Non perturbative region in Parton Branching

- The DGLAP equation with Sudakov form factors
  - what is needed ?
  - the large z-region
- PDF Distributions and TMD distributions

The DGLAP equation and Sudakov form-factor

- DGLAP equation:

$$t\frac{\partial}{\partial t}f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}P_+(z)\, f\left(\frac{x}{z},t\right)$$

- what about soft divergencies - treated with "plus" prescription

$$\frac{1}{1-z} \to \frac{1}{1-z}_+ \qquad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

$$\begin{aligned} t\frac{\partial}{\partial t}f(x,t) &= \frac{\alpha_s}{2\pi}\int \frac{dz}{z}P_+(z)\, f\left(\frac{x}{z},t\right) \\ &= \int \frac{dz}{z}\frac{\alpha_s}{2\pi}P(z)\, f\left(\frac{x}{z},t\right) - \frac{\alpha_s}{2\pi} f\left(x,t\right)\int dz P(z) \end{aligned}$$

Formulation with Sudakov form-factor

$$\Delta_s = \exp\left\{ - \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int dz \, \frac{\alpha_s}{2\pi} P(z) \right\}$$

$$\frac{\partial e^{-ax}}{\partial x} = -e^{-ax} \frac{\partial a(x)}{\partial x}$$

$$\frac{\partial \Delta_s}{\partial \mu^2} = - \Delta_s \left\{ \frac{1}{\mu^2} \int dz \, \frac{\alpha_s}{2\pi} P(z) \right\}$$

$$\mu^2 \frac{\partial f}{\partial \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f\left(\frac{x}{z}, \mu^2\right) + f(x, \mu^2) \frac{\mu^2}{\Delta_s} \frac{\partial \Delta_s}{\partial \mu^2} \Bigg| \cdot \frac{1}{\Delta_s}$$

$$\boxed{\mu^2 \frac{\partial (f/\Delta_s)}{\partial \mu^2} = \frac{\alpha_s}{2\pi} \int \frac{dz}{z} P(z) \frac{f(x/z, \mu^2)}{\Delta_s}}$$

DGLAP evolution again....

- differential form:

$$t\frac{\partial}{\partial t}f(x,t)=\int\frac{dz}{z}\frac{\alpha_s}{2\pi}P_+(z)\,f\left(\frac{x}{z},t\right) + \int P_{qg}$$

$$\Delta_s(t)=\exp\left(-\int_x^{z_{max}}dz\int_{t_0}^{t}\frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

- differential form using f/Δ_s with

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)}=\int\frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)}f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t)=f(x,t_0)\Delta_s(t)+\int\frac{dz}{z}\int\frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z)f\left(\frac{x}{z},t'\right)$$

no – branching probability from t_0 to t

DGLAP evolution: lessons learned

- Splitting functions, which have plus prescription (have singularity for $z \to 1$) can be treated with Sudakov form factor
  - off-diagonal splitting functions have no corresponding Sudakov (since they are finite):

$$P_{qg} = T_R \left[z^2 + (1-z)^2 \right]$$

$$P_{gq} = C_F \frac{1 + (1 - z^2}{z}$$

- CCFM in its original formulation works with any z_M, since only Pgg and Pqq are used (also in Hautmann, F. and Jung, Transverse momentum dependent gluon density from DIS precision data, Nuclear Physics B, 883(2014), 1, 1312.7875)

- For an expression using full DGLAP with off-diagonal terms, rewrite it with momentum weighted pdfs: $x f(x,\mu^2)$ instead of $f(x,\mu^2)$

momentum weighted DGLAP

- use momentum weighted PDFs: $x f(x,t)$

$$\mu^2 \frac{d x f_a(x,\mu^2)}{d\mu^2} = \sum_b \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_\mathrm{s}, z) \, \frac{x}{z} f_b\left(\frac{x}{z}, \mu^2\right)$$

  - with $P_{ab}{}^{(R)}\,(\alpha_s(t'),z)$ real emission probability (without virtual terms)
  - **must require:** $z_M \rightarrow 1$

- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor to treat non-resolvable and virtual corrections, with momentum weighted splitting function:

$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz \; z \; P_{ba}^{(R)}(\alpha_\mathrm{s}), z) \right)$$

momentum weighted DGLAP

$$t\frac{\partial\, xq}{\partial t} = \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\left[z\, P_{qq_+}\frac{x}{z}\, q(x/z,t) + z\, P_{qg}\frac{x}{z}\, g(x/z,t)\right]$$

expanding + prescription

$$= \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\underbrace{\left[z\, P_{qq}\frac{x}{z}\, q(x/z,t) + z\, P_{qg}\frac{x}{z}\, g(x/z,t)\right]}_{A} - \frac{\alpha_s}{2\pi}\, xq(x,t)\int_0^1 P_{qq}\, dz$$

$$= \frac{\alpha_s}{2\pi}\int_x^{z_M}\frac{dz}{z}\left[\; A \;\right] + \frac{\alpha_s}{2\pi}\int_{z_M}^1\frac{dz}{z}\left[\; A \;\right] - \frac{\alpha_s}{2\pi}\, xq(x,t)\int_0^1 P_{qq}\, dz$$

for $z\to 1$ for $z\to 1$

$$= \frac{\alpha_s}{2\pi}\Bigg[\int_x^{z_M}\frac{dz}{z}\left[A\right] + xq(x,t)\int_{z_M}^1 dz\, P_{qq} + xg(x,t)\int_{z_M}^1 dz\, P_{qg} - xq(x,t)\int_0^1 P_{qq}\, dz$$

momentum sum rule

$$- xq(x,t)\underbrace{\int_0^1 dz\; z\left(P_{qq_+} + P_{gq}\right)}_{0}\Bigg]$$

$$= \frac{\alpha_s}{2\pi}\Bigg[\int_x^{z_M}\frac{dz}{z}\left[A\right] + xq(x,t)\int_{z_M}^1 dz\, \cancel{P_{qq}} + xg(x,t)\underbrace{\int_{z_M}^1 dz\, P_{qg}}_{0 \text{ for } z\to 1} - \cancel{xq(x,t)\int_0^1 P_{qq}\, dz}$$

$$- xq(x,t)\int_0^{z_M} dz\, z\left(P_{qq} + P_{gq}\right) - xq(x,t)\int_{z_M}^1 dz\left(\cancel{P_{qq}} + \underbrace{P_{gq}}_{0 \text{ for } z\to 1}\right) + \cancel{xq(x,t)\int_0^1 dz\, P_{qq}}\Bigg]$$

expanding + prescription + momentum sum rule expanding everything

$$= \frac{\alpha_s}{2\pi}\left[\int_x^{z_m} \frac{dz}{z}\left(z P_{qq}\frac{x}{z} q\left(\frac{x}{z},t\right) + z P_{qg}\frac{x}{z} g\left(\frac{x}{z},t\right)\right)\right.$$

$$- x q(x,t) \int_0^{z_m} dz \left(z P_{qq} + z P_{gq}\right)$$

$$\left. + x g(x,t) \underbrace{\int_{z_m}^{1} dz\, P_{qg}}_{=0 \text{ for } z_m \to 1} - x q(x,t) \underbrace{\int_{z_m}^{1} dz\, P_{gq}}_{=0 \text{ for } z_m \to 1}\right]$$

and similar calculation for gluon density!

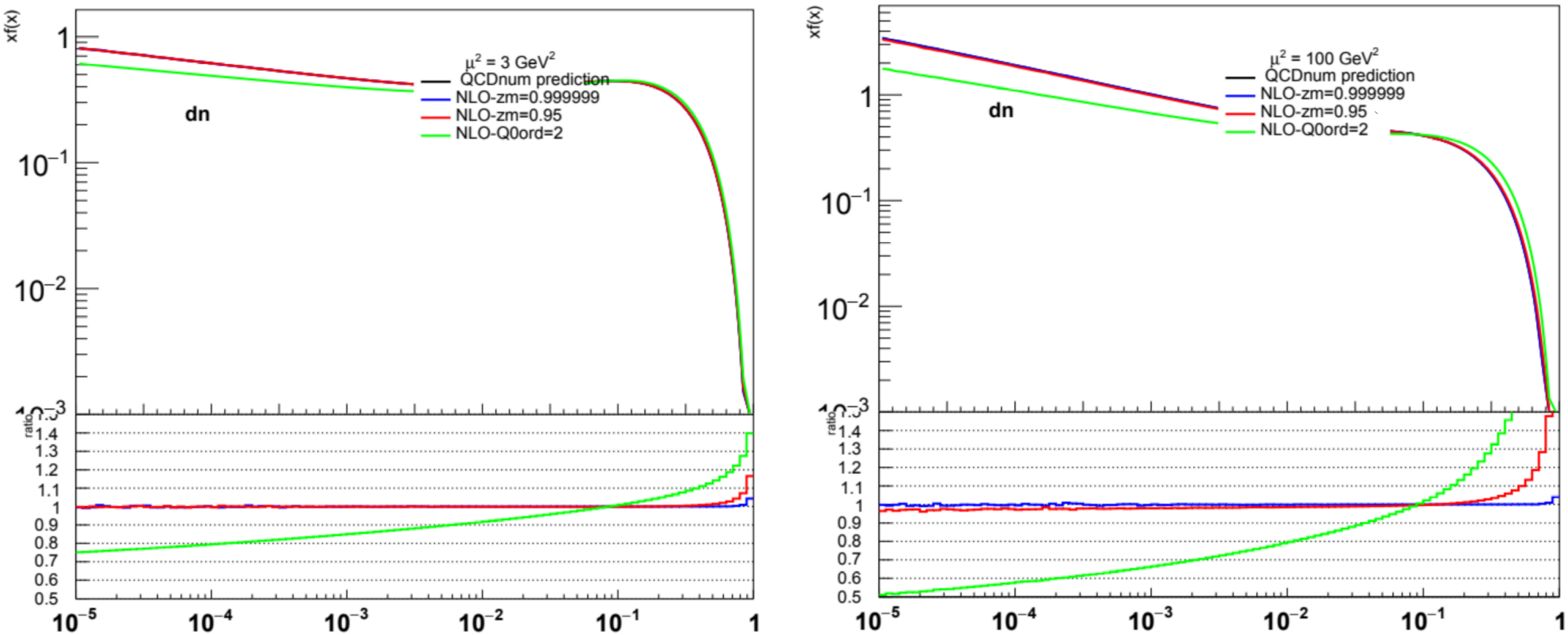
Approximations used for $z_m \to 1$:

1.) $$\int_{z_m}^{1} \frac{dz}{z}\, z\, P_{qq}\, \frac{x}{z}\, q\left(\frac{x}{z}, t\right) \longrightarrow x\, q(x,t) \int_{z_m}^{1} dz\, P_{qq}$$

2.) $$\int_{z_m}^{1} dz\, P_{qg} \longrightarrow 0$$

$$\int_{z_m}^{1} dz\, z\, P_{gq} \longrightarrow 0$$

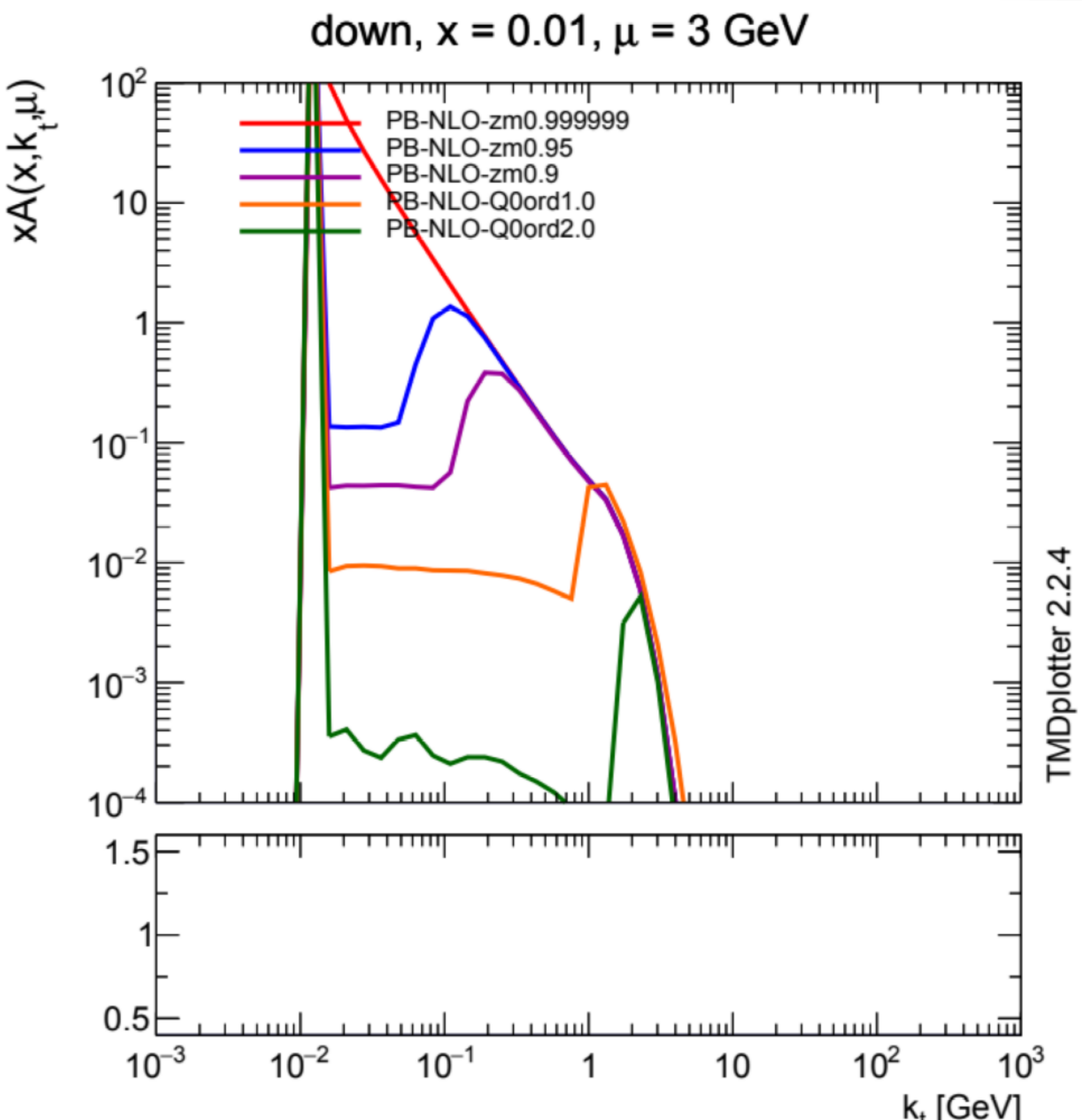
PDF distributions for different z_M

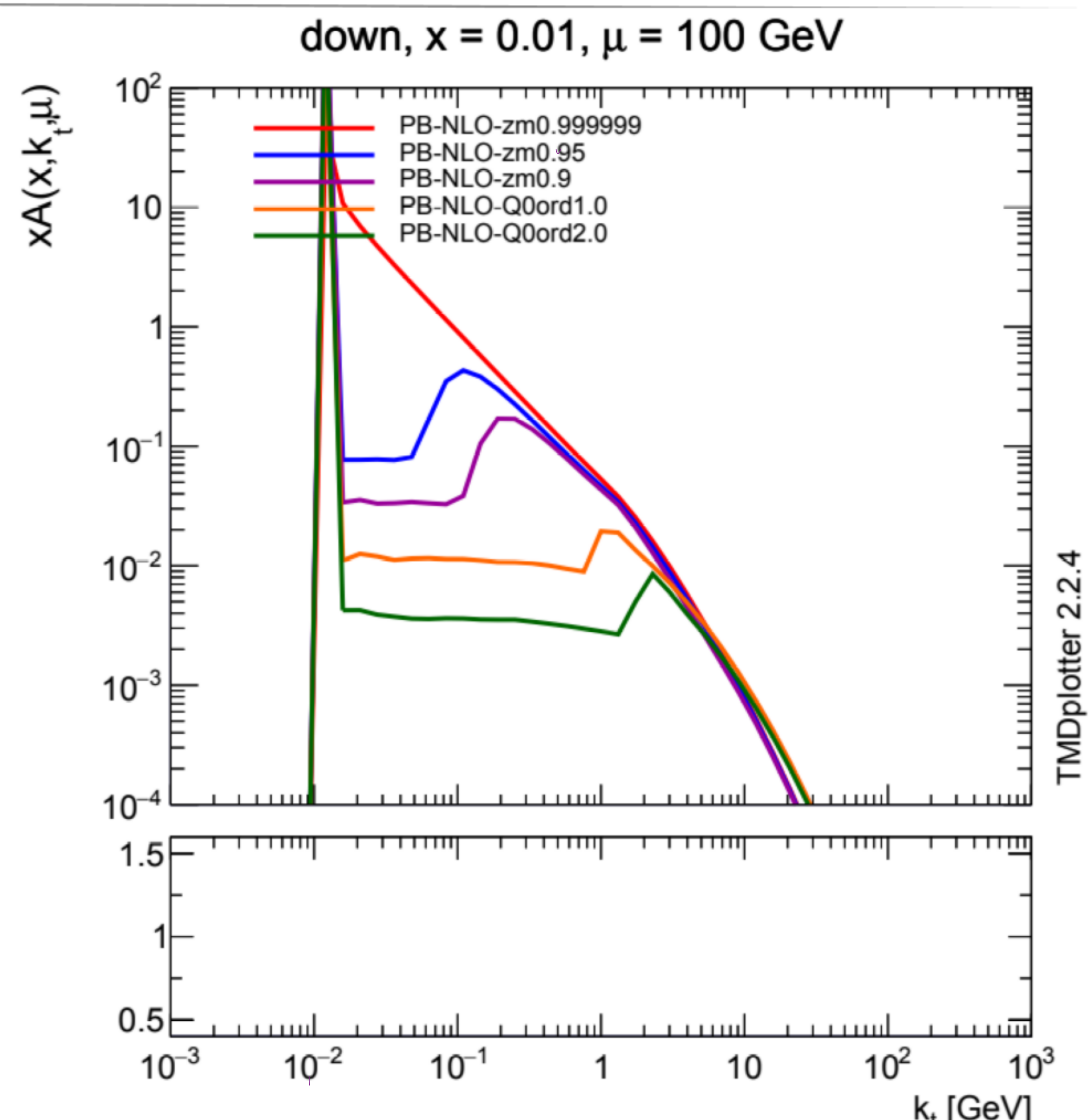


$z_m = 1 - \frac{Q_{0\,ord}}{q}$

- Only $z_M \to 1$ reproduces DGLAP (QCDnum)
  - $z_M = 0.95$ shows differences at high scales, $\mathcal{O}(\sim 5\%)$
  - dynamic z_M shows significant differences (due to missing cancellation of terms
- Proper treatment of soft region (non-perturabtive) is essential

TMD distributions for different z_M





- Distributions without intrinsic kt (qs=0.00001)
  - $z_M \to 1$ shows contributions also at very low k_T
  - for $z_M \ll 1$ significant effects appear in low kt region
    - small kt appears from $\sum_i k_{T\,i}$

Conclusion

- Treatment of soft gluons are very important in evolution equation
  - For CCFM (with only diagonal terms and with evolution of parton density) zM can have different values from 1.
  - For DGLAP with off-diagonal terms, zM → 1, otherwise some terms are not cancelled.
    - effect can be rather large in collinear distribution
    - and has signifiant effects is transverse momentum distribution at small kt
  - Using $z_M \ll 1$in momentum weighted DGLAP is not just a bad approximation, but leads to missing cancellation of important pieces and should be avoided.