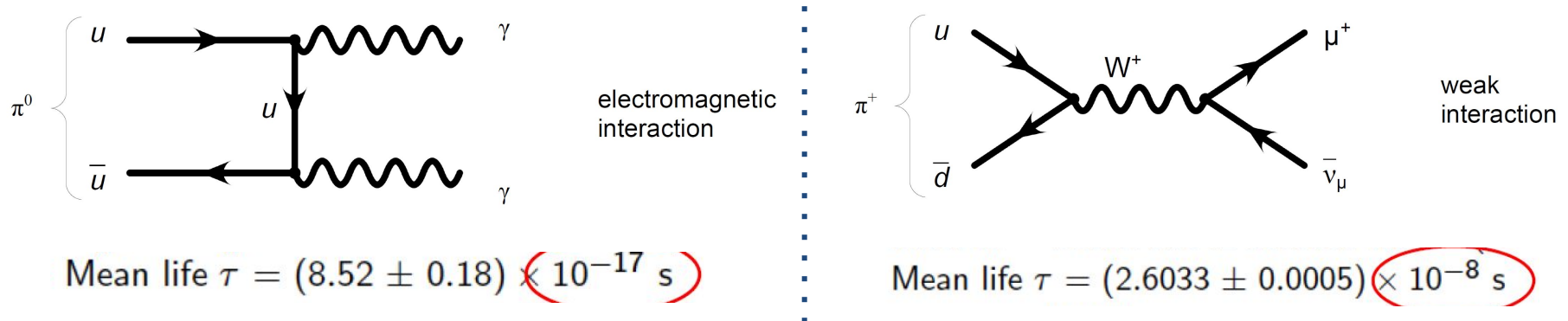
$$-m\bar{\Psi}\Psi = -m(\bar{e}_L e_R + \bar{e}_R e_L)$$

θ : angle in space rotations
 Φ : boost (time and space rotation)



What's the strange behaviour ? The τ - θ puzzle (1956)

- Additional measurements showed interactions similar like the β -decay.



- In the 50's, two particles were observed: τ^+ and θ^+ . τ - θ puzzle

$$\tau^+ \rightarrow \pi^+ \pi^+ \pi^-$$

$$P(\tau^+) = P(\pi^+ \pi^+ \pi^-) = -1$$

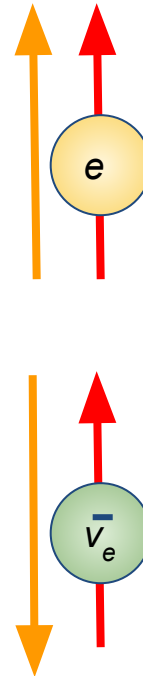
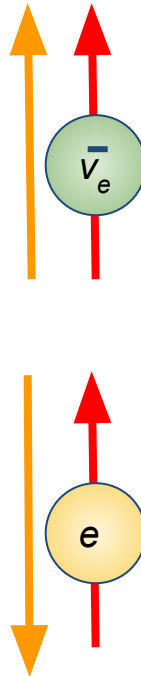
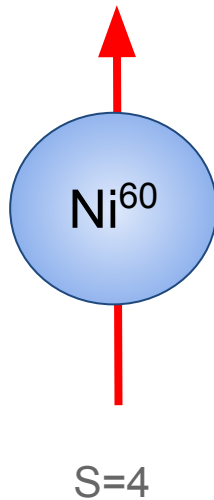
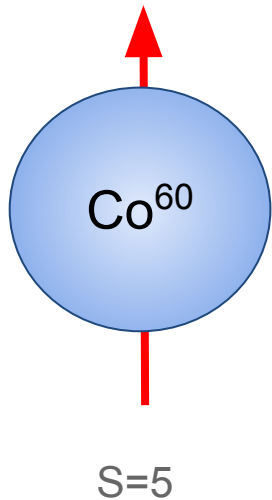
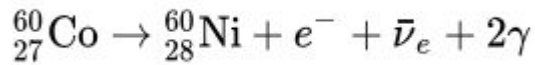
$$\theta^+ \rightarrow \pi^+ \pi^0$$

$$P(\theta^+) = P(\pi^+ \pi^0) = +1$$

- Decaying into different states and different parity Same mass, lifetime, charge, spin
- Proposal that both particles are actually the same particle (K^+) but parity violation in the interaction.

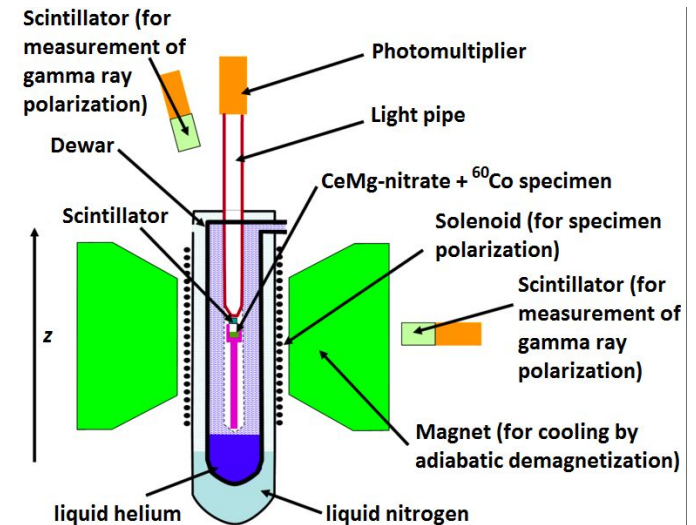
β -decays of Co^{60} : Parity violation of the weak interaction

Co^{60} atoms aligned with magnetic field



Experiment of Madame Wu

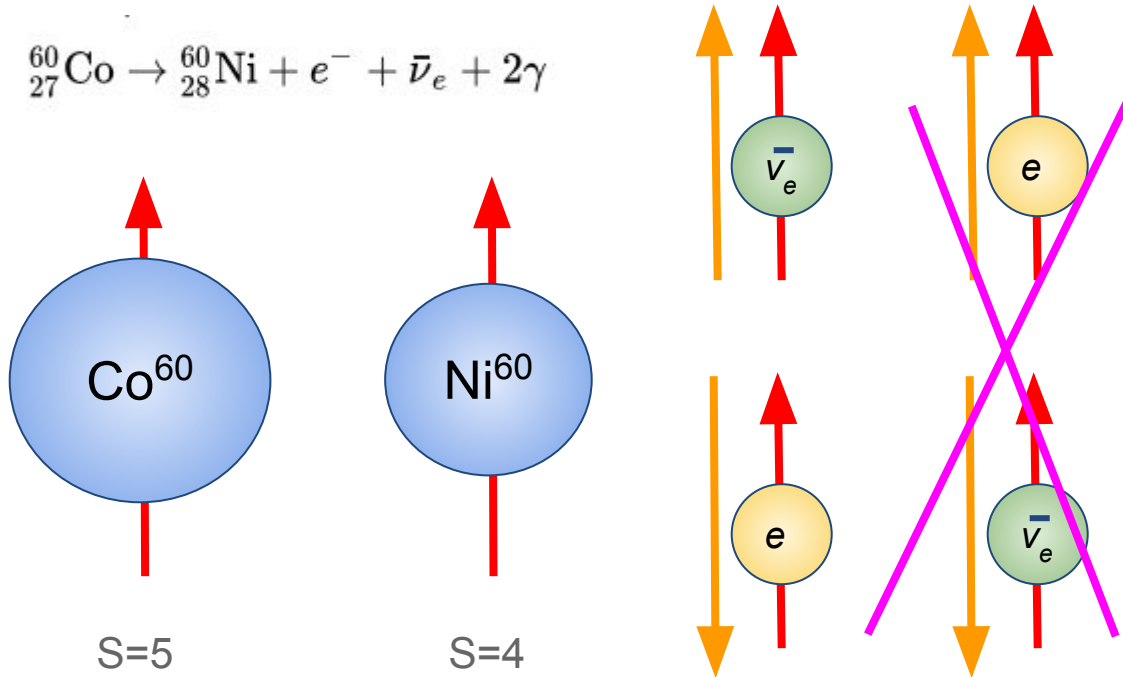
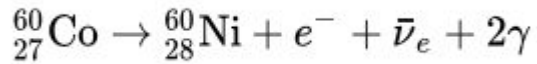
Check decays of Co^{60} into Ni^{60} . Align Co^{60} using a uniform magnetic field and reduce thermal motion with low-temperature experiment. 2γ from Ni^* de-excitation, isotropic. Anisotropy would show how Co^{60} was aligned. If parity symmetry, no preferred direction of the electron from this decay.



β -decays of Co^{60} : Parity violation of the weak interaction

Co^{60} atoms aligned with magnetic field

If parity symmetry, no preferred decay



Observed that electrons are preferentially emitted in opposite direction to nucleus spin

Weak interaction violates parity (maximally)

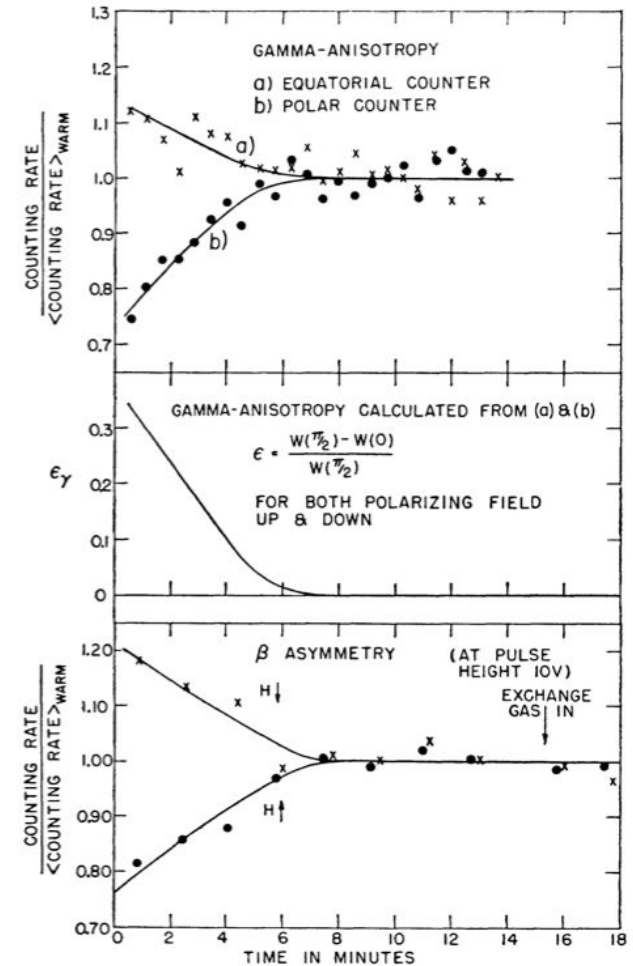


Fig. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

First proposal of a QFT for weak interactions

Feynman and Gell-Mann proposed a QFT where the force field is V-A interaction

Explanation of Wu's experiment : weak interaction only with left-handed states of particles (and right-handed anti-particles)

Vector-axial symmetry → Interaction only happens between left-handed particles (or right-handed antiparticles).

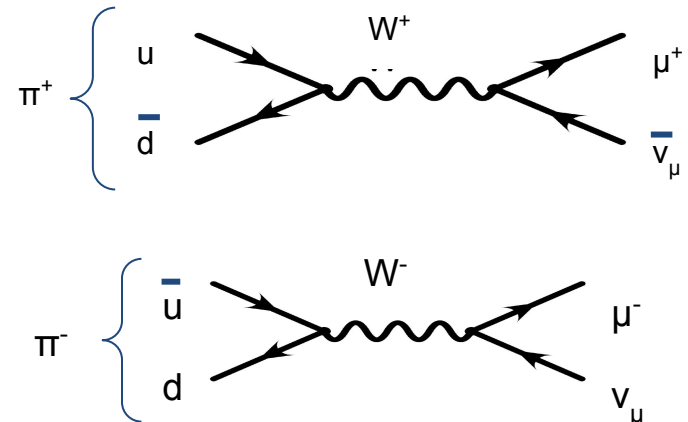
$$e_L = \frac{1}{2}(1 - \gamma_5)e$$
$$e_R = \frac{1}{2}(1 + \gamma_5)e$$

$$\mathcal{L}_\mu = \frac{G_\mu}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\lambda (1 - \gamma_5) \mu] [\bar{e} \gamma_\lambda (1 - \gamma_5) \nu_e(x)] + \text{c.c. .}$$

QFT theory still allows the CP-symmetry

Change parity of the interaction and change particles by anti-particles → Same probability.

Experiment of Wu didn't show any problem
CP-symmetry



Further problem: K_S^0 and K_L^0 and CP-violation

Experiment of Christenson-Fitch-Cronin-Turlay: [link](#)

Two neutral kaons (meson with one strange quark) were known with same mass and properties but two different lifetimes and decay types: K_S^0 and K_L^0

K_S^0 , $\tau = 9.0 \cdot 10^{-11}$ s ($c\tau = 2.7$ cm)

K_L^0 , $\tau = 5.1 \cdot 10^{-8}$ s ($c\tau = 15$ m)

- > Lifetimes typical of weak interaction
- > Experiment with a beam of neutral kaons. If beam long enough, enriched with K_L^0 .
- > If only 3π decays, no CP violation.

$$K_S^0 \rightarrow \pi^0 \pi^0 / \pi^+ \pi^- ; \quad \text{CP} = +1$$
$$K_L^0 \rightarrow \pi^0 \pi^0 \pi^0 / \pi^+ \pi^- \pi^0 ; \quad \text{CP} = -1$$

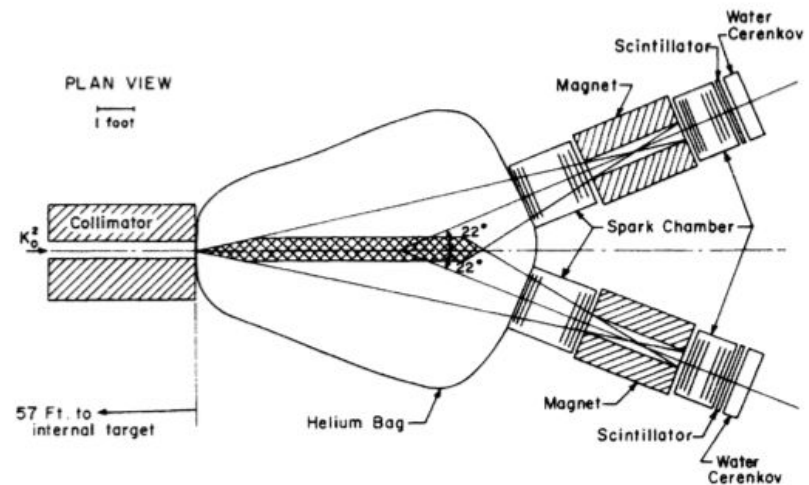


FIG. 1. Plan view of the detector arrangement.

Further problem: K_S^0 and K_L^0 and CP-violation

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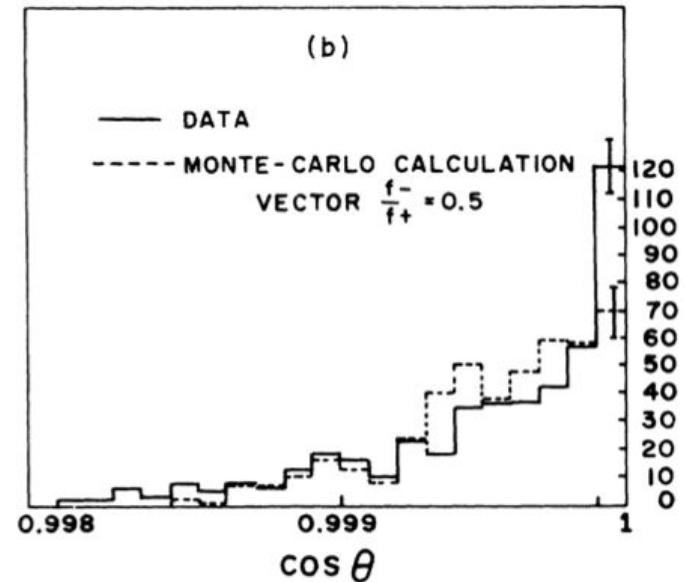
K_S^0 , $\tau = 9.0 \cdot 10^{-11}$ s ($c\tau = 2.7$ cm)

K_L^0 , $\tau = 5.1 \cdot 10^{-8}$ s ($c\tau = 15$ m)

$K_S^0 \rightarrow \pi^0 \pi^0 / \pi^+ \pi^-$; CP = +1

$K_L^0 \rightarrow \pi^0 \pi^0 \pi^0 / \pi^+ \pi^- \pi^0$; CP = -1

- > Lifetimes typical of weak interaction
- > Experiment with a beam of neutral kaons. If beam long enough, enriched with K_L^0 .
- > Observed more events than expected and associated to production of 2π !



Further problem: K^0_S and K^0_L and CP-violation

Experiment of Christenson-Fitch-Cronin-Turlay: [link](#)

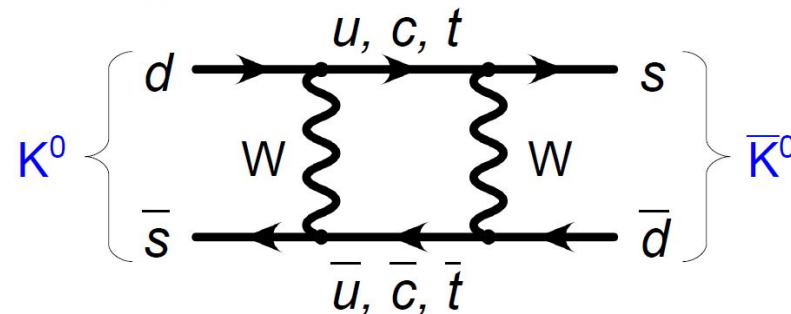
Two neutral kaons (meson with one strange quark) were known with same mass and properties but two different lifetimes and decay types: K^0_S and K^0_L

$$\begin{array}{ll}
 K^0_S, \tau = 9.0 \cdot 10^{-11} \text{ s } (c\tau = 2.7 \text{ cm}) & K^0_S \rightarrow \pi^0 \pi^0 / \pi^+ \pi^-; \quad \text{CP} = +1 \\
 K^0_L, \tau = 5.1 \cdot 10^{-8} \text{ s } (c\tau = 15 \text{ m}) & K^0_L \rightarrow \pi^0 \pi^0 \pi^0 / \pi^+ \pi^- \pi^0 \quad ; \text{CP} = -1
 \end{array}$$

- > As we know today, the reason is that K^0_S and K^0_L are actually a mixing of the strong interaction eigenstates K^0 and \bar{K}^0
- > Mass and charge of the interaction determined by QCD and QED (this is the particle we see).
- > But QCD eigenstate is not weak eigenstate. QCD eigenstate = composition of weak eigenstates \rightarrow Turn CP=1 state into CP = -1 \rightarrow **CP-violation!**

K^0 and \bar{K}^0 can turn from one into the other:

“oscillation”



Towards a QFT of weak interactions → Electroweak

➤ Problem: Divergences! Theory only valid at low energies

$$\mathcal{L}_\mu = \frac{G_\mu}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\lambda (1 - \gamma_5) \mu] [\bar{e} \gamma_\lambda (1 - \gamma_5) \nu_e(x)] + \text{c.c.} . \quad \longrightarrow \quad \sigma^{e^- + \nu_e \rightarrow e^- \nu_e} = \frac{4G_F^2}{\pi} E_{\text{CM}}^2$$

Additional problems:

- Radiative corrections divergent (but needed)
- Unitarity violated at high energy (cross-section goes to very large values)
- Loop processes such as K^0 mixing meaningless/incorrect
- Need theory which includes Feynman theory but does not diverge at high energies

1960-1968: Formulation of electroweak unification

(Glashow, Salam, Weinstein) → **massive W/Z bosons + massless γ**



Gauge theory for the weak interaction → Electroweak

A gauge theory is a QFT theory that is invariant under local transformations

Local transformation = transformation that is not the same in all space → $x+\Delta x$ with $\Delta x = f(x)$

This is the way that interactions are introduced in the SM → Find a variable/symmetry of the interaction and formulate a theory including a new gauge boson that would make the lagrangian invariant.

Lagrangian of the free Dirac field

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu \partial_\mu - mc^2)\psi$$

Invariance under phase transformations?

Physics should be similar

$$\begin{aligned}\psi(x) &\rightarrow e^{iq\Lambda(x)/\hbar}\psi(x) \\ \bar{\psi}(x) &\rightarrow \bar{\psi}(x)e^{-iq\Lambda(x)/\hbar}\end{aligned}$$

Introduce A_μ to absorb $\delta\Lambda(x)$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x) \quad D_\mu = \partial_\mu - \frac{i}{\hbar}qA_\mu$$

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu(\partial_\mu - iqA_\mu/\hbar) - mc^2)\psi$$

$$\begin{aligned}\mathcal{L} &\rightarrow \bar{\psi}e^{-iq\Lambda(x)/\hbar}(i\hbar c\gamma^\mu \partial_\mu - mc^2)e^{iq\Lambda(x)/\hbar}\psi = \\ &= \bar{\psi}(i\hbar c\gamma^\mu(\partial_\mu + iq\partial_\mu \Lambda(x)/\hbar) - mc^2)\psi\end{aligned}$$

Lagrangian of the QED (U(1))

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



Gauge theory for the weak interaction → Electroweak

Transformations are described by a Lie group of transformations → Includes the whole ensemble of NxN matrices that make a transformation possible in a space defined by N-sized vectors.

In QFT, we use unitary matrices U(N) and SU(N) (with $\det(U)=1$)

- Each transformation can be described by a set of N^2-1 matrices → generators
- In the context of QFT, one gauge boson per generator

A complex example: SU(3) or also called QCD !!

- Strong interaction behaves equally for 3 different colors → Gauge invariance from color transformations → 3 different directions : **red**, **blue** and **green**
- Transformations between 3 colors leave invariant the lagrangian
- 8 different generators → One gauge boson per generator ! 8 gluon

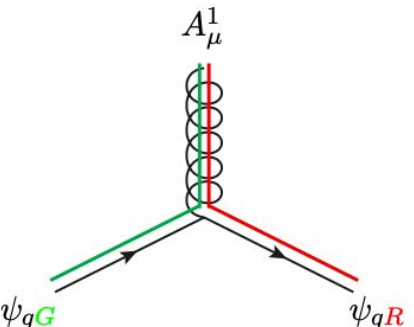
gluon

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu - ig_s t_{ij}^a A_\mu^a$$

generators of rotation in SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \end{pmatrix}$$



$$\propto -\frac{i}{2} g_s \bar{\psi}_{qR} \lambda^1 \psi_{qG}$$

$$= -\frac{i}{2} g_s \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

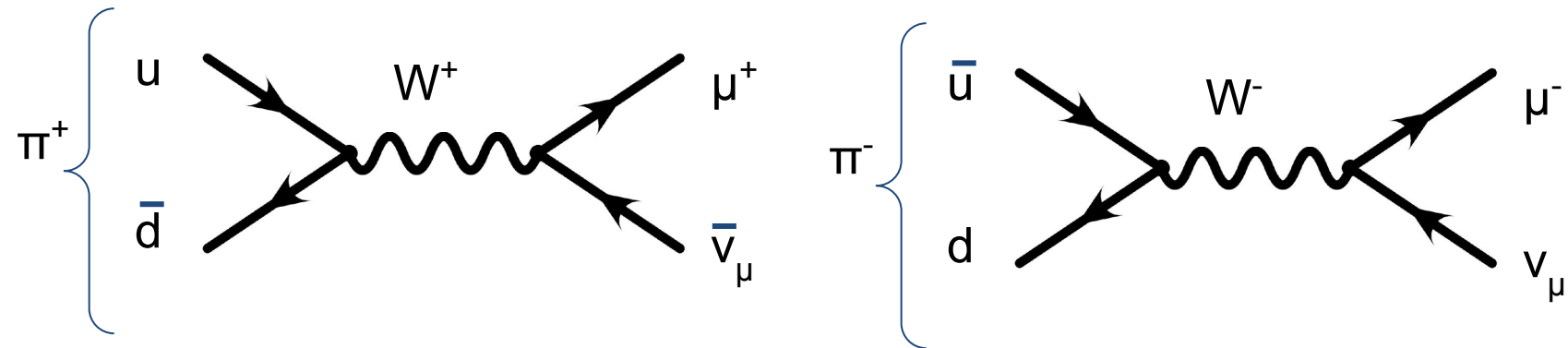


Electroweak interactions

Usually observed charged (positive and negative) interactions with long-lifetime → Propose mediated by two charged heavy bosons!

- > Consider W^+/W^- as doublets of the charge current
- > Introduce in QFT → Postulate $SU(2)_L$ symmetry acting only on the left-handed fermions.
 - Observed that weak interaction between two distinct particles → Introduction of weak isospin
 - A 3rd gauge field → Introduction of *neutral current* !
- > Try preserving $SU(2)_L$ and $U(1)_Q$ symmetry → Introduce Hypercharge Y_L to preserve $U(1)_Y$
- > Arrive at a unified interaction with massive W/Z boson + massless γ

$$\psi_L = \begin{pmatrix} e_L \\ \nu_e \end{pmatrix}$$



Electroweak unification !!!

- Glashow, Salam and Weinberg unified electromagnetic and weak interactions to **electroweak** interaction
- Gauge fields are linear combinations of B^0 (U(1)_Y weak hypercharge with coupling g'), and $W^{1,2,3}$ (SU(2)_L weak isospin with coupling g)

$$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

$$Z = \cos \theta_W W^3 - \sin \theta_W B^0$$

$$A = \sin \theta_W W^3 + \cos \theta_W B^0$$

- with the masses related (at tree level): $m_W = m_Z \cos \theta_W$ and θ_W the weak mixing angle with

$$\sin \theta_W = \frac{g'}{\sqrt{g'^2 + g^2}}$$

$$G_F^0 = \frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2 c^4} = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$$



Electroweak interactions

Fermion family	Left-chiral fermions				Right-chiral fermions			
		Electric charge Q	Weak isospin T_3	Weak hypercharge Y_W		Electric charge Q	Weak isospin T_3	Weak hypercharge Y_W
Leptons	ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	-1	No interaction, if they even exist			0
	e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-1	e_R^-, μ_R^-, τ_R^-	-1	0	-2
Quarks	u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	u_R, c_R, t_R	$+\frac{2}{3}$	0	$+\frac{4}{3}$
	d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	d_R, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

Interaction mediated	Boson	Electric charge Q	Weak isospin T_3	Weak hypercharge Y_W
Weak	W^\pm	± 1	± 1	0
	Z^0	0	0	0
Electromagnetic	γ^0	0	0	0

from wikipedia



Flavour and weak interaction: CKM matrix

- > In the 60's, together with parity and CP-violation, observed:
 - $u - d$, $e^- - \nu_e$, $\mu^- - \nu_\mu$ transitions with weak interaction had same probability to happen
 - $\Delta S = 1$ transitions had $\frac{1}{4}$ of probability of occurring than $\Delta S = 0$
- > We have seen that weak interaction eigenstates are not electric or strong interaction eigenstates \rightarrow Mixture of quarks are weak interaction
- > Nicola Cabibbo introduced mixing angle

$$\begin{bmatrix} d' \\ s' \end{bmatrix} = \begin{bmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix}$$

$$\tan \theta_c = \frac{|V_{us}|}{|V_{ud}|} = \frac{0.22534}{0.97427} \Rightarrow \theta_c = 13.02^\circ$$

Weak eigenstate

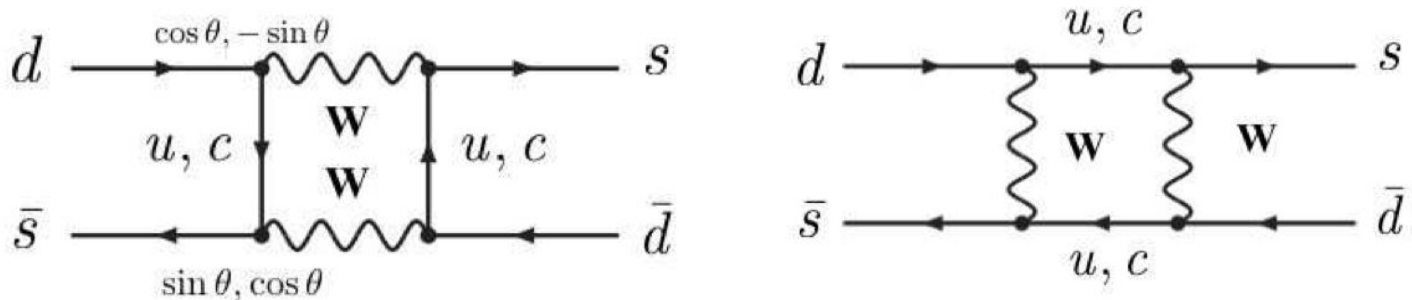
$$\left\{ \begin{array}{l} d' = \cos \theta_c d + \sin \theta_c s \\ s' = -\sin \theta_c d + \cos \theta_c s \end{array} \right.$$

Strong eigenstate



Flavour and weak interaction: CKM matrix

> **GIM:** using Cabibbo theory and GSW theory, cannot explain $K^0 - \bar{K}^0$ mixing \rightarrow Introduced the c-quark



> **CKM matrix:** Cabibbo-Kobayashi-Maskawa observed that the current Cabibbo matrix (2d and u,d,s,c) couldn't explain CP violation (can rotate 2d matrix to absorb any phase)

- Added a 3rd generation
- CKM matrix explaining mixing in charged currents with quarks

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

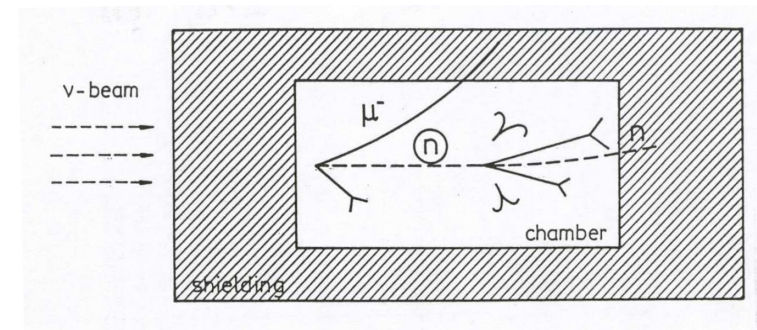
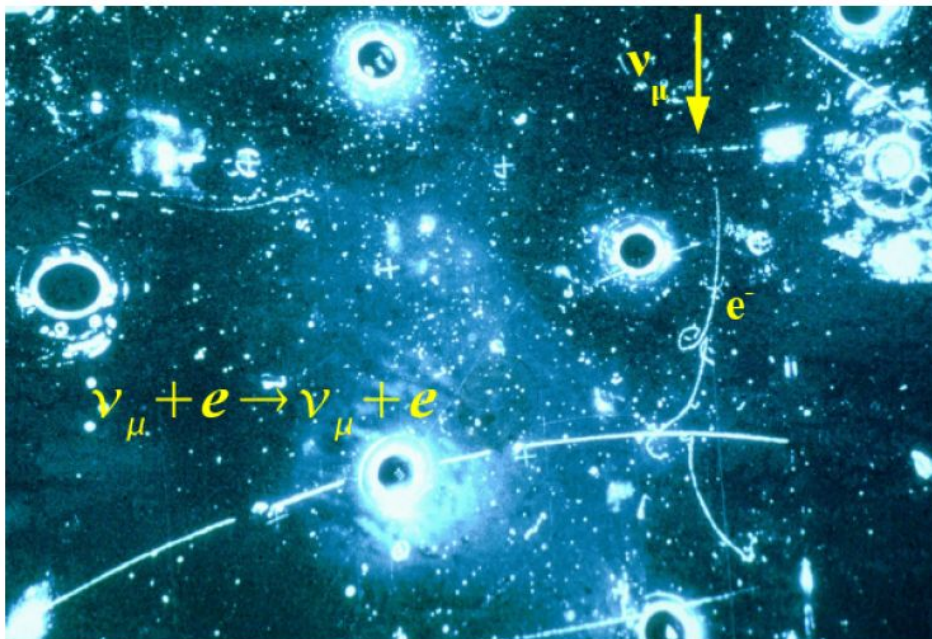
Same matrix for leptons (PMNS)

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

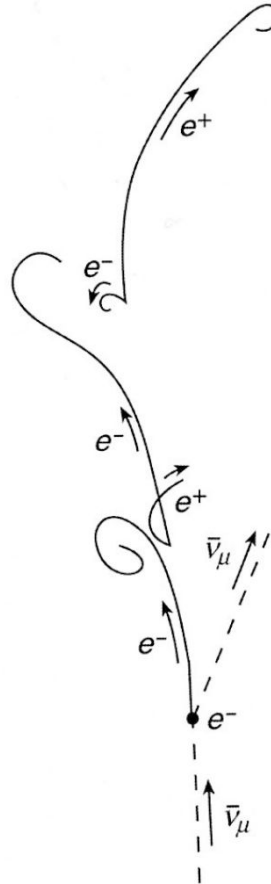
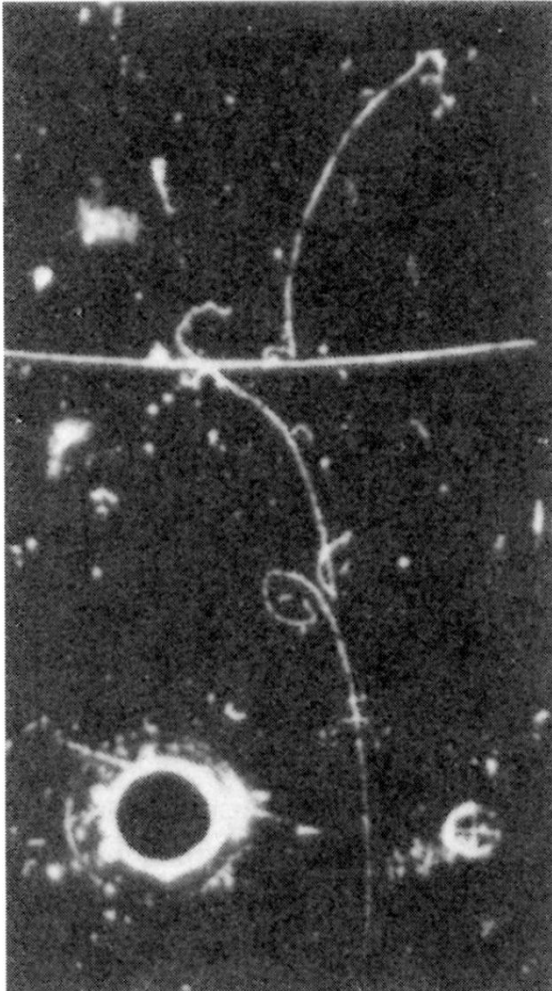
$$\mathcal{L}_C = -\frac{g}{\sqrt{2}} \left[\bar{u}_i \gamma^\mu \frac{1 - \gamma^5}{2} M_{ij}^{\text{CKM}} d_j + \bar{\nu}_i \gamma^\mu \frac{1 - \gamma^5}{2} e_i \right] W_\mu^+ + \text{h.c.},$$

Evidence of GSW validity: neutral weak interaction

- Neutral current discovered in 1973 with *Gargamelle* at CERN by observing $e\nu \rightarrow e\nu$
- Before this: no observation/indication of neutral weak currents



Evidence of GSW validity: neutral weak interaction



The first picture of a neutral weak process

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-.$$

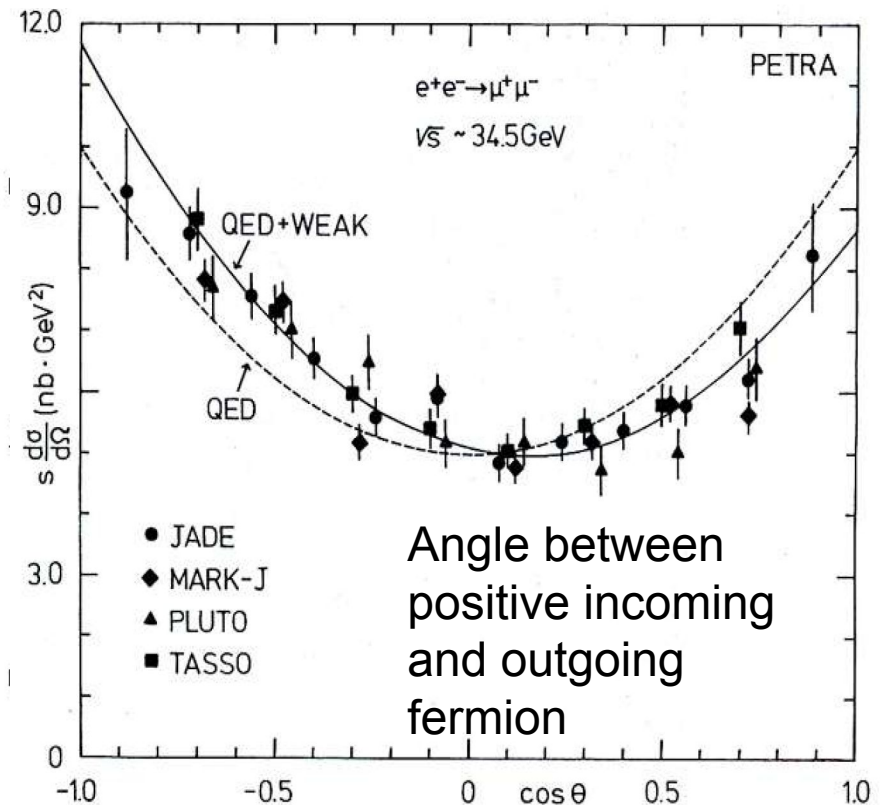
The neutrino enters from below (leaving no track), and strikes an electron, which moves upwards, emitting two photons (visible via the e^+e^- pairs from subsequent conversions)

Evidence of GSW validity: Angular relations

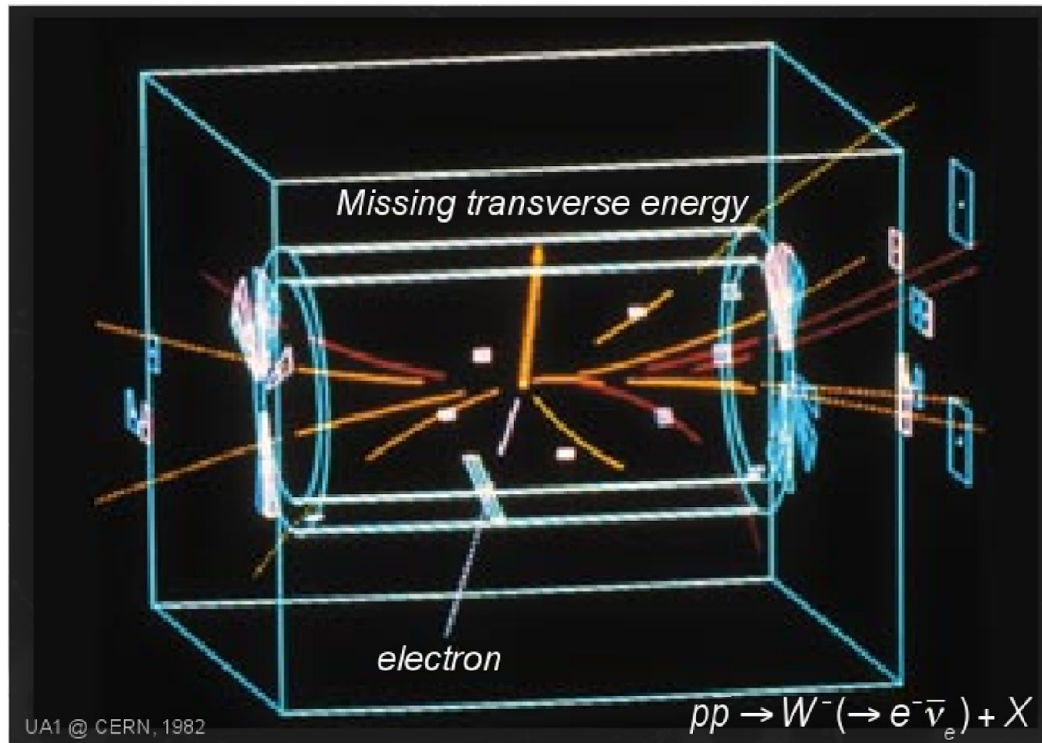
- > Angular distributions changed by electroweak interactions

$$\frac{d\sigma_0^{\text{EW}}}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \vartheta + A \cos \vartheta)$$

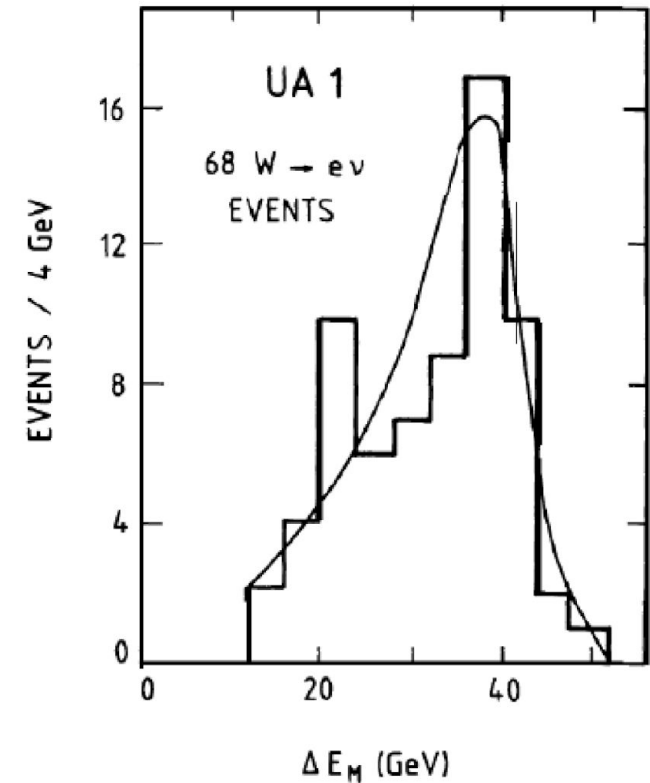
- > Total cross sections unchanged
- > Reason: V-A structure of neutral current (NC)



Evidence of GSW validity: Discovery of W boson



Missing transverse energy
in events with $E_e > 15$ GeV

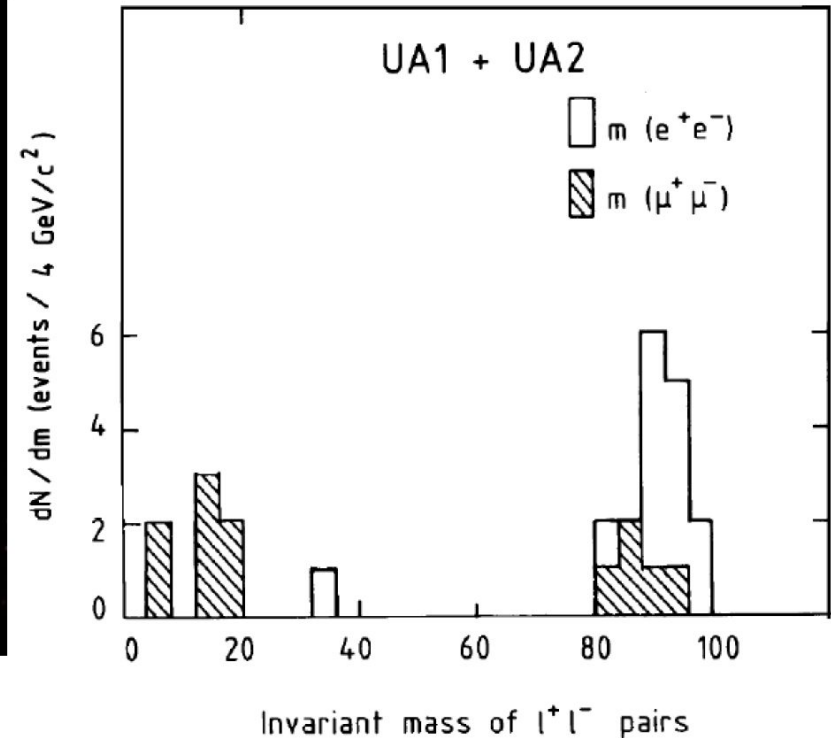
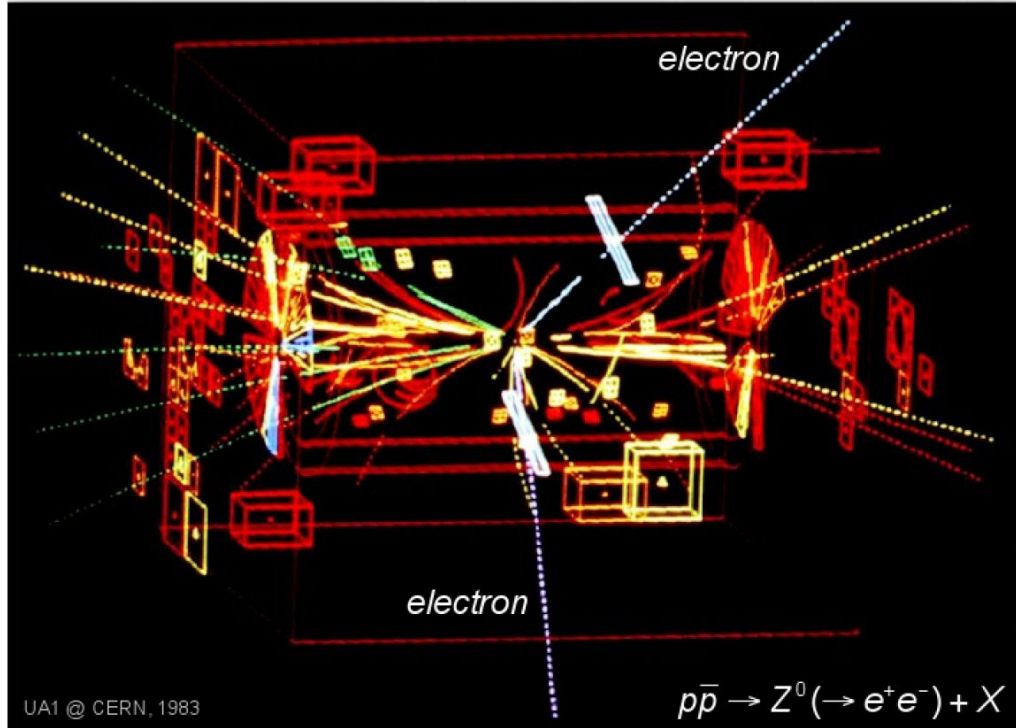


$$m_W = (80.9 \pm 1.5 \pm 2.4) \text{ GeV}$$

C. Rubbia, Nobel Lecture, 1984



Evidence of GSW validity: Discovery of Z boson

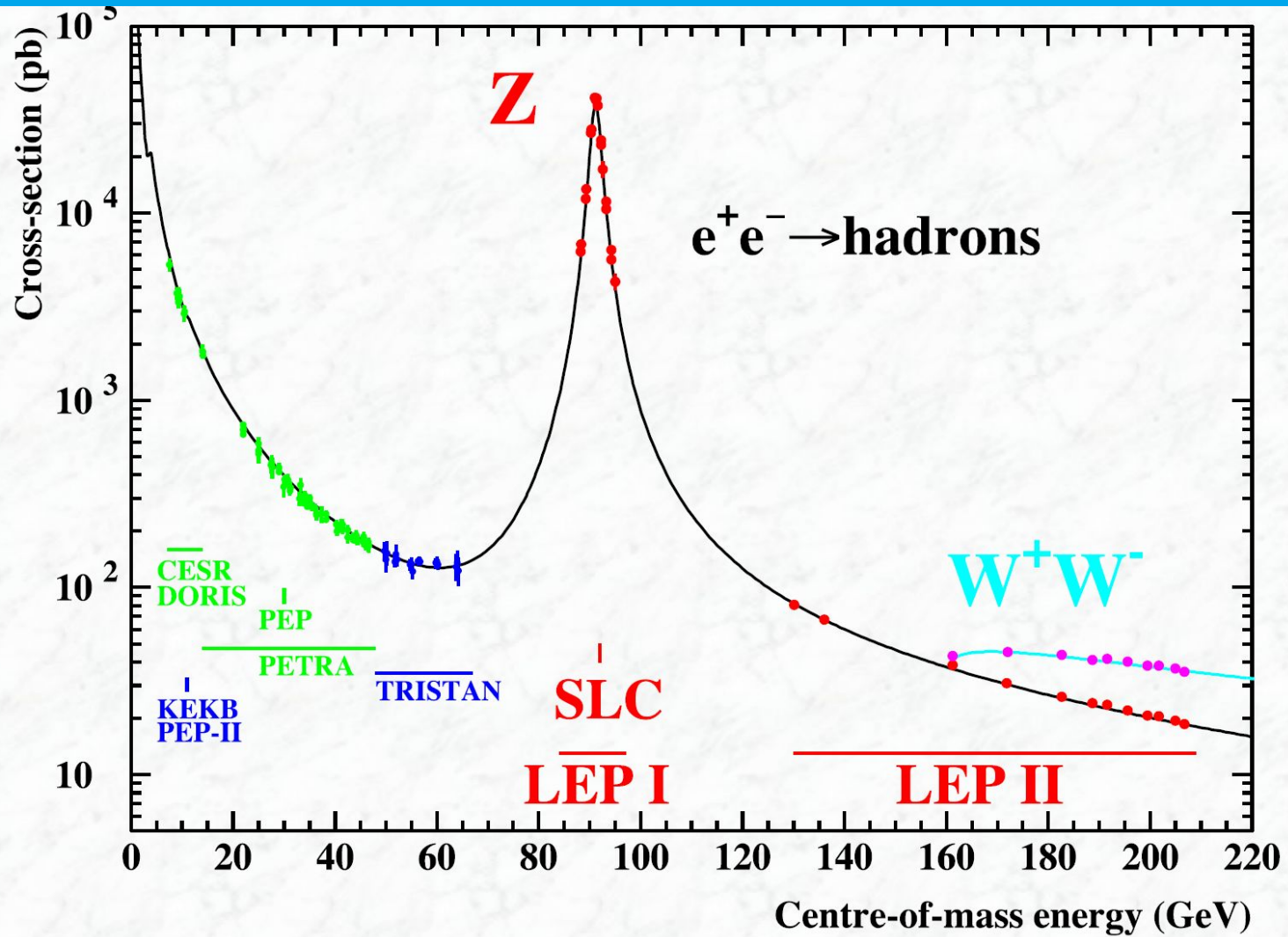


$$m_Z = (95.1 \pm 2.5) \text{ GeV}$$

- > 1983: first signals with 6 $W \rightarrow e\nu$ and 4 $Z \rightarrow ee$ events
- > 1984: Nobel prize for C. Rubbia (UA1) and S. van der Meer



EWK tests: $\sigma(e^+e^- \rightarrow W/Z)$ production at LEP

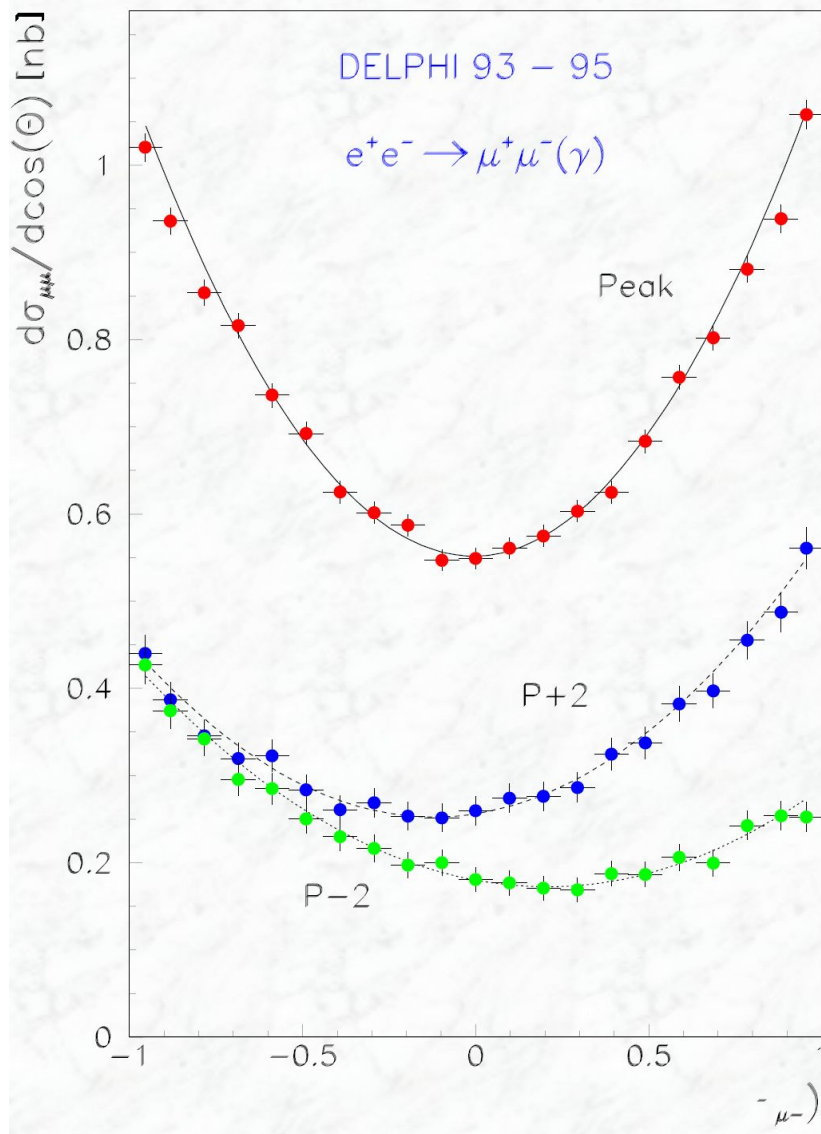


Precision tests
of the Z sector

Tests of the
W sector



LEP: Cross section of $e^+e^- \rightarrow \mu^+\mu^-$



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1 + \cos^2\theta) = (1 + \cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4 \sin^2\theta_W \cos^2\theta_W} [2g_V^e g_V^\mu (1 + \cos^2\theta) + 4g_A^e g_A^\mu \cos\theta]$$

$$F_Z(\cos\theta) = \frac{1}{16 \sin^4\theta_W \cos^4\theta_W} [(g_V^e{}^2 + g_A^e{}^2)(g_V^\mu{}^2 + g_A^\mu{}^2)(1 + \cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

Terms $\propto \cos\theta$ in $d\sigma/d\cos\theta$
 \rightarrow asymmetry

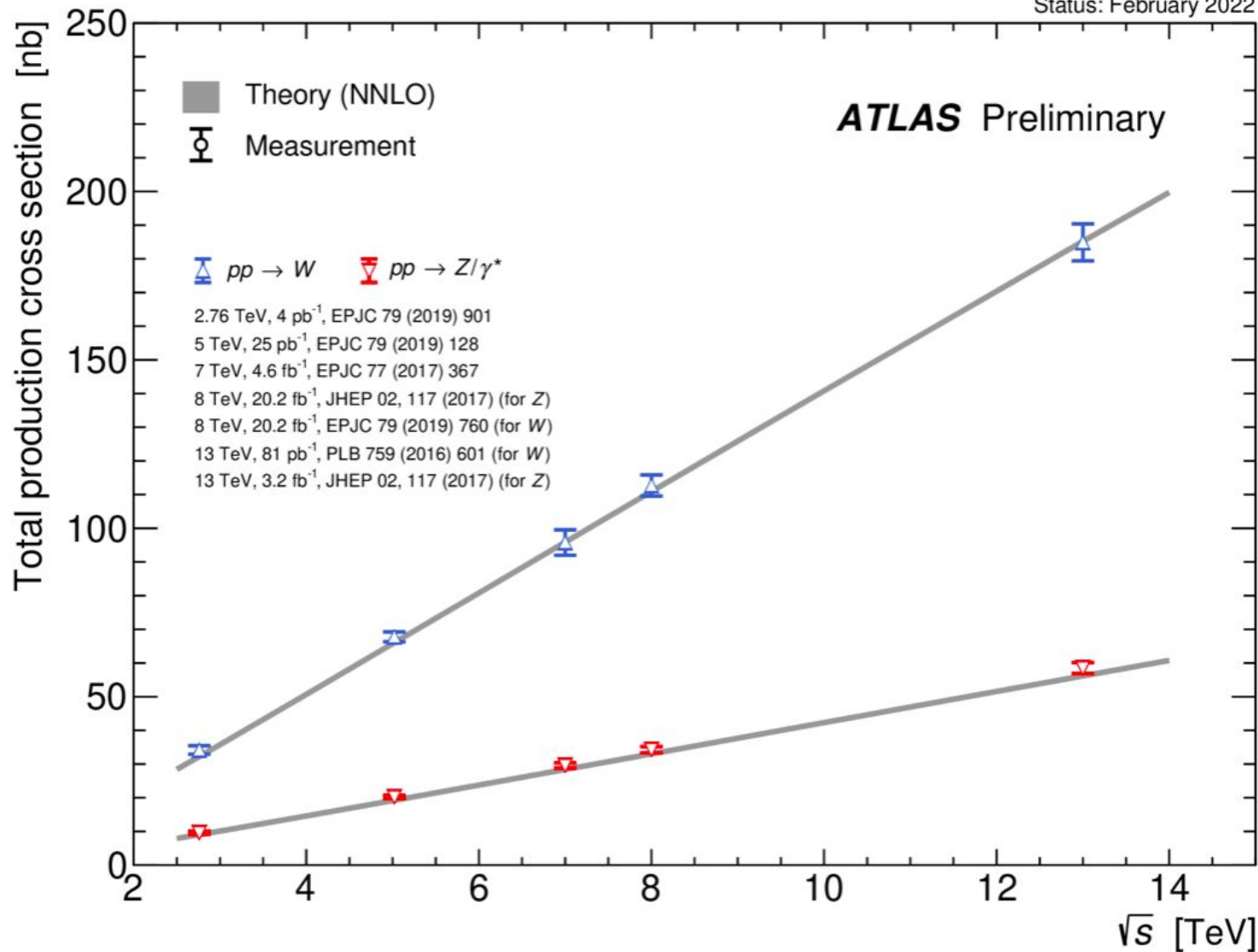
$$\sigma_{F(B)} = \int_{0(-1)}^{1(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



EWK tests: $\sigma(pp \rightarrow W/Z)$ production at LHC

Status: February 2022



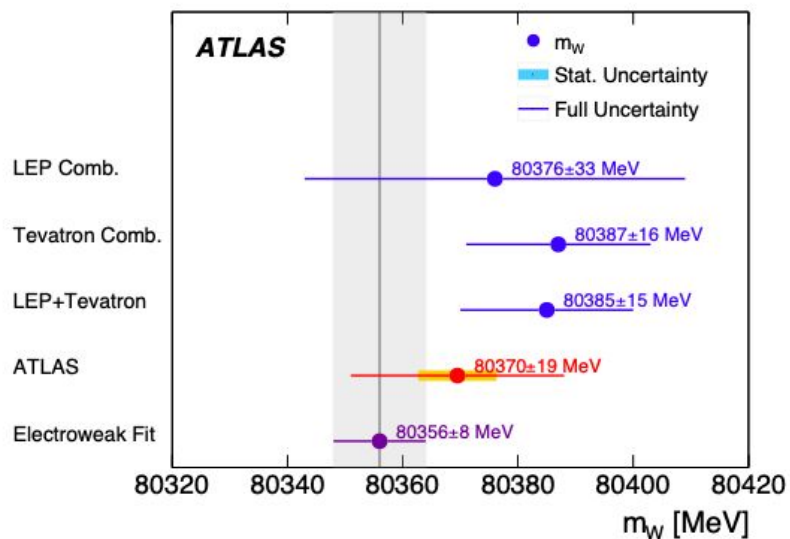
Consistent picture of electroweak parameters

LEP

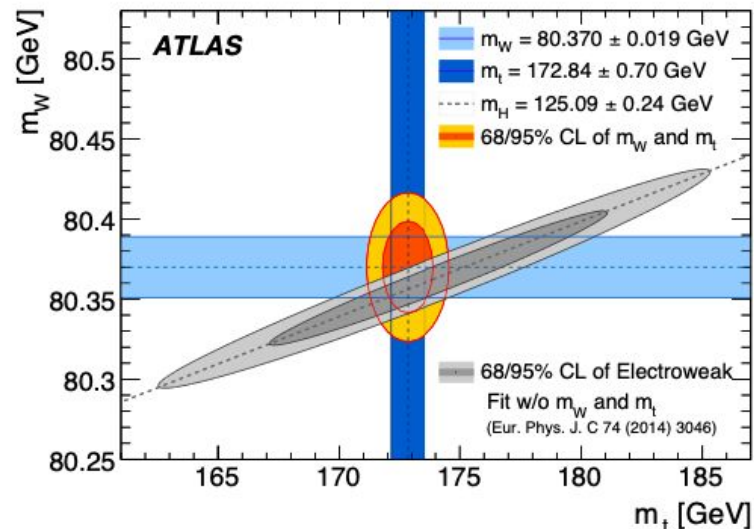


March 2012

ATLAS W-mass measurement



m_W - m_t - m_H dependence



Higgs boson mechanism

But why was the Higgs predicted?

- > **Problem with electroweak unification:**
- > Gauge invariance implied mass less gauge bosons and fermions
 - Mass terms are not allowed for gauge bosons

Lagrangian of the QED (U(1))

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

+ mass term for gauge bosons

$$+ \frac{1}{2}m^2 A_\mu A^\mu$$

Mass term + gauge transform

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x)$$

$$+ \frac{1}{2}m^2 A_\mu A^\mu \rightarrow + \frac{1}{2}m^2 A_\mu A^\mu + m^2 A_\mu \partial^\mu \Lambda + \frac{1}{2}m^2 \partial_\mu \Lambda \partial^\mu \Lambda$$

Breaking of gauge symmetry → Forbidden mass terms



How can we get massive gauge bosons → BEH mechanism

We know weak interaction must have massive gauge bosons →

Spontaneous symmetry breaking

Symmetry is formally kept in the Lagrangian, but in reality, the ground state of the theory doesn't preserve the symmetry.

- > Add scalar field with a particular potential
- > If $\mu^2 > 0$, potential follows the “Mexican” hat form
 - Minimum of potential is not for $\langle \Phi \rangle = 0$ but for:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

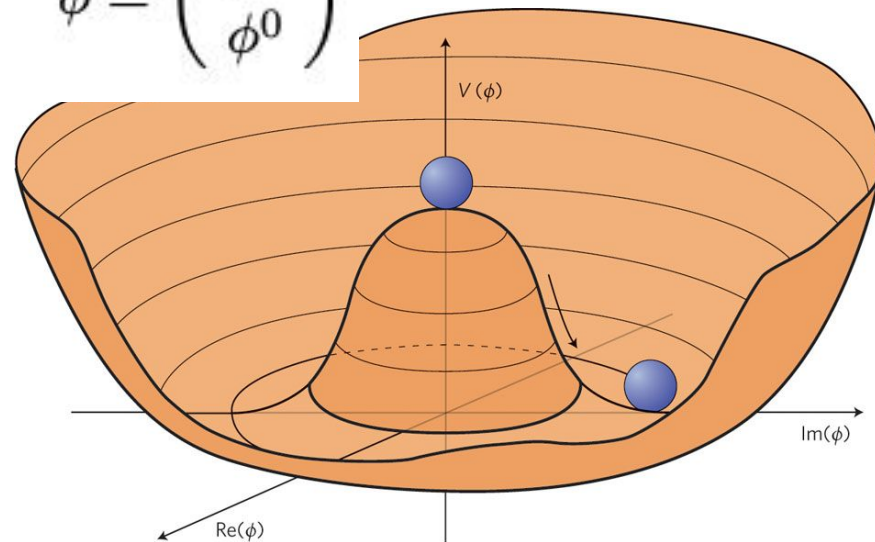
v = Vacuum expectation value (VEV)

- > Ground state of field, a certain value with $v = \mu^2/\lambda$
- > Excitations around VEV. Quanta of the field → Higgs bosons

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$



Brout-Englert-Higgs mechanism → Higgs boson

➤ When spontaneously breaking the GSW $SU(2)_L \times U(1)_Y$, got a residual symmetry $U(1)_Q \rightarrow$ Associated to QED

- 3 massive bosons (electroweak bosons)
- Massive scalar (Higgs)
- Electric charge as function of weak isospin and hypercharge.

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

$$Q = T_3 + \frac{1}{2} Y_W$$

Mass terms for gauge bosons

$$\begin{aligned} (D^\mu \phi)^\dagger (D_\mu \phi) &= \left| \left(\partial_\mu + \frac{i}{2} g \tau^k W_\mu^k + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left| \left(g \tau^k W_\mu^k + g' B_\mu \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left| \begin{pmatrix} g W_\mu^1 - i g W_\mu^2 \\ -g W_\mu^3 + g' B_\mu \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g W_\mu^3 - g' B_\mu)^2 \right]. \end{aligned}$$

Mass term for fermions

$$\mathcal{L}_{Yuk} = f_e \bar{l}_L \phi e_R + f_u \bar{q}_L \tilde{\phi} u_R + f_d \bar{q}_L \phi d_R + h.c.$$

$$\mathcal{L}_{Yuk} = \frac{f_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) + \frac{f_u v}{\sqrt{2}} (\bar{u}_L u_R + \bar{u}_R u_L) + \frac{f_d v}{\sqrt{2}} (\bar{d}_L d_R + \bar{d}_R d_L)$$



Where is it ? The Standard Model's biggest triumph

Even before the direct discovery, indirect constraints on Higgs mass through connections with W and top

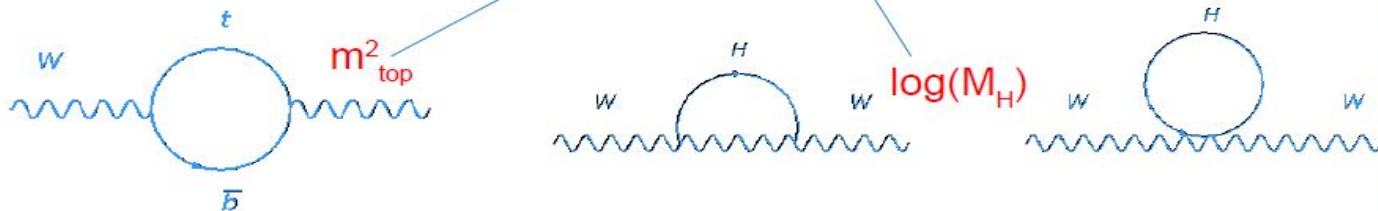
$$m_i = -\frac{f_i v}{\sqrt{2}}, \quad i = e, u, d$$

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

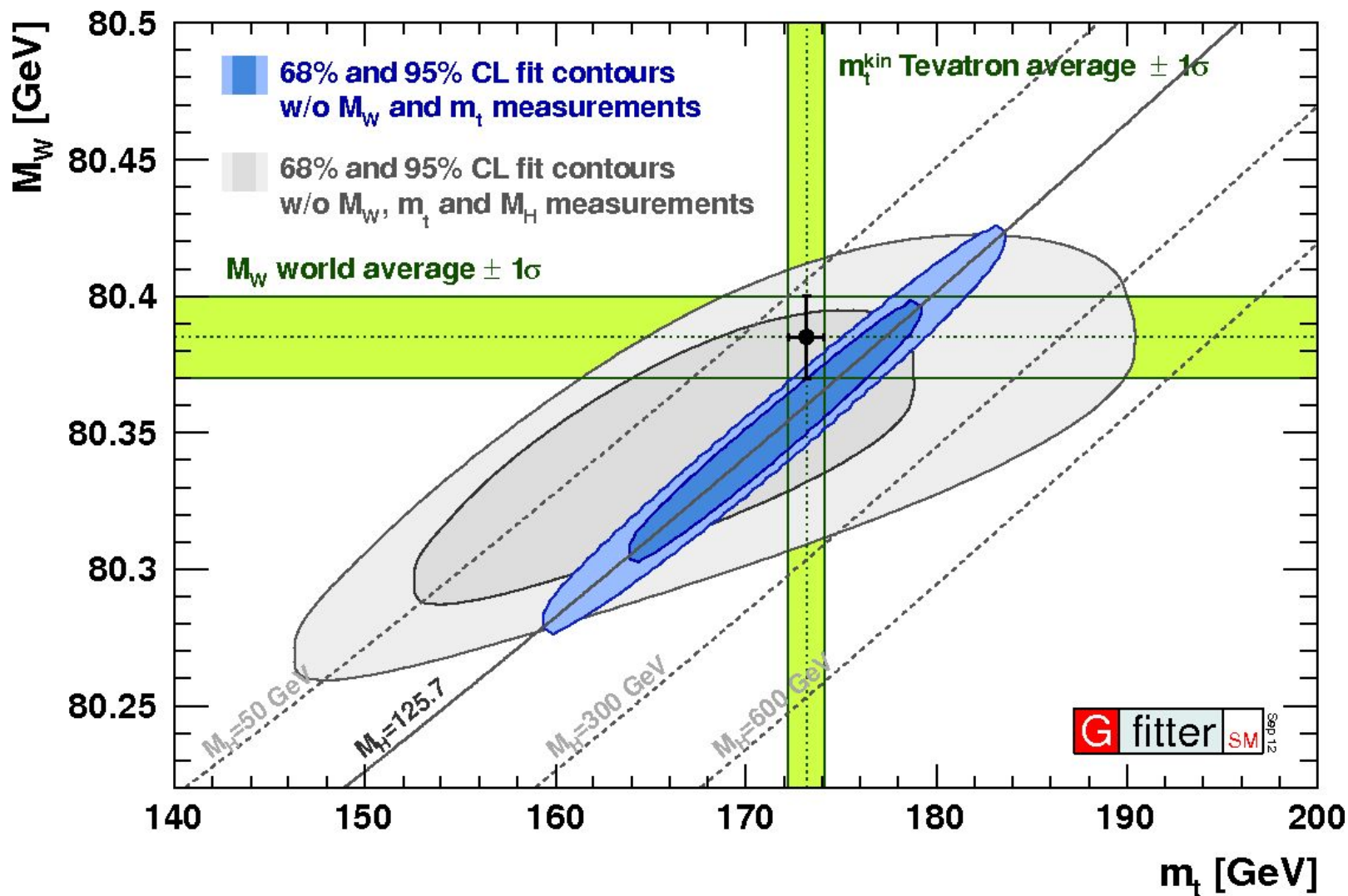
$$m_W = \frac{g v}{2}$$

$$m_Z = \frac{m_W}{\cos \theta_W}$$

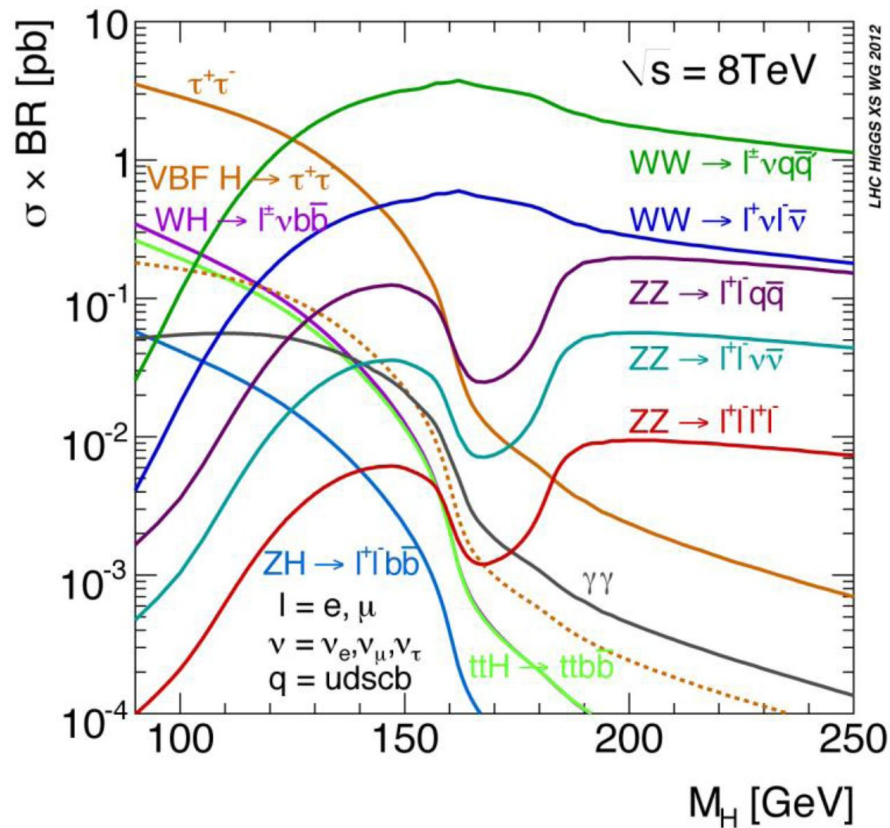
$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F} \right)^{1/2} \frac{1}{\sin \theta_W \sqrt{1 - \Delta r}} \quad \text{radiative corrections}$$



LEP: Quantum corrections and the Higgs



Higgs



Low mass ($\lesssim 140$ GeV)

$H \rightarrow \gamma\gamma$

Rare decay, but distinct signal

$H \rightarrow \tau\tau$

Enhanced in MSSM, also contributes to SM search

$H \rightarrow b\bar{b}$

Main search channel at LEP and Tevatron, important to study Higgs properties

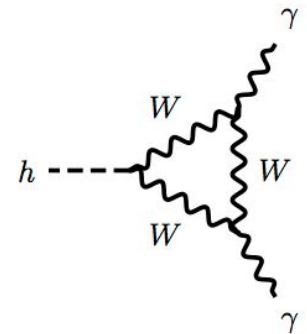
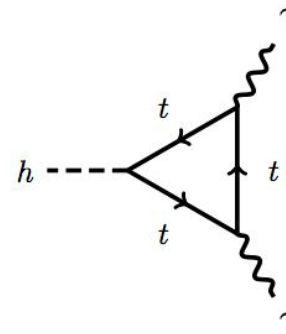
Intermediate and large m_H ($\gtrsim 130$ GeV)

$H \rightarrow WW$

Large signal yield

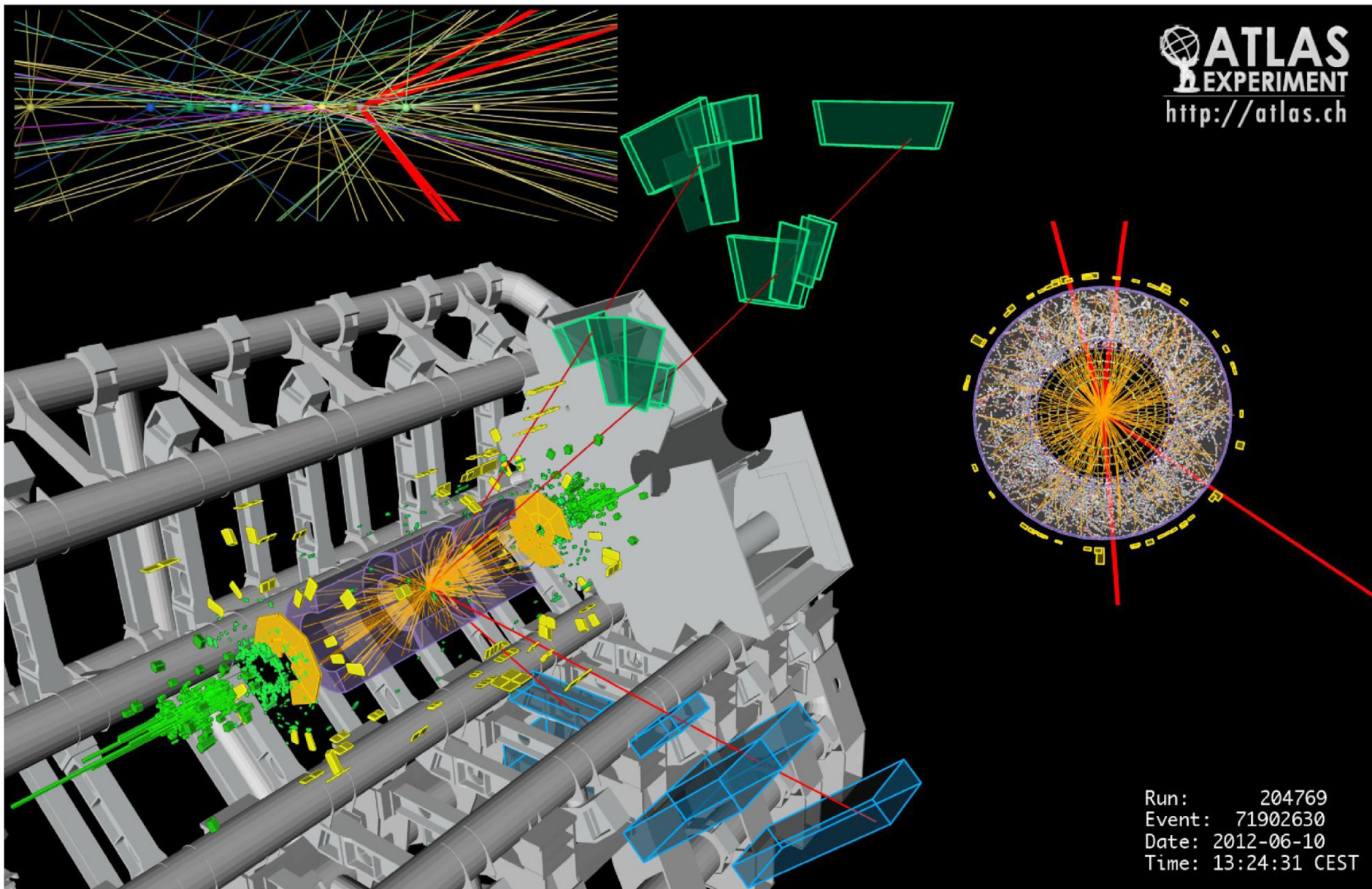
$H \rightarrow ZZ$

Very clean signal if both $Z \rightarrow ll$



Higgs

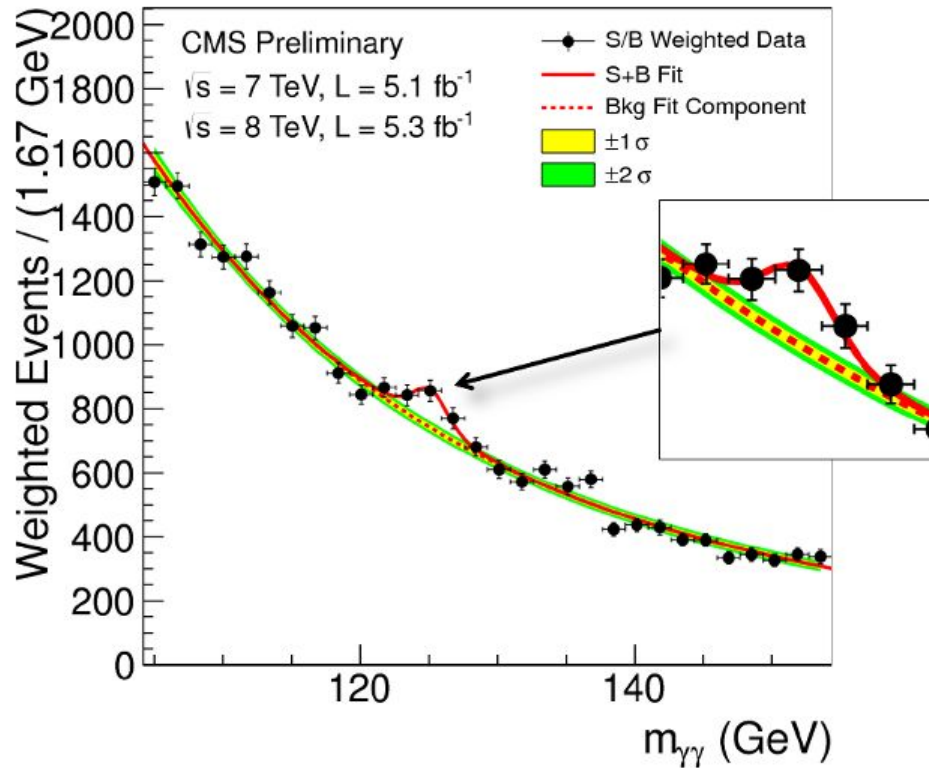
 **ATLAS**
EXPERIMENT
<http://atlas.ch>



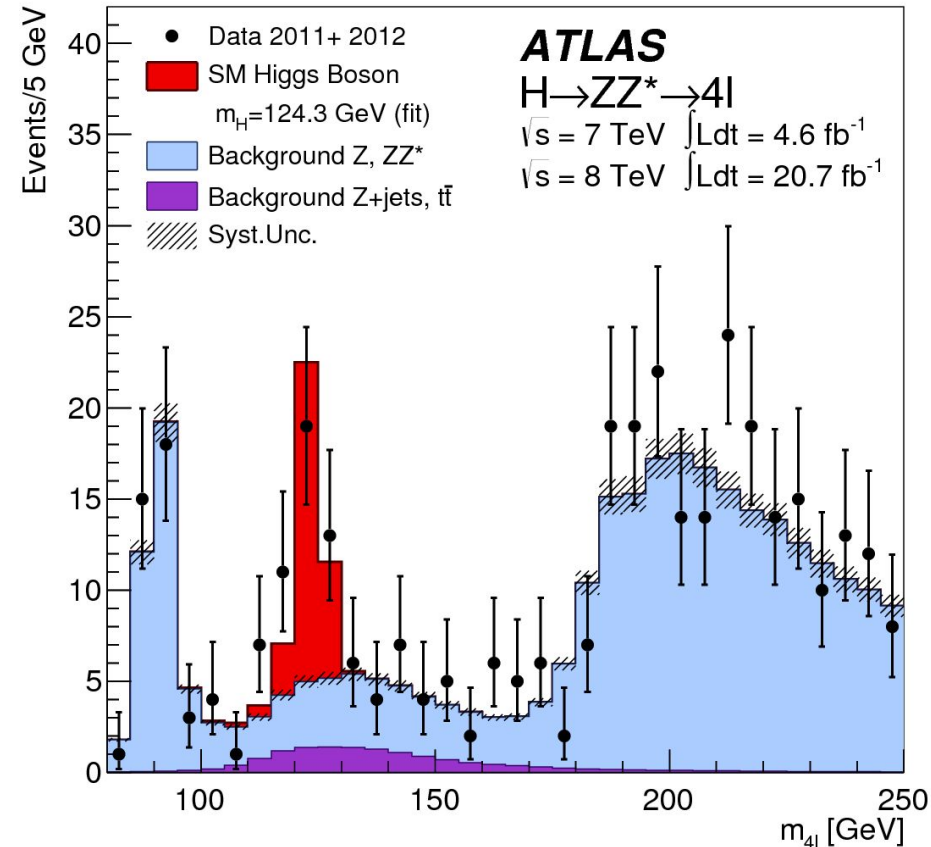
Run: 204769
Event: 71902630
Date: 2012-06-10
Time: 13:24:31 CEST

Higgs

$H \rightarrow \gamma\gamma$



$H \rightarrow ZZ$



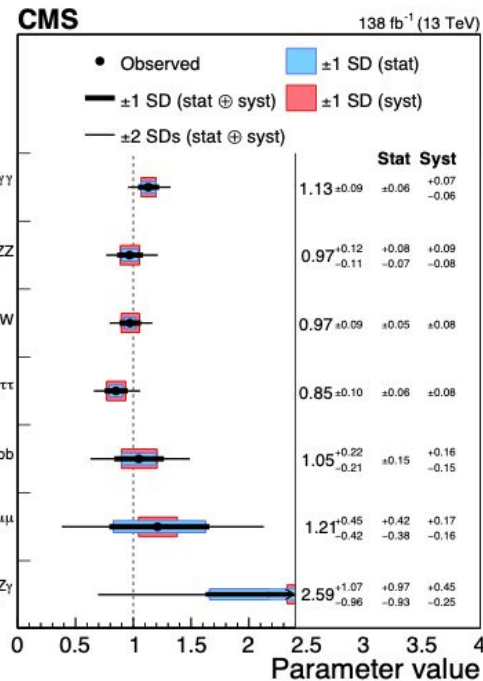
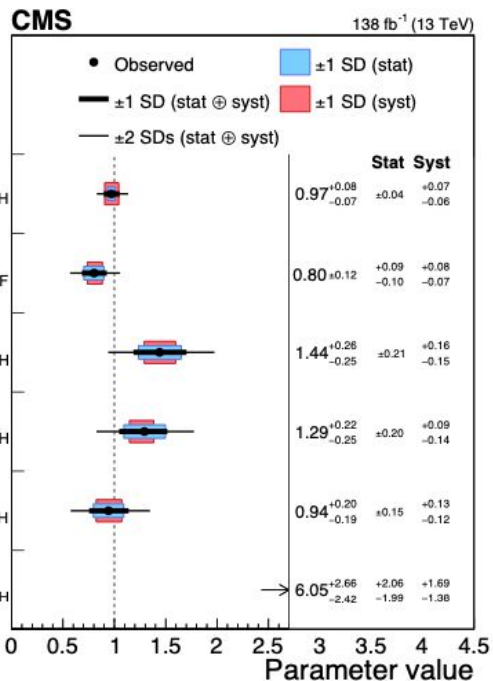
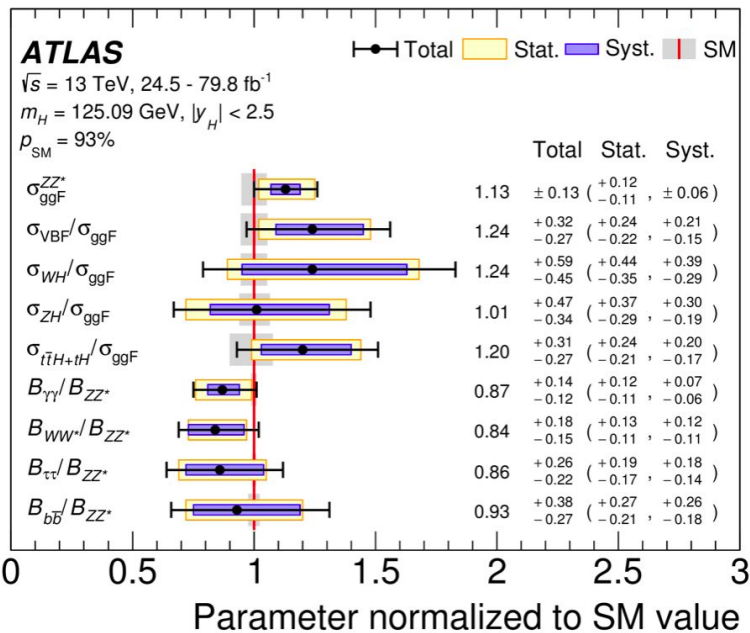
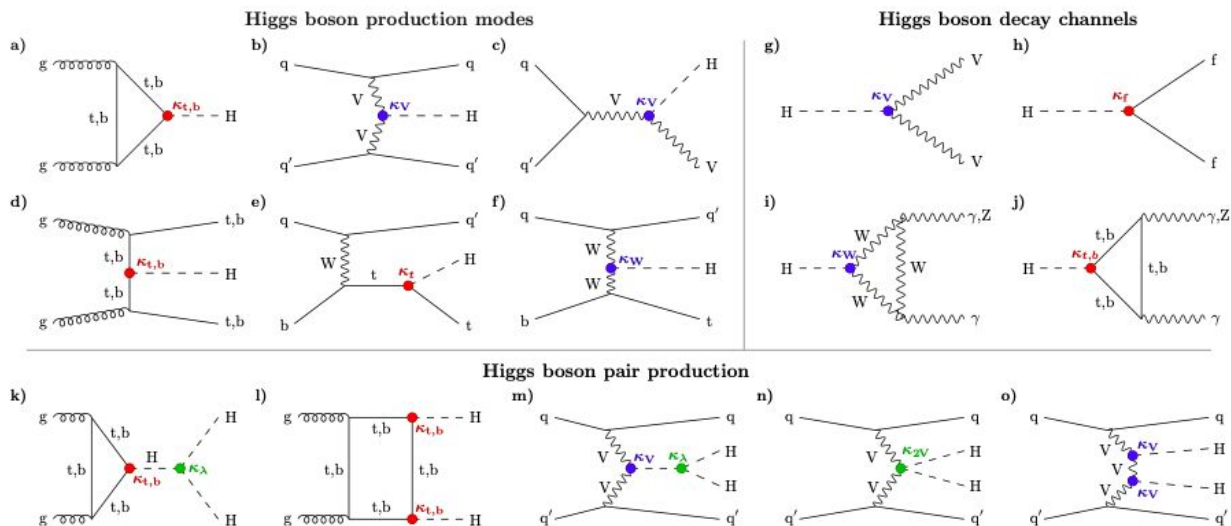
LHC is running at 13 TeV since 2015 → much bigger sample will be corrected in the next year → precision Higgs physics, possibility to discover the open $t\bar{t}H$ coupling



The last missing piece in the Standard Model



The last missing piece in the Standard Model



Beyond Standard Model

The Standard Model: Free parameters

The standard model establish relations between different parameters

However, some of its parameters cannot be known a priori → Experiments

- Particle masses
- CKM parameters
- Gauge couplings at a given energy: strength of forces
- CP properties of QCD
- Parameters of electroweak symmetry breaking: v and m_H

Parameters of the Standard Model [hide]				
#	Symbol	Description	Renormalization scheme (point)	Value
1	m_e	Electron mass		0.511 MeV
2	m_μ	Muon mass		105.7 MeV
3	m_τ	Tau mass		1.78 GeV
4	m_u	Up quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	1.9 MeV
5	m_d	Down quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	4.4 MeV
6	m_s	Strange quark mass	$\mu_{\overline{MS}} = 2 \text{ GeV}$	87 MeV
7	m_c	Charm quark mass	$\mu_{\overline{MS}} = m_c$	1.32 GeV
8	m_b	Bottom quark mass	$\mu_{\overline{MS}} = m_b$	4.24 GeV
9	m_t	Top quark mass	On shell scheme	173.5 GeV
10	θ_{12}	CKM 12-mixing angle		13.1°

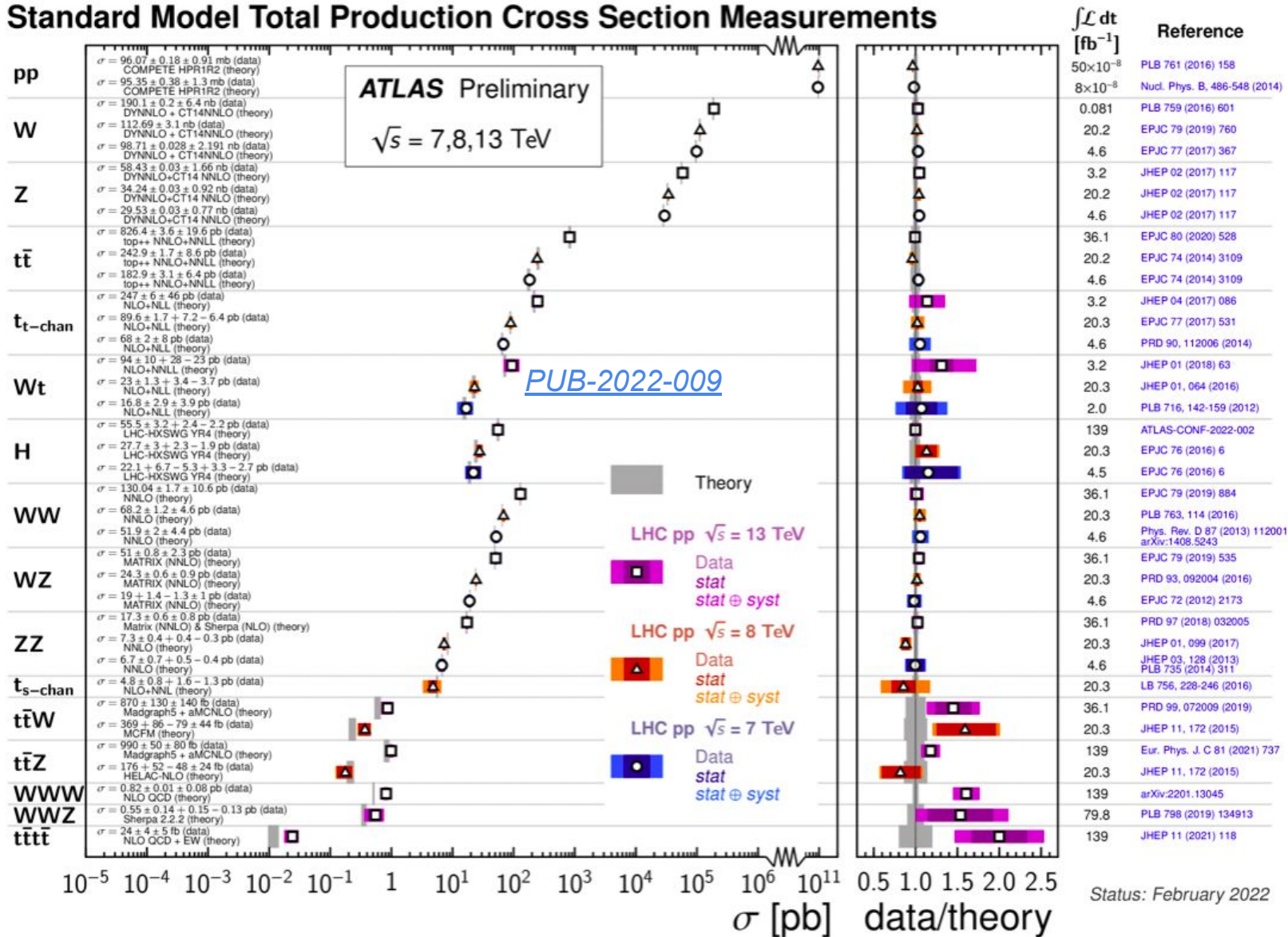
11	θ_{23}	CKM 23-mixing angle		2.4°
12	θ_{13}	CKM 13-mixing angle		0.2°
13	δ	CKM CP violation Phase		0.995
14	g_1 or g'	U(1) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.357
15	g_2 or g	SU(2) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.652
16	g_3 or g_s	SU(3) gauge coupling	$\mu_{\overline{MS}} = m_Z$	1.221
17	θ_{QCD}	QCD vacuum angle		~0
18	v	Higgs vacuum expectation value		246 GeV
19	m_H	Higgs mass		125.09 ± 0.24 GeV



The Standard Model: Extremely predictive

Once parameters are known, everything else is “fixed”

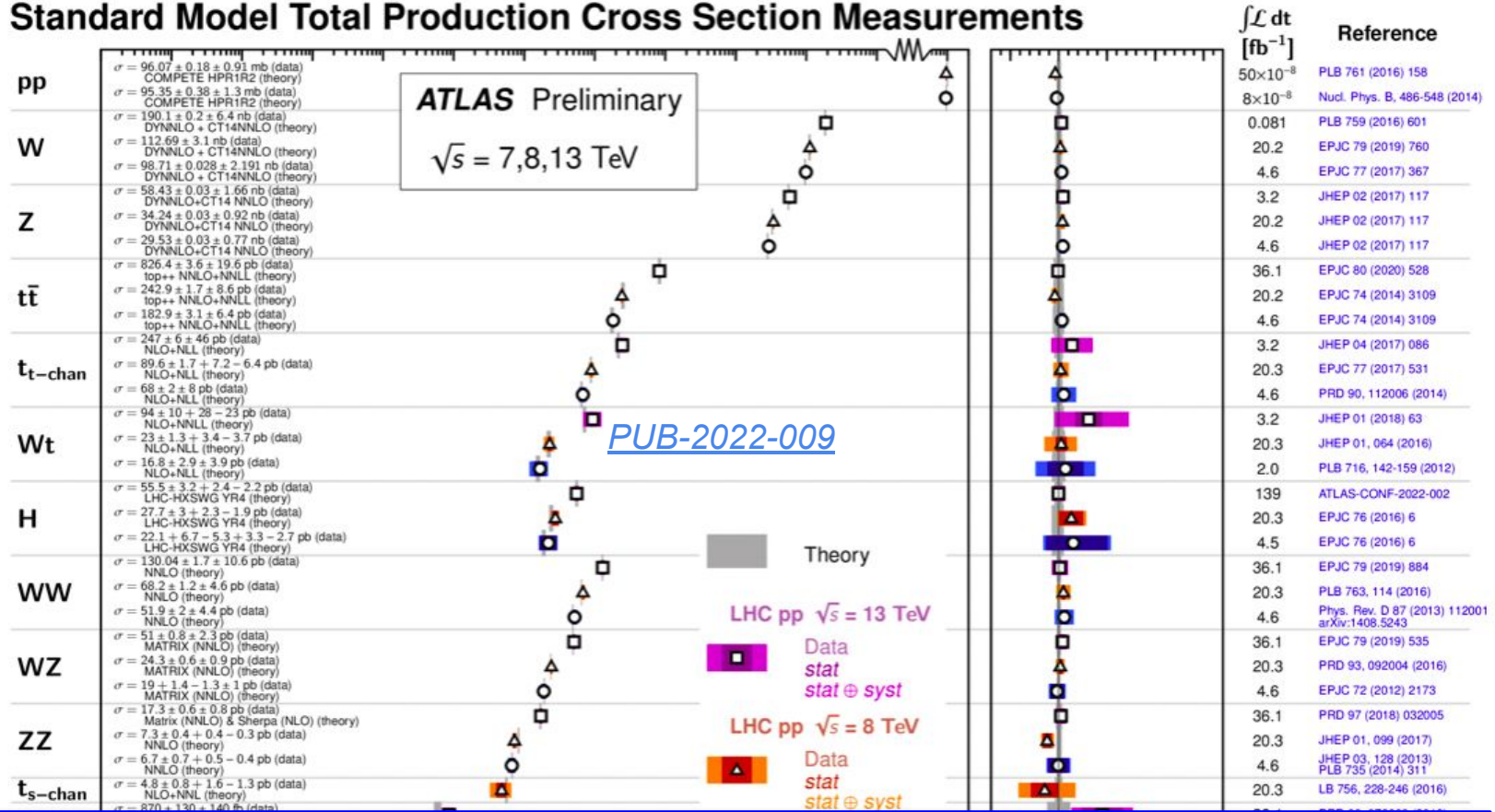
Standard Model Total Production Cross Section Measurements



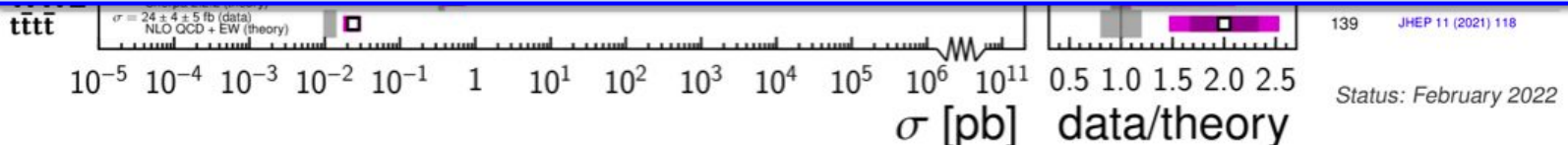
The Standard Model: Extremely predictive

Once parameters are known, everything else is “fixed”

Standard Model Total Production Cross Section Measurements



More and better in the LHC Physics talks by Evgeniya Cheremushkina !



What is missing ? Beyond Standard Model Physics

Standard Model of Elementary Particles

		three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)	
		I	II	III	I	II	III		
mass		=2.2 MeV/c ²	=1.28 GeV/c ²	=173.1 GeV/c ²	=2.2 MeV/c ²	=1.28 GeV/c ²	=173.1 GeV/c ²	0	=124.97 GeV/c ²
charge		$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
spin		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		u up	c charm	t top	\bar{u} antiup	\bar{c} anticharm	\bar{t} antitop	g gluon	H higgs
	QUARKS	d down	s strange	b bottom	\bar{d} antidown	\bar{s} antistrange	\bar{b} antibottom	γ photon	GAUGE BOSONS VECTOR BOSONS
		e electron	μ muon	τ tau	e⁺ positron	$\bar{\mu}$ antimuon	$\bar{\tau}$ antitau	Z Z ⁰ boson	
	LEPTONS	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	$\bar{\nu}_e$ electron antineutrino	$\bar{\nu}_\mu$ muon antineutrino	$\bar{\nu}_\tau$ tau antineutrino	W⁺ W ⁺ boson	W⁻ W ⁻ boson
		W⁺ W ⁺ boson	W⁻ W ⁻ boson					SCALAR BOSONS	

Is the SM complete ?

Presented the SM during the last days. Very successful and predictive theory but

- We know gravity to be one interaction of nature. Why is it not included ?
- Naturalness problem
- Hierarchy problem
- Matter-antimatter asymmetry

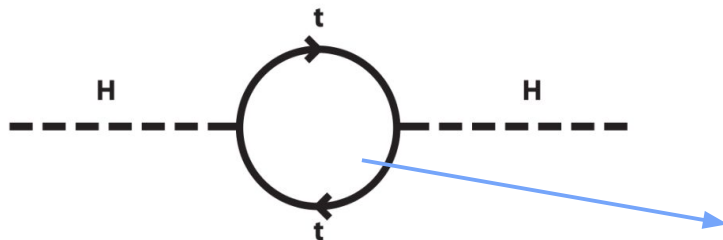
Naturalness problem (some might call it hierarchy)

Mass corrections to the Higgs

As we have seen, some Feynman diagrams might diverge and renormalization of couplings and masses helps to remove these divergences.

- > Correction to the Higgs mass include loops with creation of fermions.
- > Cannot absorb this correction. Dependent on cut-off Λ .

Radiative correction to Higgs mass very large, if no other new physics of mass Λ



$$\Delta m_h^2 \sim \frac{3}{4\pi^2} \left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8 \cos^2 \theta_W} + \lambda \right) \Lambda^2$$

$$M < \left(\frac{10\%}{\text{tuning}} \right) 1 \text{ TeV}$$

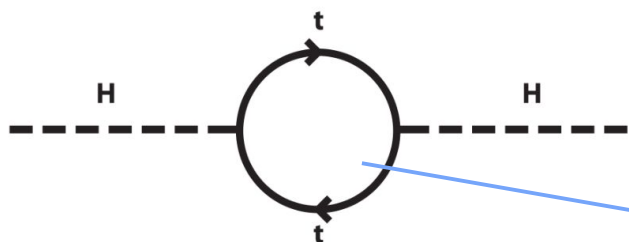
Naturalness problem (some might call it hierarchy)

Mass corrections to the Higgs

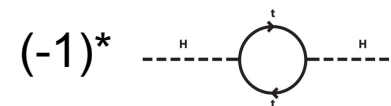
As we have seen, some Feynman diagrams might diverge and renormalization of couplings and masses helps to remove these divergences.

- > Correction to the Higgs mass include loops with creation of fermions.
- > Cannot absorb this correction. Dependent on cut-off Λ .
- > Very typical new theory to solve Naturalness problem : Supersymmetry !

Radiative correction to Higgs mass very large, if no other new physics of mass Λ



$$\Delta m_h^2 \sim \frac{3}{4\pi^2} \left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8 \cos^2 \theta_W} + \lambda \right) \Lambda^2$$



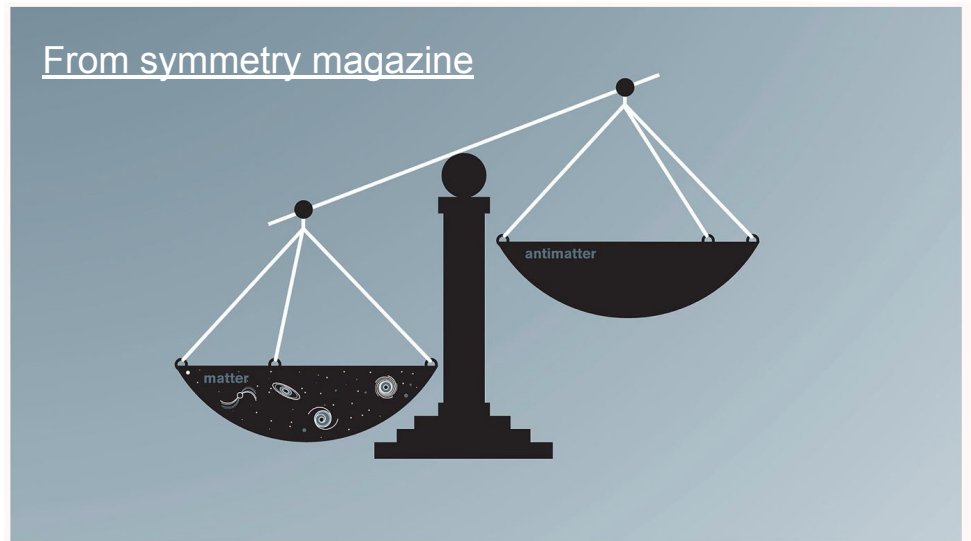
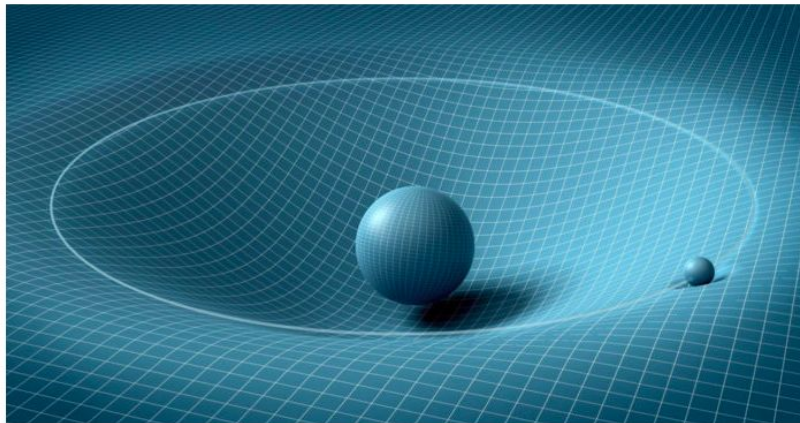
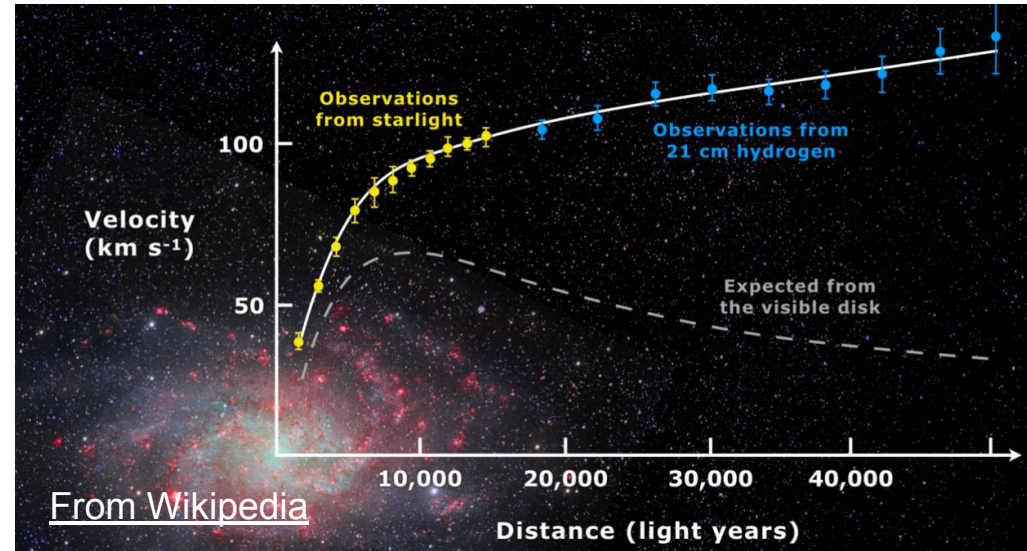
A super-partner of the top (boson) would generate same correction but with negative value \rightarrow Cancellation

Gravity, dark matter, matter-antimatter asymmetry,

More missing pieces

- Gravity: non-renormalizable theory
- Dark Matter: no candidate particle in SM
- We live in a matter dominated Universe. CP violation in EWK and CKM/PMNS cannot explain it. Why ?
- Strong CP problem

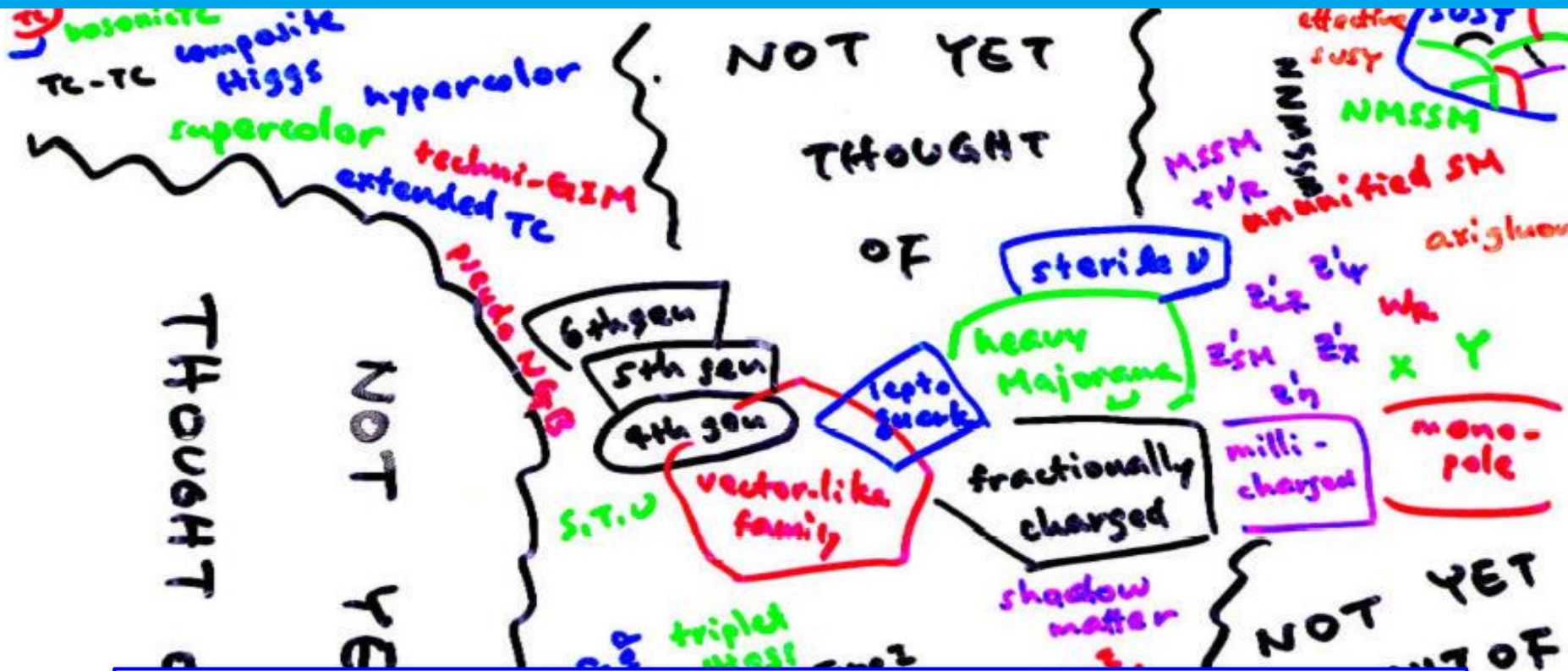
And many more missing pieces !



So what else is out there?



So what else is out there?



More and better in the BSM talks in September by Marco Rimoldi !

Backup

Handed-ness and hadronic decays: The Pion

$$\boxed{\pi^0}$$

$$\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2} \quad I^G(J^{PC}) = 1^-(0^{-+})$$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)

$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV

→ Mean life $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s (S = 1.2)

$c\tau = 25.5$ nm

π^0 DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
→ 2γ	(98.823 ± 0.034) %	S=1.5	67
$e^+e^- \nu$	(1.174 ± 0.035) %	S=1.5	67

$$\boxed{\pi^\pm}$$

$$\pi^+ = u\bar{d}, \pi^- = \bar{u}d \quad I^G(J^P) = 1^-(0^-)$$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)

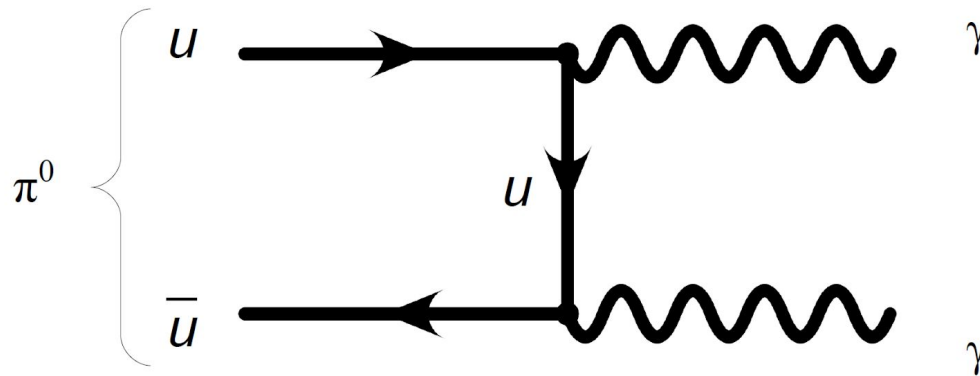
→ Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)

$c\tau = 7.8045$ m

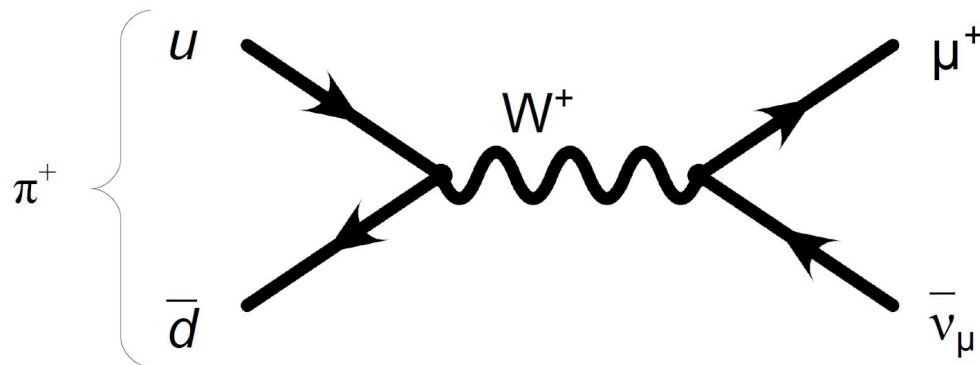
π^\pm DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
→ $\mu^+ \nu_\mu$	[b] (99.98770 ± 0.00004) %		30
$e^+e^- \nu$	[c] (2.00 ± 0.25) × 10 ⁻⁴		30



Handed-ness and hadronic decays: The Pion



electromagnetic
interaction



weak
interaction

Handed-ness and hadronic decays: The Pion



$$I^G(J^P) = 1^-(0^-)$$

$$\text{Mass } m = 139.57018 \pm 0.00035 \text{ MeV} \quad (S = 1.2)$$

$$\text{Mean life } \tau = (2.6033 \pm 0.0005) \times 10^{-8} \text{ s} \quad (S = 1.2)$$

$$c\tau = 7.8045 \text{ m}$$

⋮

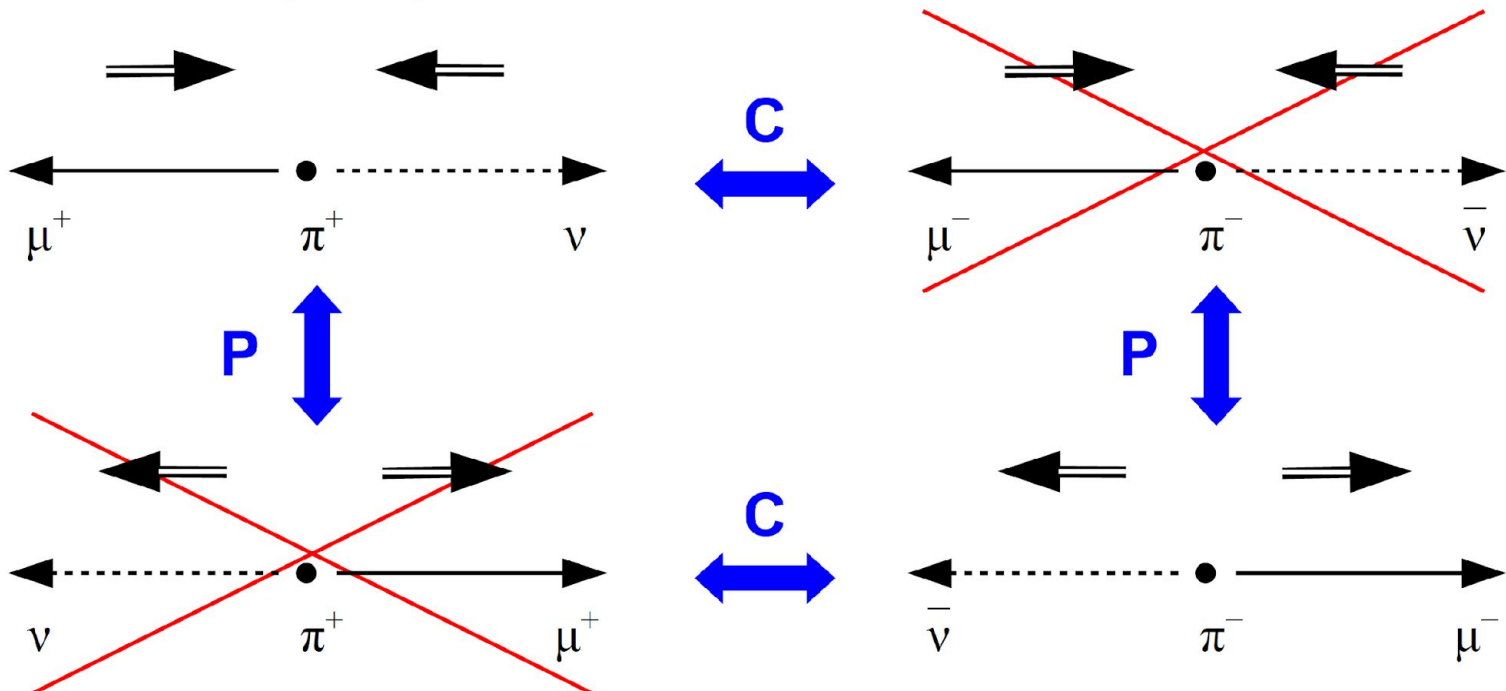
π^+ DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	P (MeV/c)
$\mu^+ \nu_\mu$	[b] $(99.98770 \pm 0.00004) \%$		30
$\mu^+ \nu_\mu \gamma$	[c] $(2.00 \pm 0.25) \times 10^{-4}$		30
$e^+ \nu_e$	[b] $(1.230 \pm 0.004) \times 10^{-4}$		70
$e^+ \nu_e \gamma$	[c] $(7.39 \pm 0.05) \times 10^{-7}$		70
$e^+ \nu_e \pi^0$	[c] $(1.036 \pm 0.006) \times 10^{-8}$		4
⋮			

why is the decay to muon and neutrino so much more likely than the decay to electron and neutrino, although the muon is much heavier than the electron?



Handed-ness and hadronic decays: The Pion

- neutrino is left-handed, π has spin 0
 \Rightarrow charged lepton also has to be left-handed, which is the “wrong” spin
- the heavier the charged lepton, the less suppressed is the wrong helicity, proportional to $(1-v/c)$



- left-handedness of neutrinos also means that weak interaction violates C, but CP can be conserved (and indeed CP violation is much smaller)

LEP: Cross section of $e^+e^- \rightarrow \mu^+\mu^-$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta) \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + F_Z(\cos\theta) \frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

γ

γ/Z interference

Z

vanishes at $\sqrt{s} \approx M_Z$

$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1 + \cos^2\theta) = (1 + \cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4 \sin^2\theta_W \cos^2\theta_W} [2g_V^e g_V^\mu (1 + \cos^2\theta) + 4g_A^e g_A^\mu \cos\theta]$$

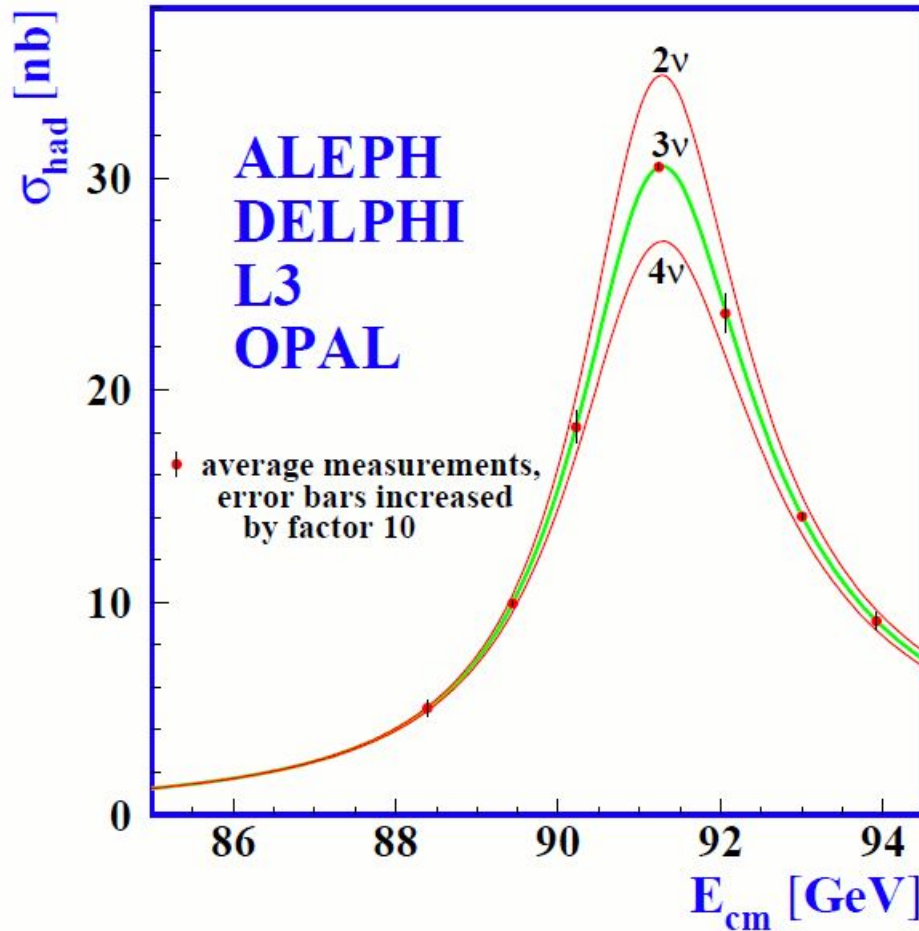
$$F_Z(\cos\theta) = \frac{1}{16 \sin^4\theta_W \cos^4\theta_W} [(g_V^e)^2 + (g_A^e)^2] (g_V^\mu)^2 + (g_A^\mu)^2 (1 + \cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

$\alpha = \alpha(m_Z)$: running electromagnetic coupling [$\alpha(m_Z) = \alpha / (1 - \Delta\alpha)$ with $\Delta\alpha \approx 0.06$]

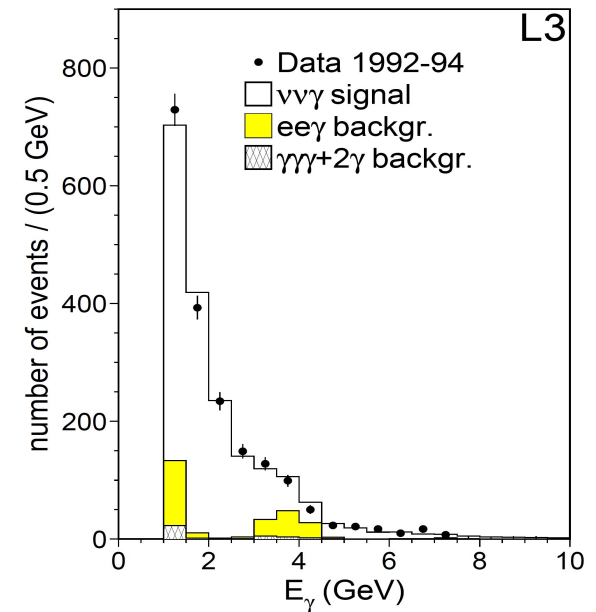
$g_V, g_A = c_V, c_A$: effective coupling constants (vector and axial vector)



LEP: Number of light neutrinos



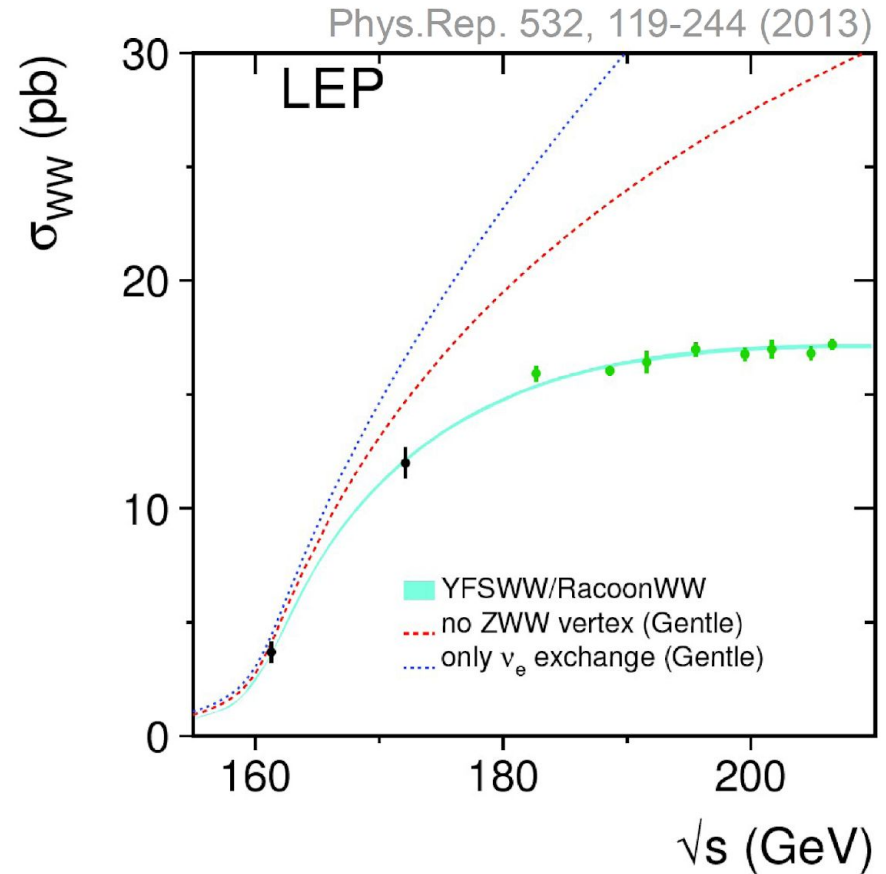
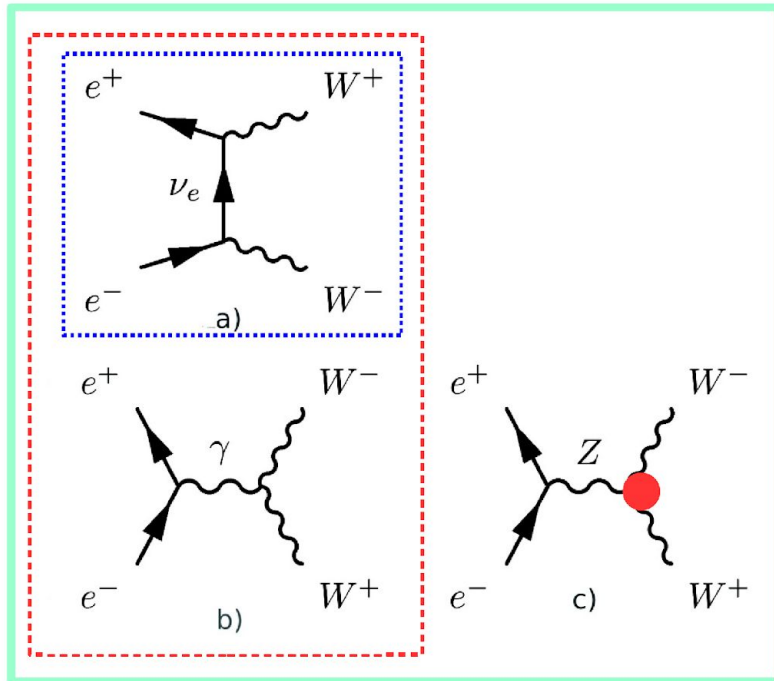
➤ Data selected using invisible Z decays with photon radiation



➤ Width of Z boson depends of the number of decay channels

LEP: WW production and the TGC vertex

- LEP also proved self-interaction of weak bosons through indirect measurement of triple gauge coupling vertex
- Interference between all three diagrams leads to “safe” energy behavior



Gauge invariance and Noether's theorem

Noether's theorem (informal version):

If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time.

quantity	interaction			invariance
	strong	elm.	weak	
energy	yes	yes	yes	translation in time
momentum	yes	yes	yes	translation in space
angular momentum	yes	yes	yes	rotation in space

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0,$$

As in classical mechanics, equations of motions (e.g. Dirac equation) can be obtained from Lagrangian (or rather: can construct Lagrangian to further analyse equations of motions)



Gauge invariance and Noether's theorem

- > Invariance of field equations describing electromagnetic and weak interaction (→ Lagrangian)
- > Gauge (phase) transition possible (without changing effect of interaction) for electron (→ U(1) symmetry: $U(A)U(B) = U(B)U(A)$):

>

$$\psi(x) \rightarrow e^{i\alpha} \psi(x),$$

$$\partial_\mu \psi \rightarrow e^{i\alpha} \partial_\mu \psi,$$

$$\bar{\psi} \rightarrow e^{-i\alpha} \bar{\psi}.$$

- > Apply to field questions



- > These extra phase terms cancel in the equations and we are left with:

$$\partial_\mu j^\mu = 0,$$

- > Global gauge invariance → α fixed to one value everywhere

> Conservation of electrical charge!



Gauge invariance and Noether's theorem

- More general: local gauge invariance, α depends on space points



- With some tweaking to make this gauge invariance work, we arrive at the Lagrangian for the QED (Quantum electrodynamics):

➤



- Note: This was done posteriori (i.e. after QED was already known)
- One take away message however for theorists: It only works, if the photon is massless (and this also explains problems with the weak boson masses)
- Works well also for QCD (and naturally requires gluon self-interaction)
- **Find a solution that allows for gauge invariance in EWK interactions including massive W and Z bosons**

