Weizmann Institute

# Degeneracies in Cosmography Analyses MMS annual meeting 2023

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#### Measuring H<sub>o</sub> with lensing



- TDCOSMO collaboration (COSMOGRAIL, HoLiCOW, STRIDES, SHARP, COSMICLENS)
- Unaccounted galactic density profile features (core? Nearby group?): internal mass sheet degeneracy

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 Large scale structure "on the way": external mass sheet degeneracy (weak lensing)



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Wong et al. 2019



- Small recap: how to measure H<sub>o</sub> from gravitational lensing
- Large scale structures
- Non-trivial dark matter dynamics (Ultralight dark matter)
- Nearby group

### Strong Gravitational Lensing in Elliptical Galaxies





- $\hat{\alpha} = 2 \int \nabla_{\perp} \Phi \, d\lambda \implies \vec{\beta} = \vec{\theta} \frac{D_{\rm LS}}{D_{\rm S}} \hat{\alpha}(\vec{\theta})$
- Convergence  $\kappa \sim \int dz \rho$ .
- Lens model + time delay measurement

$$\Delta t_{ij} \propto rac{1}{H_0}$$
 .

• Degeneracies: source position and mass of galaxy unknown

$$\vec{\beta} 
ightarrow \lambda \vec{\beta} \,, \, \kappa 
ightarrow \lambda \kappa + (1 - \lambda) \implies H_0 
ightarrow \lambda H_0$$

#### Strong Gravitational Lensing in Elliptical Galaxies



credit: Wikipedia

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• Degeneracies: source position and mass of galaxy unknown

$$\vec{\beta} \to \lambda \vec{\beta} \,, \, \kappa \to \lambda \kappa + (1 - \lambda) \implies H_0 \to \lambda H_0$$



from V. Bonvin et al (2016)

#### Strong Gravitational Lensing in Elliptical Galaxies



• 
$$\hat{\alpha} = 2 \int \nabla_{\perp} \Phi \, \mathrm{d}\lambda \implies \vec{\beta} = \vec{\theta} - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \hat{\alpha}(\vec{\theta})$$

- Convergence  $\kappa \sim \int dz \rho$ .
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$$\Delta t_{ij} \propto rac{1}{H_{
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Degeneracies: source position and mass of galaxy unknown

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from V. Bonvin et al (2016)

#### Effects of Large Scale Structure



External convergences  $\kappa$  are unobservable, can be removed from modeling (not completely true in multiple source systems!)

•  $\kappa^{\rm S}$  works as a MSD,  $\kappa^{\rm LS}$  and  $\kappa^{\rm L}$  can be reabsorbed in lens parameters

• Bias in time delays:

$$\frac{H_{\rm o}^{\rm inferred}}{H_{\rm o}^{\rm true}} = \frac{1 - \kappa^{\rm LS}}{1 - \kappa^{\rm L}}$$

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#### Internal Mass sheet degeneracy







 Possible cores from non-trivial DM models, e.g. Ultralight dark matter, cores

$$\lambda_{\rm deBroglie} = rac{1}{mv} \sim 20 \, {\rm kpc} rac{1 imes 10^{-24} \, {\rm eV}}{m}$$

- Effect of galaxy group. Galaxy kinematics can constrain how much mass in the group.
- Kinematics measurements fundamental to constrain MSD, reminding that exact MSD ≠ approximate MSD

$$M \rightarrow (1-\kappa_{\rm c}) M + M_{\rm c} \;,\; M_{\rm c} \sim \kappa_{\rm c} \frac{r^3}{r_{\rm c}} \label{eq:mass_state}$$

#### Multi-source systems and MSD

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019





- External convergences and convergences from approximate cores (from non-trivial DM or groups) are unobservable, in single source systems
- For multiple source systems, observable (*C<sub>i</sub>* involves ratio of angular diameter distances):

$$\gamma_{1i} := \delta \kappa_{1i}^{\mathrm{S}} - \delta \kappa_{1i}^{\mathrm{LS}} + \kappa_{\mathrm{c1}} \left( 1 - \mathsf{C}_{i} \right)$$

 If γ<sub>1i</sub> is too large to be explained by external convergences, it might be an indication of a non-zero, unaccounted κ<sub>c</sub>!

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### Discrete flexion degeneracy from a NFW group



- Lens galaxies often reside in groups
- First order α expansion ~ convergence; second order: flexion F, G.
- $F \sim {\rm e}^{{\rm i}\phi} F_O, G \sim {\rm e}^{3{\rm i}\phi} G_O$
- Falling into the false center yields a bias in H<sub>o</sub> (due to the different convergence inferred)
- Measuring the velocity dispersion of galaxies ⇒ find a prior on mass, mitigating the MSD.







- Challenge: modeling degeneracies; important to point out all the shortcomings to lensing collaborations, in other to achieve an *accurate* H<sub>o</sub> measurement.
- Promising! New data arriving, including precise stellar kinematics, new lensed systems etc.
- Effects from observer-lens and lens-source sightlines *must* be taken into account for a non-biased inference of time delays and non-biased stellar kinematics constraint.
- With multi-source systems, you might spot the core (still not possible yet).
- The presence of a group nearby can bias the H<sub>o</sub> inference if the flexion degeneracy is not taken into account.
- With an H<sub>o</sub> prior, we can measure galactic features like large cores, difficult to spot otherwise, and a better characterization of the weak lensing field

## The multilens equation

The tidal approximation



Lens equation in tidal approximation

$$\vec{eta} = (\mathbb{I} - M^{\mathrm{S}})\vec{ heta} - (\mathbb{I} - M^{\mathrm{LS}})\vec{lpha}((\mathbb{I} - M^{\mathrm{L}})\vec{ heta})$$

Degeneracy ("revised" MSD)

$$\begin{split} \mathbf{1} &- \mathbf{M}^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}} (\mathbf{1} - \mathbf{M}^{\mathrm{R}}), \\ & \vec{\beta} \longmapsto \lambda_{\mathrm{S}} \vec{\beta}, \\ & \vec{\alpha} (\vec{\theta}) \longmapsto \lambda_{\mathrm{S}} \lambda_{\mathrm{LS}}^{-1} \vec{\alpha} (\lambda_{\mathrm{L}}^{-1} \vec{\theta}), \\ & \Psi (\vec{\theta}) \longmapsto \lambda_{\mathrm{S}} \lambda_{\mathrm{LS}}^{-1} \lambda_{\mathrm{L}} \Psi (\lambda_{\mathrm{L}}^{-1} \vec{\theta}) \end{split}$$

Choose

$$\lambda_{\mathrm{S}} = \frac{1}{1-\kappa^{\mathrm{S}}}, \quad \lambda_{\mathrm{LS}} = \frac{1}{1-\kappa^{\mathrm{LS}}}, \quad \lambda_{\mathrm{L}} = \frac{1}{1-\kappa^{\mathrm{L}}},$$

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external convergence removed from the modeling

#### Interpreting the Mass Sheet Degeneracy

#### Springer et al 2005



Suvu et al 2010





- Change  $\lambda_S$  : changing one's mind about the true  $\kappa^S$
- •

$$\kappa^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}} \kappa^{\mathrm{R}} + (1 - \lambda_{\mathrm{R}})$$
$$\Gamma^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}} \Gamma^{\mathrm{R}}$$

Time delays do change!

$$\begin{split} \Delta \tau &\to \lambda_{\rm S} \lambda_{\rm LS}^{-1} \lambda_{\rm L} \Delta \tau \\ {\rm H_o} &\to \lambda_{\rm S} \lambda_{\rm LS}^{-1} \lambda_{\rm L} {\rm H_o} \end{split}$$

- Estimate  $\kappa^{\rm S}$  via ray-tracing through Millennium Simulation and characterization of the lens field
- Degeneracy is limited by priors on weak lensing quantities and constraints on mass of lens galaxy (stellar kinematics)

#### Multi-source systems and MSD

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019





• New lens equation (ignoring lens-lens coupling)

$$\begin{split} \vec{\beta_i} = & (\mathbb{I} - M_i^{\mathrm{S}})\vec{\theta} - (\mathbb{I} - M_i^{\mathrm{LS}})C_i\vec{\alpha_1}((\mathbb{I} - M^{\mathrm{L}})\vec{\theta}) \\ & C_i := & \frac{D_{\mathrm{S}_1}D_{\mathrm{LS}_1}}{D_{\mathrm{LS}_1}D_{\mathrm{S}_1}} , \ \left(\hat{\alpha}\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} =: \vec{\alpha}\right) \end{split}$$

- Observable:

$$C_{i} \frac{\left| \vec{\alpha}_{1}^{\mathrm{model}}(\vec{\theta}) \right|}{\left| \vec{\alpha}_{2}^{\mathrm{model}}(\vec{\theta}) \right|} \approx \left( 1 + \delta \kappa_{12}^{\mathrm{S}} - \delta \kappa_{12}^{\mathrm{LS}} \right)$$