GBP Main Meeting

Ptarmigan LMA Simulations

K Fleck - 21/09/2022

Ptarmigan LMA simulations
Simulation parameters

- Laser
  - $\lambda = 0.8$ μm
  - $\tau_{FWHM} = 30$ fs
  - $E_L = 1.2$ J
  - Linear polarisation
- Electrons
  - $1.5 \times 10^9\, e^-$
  - 16.5 GeV
  - $\Theta_{rms} = 8.672$ μrad
  - $r_b = 5.0$ μm

Radiation reaction = on, pair production = off

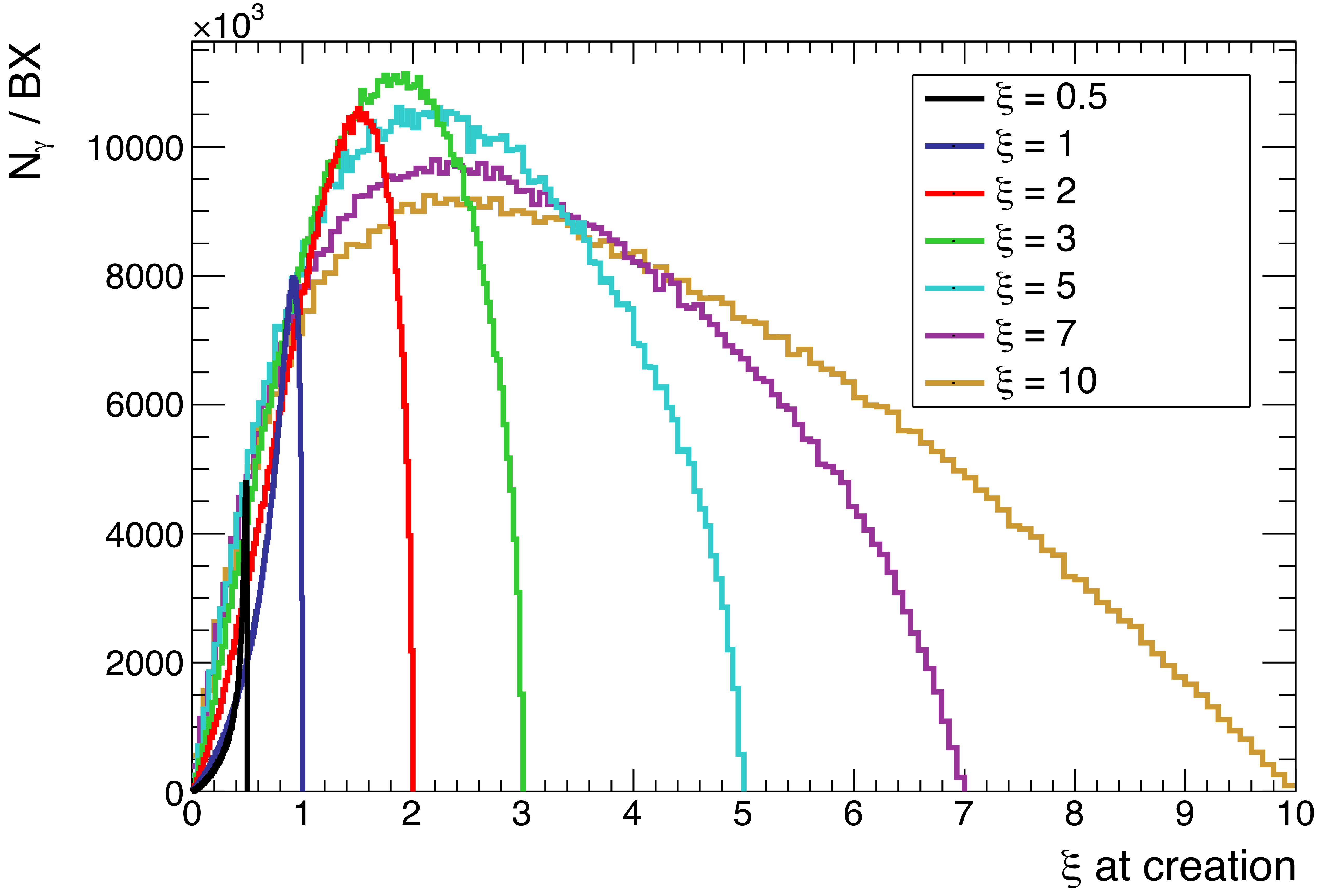
Ptarmigan LMA simulations

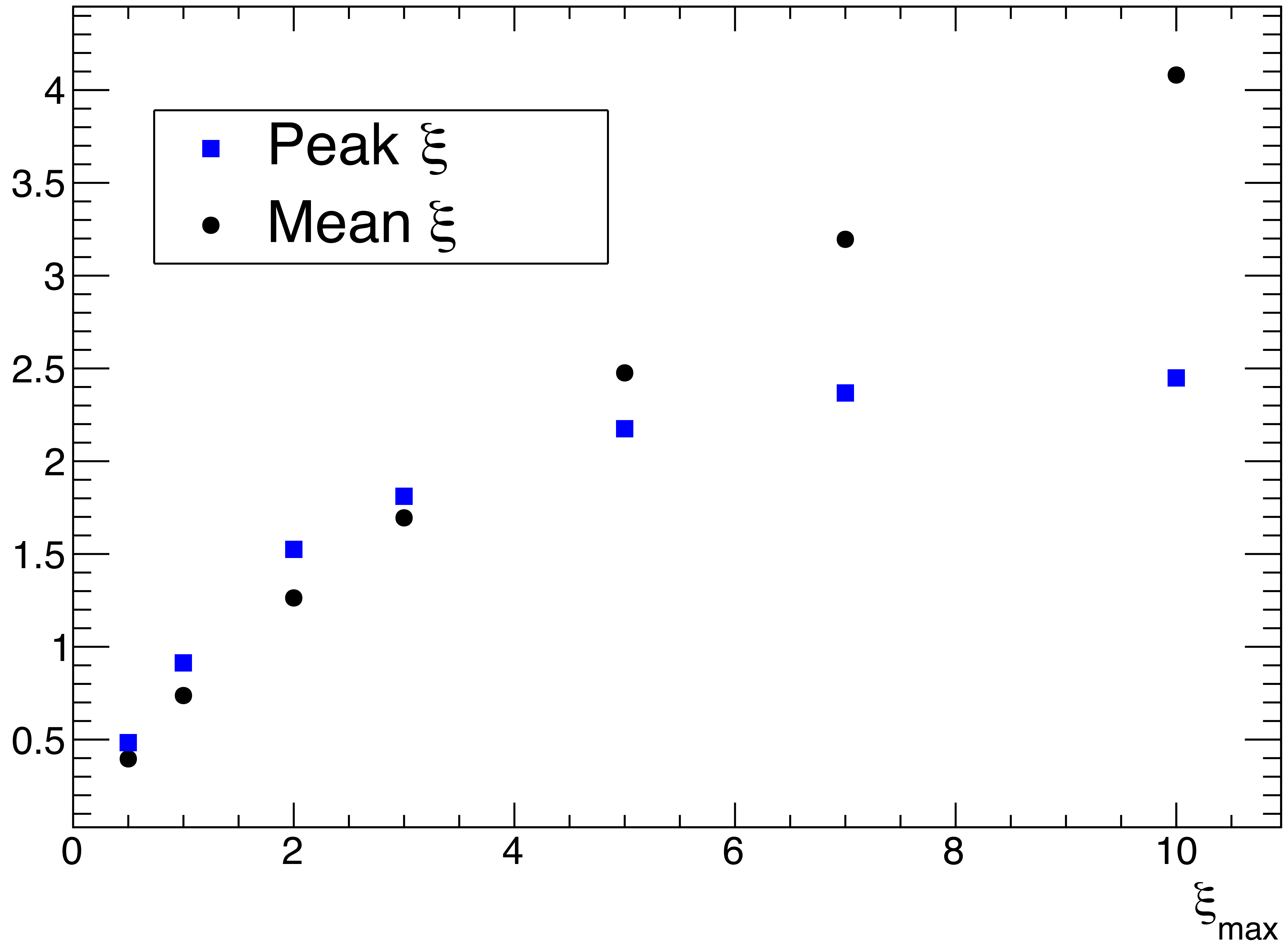
Simulation processing

Nominal ξ	Number of entries processed (1e8)	Number of entries in 10 BX (1e8)	Number of BXs processed
0.5	9.22	9.22	10.0
1.0	28.91	28.91	10.0
2.0	15.35	62.23	2.47
3.0	14.79	75.99	1.95
5.0	5.87	76.01	0.77
7.0	14.05	68.13	2.06
10.0	15.35	58.31	2.63

Ptarmigan LMA simulations

Laser intensity at creation

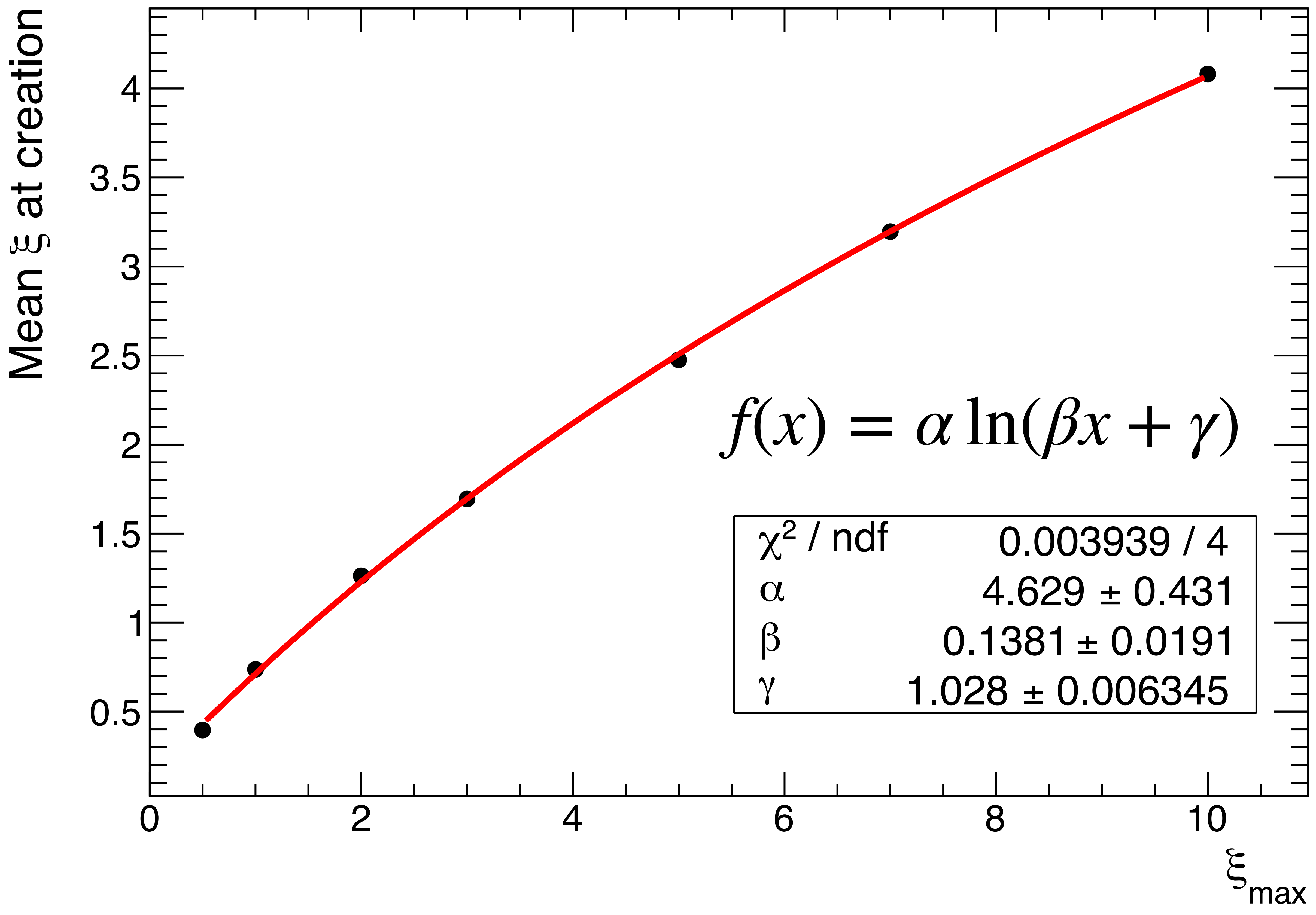


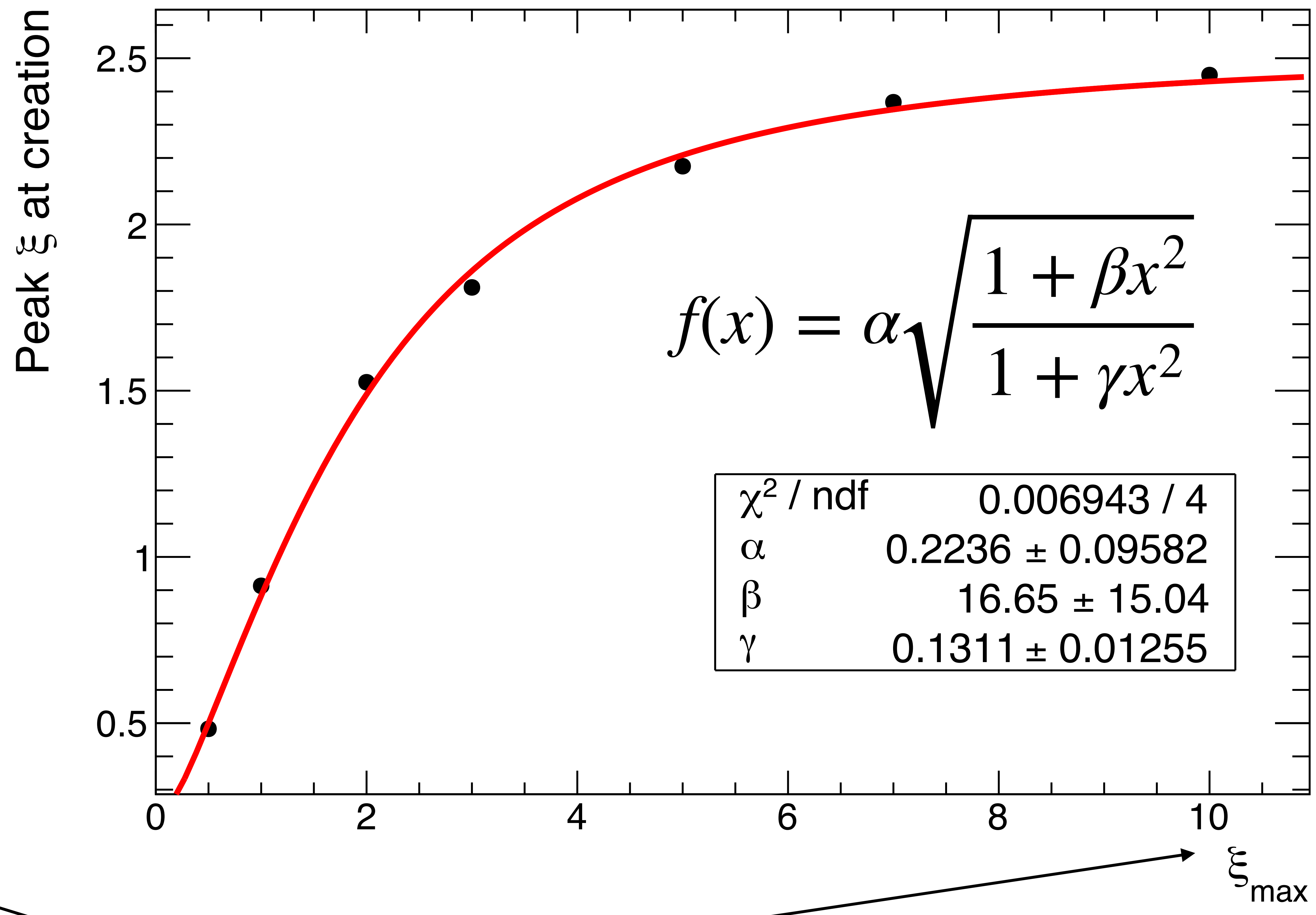


Ptarmigan LMA simulations

Laser intensity at creation

Functional form of fit taken from Blackburn et. al. 2020

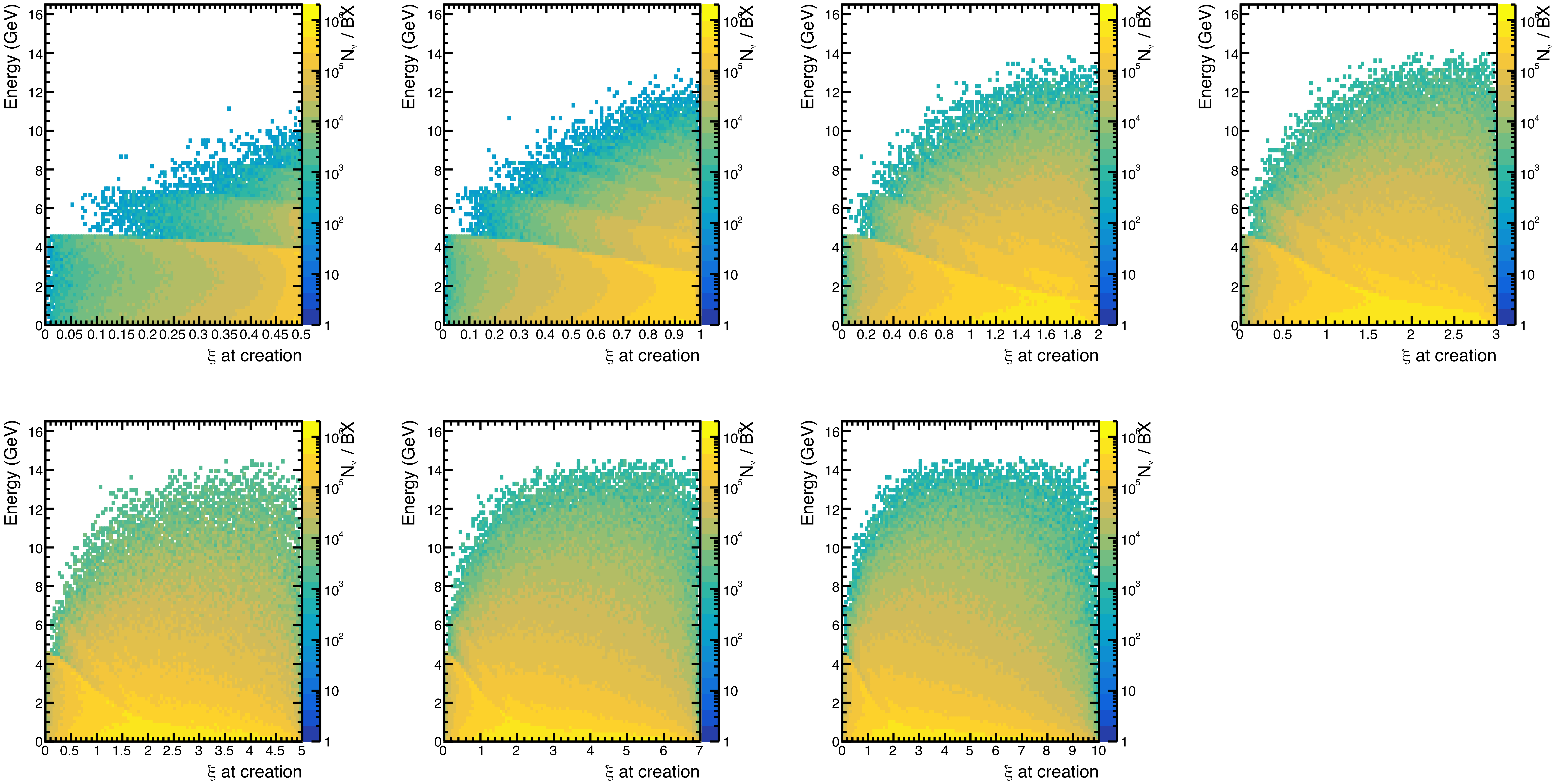




Same as nominal ξ

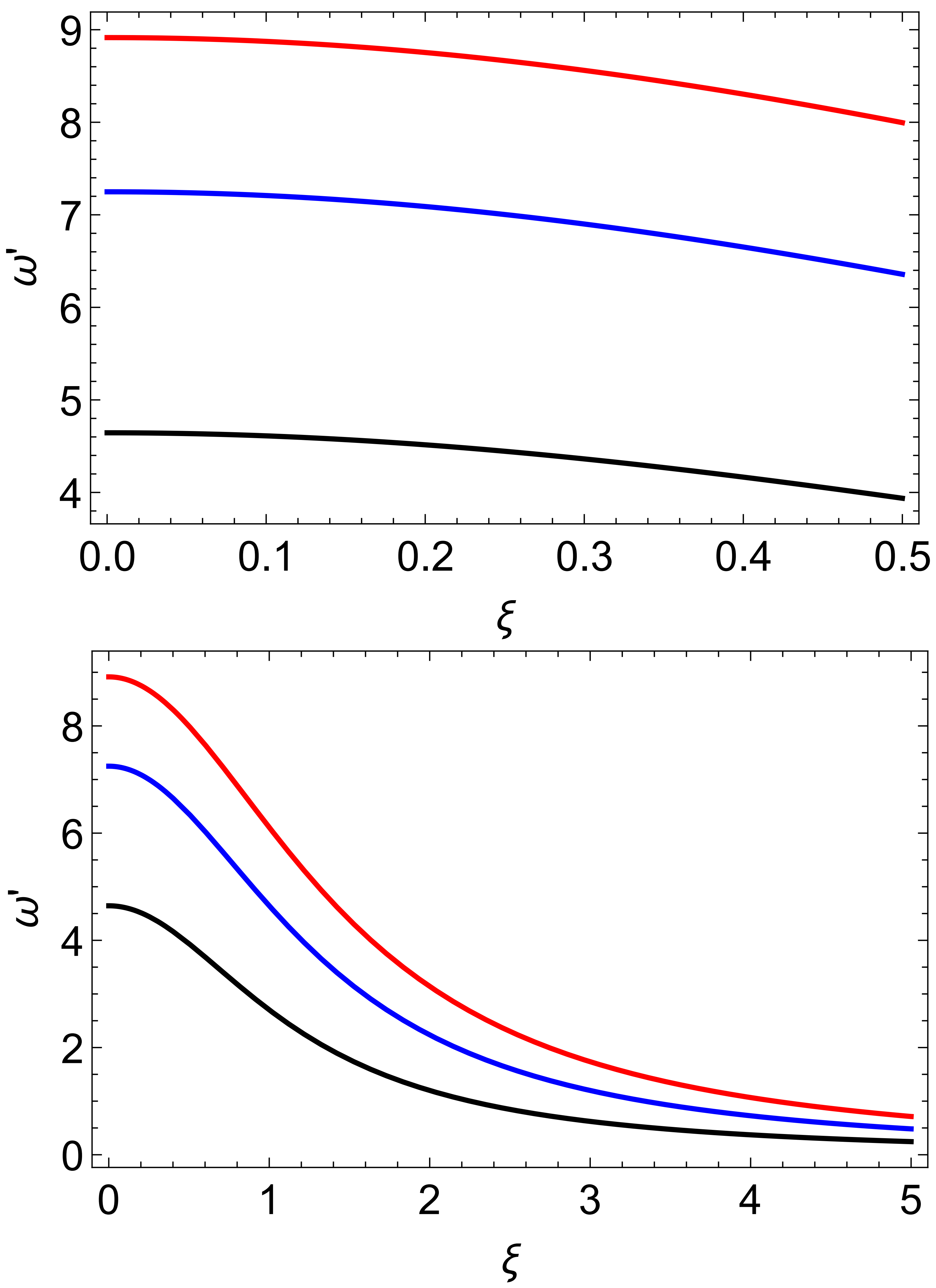
Ptarmigan LMA simulations

Laser intensity at creation with energy



Ptarmigan LMA simulations

Compton harmonics for IPWs



General case

$$\omega' = \frac{\nu k \cdot p}{(p + \nu k) \cdot n'}$$

$$\nu = \frac{k' \cdot p}{k \cdot p'} = \frac{k^{'-} + p^{'-} - p^-}{k^-}$$

Infinite plane wave case

$$\nu \to \nu_n = n - \frac{\xi^2}{4\eta} \frac{s}{1-s}$$

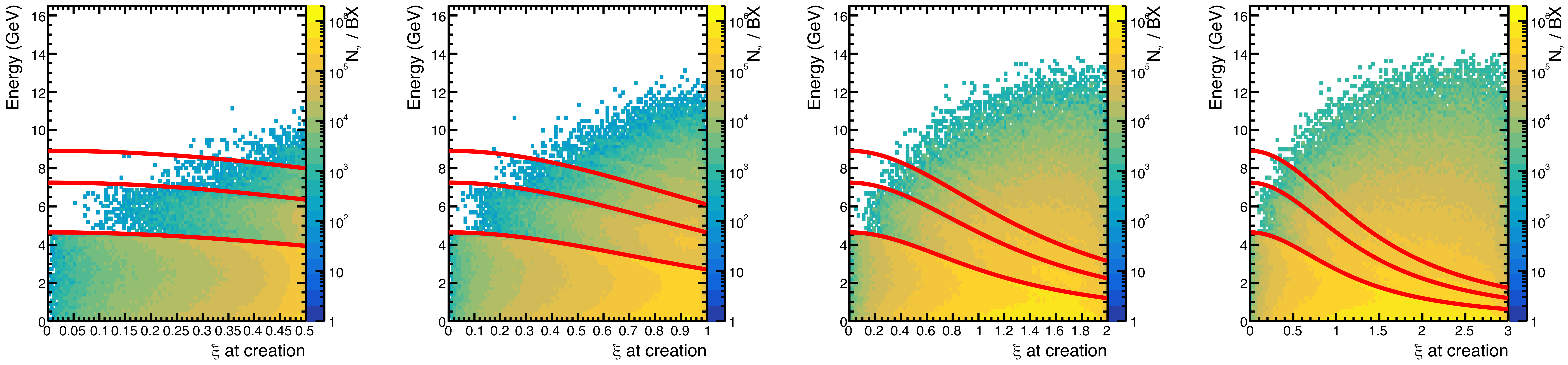
$$\omega'(n) = \frac{n\omega e^{2\zeta}}{1 + 2n\frac{\omega}{m}e^{\zeta} + \xi^2}$$

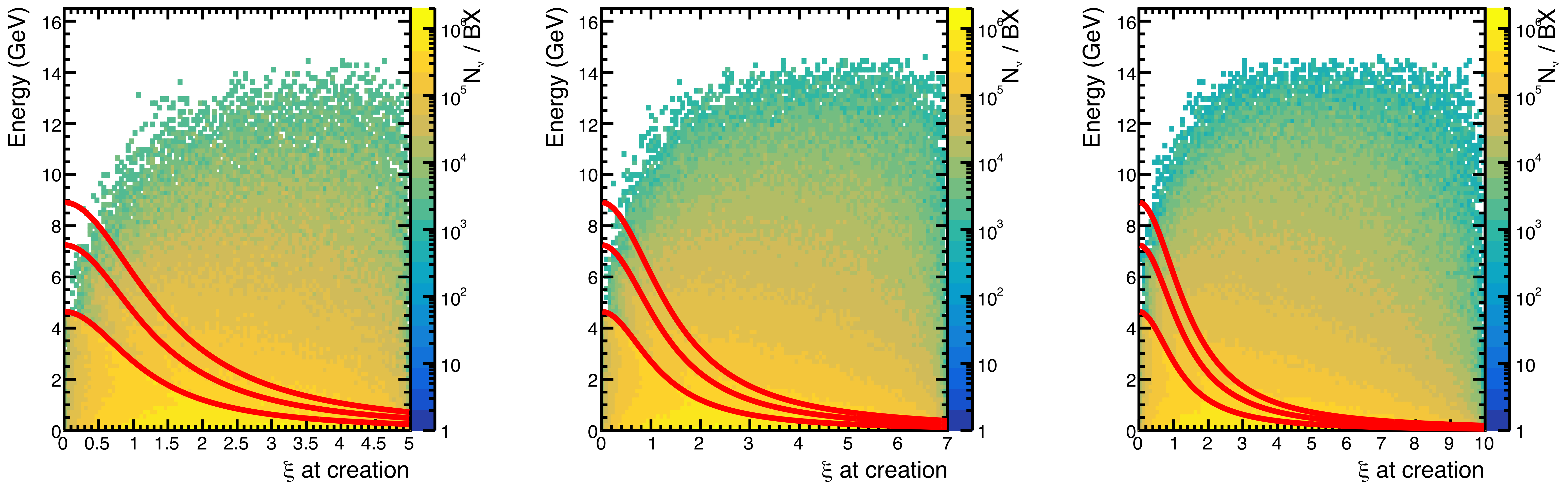
$$\zeta = \operatorname{arccosh} \gamma$$

Ptarmigan LMA simulations

Laser intensity at creation with energy with harmonics

Same plots as slide 6 but with first, second and third IPW harmonics overlayed

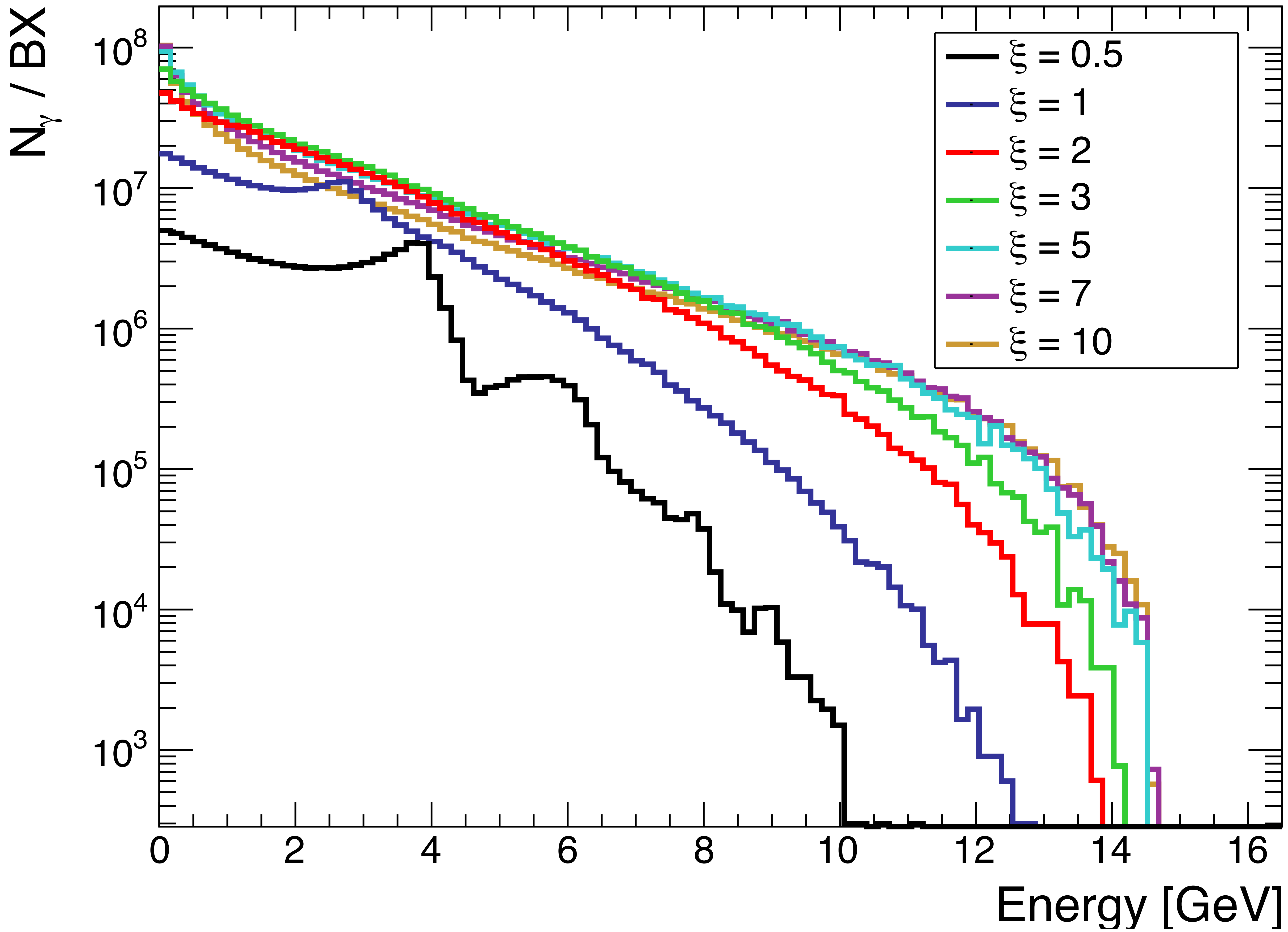




Broadening of harmonics in a focused beam is due to electrons seeing a range of ξ values

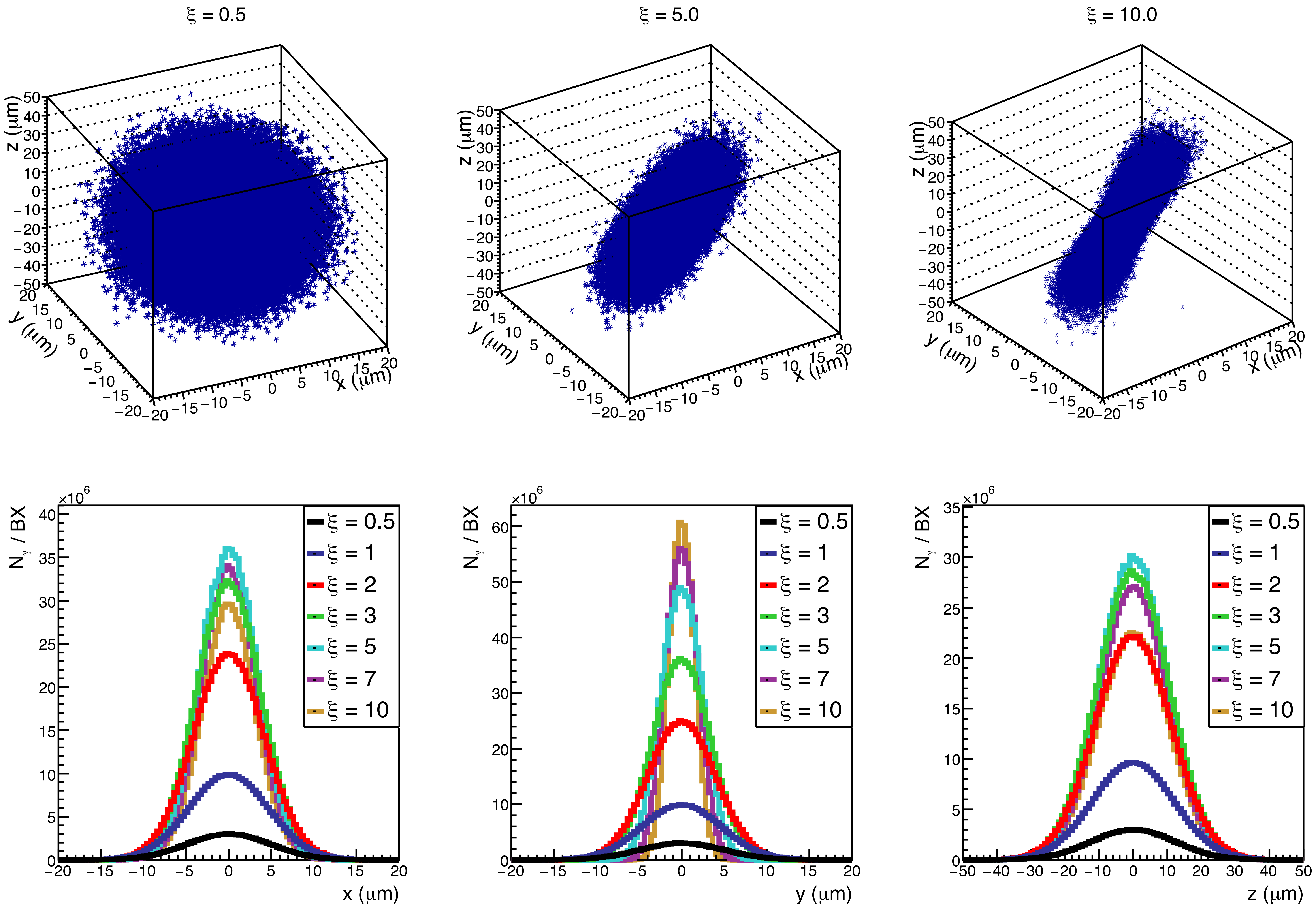
Ptarmigan LMA simulations

Energy spectra



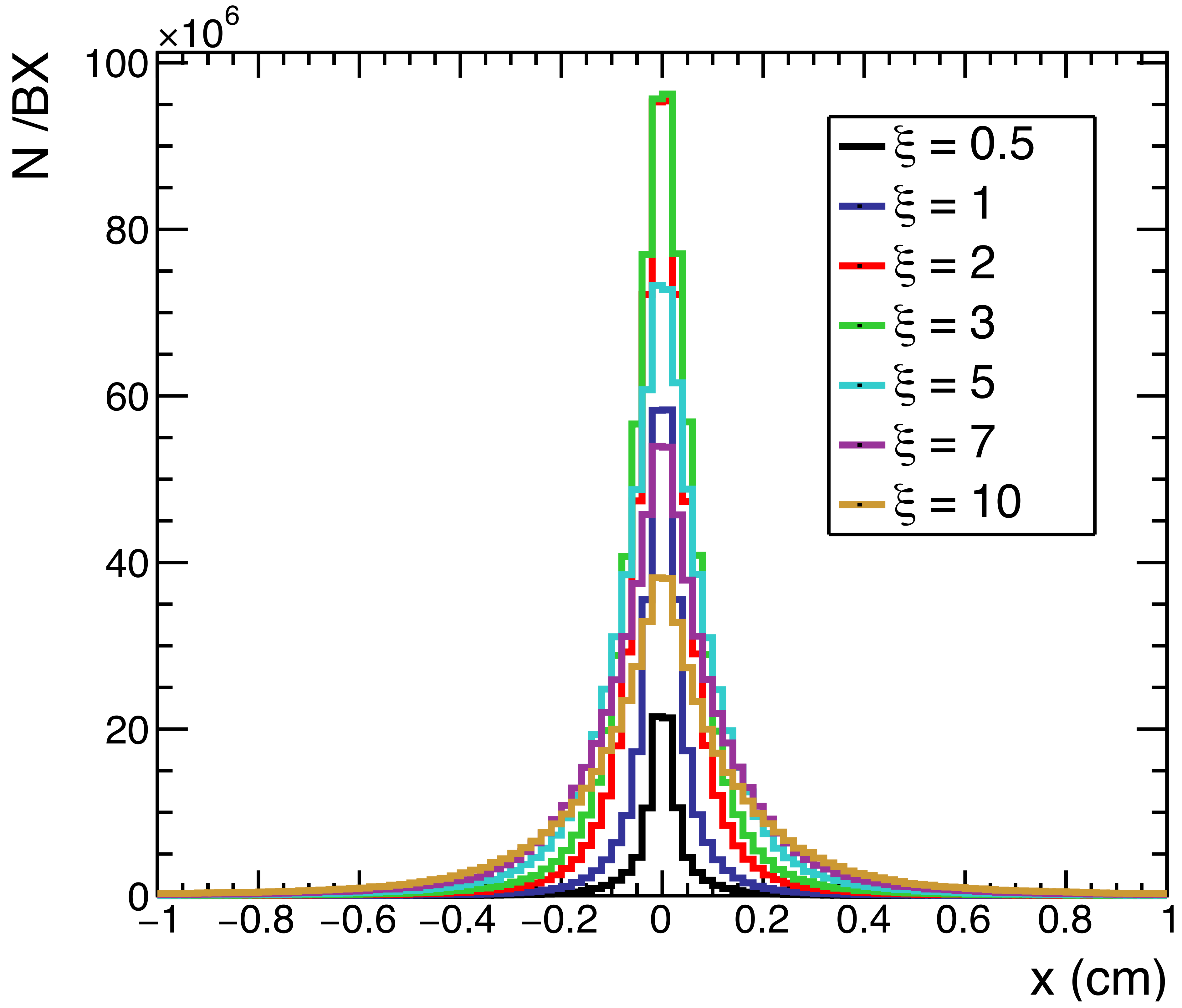
Ptarmigan LMA simulations

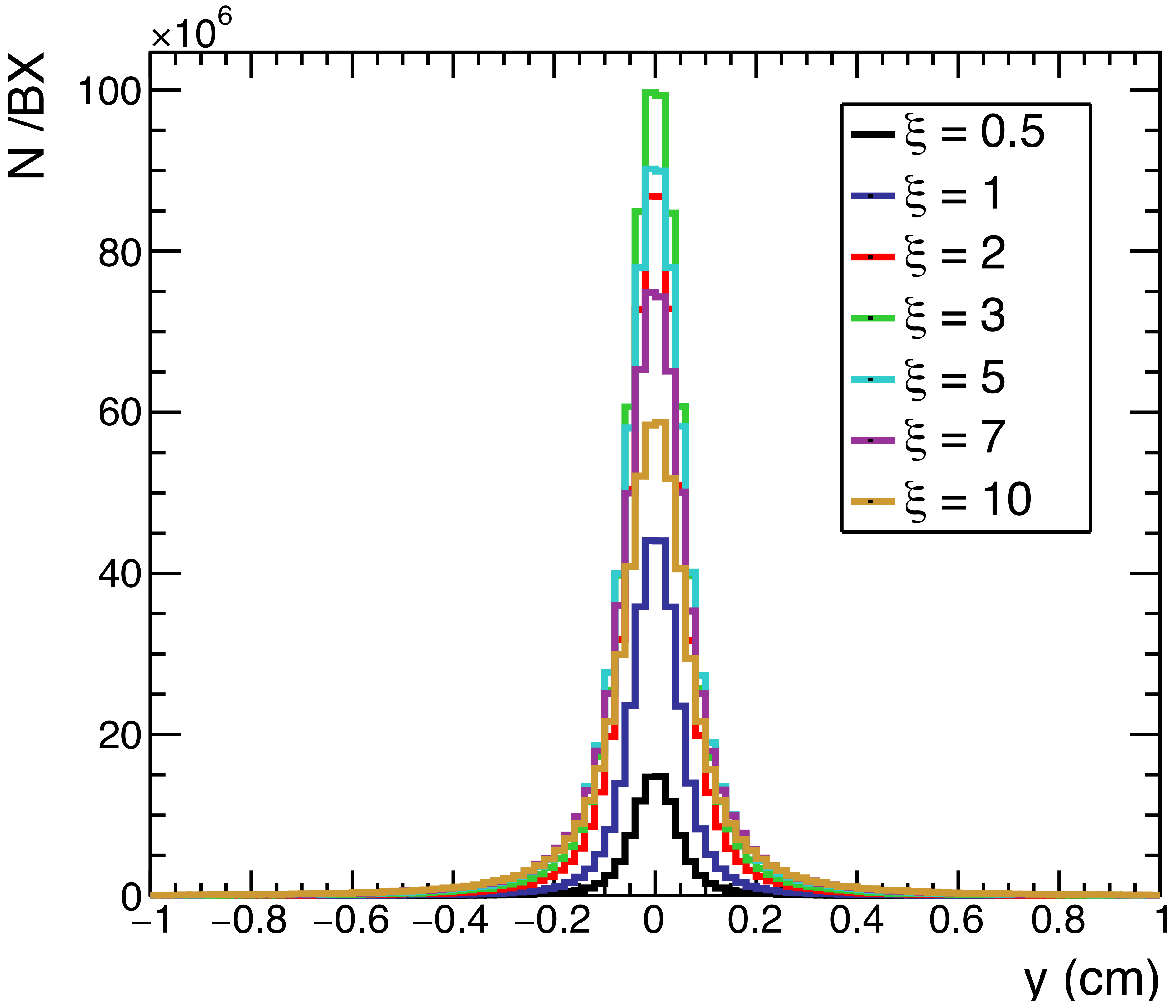
Spatial distribution at creation



Ptarmigan LMA simulations

Spatial distribution at profiler

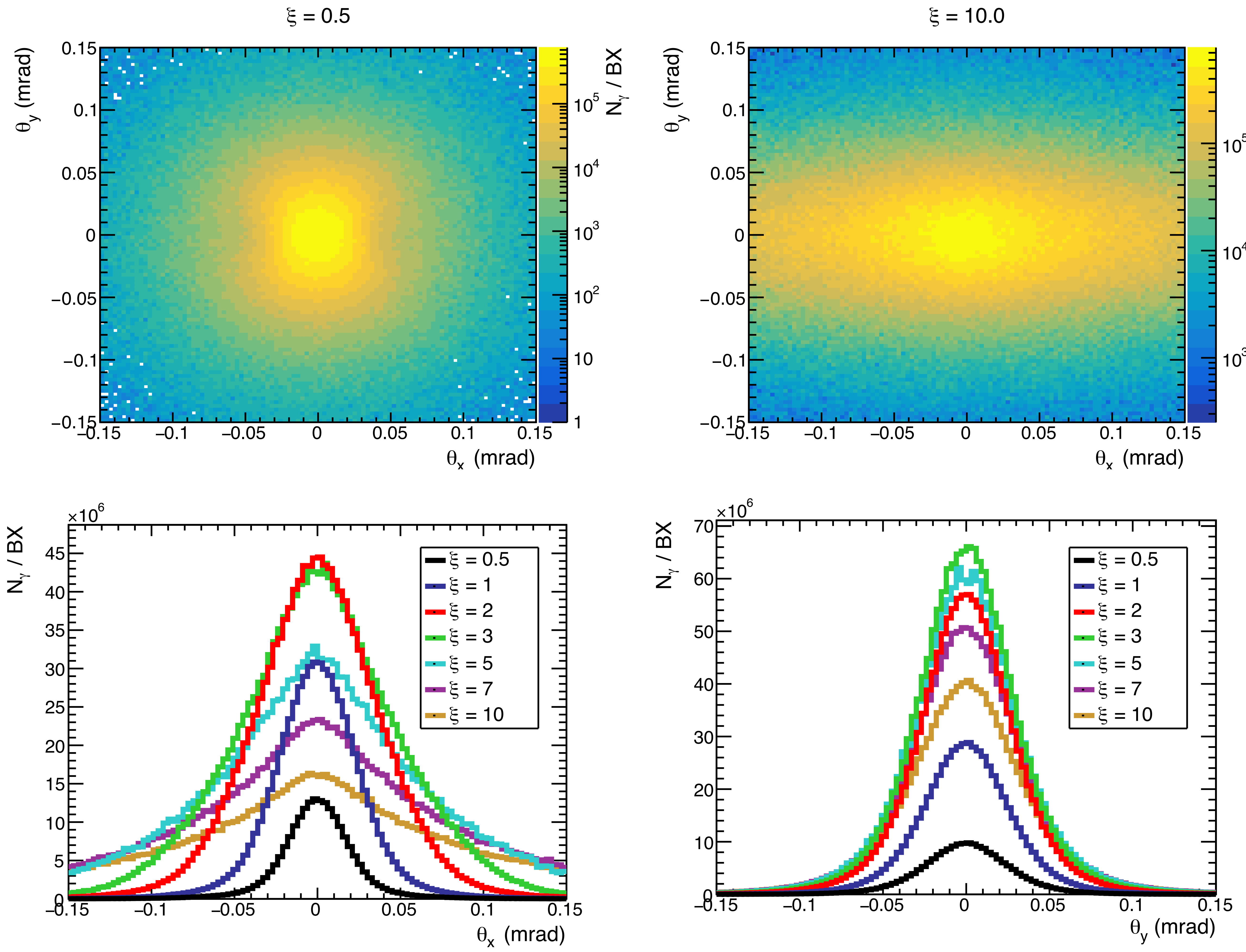




Projection taken at z = 11.5 m downstream of IP

Ptarmigan LMA simulations

Energy weighted radiation profile



Ptarmigan LMA simulations

Inference of laser intensity

- Inference of laser intensity follows Blackburn et. al. 2020

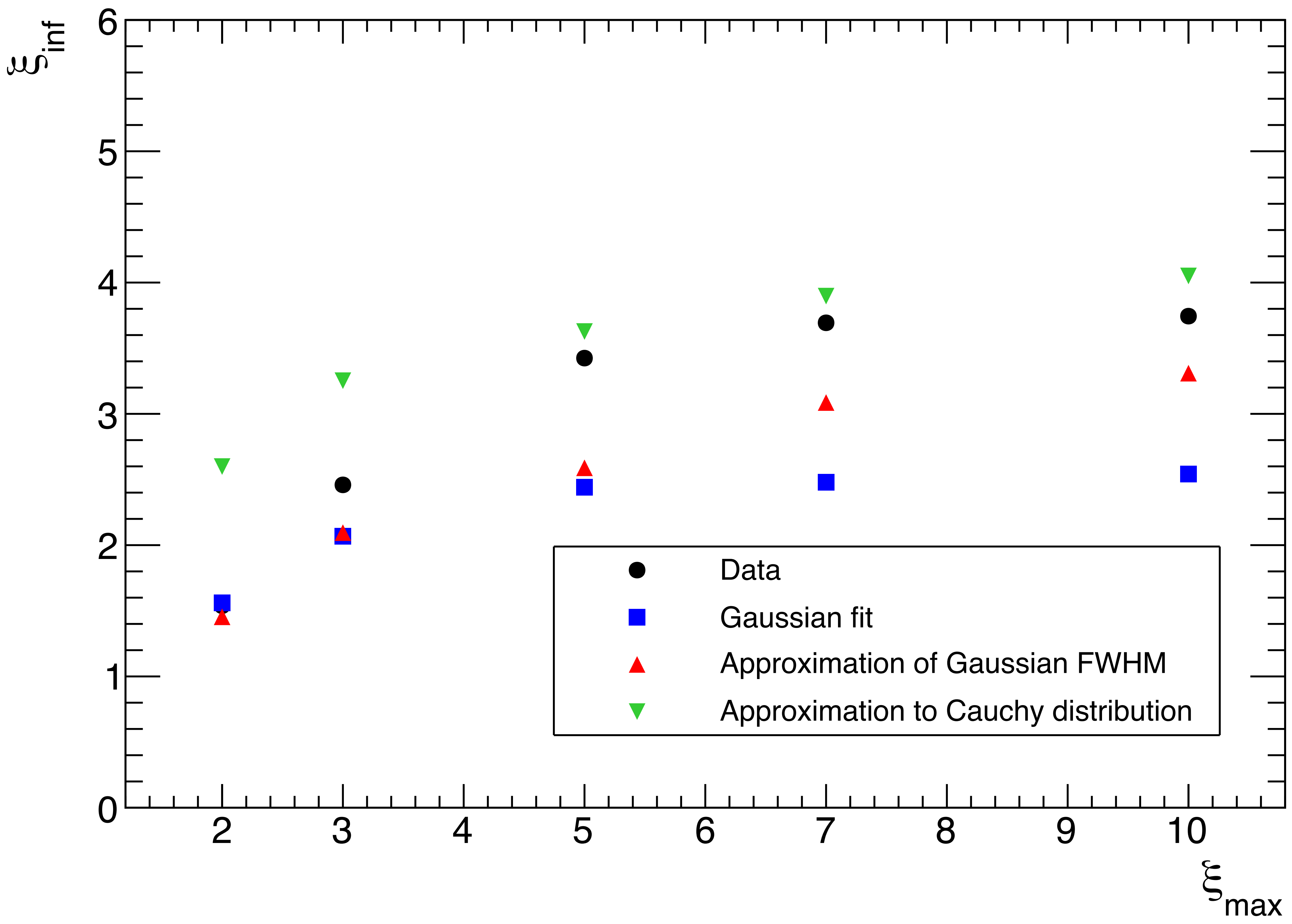
$$\xi = g(\rho)\, \xi_{inf} = \xi_{inf} \sqrt{\frac{1 + 8\rho^2}{1 + 4\rho^2}} \quad \rho = \frac{r_b}{w_0}$$

$$\xi_{inf}^2 = 4\sqrt{2} \langle \gamma_i \rangle \langle \gamma_f \rangle (\sigma_{\parallel}^2 - \sigma_{\perp}^2)$$

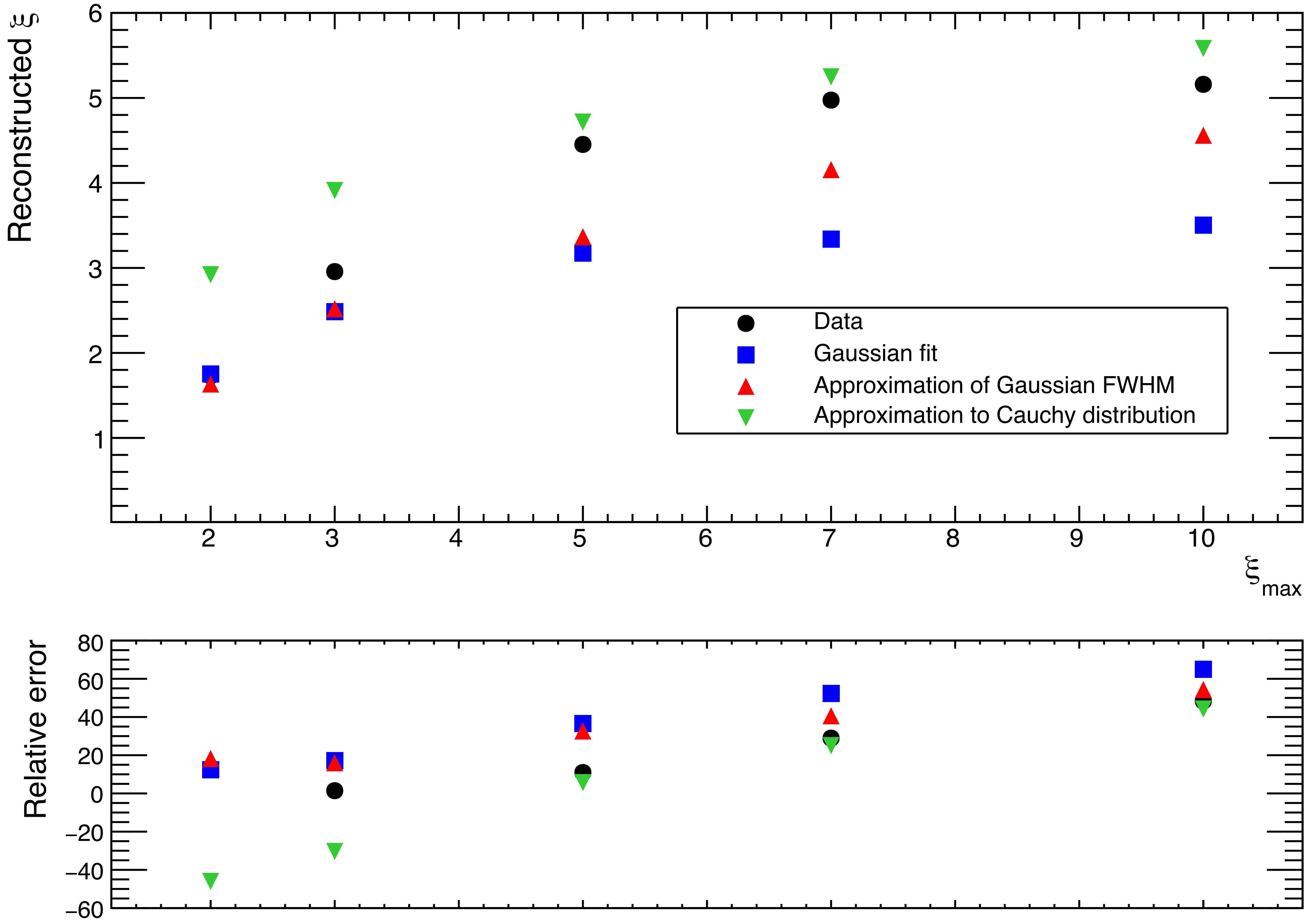
- Variance of angular profiles constructed using different methods:
    - Variance of data
    - Variance of Gaussian fit
    - Variance of approximation to Cauchy fit (see slides 15-17)
    - Approximation of variance assuming a Gaussian FWHM
- Fittings are performed on a central region of width 0.06 mrad

Ptarmigan LMA simulations

Inference of laser intensity



Inferred xi plot seems reasonable based on slide 4

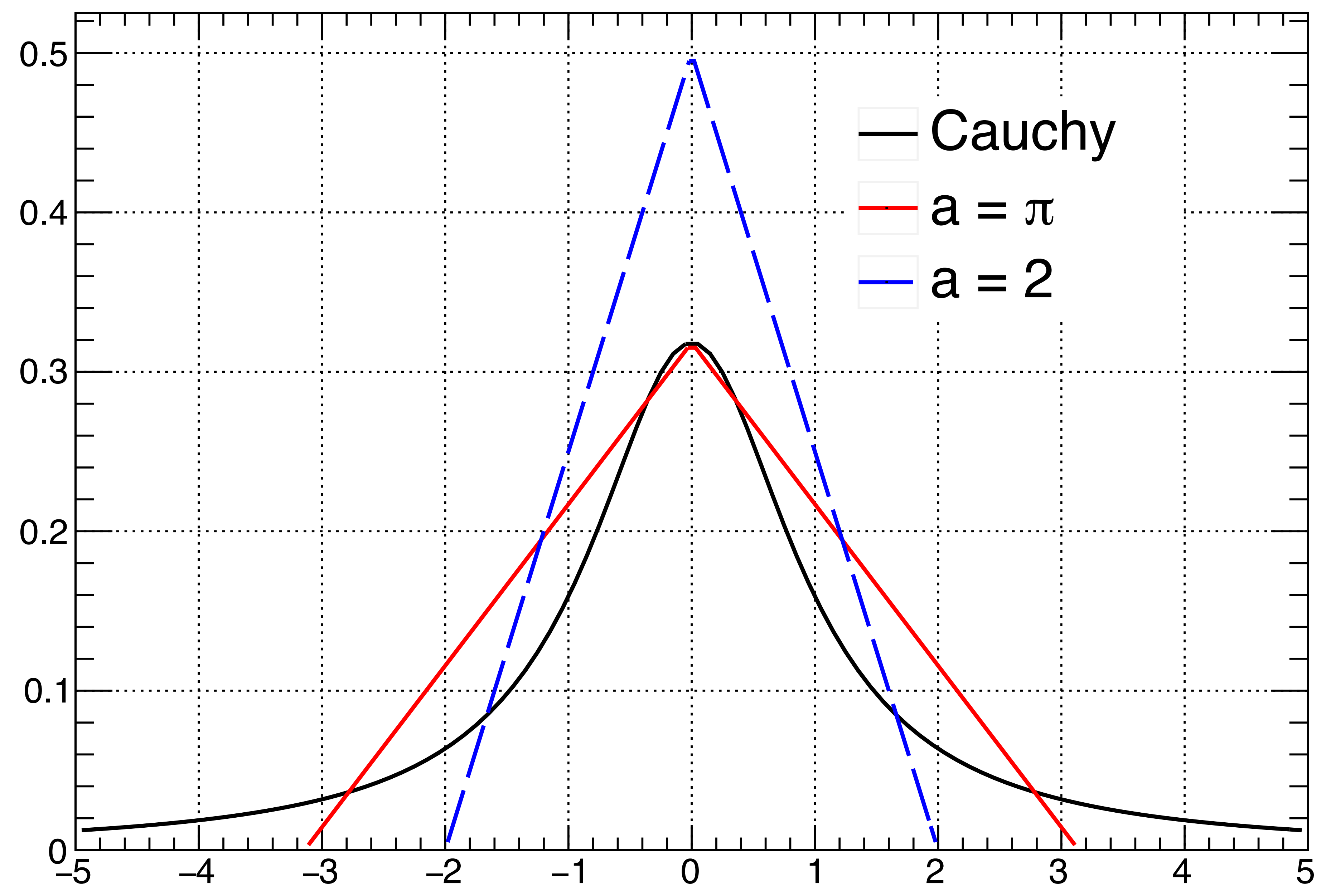


Reconstruction needs more work - alright for low xi but completely off for larger xi

Additional slides

Extra

Approximation of Cauchy distribution



- Standard Cauchy distribution is given as

$$X \sim \text{Cauchy}(0,1) \Rightarrow f_X(x) = \frac{1}{\pi}\frac{1}{1+x^2}$$

- A first order approximation to this is a triangle function of the form

$$g(x) = \begin{cases} \frac{1}{a}\left(1 - \frac{|x|}{a}\right) & \text{for } -a \leq x \leq a \\ 0 & \text{elsewhere} \end{cases}$$

- a is determined by the requirements of the approximation; two are considered here:
  - Peak values of distributions coincide $\Rightarrow \; a = \pi$
  - Function must pass through FWHM points $\Rightarrow \; a = 2$

Extra

Triangular distribution

- The second moment of a distribution is defined as

$$m_2 = \mathbb{E}_X[X^2] = \int_{\mathbb{R}} dx\, x^2 f_X(x)$$

- Using $f_X(x) = g(x)$ from previous slide

$$m_2 = \frac{a^2}{6} \text{ if } a > 0$$

- Hence, the RMS for each value of a is:
  - $\text{rms} = \frac{\pi}{\sqrt{6}}$ for $a = \pi$
  - $\text{rms} = \sqrt{\frac{2}{3}}$ for $a = 2$

Extra

General triangular distribution

- For a Cauchy distribution with general width parameter γ, the PDF is

$$X \sim \text{Cauchy}(0,\gamma) \Rightarrow f_X(x) = \frac{1}{\pi\gamma}\frac{1}{1+\left(\frac{x}{\gamma}\right)^2}$$

- Triangular approximation can be obtained by the substitution $x \to \dfrac{x}{\gamma}$ or equivalently $a \to \gamma a$:

$$g(x) = \begin{cases} \frac{1}{a\gamma}\left(1 - \frac{|x|}{a\gamma}\right) & \text{for } -a\gamma \le x \le a\gamma \\ 0 & \text{elsewhere} \end{cases}$$

- Hence, the second moments are given by $m_2 = \dfrac{a^2\gamma^2}{6}$ so
    - $\text{rms} = \dfrac{\pi\gamma}{\sqrt{6}} \approx 1.283\,\gamma$ for $a = \pi$
    - $\text{rms} = \gamma\sqrt{\dfrac{2}{3}} \approx 0.816\,\gamma$ for $a = 2$