Physics of Particle Showers for High-Energy Calorimetry

Part I: Photons and Electrons

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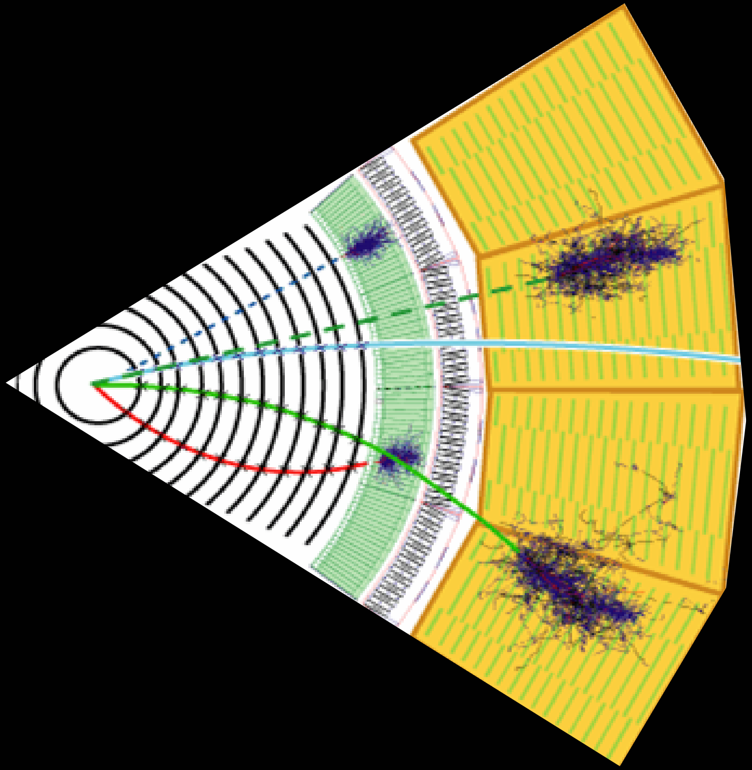
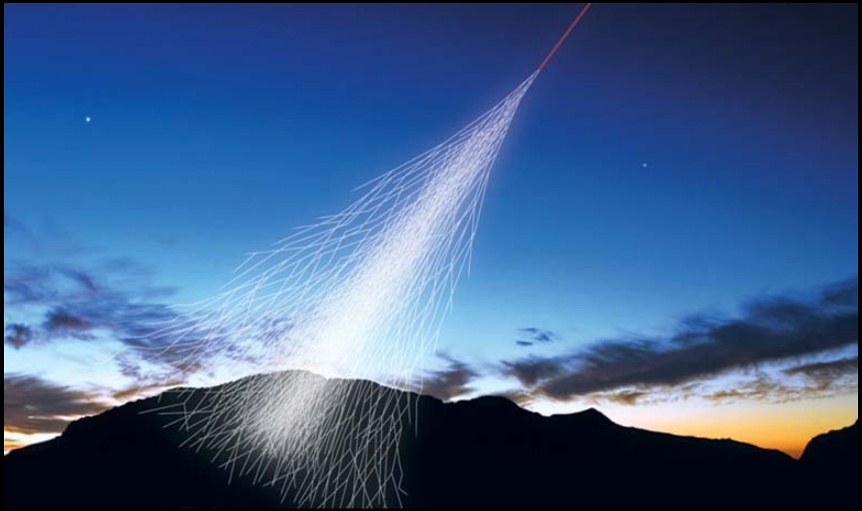
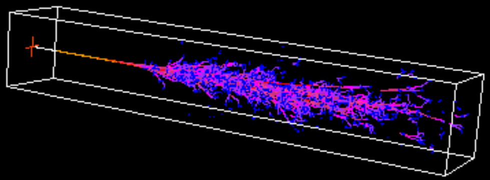






High-Energy Calorimetry: Physics of Particle Showers

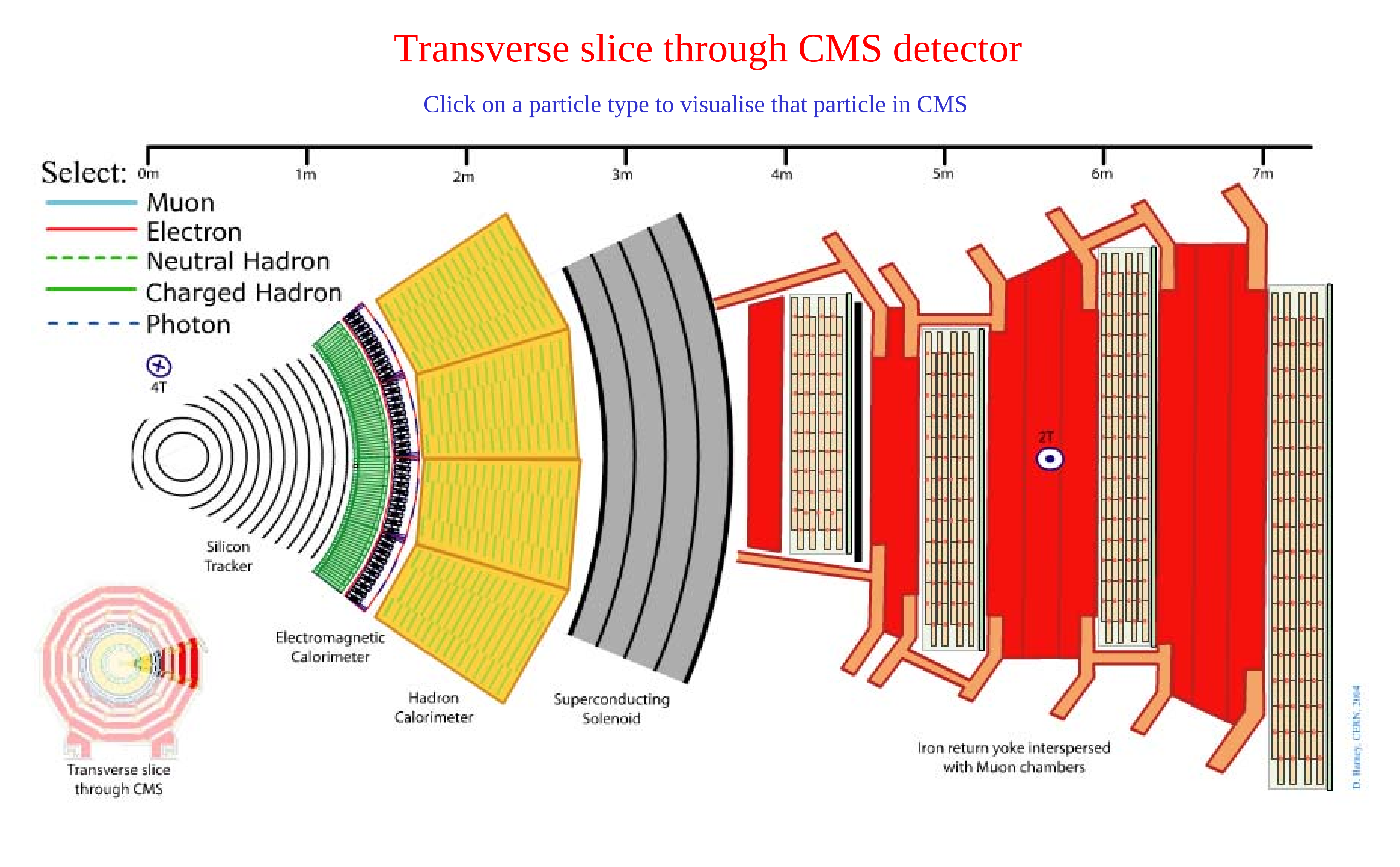
1. Introduction
2. Electromagnetic Showers
3. Construction Criteria for Calorimeters
4. Electromagnetic Calorimeters

} 1st lecture (EM)

5. Hadron Showers
6. Hadron Calorimeters
7. Examples for Calorimeters for Hadrons and Jets

} 2nd lecture (HAD)

1. Introduction



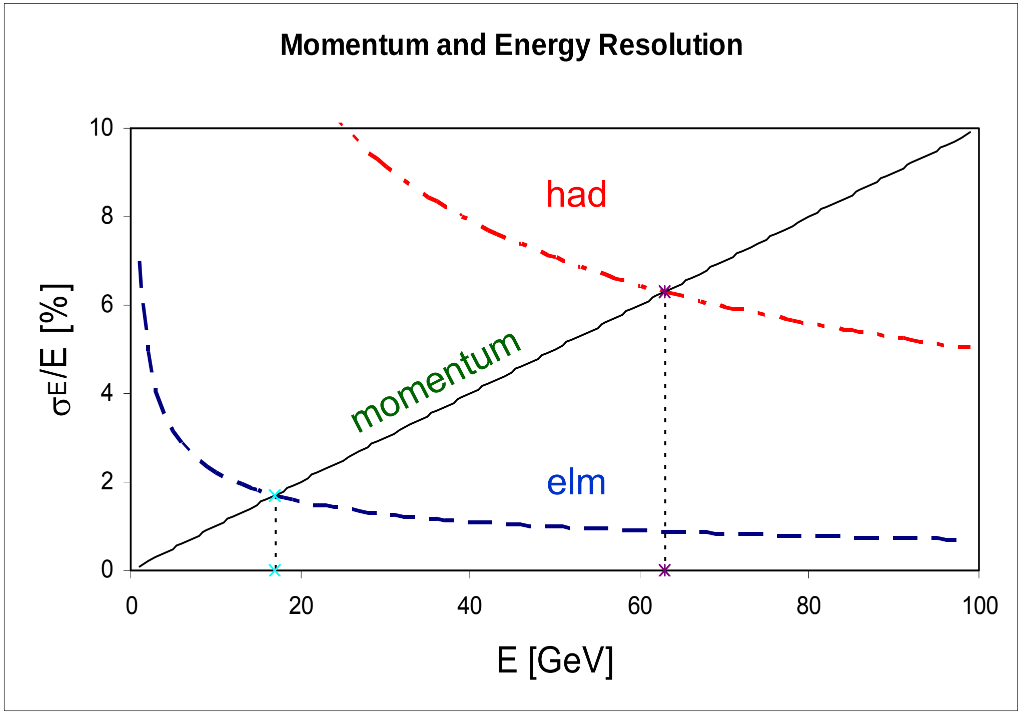
Momentum and Energy Resolutions

Momentum

$$\frac{\sigma_p}{p} \approx 0.1 \ldots 1\,\% \cdot p/\mathrm{GeV}$$

Energy

$$\frac{\sigma_E}{E} \approx \begin{cases} \dfrac{2\text{–}15\,\%}{\sqrt{E/\mathrm{GeV}}} & \text{elm.} \\[2ex] \dfrac{35\text{–}120\,\%}{\sqrt{E/\mathrm{GeV}}} & \text{had..} \end{cases}$$



2 Electromagnetic Showers

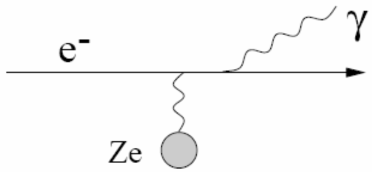
2.1 Model of Shower Development

2.2 Characteristic Size of Electromagnetic Showers

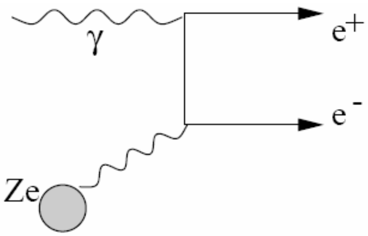
  2.2.1 Longitudinal Shower Profile

  2.2.2 Lateral Shower Profile

Electromagnetic Showers



Bremsstrahlung



pair production

$$\sigma_{paar,brems} \sim Z^2$$

$$\theta \sim \frac{1}{\gamma} = \frac{m_e}{E}$$

common scale: **radiation length**

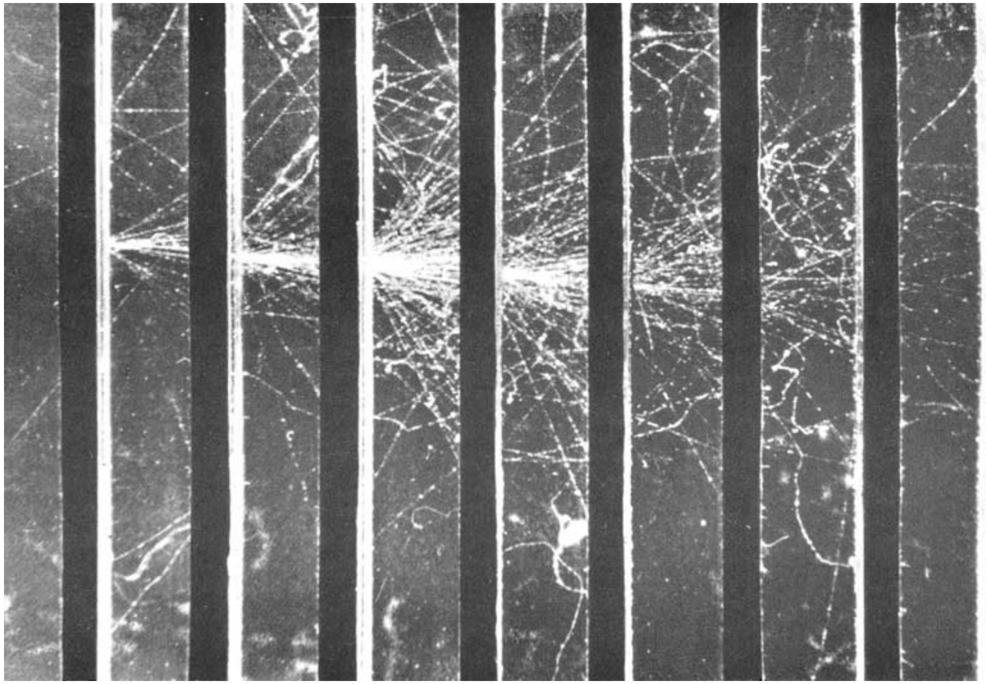
photons:

$$I_\gamma(x) = I_0\, e^{-\frac{x}{\lambda}}$$

$$\lambda \approx \frac{9}{7} X_0$$

electrons:

$$\frac{dE}{E} = \frac{dx}{X_0}$$



Shower Parameters: radiation length and critical energy

radiation length

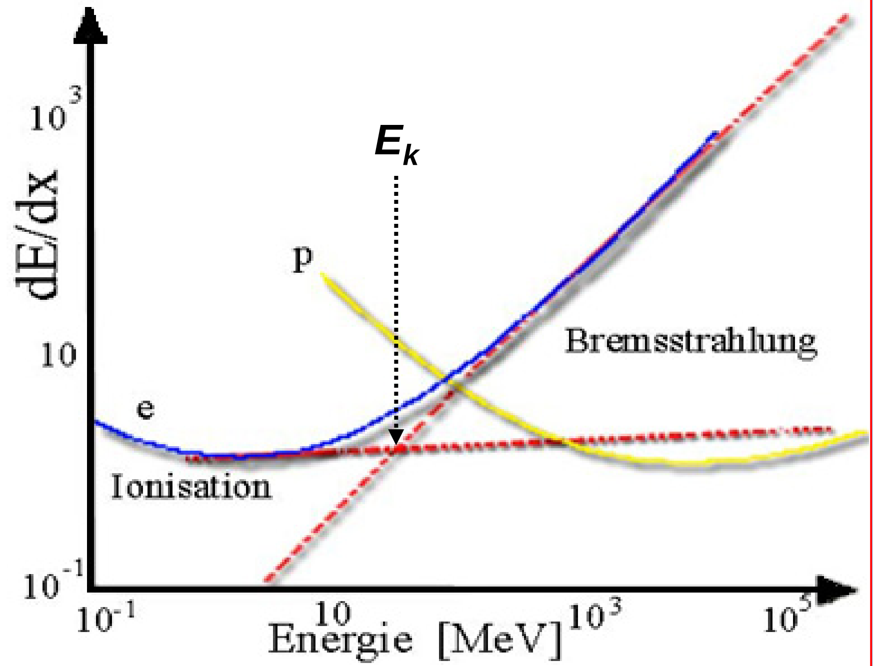
$$\left(\frac{dE}{dx}\right)_{rad} = -\frac{E}{X_0} \quad \Rightarrow \quad E(x) = E_0 \cdot e^{-x/X_0}$$

$$\frac{1}{\rho X_0} \approx \underbrace{4\alpha r_e^2 N_A}_{(716\ \frac{\mathrm{g}}{\mathrm{cm}^2})^{-1}} \ \frac{1}{A}\, Z(Z+1)\, \ln\frac{287}{\sqrt{Z}} \sim Z^2$$

critical energy

$$\frac{dE}{dx}\Big|_{ion}(E_k) = \left(\frac{dE}{dx}(E_k)\right)_{rad} \approx -\frac{E_k}{X_0}$$

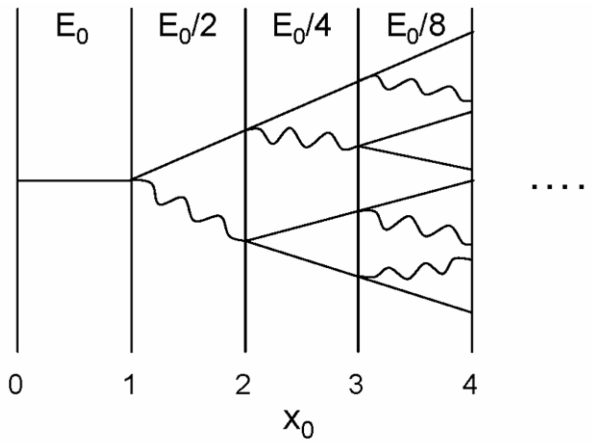
$$E_k \approx \frac{610\,\mathrm{MeV}}{Z+1.24} \sim \frac{1}{Z}$$ (solids)



Simplified Model: Rossi's 'Approximation B'

Assumptions:

- only brems. and pair p. (asymptotic cross sections)
- only ionisation when E_k is reached $dE/dx = E_k/X_0$
- shower 1-dim, no multiple scattering



total number : $N_{tot} \approx \frac{E_0}{E_k}$,

total track length : $S_{tot} \approx \frac{E_0}{E_k} \cdot X_0$

after t steps : $N = 2^t$ and $E_{e,\gamma} = \frac{E_0}{2^t}$

at the end : $E_{e,\gamma} = E_k = \frac{E_0}{2^{t_{max}}}$

$\Rightarrow$ $N_{max} = \frac{E_0}{E_k}$ and $t_{max} = \frac{\ln E_0/E_k}{\ln 2}$

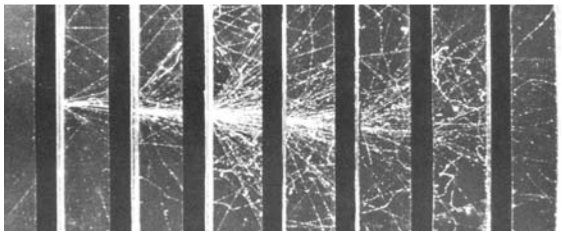
Important result: $N_{max} \sim E$ and $t_{max} \sim \ln E + const$

Basis of calorimetry: - Signal ~ N_{max} ~ E $\Rightarrow$ linearity

- shower size ~ ln E $\Rightarrow$ makes calorimeters practical

Realistic Showers

Many experimental studies of shower development



supported by realistic simulations :

Geant4, EGS (= Electron Gamma Shower), Fluka, ...

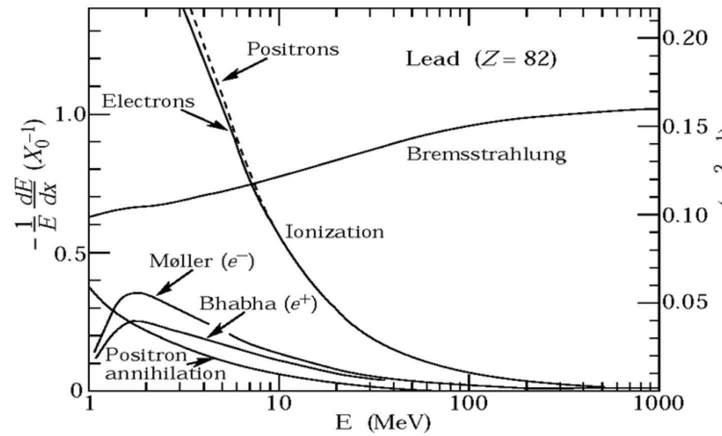
include all processes, specifically also

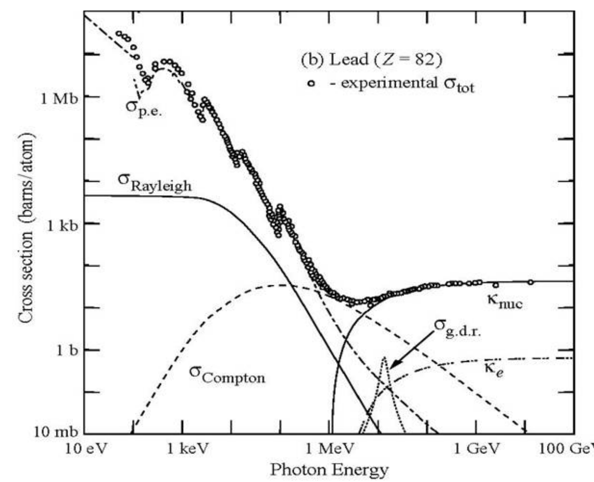
Compton, photo effect, multiple scattering

Simulations became very reliable (for e.m. showers!)

– **but watch the E_{cut}'s and step sizes.**

Reasonable parameterizations for design studies are possible.





Use of Review of Particle Physics (PDG)

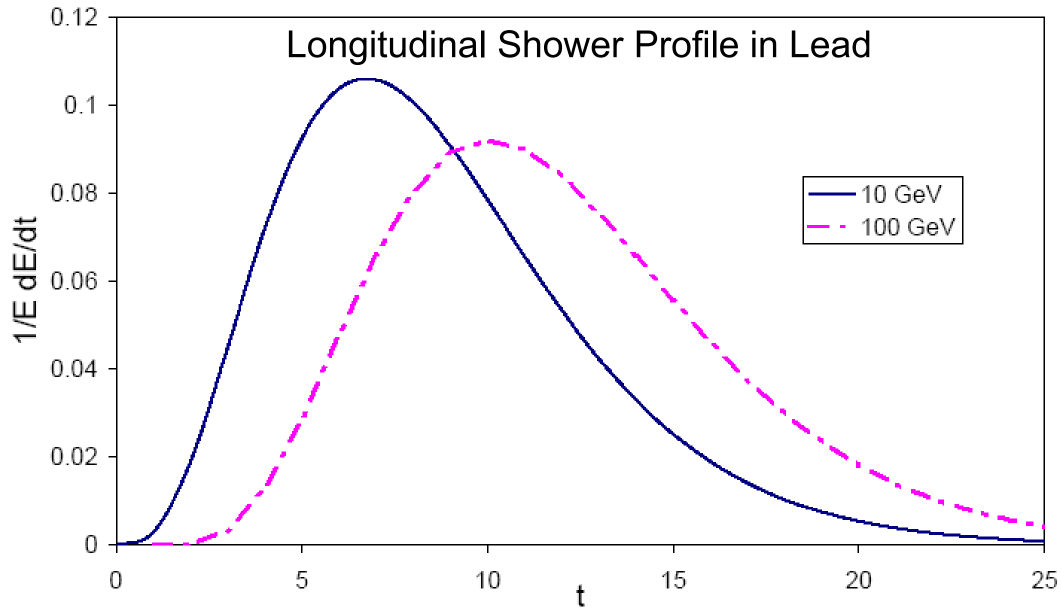
Experimental Methods and Colliders

- Accelerator physics of colliders (rev.)
- High-energy collider parameters (rev.)
- Passage of particles through matter (rev.)
- Particle detectors for accelerators (rev.)
- Particle detectors for non-accelerator physics (new)
- Radioactivity and radiation protection (rev.)
- Commonly used radioactive sources

Longitudinal Shower Profile

'Longo formula'

$$\frac{dE}{dt} = E_0 \frac{b^{\alpha+1}}{\Gamma(\alpha+1)} t^{\alpha} e^{-bt}$$



$$t_{max} = \frac{\alpha}{b} = \ln \frac{E_0}{E_k} + \begin{cases} -0.5 & \text{(electrons)} \\ +0.5 & \text{(photons)} \end{cases}$$

$b \approx 0.5$

$$t^{98\%} \approx t_{max} + 13.6 \pm 2.0$$

Lateral Shower Profile

pair, brems $\theta \sim \frac{1}{\gamma} = \frac{m_e}{E}$ very small

Molière scattering $\theta_{rms} \approx \frac{E_s}{p\,c\,\beta}\sqrt{\frac{x}{2\,X_0}}$ $(E_s = 21.2\,\mathrm{MeV})$

⇒ strong correlation with energy

Important for lateral shower size:

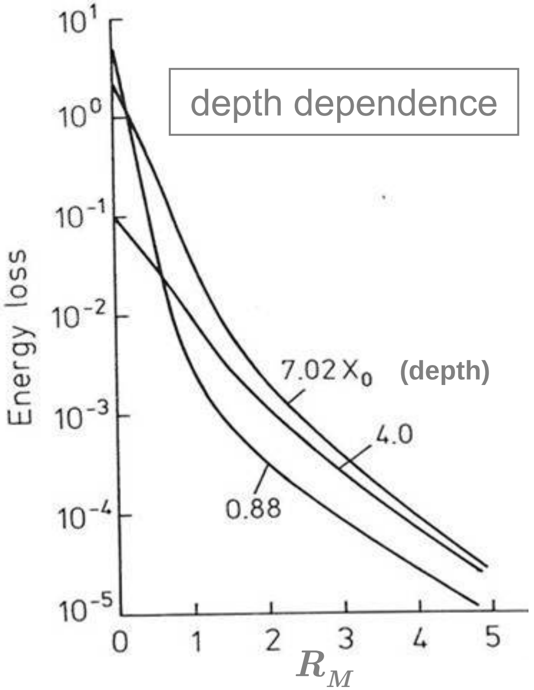
multiple scattering, Compton and photo effect

(neglected in Rossi's approxiamtion B)

Molière radius

$$R_M = \frac{E_s}{E_k} \cdot X_0$$

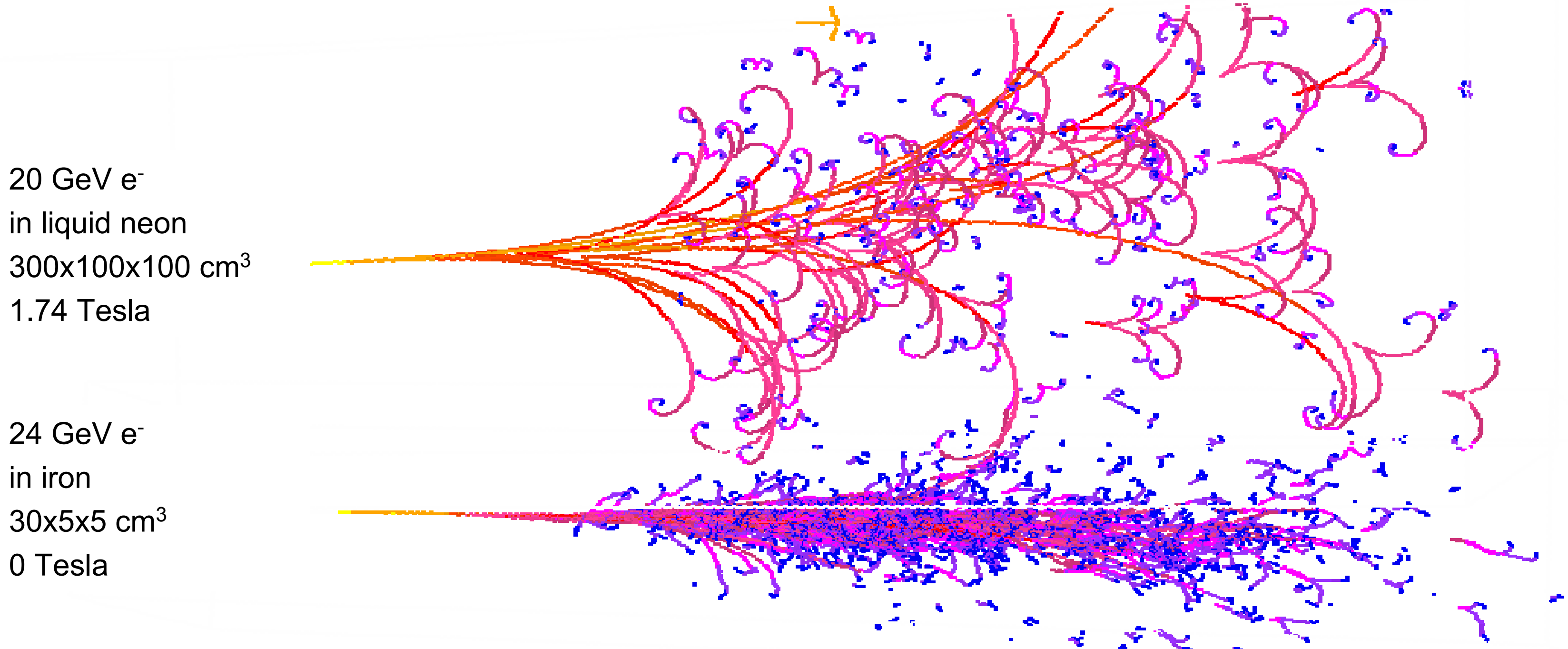
$$R_M \sim \frac{1}{Z} \quad \text{but} \quad \frac{R_M}{X_0} \sim Z$$



R/R_M	1	2	3.5
$\Delta E/E_0$ [%]	90	95	99

Shower Size Parameters

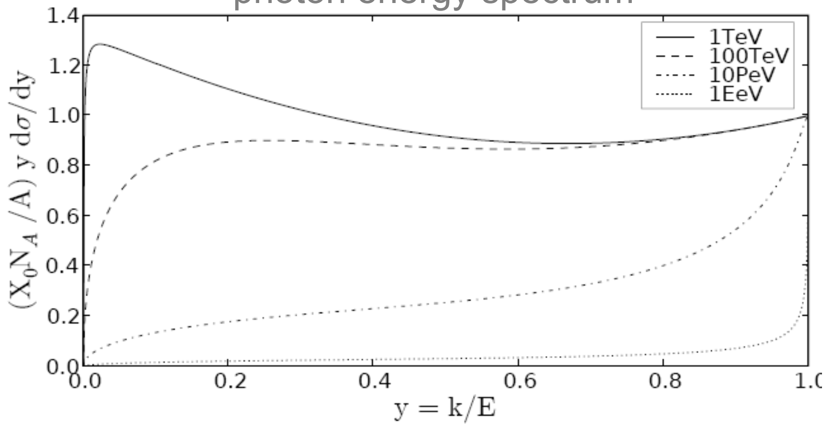
Material	Z	X_0 [cm]	E_k [MeV]	t_{max} 10 GeV	t_{max} 100 GeV	$t^{98\%}$ 10 GeV	$t^{98\%}$ 100 GeV	R_M [cm]	R_M/X_0
H_2O	1,8	36.1	92.0	4.2	6.5	17.8	20.1	8.3	0.23
LNe	10.0	24.0	55.0	4.7	7.0	18.3	20.6	9.2	0.39
Al	13.0	8.9	43.0	4.9	7.3	18.5	20.9	4.5	0.51
Fe	26.0	1.8	22.0	5.6	7.9	19.2	21.5	1.8	1.02
Pb	82.0	0.6	7.3	6.7	9.0	20.3	22.6	1.6	2.86

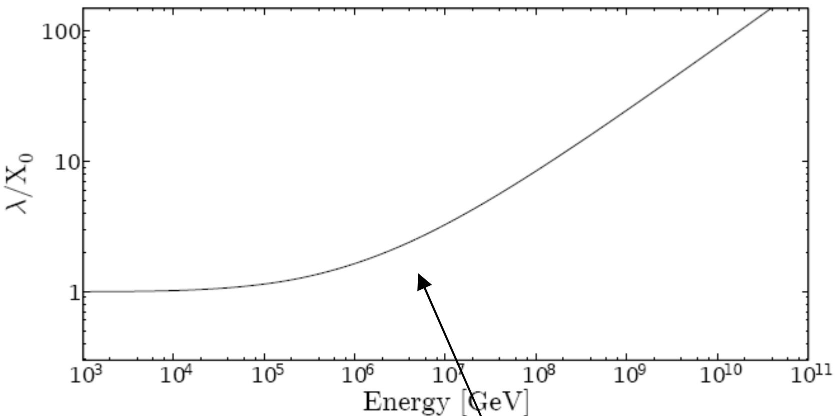


[showers: http://www.mppmu.mpg.de/~menke/elss/home.shtml]

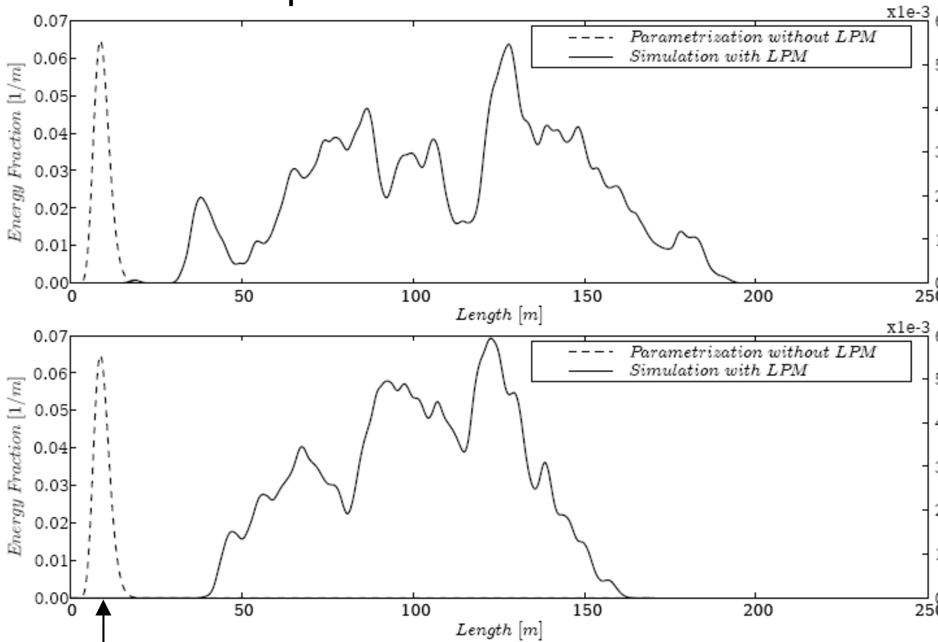
LPM Effect

photon energy spectrum





examples of 10 EeV electrons in ice



shower without LPM effect

wavelengths of sh.particles become longer than distance between atoms

$\Rightarrow$ multiple scattering distorts coherence

3 Construction Criteria for Calorimeters (1)

- Construction Features:

homogeneous vs. sampling,

passive = absorber – active = readout

modular, (non-)pointing geometry, trigger towers

hermeticity

- Size and Granularity of a Calorimeter

small leakage: long. $t^{98\%}$, lateral ~2 R_M, modularity ~R_M,

pre-sampler and tail catcher

- Energy Resolution:

fine sampling, efficient RO,

low noise,

small leakage,

low mechanical and electronic tolerances,

small intercalibration errors, ...

Construction Criteria for Calorimeters (2)

- Position and Direction Resolution

  granularity $\leq R_M$

- Signal Collection and Time Resolution

  for triggering event separation, (scint ~ ns, LAr ~ μs, ...)

- Linearity

  no leakage, no saturation, high electronic dynamic range

- Calibration

  external beams with MC and in situ calibration (kinematics, rad.act. sources)

- Radiation Hardness

  material and electronics, issue mainly in high intensity hadron machines

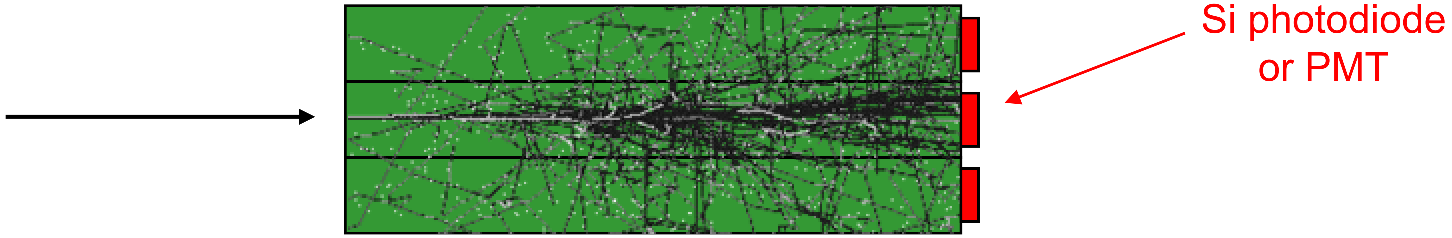
4 Electromagnetic Calorimeters

Calorimeter Types

2 general classes:

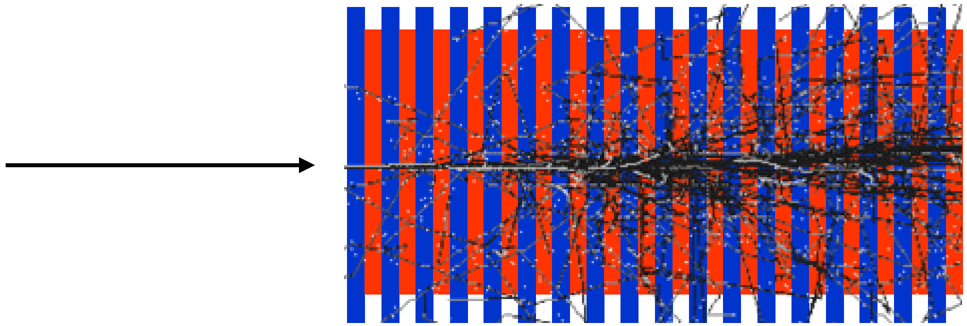
Homogeneous calorimeters:

Single medium as both absorber and detector, such as anorganic crystal scintillators (NaI, CsI, BGO, $PbWO_4$ ....), lead glass, liquid Xe or Kr, ...



Sampling calorimeters:

Layers of passive absorber (such as Pb, Cu, Fe, ...) and active detector layers such as scintillator, Si, liquid argon (LAr), PWC, ...



Material Properties for Mixtures

Define the **weight fractions** (for sandwich with l_i thicknesses):

$$\frac{1}{\rho} = \sum_i \frac{w_i}{\rho_i} \qquad \text{and} \qquad w_i = \frac{l_i \rho_i}{\sum_j l_j \rho_j}$$

radiation length:

$$\frac{1}{X_0} = \sum_i \frac{w_i}{X_{0,i}}$$

$$X_0,\ X_{0i},\ R_M \text{ in } [\frac{\text{g}}{\text{cm}^2}]$$

Moliere radius:

$$\frac{1}{R_M} = \frac{1}{E_s} \sum_i \left(w_i \frac{E_{k,i}}{X_{0i}} \right)$$

More generic: how to calculate mean free pathes in mixtures?

$$\text{reaction rate}: \quad \frac{dN}{N} = -\underbrace{n\,\sigma}_{1/\lambda}\,dx \quad \Rightarrow \quad N = N_0\,\mathrm{e}^{-\frac{x}{\lambda}}$$

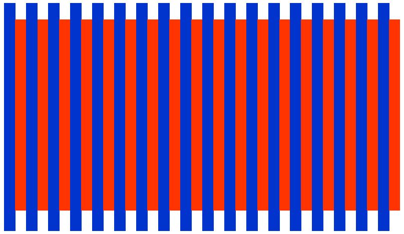
$$\text{target density}: \quad n = \frac{N_A}{A}\,\rho$$

$$\text{meanfreepath}: \quad \lambda = \frac{1}{n\,\sigma}$$

mixture (e.g. 2):

$$\bar{n}\,\bar{\sigma}\,dx = \frac{n_1\,\sigma_1\,l_1 + n_2\,\sigma_2\,l_2}{l_1 + l_2}\,dx$$

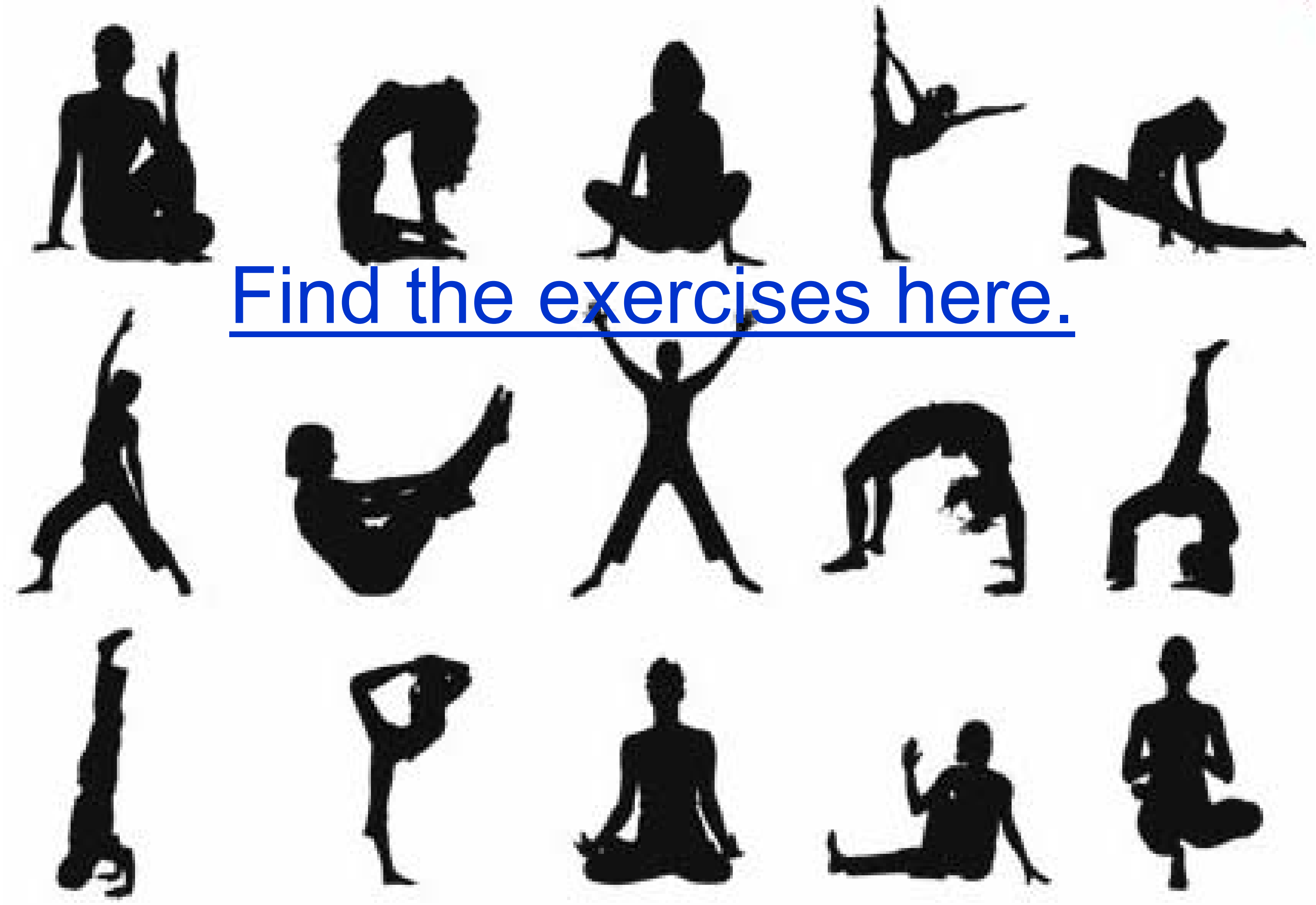
$$\frac{1}{\lambda} = \frac{\frac{1}{\lambda_1}\,l_1 + \frac{1}{\lambda_2}\,l_2}{l_1 + l_2}$$



$$\text{defining}: \quad \bar{\rho} = \frac{\rho_1\,l_1 + \rho_2\,l_2}{l_1 + l_2} \quad \text{and} \quad \lambda' = \bar{\rho}\,\lambda \quad \text{etc.} \quad \text{finally gets}:$$

$$\frac{1}{\lambda'} = \frac{\frac{1}{\lambda'_1}\,\rho_1\,l_1 + \frac{1}{\lambda'_2}\,\rho_2\,l_2}{\rho_1\,l_1 + \rho_2\,l_2}$$

Exercises (EM)



Find the exercises here.

Energy Resolution (general)

$$\frac{\sigma_E}{E} = \sqrt{\frac{a^2}{E} + \frac{b^2}{E^2} + c^2} =: \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c$$

sampling | noise | const

limiting at — low energy (noise) — high energy (const)

Examples:

H1 LAr Calorimeter (BBE = 'backward barrel electromagnetic')

a = 9.4% √GeV   *b* = 230 MeV   *c* = 1.1%

100-GeV electrons: a/√E = 0.9 %; b/E ≈ 0; c =1.1 % ⇒ $\sigma_E/E \cong 2$ %

CMS Crystal Calorimeter:

a = 2.8% √GeV   *b* = 120 MeV   *c* = 0.3%

100-GeV electrons: a/√E = 0.3 %; b/E ≈ 0; c =0.3 % ⇒ $\sigma_E/E \cong 0.4$ %

Properties of Selected EM Calorimeters

type	X_0	R_M	distance from IR	face area of a cell	thickness/X_0 passive layer	thickness/X_0 total	resolution (E in GeV) $\sigma(E)/E$ A, [%/GeV],	B [MeV]	C [%]	σ_θ [mrad]	experiment
	[cm]	[cm]	[cm]	[cm^2]							
homogeneous Calorimeter											
NaI(Tl)	2.59	4.8	25.4	12.9	-	15.7	$2.8/\sqrt[4]{E}$	~ 0.05	-	26-35	C. Ball
CsI(Tl)	1.85	3.5	92	4.7× 4.7	-	16	$2.3/\sqrt[4]{E}$	0.15	1.35	$4.2/\sqrt{E}$	BaBar
BGO	1.12	2.3	50	2×2	-	22	$\sim 2/\sqrt{E}$	-	-	~ 10	L3
Pb glass	2.54	3.5	245	10×10	-	25	$6.3/\sqrt{E}$	11	0.2	4.5	OPAL
$PbWO_4$	0.89	2.0	130	2.2 × 2.2	-	25.8	$2.8/\sqrt{E}$	120	0.3	~ 0.7	CMS
LKr	4.7	5.9	∼100 m	2.0 × 2.0	-	27	$3.2/\sqrt{E}$	90	0.42	$\sigma_x \approx 1\,\mathrm{mm}$	NA 48
Sampling Calorimeter											
Pb/Szi	3.2	5.0	230	10×10	0.18	12.5	$6.5/\sqrt{E}$	< 10	7.2	$6.5/\sqrt{E}$	ARGUS
Pb/LAr	1.1	2.66	90	10-100	0.42	20-30	$11/\sqrt{E}$	150	0.6	?	H1
Pb/Szi	1.7	4.15	1350	5.59×5.59	0.54	20	$11.8/\sqrt{E}$	-	1.4	$1.0/\sqrt{E} \oplus 0.2$	HERA-B
Pb/LAr	∼2	∼4.1	150	14.7×0.47	∼0.4	22.5	$10\sqrt{E}$	190	0.5-0.7	$50 - 75/\sqrt{E}$	ATLAS
U/Szi	0.56	1.66	120	115-200	1.0	25	$18\%/\sqrt{E}$	-	-	8?	ZEUS

Missing entries mean that the numbers are not provided by the experiment (may be negligible).
Numbers are given for 'typical' parts in a detector, e.g. central detector; for HERA-B it is 'Middle ECAL'
For ATLAS LAr barrel with about 50°incidence on accordeon structure assumed; face area for presampler

Requirements and Solutions
(to be compromised)

requirement	optimal solution
energy resolution	homogeneous calorimeter or high sampling frequency
space and position resolution	high granularity
electron-hadron separation	longitudinale segmentation
hermetic coverrage	dense, little frame space and utility shafts
energy resolution at high energy	small constant term (tolerances, intercalibration, ..)
same energy scale for EM and HAD ($e/h \approx 1$)	in hardware $\Rightarrow$ EM = HAD technology (compromises EM).
jet resolution	granularity, matching of EM and HAD Hadronkalorimeter
linearity	no leakage (sufficient depth), sufficient dynamical range of signal electronics.
absolute calibration	combination of test beams, in situ and simulation
low cost	low number of electronic channels $\sim$ volume ($\Rightarrow$ size of tracking devices)

Homogeneous Calorimeters

Scintillators:

- Anorganic crystals like NaJ(Tl), CsJ(Tl) oder BGO.
  Typical resolutions:

$$\frac{\sigma_E}{E} \approx \frac{3\%}{\sqrt{E/\mathrm{GeV}}} \quad \text{or often} \quad \frac{\sigma_E}{E} \approx \frac{3\%}{\sqrt[4]{E}}$$

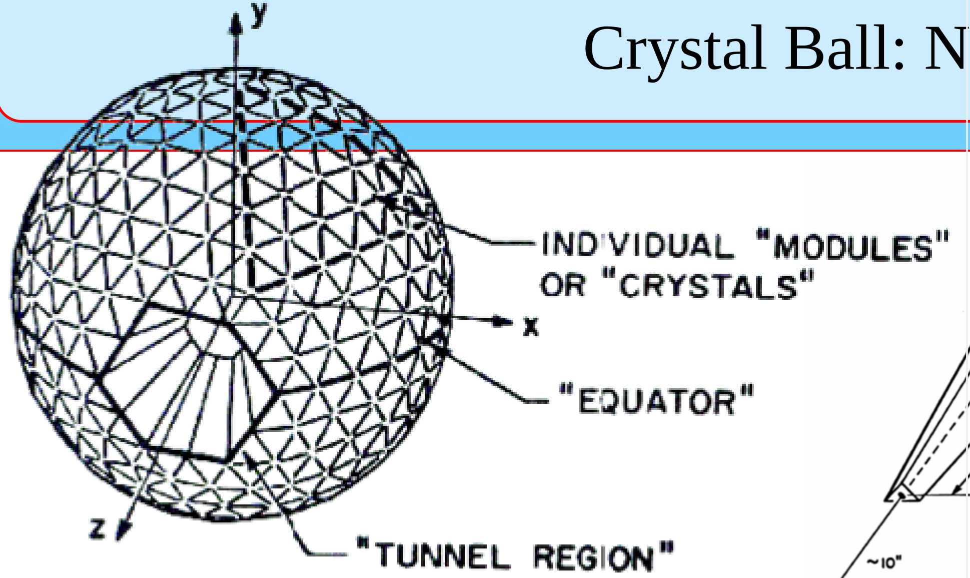
- liquid nobel gases with high Z (krypton, xenon).

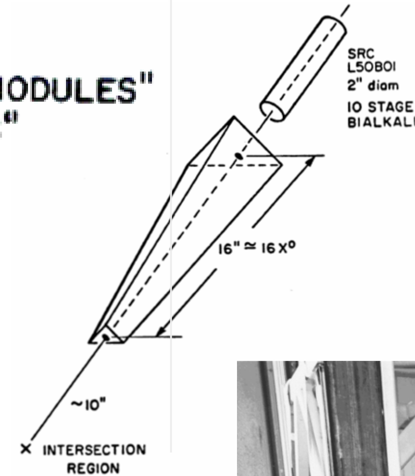
Cherenkov-Detectors

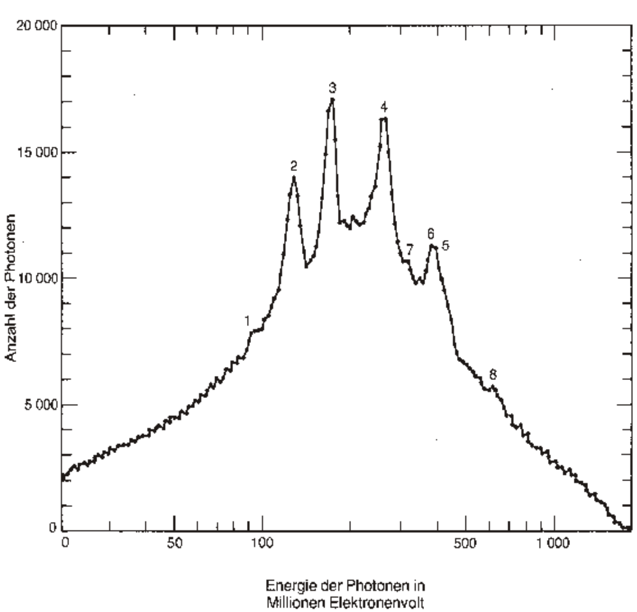
- (Pb glass, Pb F_2, . . . , ice, water, . . .).
  Typical resolutions :

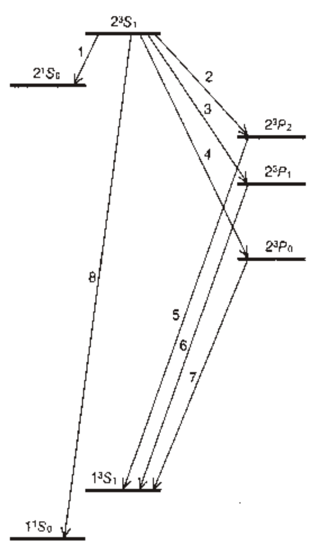
$$\frac{\sigma_E}{E} \approx \frac{5\text{–}10\%}{\sqrt{E/\mathrm{GeV}}}$$

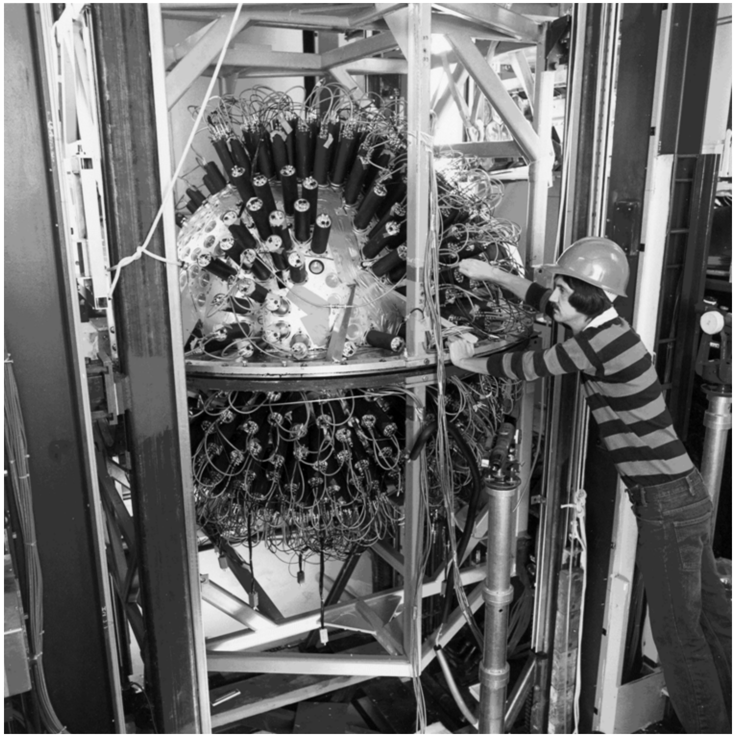
Crystal Ball: NaJ(Tl)





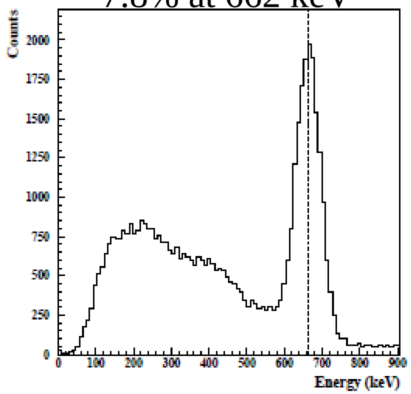






Crystal Calibration with Sources

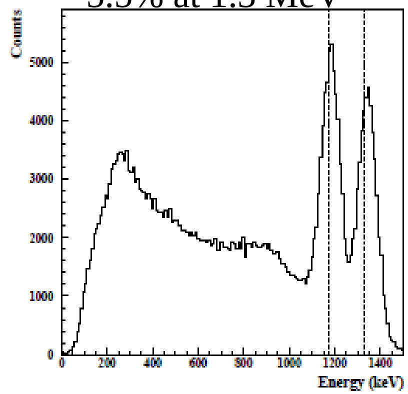
7.8% at 662 keV



(a) ^{137}Cs

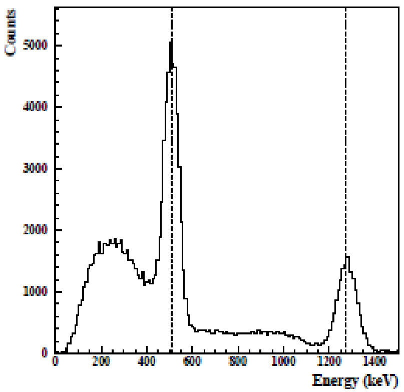
5.5% at 1.3 MeV

Intrinsic energy resolution of a single crystal

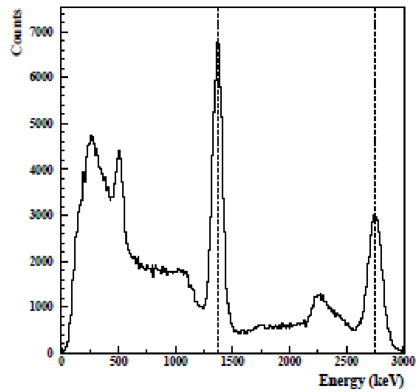


(b) ^{60}Co

Crystal Ball



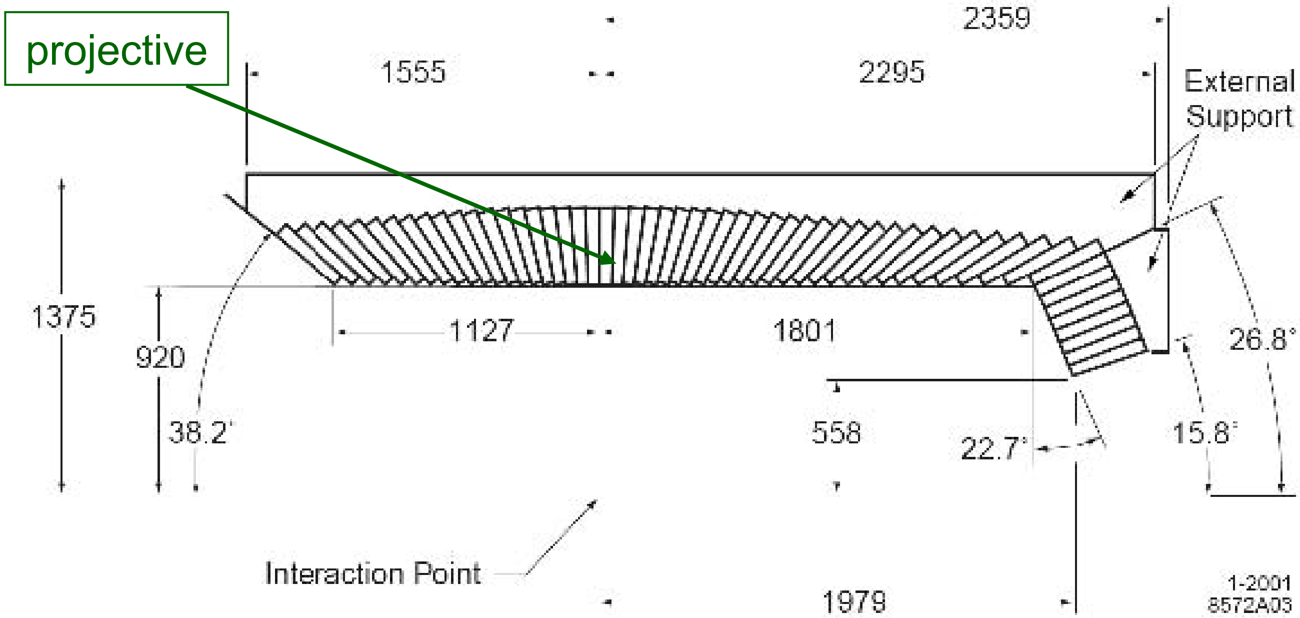
(c) ^{22}Na

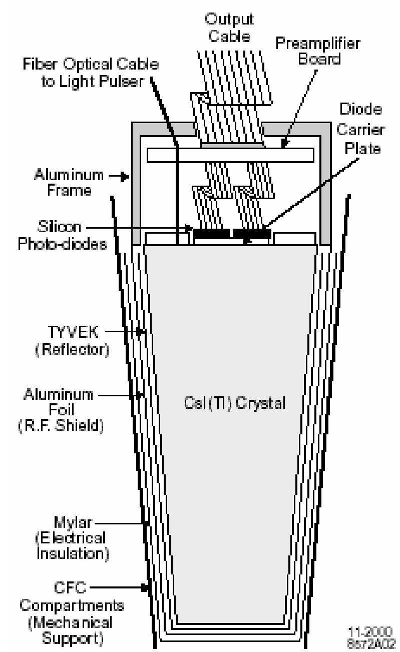


(d) ^{24}Na

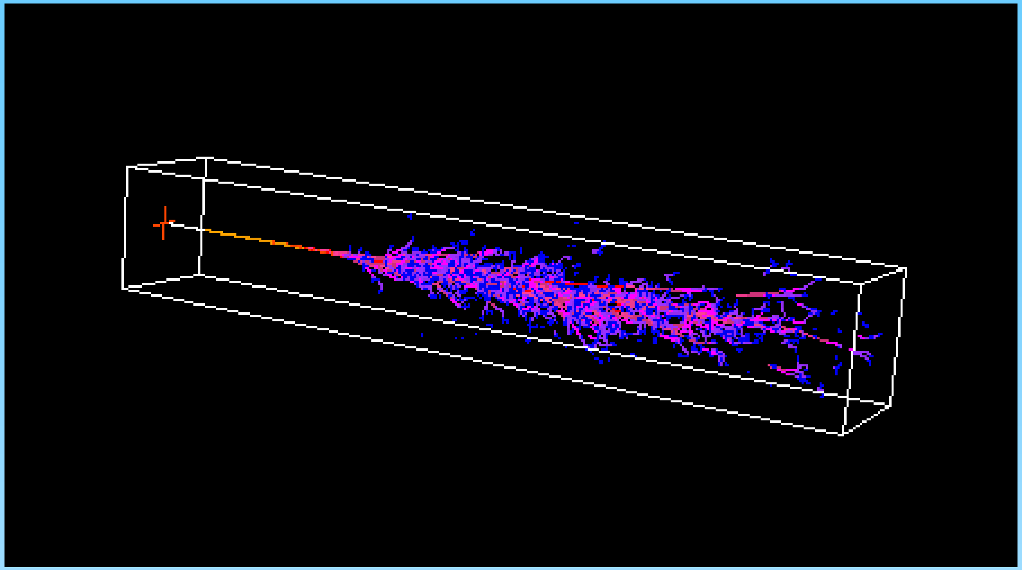
Noise as low as O(100 keV) can be reached in crystals
$\Rightarrow$ low energy calibration with sources possible

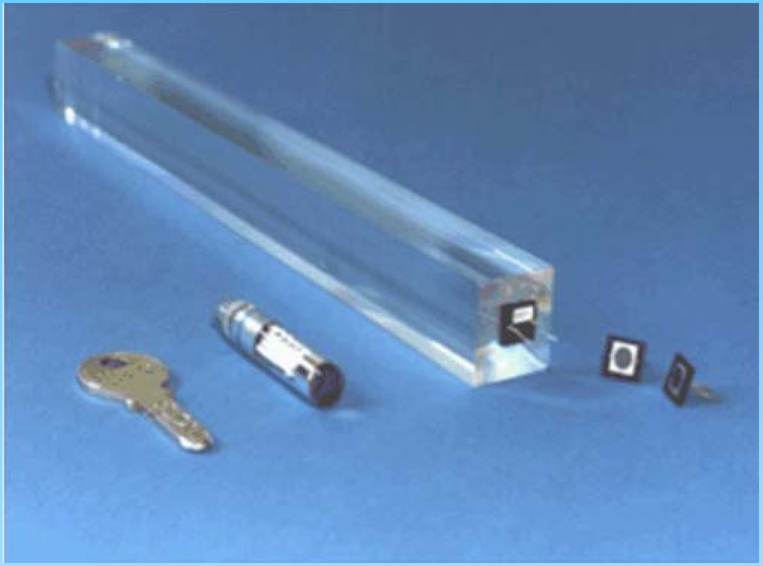
Crystal-Calorimeter: BaBar CsI(Tl) Calorimeter

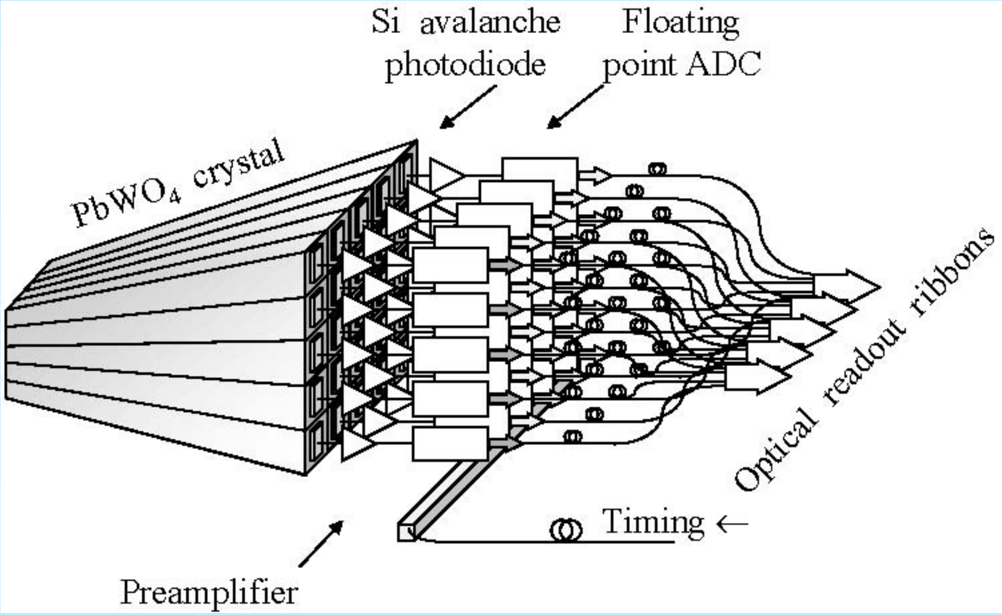


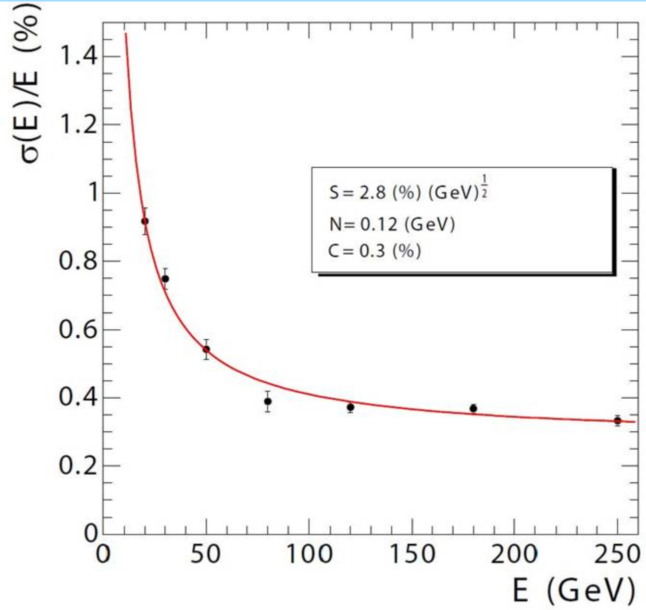


Electromagnetic Showers in Scintillation Crystals

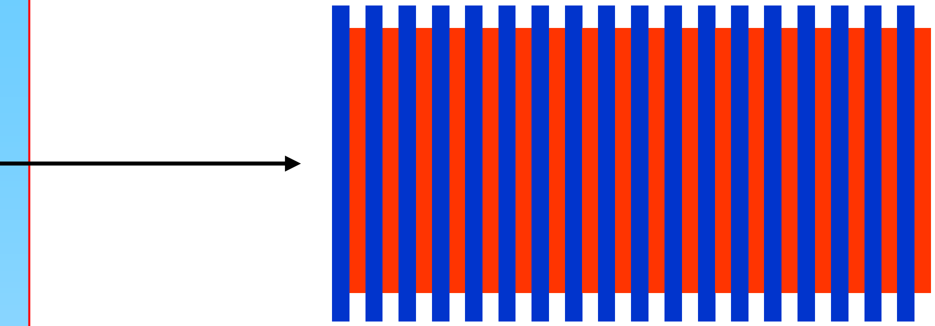








Sampling Calorimeters



$$x_p = d_p X_0^p \quad \text{passive layer (absorber)}$$
$$x_a = d_a X_0^a \quad \text{active layer (detector)}$$

$$f_s = \frac{E_{vis}}{E_{dep}} \approx \mathrm{O}(10\%)$$

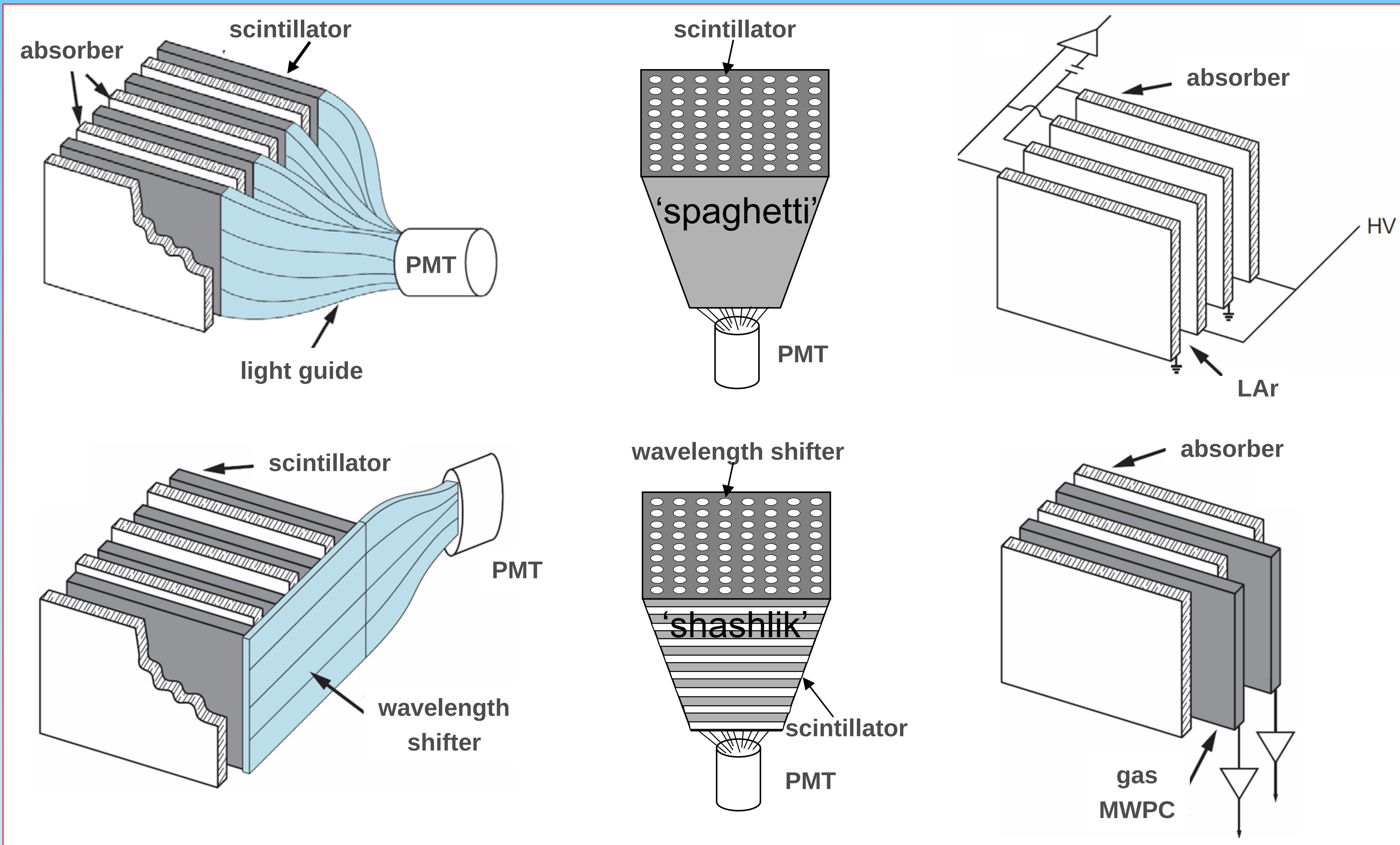
- sandwich calorimeter: absorber plates with high Z (e.g. Pb, W, U, . . .) alternate with low Z detector layers.

readout methods:

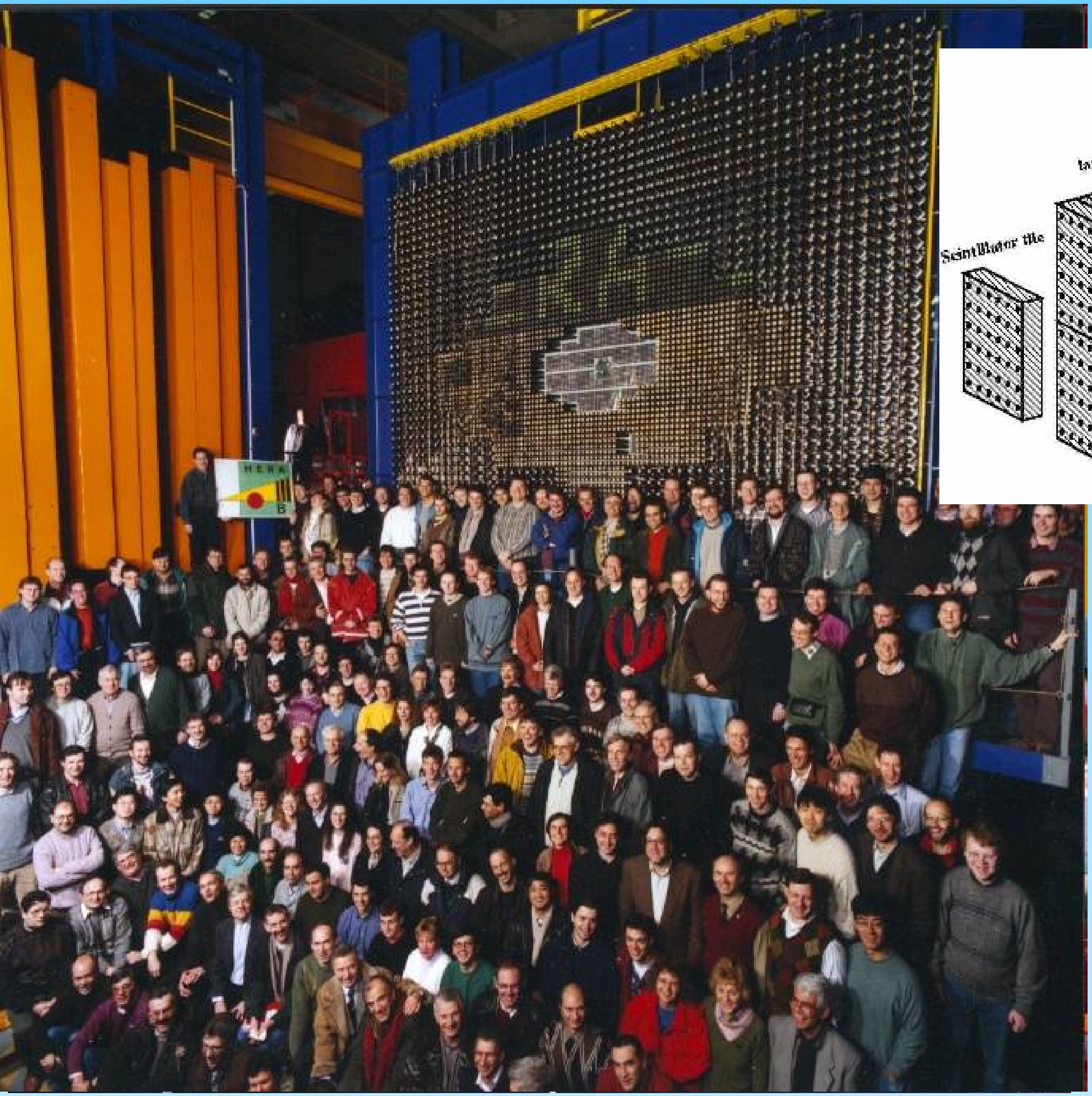
- scintillator with wavelength shifter readout (plates, fibers, ...)
- proportional chambers (PWC, MWPC, streamer tubes, ...)
- ionization chambers (LAr, ..)

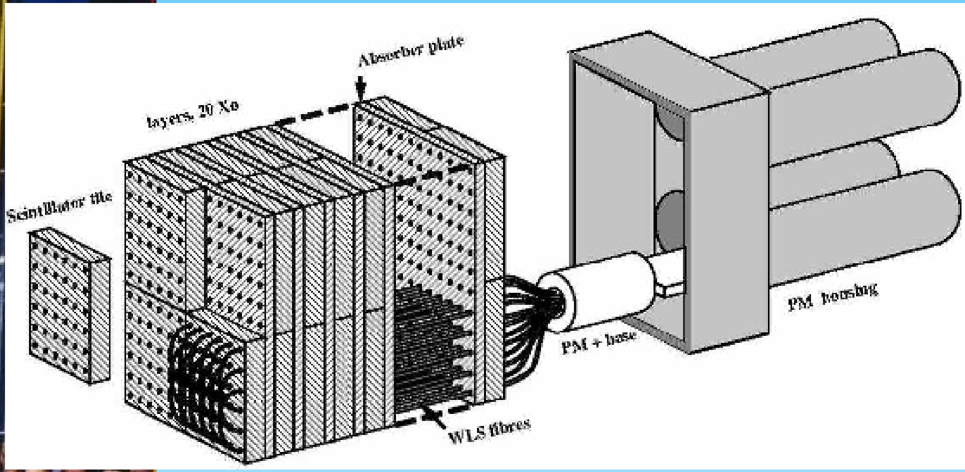
- Spaghetti calorimeter (SpaCal): scintillating fibers embedded in lead

Readout of Sampling Calorimeters

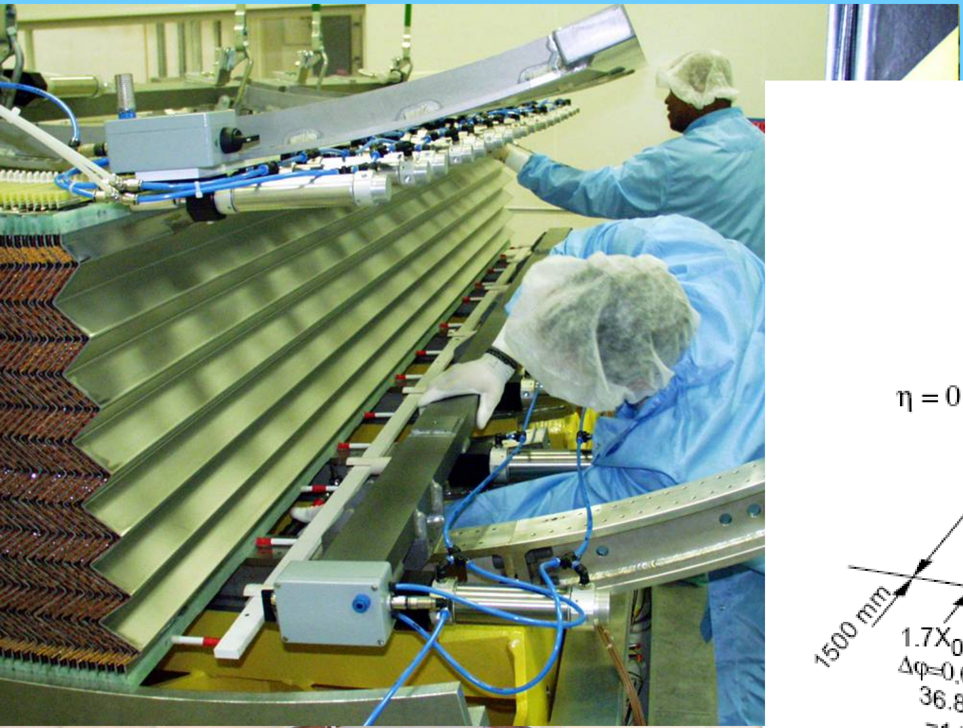


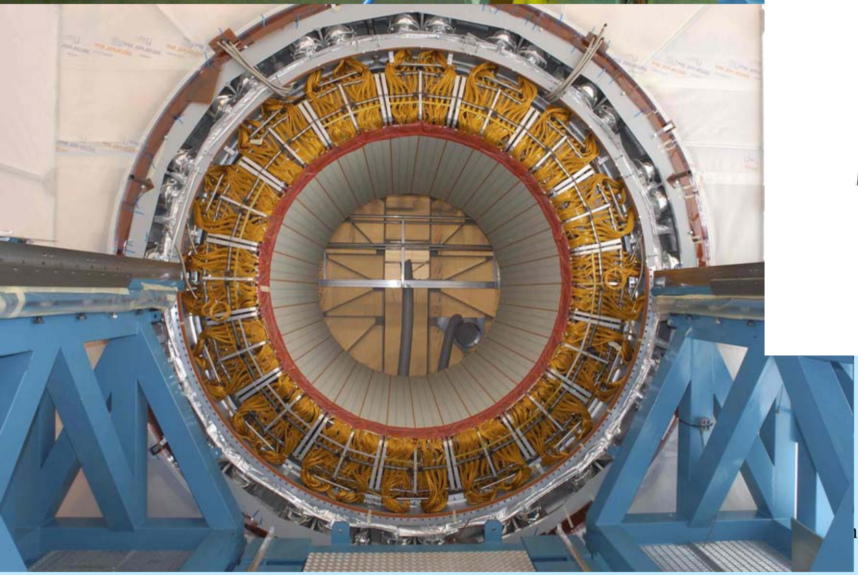
HERA-B ECAL

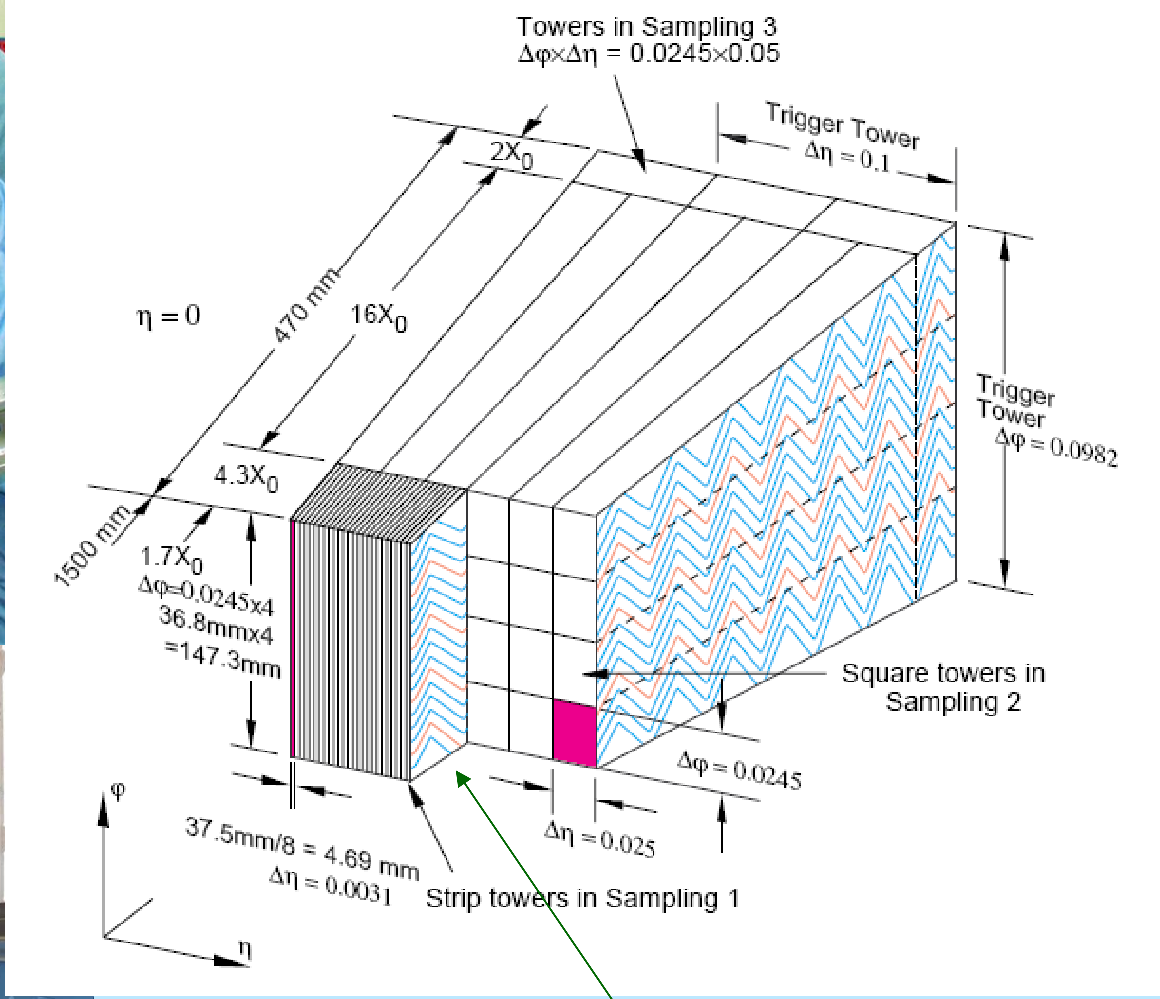



Absorber plate
layers, 20 Xo
Scintillator tile
PM housing
PM + base
WLS fibres

ATLAS Accordion Calorimeter

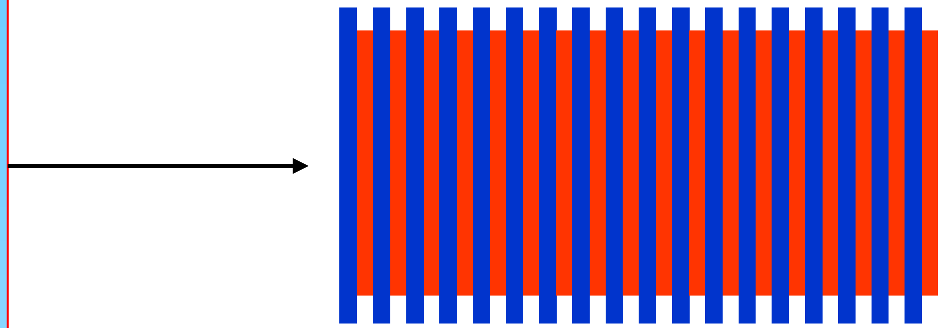






pre-sampler for O(r.l.) material in front

Sampling Fluctuations and Energy Resolution



Poisson statistics of particles crossing the active layer :

$$\frac{\sigma_E}{E} \approx \frac{\sqrt{N_{vis}}}{N_{vis}} = \frac{1}{\sqrt{N_{vis}}}$$

Rossi's approximation:

$$N_{tot} \approx \frac{E_0}{E_k} \quad \Rightarrow \quad S_{tot} \approx \frac{E_0}{E_k} X_0 \quad \Rightarrow \quad N_{vis} \approx \frac{S_{tot}}{d_p X_0} = \frac{E_0}{d_p E_k}$$

- S_{tot}: total path length
- $x_p = d_p X_0$ absorber thickness

$$\left(\frac{\sigma_E}{E}\right)_{samp} = \frac{1}{\sqrt{N_{vis}}} = \sqrt{\frac{d_p E_k}{E}} = 3.2\,\%\sqrt{\frac{d_p E_k/\mathrm{MeV}}{E/\mathrm{GeV}}}$$

- $\sqrt{\frac{d_p E_k}{E}}$: independent of active layer!
- $E_k \sim \frac{1}{Z}$

$$8.6\%\sqrt{d_p/E} \quad \text{for} \quad \mathrm{Pb} \quad (N_{vis} = 136\,E/d_p)$$

$$15\%\sqrt{d_p/E} \quad \text{for} \quad \mathrm{Fe} \quad (N_{vis} = 45\,E/d_p)$$

← experimentally usually worse. Why?

More Realistic Sampling Term

Important indication of a problem:

$$'\mathrm{Rossi\ B}' : \frac{e}{mip} \quad =: \quad \frac{\epsilon_e}{\epsilon_{mip}} =\approx 1$$

$$\mathrm{Exp.} : \quad \frac{e}{mip} \quad < \quad 1 \qquad (\mathrm{typical} \ \approx 0.5 - 0.7)$$

$$\epsilon = \frac{\mathrm{signal}}{E_{dep}}$$

Revise assumptions:

- ionisation is not always E_k/X_0 *and* not energy independent
- contribution of particles with $E \ll E_k$ substantial (MeV $e^{\pm}$ with large dE/dx)
- photo- and Compton effect is not negligible and pair p. ≠ const
- correlations of particles: pair production, a particle crosses many layers
- shower is not 1-dim (multiple scatt., photo- and Compton effect)

in the following assumed to be corrected for

consider only intrinsic fluctuations (e.g. no fluctuations of readout quanta)

Modifications of the Basic Formula

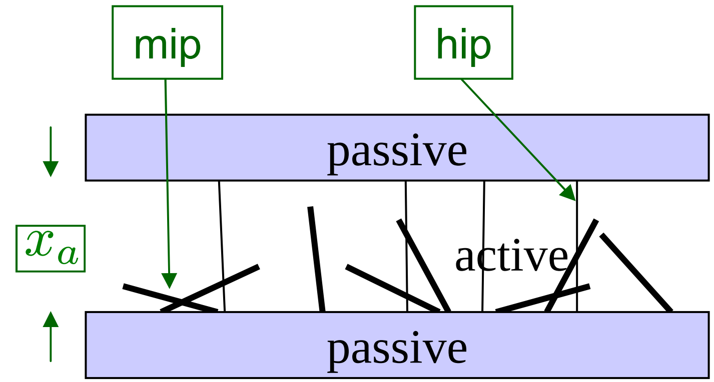
Effect of cut-off energy (e.g. C-threshold):

$$N'_{vis} = F(\xi)\, N_{vis}$$

$$\xi \sim \frac{E_{cut}}{E_k}$$

effective threshold by active layer thickness x_a

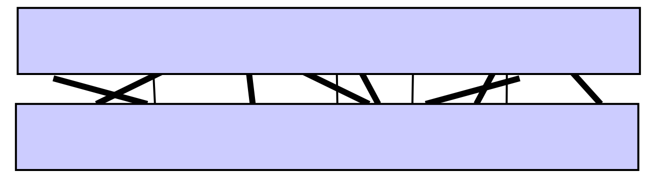
$$E_{cut} \approx \frac{x_a}{2} \frac{dE}{dx} \Big|_{E_{cut}}$$



Effect of scattering:

$$d'_p = \frac{d_p}{\langle \cos\theta \rangle}$$

strong <cos θ> - E correlation $\Rightarrow$ E_{cut} dependence
more scattering at higher Z $\Rightarrow$ more in passive layer

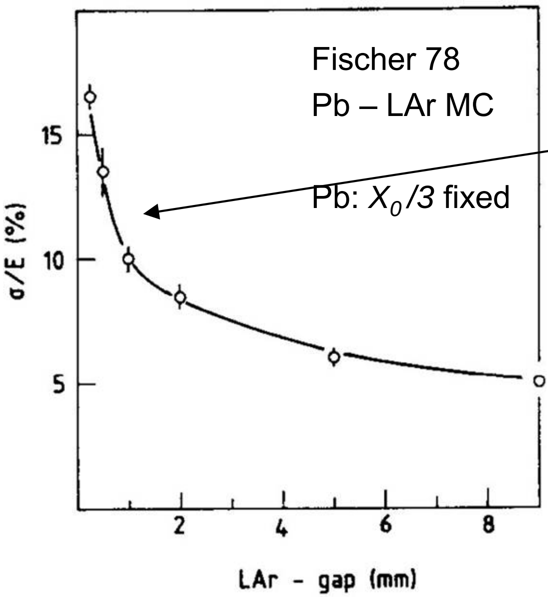


$$\left(\frac{\sigma_E}{E}\right)_{samp} = 3.2\,\% \sqrt{\frac{d_p E_k/\mathrm{MeV}}{F(\xi)\,\langle\cos\theta\rangle\, E/\mathrm{GeV}}} = a' \sqrt{\frac{d_p}{E/\mathrm{GeV}}}$$

[Amaldi 81]

pass. Med.	t	x_a $\left[\frac{\mathrm{g}}{\mathrm{cm}^2}\right]$	E [GeV]	a'_{exp} [%]	E_{cut} [MeV]	ξ	$\frac{1}{\sqrt{F(\xi)}}$	$\frac{1}{\sqrt{\langle\cos\theta\rangle}}$	a'_{theo} [%]	$N_{vis}\cdot t$ $\left[\frac{1}{\mathrm{GeV}}\right]$
Al	1.0	3.0	10-50	20	3.0	0.168	1.16	1.00	23.0	19
Fe	0.3-1.5	0.65	0.2-2.5	16.9	0.65	0.068	1.09	1.03	16.1	39
Pb	0.3-1.5	1.3	0.2-2.5	12.6	1.3	0.328	1.21	1.29	13.2	57

Active Layer Thickness

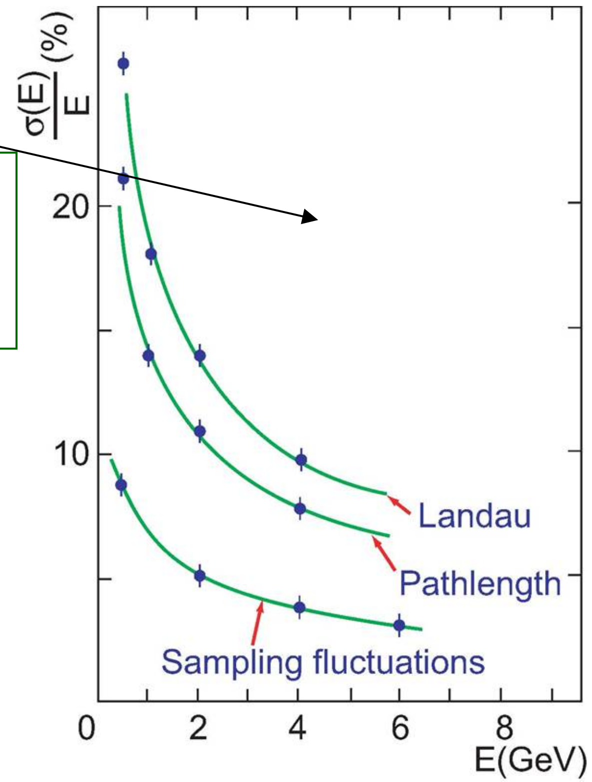


x_a in $\frac{\mathrm{g}}{\mathrm{cm}^2}$ matters

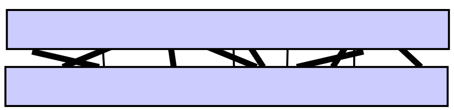
not critical for LAr

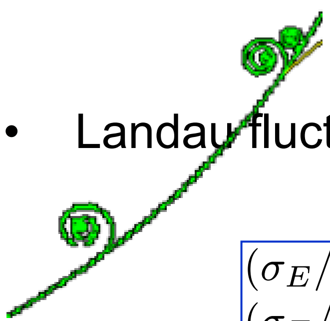
unavoidable for gaseous det.

$\rho_{gas} \approx \rho_{solid} / 1000$



- pathlength fluctuations:





- Landau fluctuations: $\left(\frac{\sigma_E}{E}\right)_{\mathrm{L.}} \simeq \frac{1}{\sqrt{N'_{vis}}} \frac{3}{\ln(1.3 \cdot 10^4 \cdot \Delta E/\mathrm{MeV})}$ $\quad (\Delta E = \langle \frac{dE}{dx} x_a \rangle)$

$$(\sigma_E/E)_{\mathrm{L.}} \approx 20\% \, (\sigma_E/E)_{\mathrm{samp}} \quad \text{for } \Delta E = 1\,\mathrm{MeV} \text{ (solid, liquid)}$$
$$(\sigma_E/E)_{\mathrm{L.}} \approx 87\% \, (\sigma_E/E)_{\mathrm{samp}} \quad \text{for } \Delta E \simeq 10^{-3}\,\mathrm{MeV} \text{ (gaseous)}$$

Summary Stochastic Term in EM Resolution

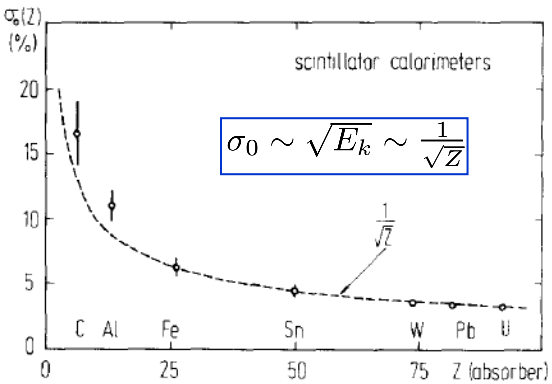
Basic (`approx. B') formula

$$\left(\frac{\sigma_E}{E}\right)_{samp} = \frac{1}{\sqrt{N_{vis}}} = \sqrt{\frac{t\, E_k}{E_0}}$$

An effective formula accounts for both active and passive layer:

$$\frac{\sigma_E}{E} = \frac{\sigma_0}{\sqrt{E}} \frac{t^\alpha}{s^\beta}$$

$$x_p = t \cdot X_0^p \qquad x_a = s \cdot X_0^a$$



has to be corrected by:

- quantum fluctuations
- energy thresholds of particle signals
- multiple scattering
- pathlength fluctuations
- Landau fluctuations

[Del Peso&Ros]

passiv	aktiv	σ_0 [%]	α	β
C	Szin.	16.48±2.50	0.72±0.03	0.16±0.02
Al	Szin.	11.02±1.21	0.70±0.03	0.15±0.02
Fe	Szin.	6.33±0.52	0.62±0.03	0.21±0.02
Sn	Szin.	4.53±0.32	0.65±0.03	0.25±0.03
W	Szin.	3.61±0.17	0.70±0.03	0.29±0.03
Pb	Szin.	3.46±0.19	0.67±0.03	0.29±0.03
U	Szin.	3.28±0.15	0.67±0.03	0.30±0.03
Pb	Si	5.04±0.20	0.66±0.03	0.24±0.03
Pb	LAr	6.49±0.31	0.62±0.03	0.19±0.03

Noise and Constant Term in Resolution

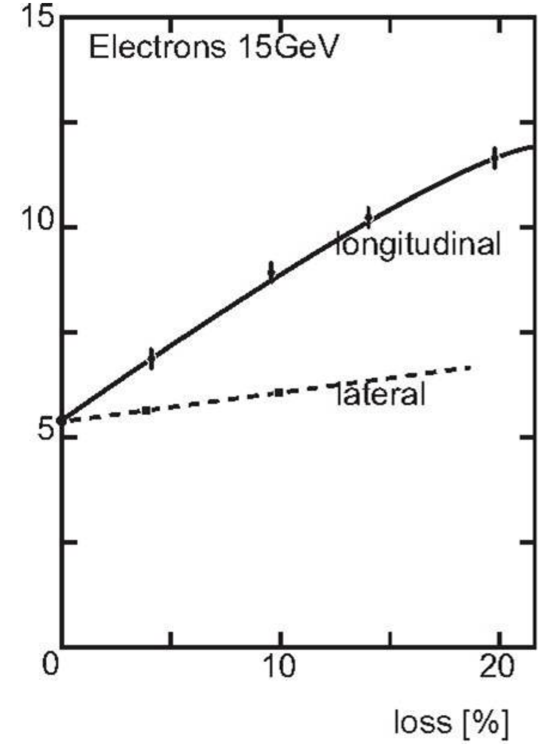
Noise ~1/E: dominates at low energy;

- electronic : O(10 keV) for PMT readout of crystals to O(100 MeV) for ionisation chamber readout (keep capacitances small for charge collection detectors).
- pile-up noise in high intensity experiments (LHC, ..)

Term ~ const: dominates at high energy

- leakage,
- mechanical and electronic tolerances,
- intercalibration errors, ..

$$\left(\frac{\sigma_E}{E}\right) \approx \left(\frac{\sigma_E}{E}\right)_{f_{\text{leak}}=0} \cdot \left(1 + 2 \cdot \sqrt{E(GeV)} \cdot f_{\text{leak}}\right)$$



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