Lagrangian sets in turbulent flows

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SFT Meeting, 25.08.2022

Lagrangian coherence

Given N particle trajectories at discrete times Γ = *{0, 1, …, T}*

$$x_i(t) \text{ with } I = 1, \ldots, N \text{ and } t = 0, \ldots, T$$

Coherent set: subset of particles which remain close to each other over defined time span

- **Spatio-temporal clustering on particle trajectories**

Weighted network ansatz:

instantaneous adjacency matrix $A_t = 1,\ d_{ij}(t) \leq \varepsilon \mid 0,\ else$

network weight matrix $W = \Sigma_{t \text{ in } \Gamma}\ A_t$

Clustering on the eigenvectors associated with the largest eigenvalues of

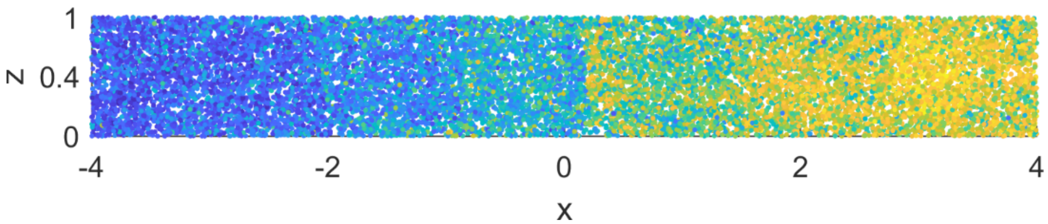
$L = D^{-1/2} W D^{-1/2}$ where D is the diagonal degree matrix with $d_{ii} = \Sigma_j w_{ij}$.

[1] Shi, Malik 2000

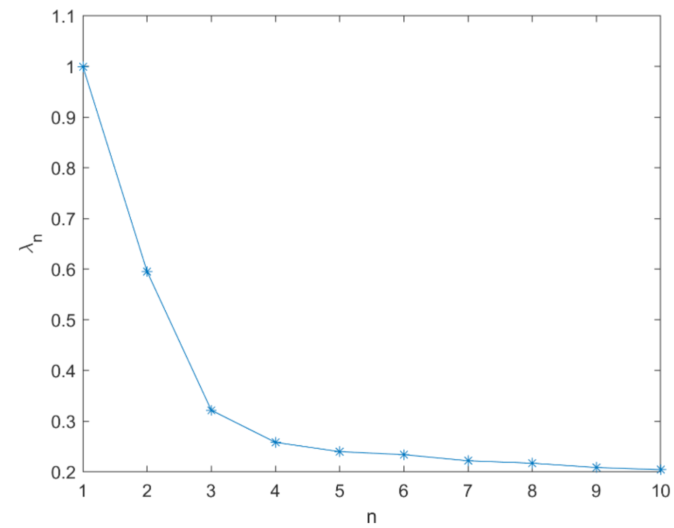
2d Rayleigh-Bénard Convection

Static clustering

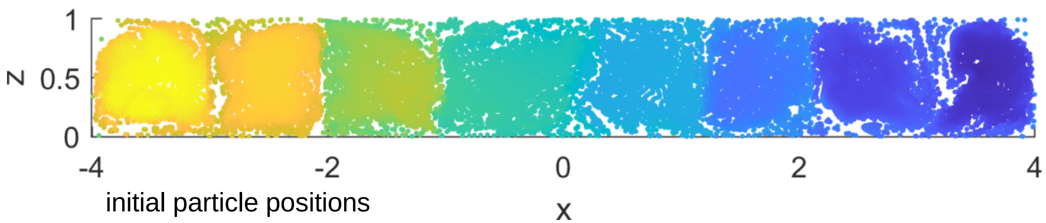
Clustering over long time span



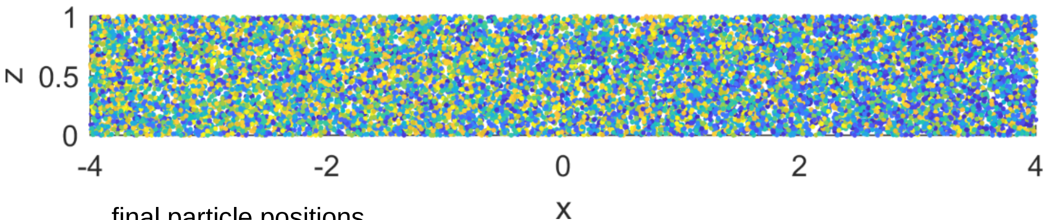
final particle positions



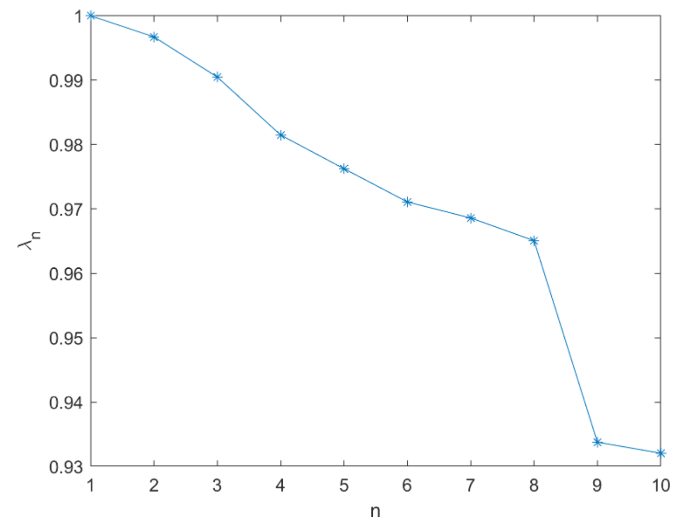
Clustering over short time span



initial particle positions



final particle positions



Dynamic Community Discovery[2]

DCD		
instant optimal	temporal trade-off	cross-time
dependent on current state	dependent on current and previous states	dependent on all current and future states

amount of dynamic information

[2] Rossetti, Cazabet 2018

Evolutionary clustering 1

Aim: smooth variation of the network weight matrix

1) Consider shorter time periods τ centered at t, i.e. $\Gamma(t,\tau) = \{t-\tau/2, \ldots, t+\tau/2\}$

2) **Evolutionary network approach:** time-dependent weight matrix

$$W_t = \Sigma_{s \text{ in } \Gamma(t,\tau)} A_s$$

3) Evolutionary clustering on the eigenvectors associated with the largest eigenvalues of $L_t = D_t^{-1/2} W_t D_t^{-1/2}$. [3]

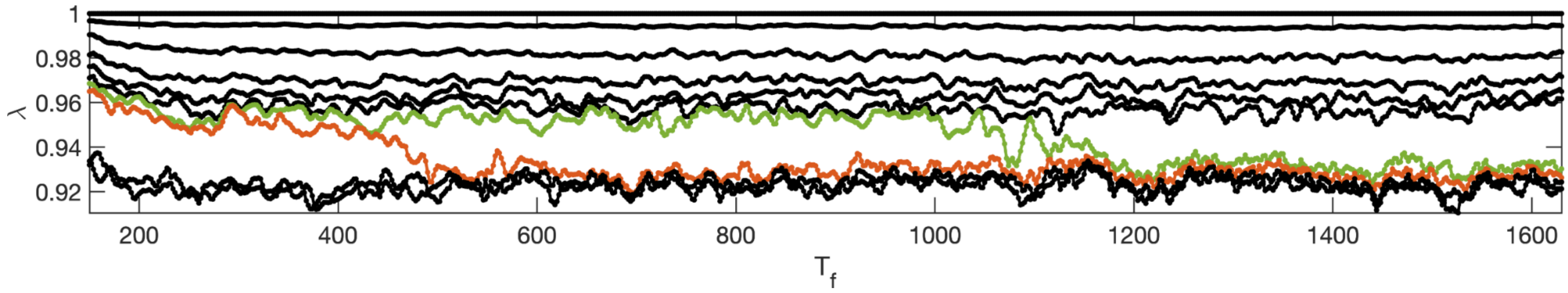
4) Extract particles with large cluster membership likelihood using the **S**parse **E**igen**B**asis **A**pproximation[4].

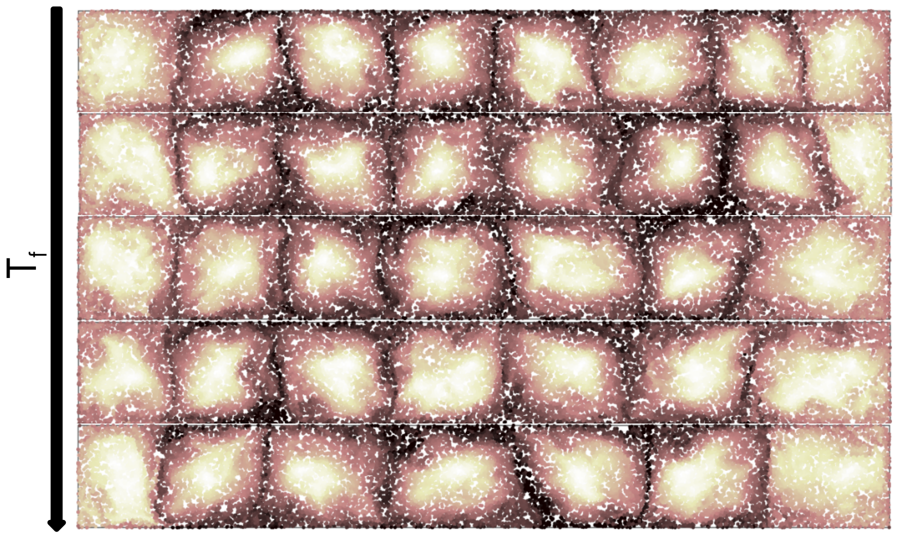
[3]Chi et al. 2007

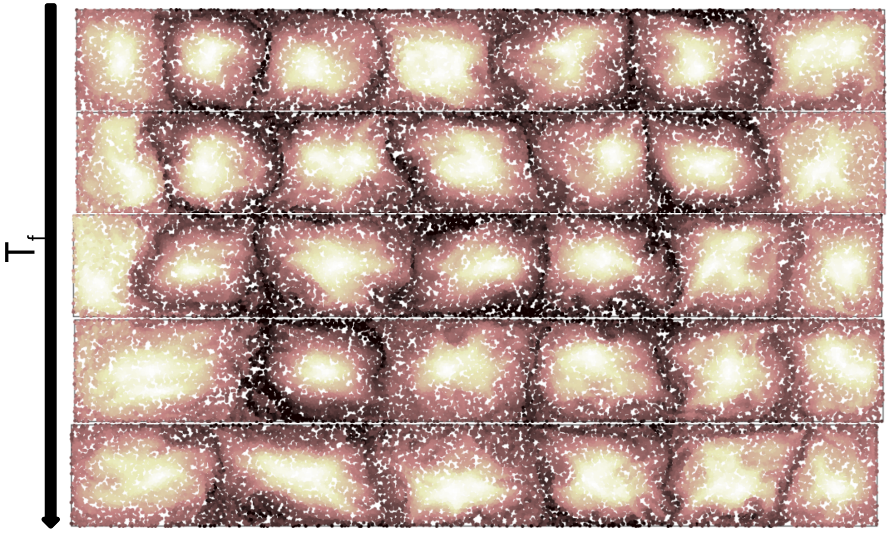
[4]Froyland et al. 2019

2d Rayleigh-Bénard Convection

Evolutionary clustering

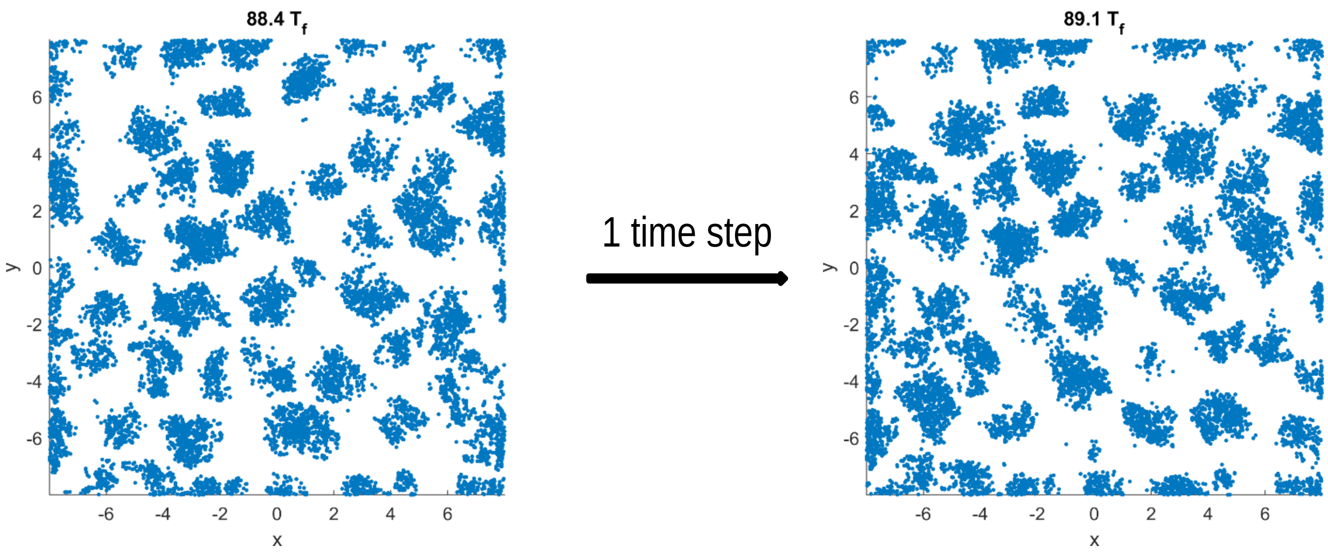






3d Rayleigh-Bénard Convection

- Large number of particles (> 60000)
- Physical estimate on number of sets: ~ 80
- Clustering algorithms may not be stable



Evolutionary clustering 2

Aim: smooth variation of the clustering

1) Create a matrix that incorporates the current clustering with factor α and the previous clustering with factor (1-α) [3].

2) Solve the eigenvalue problem of the above matrix.

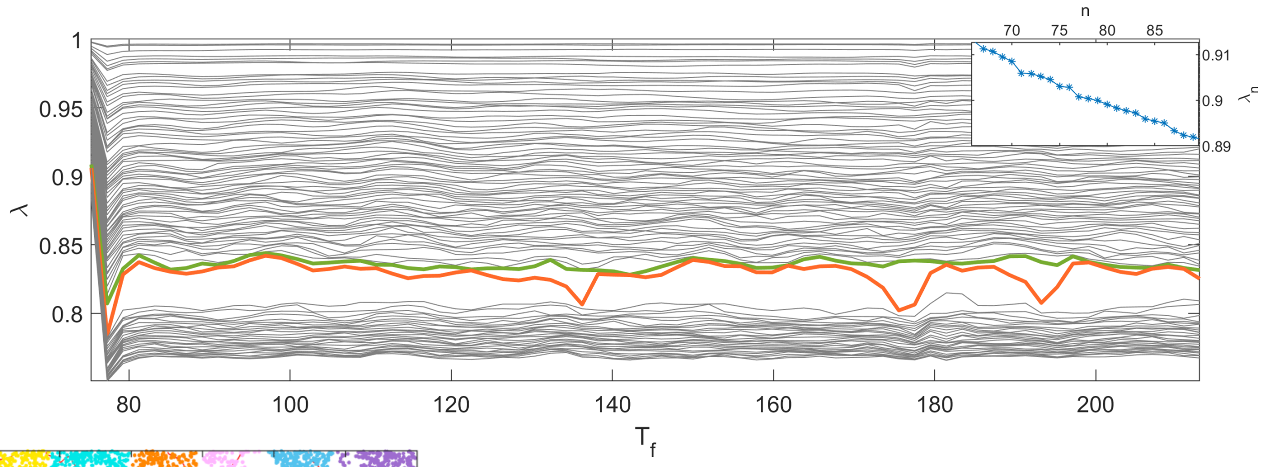
3) Create a sparse approximation of the eigenspace using SEBA.

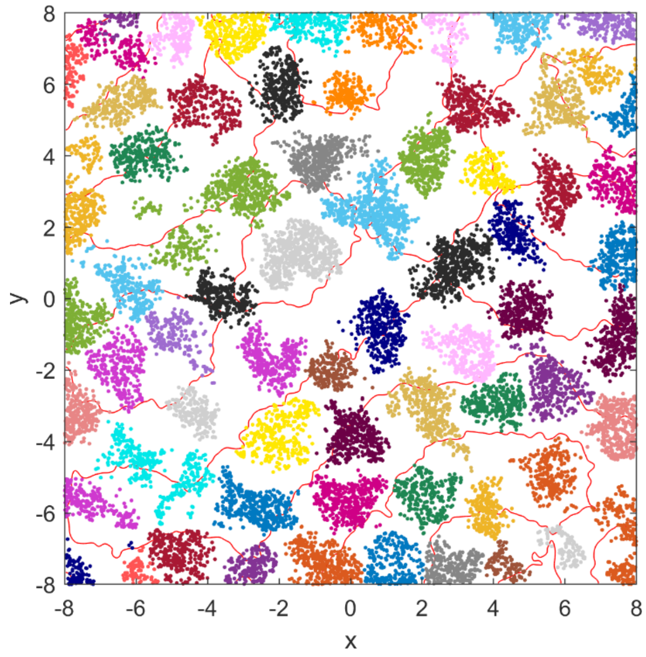
4) Extract particles with large cluster membership likelihood.

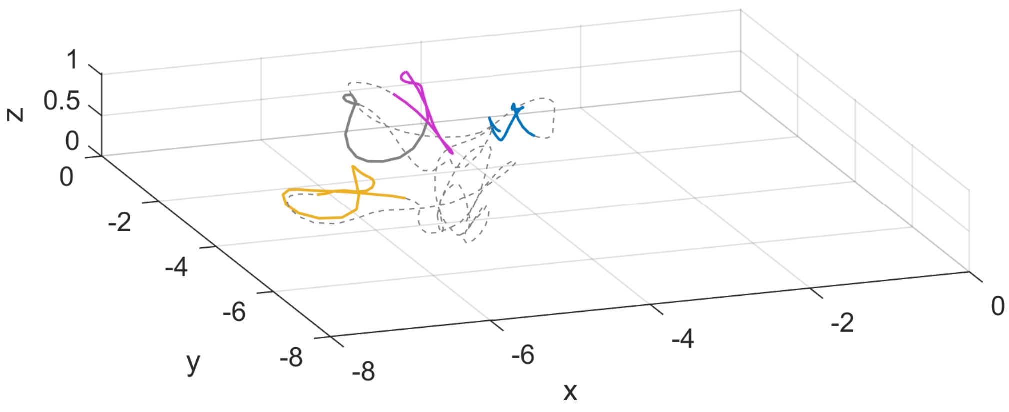
[3]Chi et al. 2007

3d Rayleigh-Bénard Convection

Evolutionary clustering







Back-up